NOTE

Profit and value in a random system: interpretation of professor Schefold's 2016 article

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Abstract
This note discusses Professor Schefold's 2016 article. It checks the mathematical treatment of transformation problem in the article. It also reviews the interpretation of Marx's mathematical manuscripts.

Keywords Marx · Transformation problem · Random matrices · Mathematics · Differential

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1 Introduction

Clearly, the transformation problem of finding a rule to transform labor value to production price has long been one of the most important questions in Marxian economics. However, some do not consider it a real problem because both the labor value and production price can be independently deduced from input coefficient matrices.

Professor Bertram Schefold's (2016) article provided us another viewpoint to consider the transformation problem. His approach uses random matrices as a mathematical tool. This note tries to interpret the mathematical and methodological implications of the article.

The latter half of Schefold's article argues about Marx's treatment of differential calculus. This note also compares Marx's treatment with Hegel's and Leibniz's views; further, it will clarify the overall implications of Professor Schefold's method.

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2 Transformation Problem and Random Matrices

Professor Schefold's model of the transformation problem consists of Sraffian and Marxian parts. The Sraffian part is almost the same as the standard commodity system of Sraffa (1960). The Sraffa vector refers to the standard commodities. First, we consider a normal production system, as represented by Eq. (1).

$$y = Ay + b + s \tag{1}$$

Here y, A, b, and s are the activity vector, input coefficient matrix, real wage vector, and surplus production vector, respectively.

$$y^* = (1 + R)Ay^* \tag{2}$$

Next, we consider eigenvalues and eigenvectors of the input coefficient matrix A. In Eq. (2), R is the maximum profit rate that corresponds to the Frobenius root. We can derive the Sraffa vector y^* as the right-hand side eigenvector of the input coefficient matrix. The term "Sraffa vector" is Schefold's usage of words. However, because it corresponds to Sraffian standard commodities, the usage is appropriate.

Now, we define the vector m whose elements are deviations from the real activity level of the standard Sraffa vector to obtain Eq. (3).

$$m = y - y^* \tag{3}$$

In contrast to the Sraffa vector, Professor Schefold defines a Marx vector as well. As the Sraffa vector refers to the Sraffian standard commodities, it can be deduced from a quantity equation. By contrast, the Marx vector is deduced from a price equation similar to Eq. (4).

$$p = (1 + r)pA + wl, \tag{4}$$

where p, r, w, and l are the production price vector, profit rate, wage rate, and labor input vector, respectively. We again consider eigenvalues and eigenvectors of the input coefficient matrix.

$$p^* = (1 + R)p^*A \tag{5}$$

Here, p^* is the left-hand side eigenvector of the input coefficient matrix. Schefold calls this eigenvector the Marx vector. It is, however, difficult to understand why this vector is named after Marx. In Eq. (5), the maximum profit rate is valid. In other words, the wage rate here is zero. This is in complete contradiction to Marx's opinion. We, however, infer the reason for Professor Schefold's naming as follows. In his analysis, Marx concentrated on the relation between labor value and production price. This situation is almost similar to Sraffa's approach, in which he mainly considered the activity level of production. So, we may call the vector of standard prices the Marx vector, whereas that of standard commodities is called the Sraffa vector.

We define the vector v whose elements are deviations of the labor input vector from the Marx vector p^* as in Eq. (6). This is, however, more difficult to understand than the case above. In Eq. (3), we defined the vector m. When the vector becomes a zero vector, the real activity level will coincide with the standard one.

$$v = l - p^* \tag{6}$$

In contrast to Eq. (3), Eq. (6) insists on the coincidence of the labor input vector and the standard prices.[1] This, however, seems very strange. When the maximum profit rate is valid, it is not the labor input vector, but the actual price vector that coincides with the standard prices. In the case of the vector m defined in Eq. (3), the maximum profit rate leads to the coincidence of the actual activity of production and the standard commodities.

We may suppose that Professor Schefold aims at the coincidence of labor value and the standard price. Labor value is, however, the summation of living labor and dead labor. However, the labor input vector indicates only living labor. Thus, a riddle remains. We will accept the definition by Professor Schefold once and reconsider the riddle after examining his mathematical explanation.

From Eq. (4), we can derive Eq. (7) as follows.

$$p = wl[I - (1 + r)A]^{-1} \tag{7}$$

Using the mathematical formula, we can obtain Eq. (8). In Eq. (8), μ_i is the eigenvalue. We find that only μ_1 is dominant, i.e., it is the Frobenius root. The other eigenvalues are all non-dominant.

$$p = w \sum_i \frac{1}{1 - (1 + r)\mu_i} p_i \tag{8}$$

Here $\mu_1 = 1/(1 + R)$. If we suppose $\mu_2 = \ldots = \mu_n = 0$, we obtain Eq. (9). Here, n is the dimension of vectors. The assumption is really crucial in Schefold's argument. The argument requires that the input coefficient matrix should be a random matrix.[2] The elements are changing randomly in random matrices. Further, non-dominant eigenvalues disappear when their average is taken.

$$p = w \left[\frac{1}{1 - (1 + r)\mu_1} p_1 + p_2 + \cdots + p_n \right] \tag{9}$$

Because $p_1 = p^*$ and $p_2 + \cdots + p_n = v$ in Eq. (9), we finally obtain Eq. (10).

$$p = w \left[\frac{p^*}{1 - \frac{1+r}{1+R}} + v \right] \tag{10}$$

Then, we normalize the production price vector to obtain Eq. (11).

$$1 = \bar{p}y = \bar{w} \left[\frac{p^* y^*}{1 - \frac{1+r}{1+R}} + vm \right] \tag{11}$$

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[1] Here we call the left-hand side eigenvector of the input coefficient matrix the "standard prices." Needless to say, we follow Sraffa (1960) in calling the right-hand side eigenvector the standard commodities.

[2] Random matrices were first introduced into statistical theory in the 1920s. Wigner (1956) investigated these in the 1950 s for their application in nuclear physics. Random matrices have random numbers as their elements. Dyson (1962) built a Brownian movement model using random matrices. The introduction of random matrices into quantum mechanics in the 1980 s aroused mathematicians' interest in the eigenvalues of random matrices.

The bars over the letters p and w refer to the normalization. Because $cov(v, m)=0$, we obtain Eq. (12).

$$vm = n\bar{v}\bar{m} \tag{12}$$

The bars over the letters v and m refer to the average value. We obtain Eq. (13) by substituting Eq. (12) in Eq. (11).

$$\bar{w} = \cfrac{1}{\cfrac{p^*y^*}{1-\frac{1+r}{1+R}} + n\bar{v}\bar{m}} \tag{13}$$

If we assume that the average value of v is 0, Eq. (14) follows. Here, Professor Schefold takes the average value of v as 0. This assumption is, however, again of very great significance. Though this assumption looks very simple and subtle, it actually fixes the random matrix's character. The matrix cannot avoid becoming a very trivial one.

$$\bar{w} = \frac{1 - \frac{1+r}{1+R}}{p^*y^*} \tag{14}$$

When $r=R$, the normalized wage rate is zero.
As in the wage rate, we can calculate the total profit from Eq. (15).

$$\varPi = \bar{p}s = \bar{w}\left[\frac{p^*}{1-\frac{1+r}{1+R}} + v\right]s = \frac{1 - \frac{1+r}{1+R}}{p^*y^*}\left[\frac{p^*}{1-\frac{1+r}{1+R}} + v\right]s = \frac{1}{p^*y^*}\left[p^*s + \left(1 - \frac{1+r}{1+R}\right)vs\right] \tag{15}$$

Because $cov(v, s)=0$, Eq. (16) follows. The bars over the letters v and s refer to the average value.

$$vs = n\bar{v}\bar{s} \tag{16}$$

We obtain Eq. (17) by substituting Eq. (16) in Eq. (15).

$$\varPi = \frac{1}{p^*y^*}\left[p^*s + \left(1 - \frac{1+r}{1+R}\right)n\bar{v}\bar{s}\right] \tag{17}$$

We assume that the average value of v is zero again to obtain Eq. (18).

$$\varPi = \frac{p^*s}{p^*y^*} \tag{18}$$

Even if the average value of v is not zero, Eq. (18) is obtained when $r=R$ or $r=0$.

The transformation problem has been treated as a gradual recalculation of the production price from labor value. The substantial backbone of these formal procedures has been supposed to be confirmed by the following three propositions: total value = total production price; total surplus value = total profit; and total value product = total income. As is well known, only one of the three propositions holds.

Professor Schefold's approach has significance in that it can show the proposition "total surplus value = total profit" generally holds, on average. Thus, it can explore

a new aspect of the transformation problem. The approach tells us not only that the value system works in a deeper dimension than the production price system, but also the two systems interact quantitatively at the same time.

We, however, have to say that Professor Schefold's approach is not so successful in terms of its mathematical meaning. His conclusion, in fact, depends on special and trivial random matrices. Further, there is the riddle about using the vector v, which refers to the difference between labor input vector and the standard prices. On checking, we find that the assumption of $v = 0$ is most crucial in Professor Schefold's model. The vector is necessary only for this assumption. The necessity arises from the mathematical treatment; however, the economic interpretation is difficult.

3 Differential calculus in Marx

Now, let us move to the part of Professor Schefold's view on Marx and mathematics. Professor Schefold discusses here Marx's approach to the transformation problem. Marx's liking for mathematics, especially differential calculus, is famous. Mathematics had a deep influence on Marx's style of thinking. Professor Schefold calls Marx's method a kind of dialectical logics. He also insists that we should compare the thoughts of Marx and Hegel on the infinitesimal.

Professor Schefold notices that Marx compared the "apparent contradiction" in the transformation problem with the difficulty of understanding differential calculus from the viewpoint of elementary algebra.[3]

> The long way followed by Marx to get to his theory of prices of production is curiously compared with the way leading from elementary algebra to the idea that 0/0 could represent a 'real magnitude' (Schefold 2016, p. 183).

Professor Schefold further argues that Marx's attitude to differential calculus is clarified by a careful examination of his mathematical manuscript on the one hand and by its comparison with Hegel's view in his *Logics* on the other hand.

Leibniz is said to have been ambiguous on the concept of the infinitesimal. Considering the antipathy of the contemporary scholars against such a novel idea, he defined dx or dy as finite quantities and described them as being less than any given quantity in his article written in Latin (Leibnitz 1684). By so doing, Leibniz tried to persuade people to understand and accept the new idea. It was Hegel who extended the idea of the infinitesimal in philosophy.

Hegel attached a deep philosophical meaning to this mathematical concept. He treated the infinitesimal as something between "being and nothing". He called it an "intermediate state". However, Marx's understanding is not so philosophical. He was particularly interested in the fact that $\Delta y/\Delta x = dy/dx$ in the linear case, whereas $\Delta y/\Delta x \neq dy/dx$ in the non-linear case. Marx also borrowed the Leibniz rule.

$$\frac{dyz}{dx} = z\frac{dy}{dx} + y\frac{dz}{dx} \tag{19}$$

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[3] Schefold (2016), p. 183, quoted Marx's text from MEGA^2 II, 9, pp. 264–5.

In Eq. (19), the right-hand side no longer defines the left-hand side. If we eliminate all dx terms from the both sides, we get Eq. (20).

$$\mathrm{d}yz = z\mathrm{d}y + y\mathrm{d}z \tag{20}$$

In Eq. (20), we cannot regard dy, and dz as an "intermediate state" between "being and nothing". They must be interpreted as a "real magnitude". However, it does not require a specially arranged metaphysics. According to Professor Schefold, Marx's approach was different from that of Hegel in that Marx regarded the differentials not as magnitudes of the infinitesimal, but as operators. Marx wrote that the equation is, thus, only a symbolic indication of the operations to be performed.

Admitting the incompleteness of the publication of Marx's mathematical manuscript, Professor Schefold makes the tentative judgment as follows.

> My impression is that he seems to have kept his distance from the axiomatic method. Mathematics in Marx do not appear like construct, resulting from different conceptions defined by axioms, but it is a coherent realm of objects, to be analysed by experience and research—hence the 'intermediate terms', which are there to connect algebra and the infinitesimal calculus (Schefold 2016, p. 186).

Because Marx intrinsically tried to analyze economic reality, his mathematics did not swerve into "pathological" cases of mathematical details. His method of mathematics was healthy, operational, and operable.

4 Conclusions

We have discussed how we can connect the different dimensions of labor value and production price in the transformation problem. Originally, Marx tried to solve the problem by repeating the calculation a very large number of times. It is also a challenge to the infinity in the opposite direction of the infinitesimal.

However, connecting quantitatively two entities in different dimensions is logically impossible. Professor Schefold tried to make this possible by considering the average state of variable realities. He utilized the mathematics of random matrices for the representation. We, however, find that the result finally turned out to be a trivial case.

We, nonetheless, can appreciate Professor Schefold's proposal. Stating that the two systems coincide, on average, is almost the same as saying that the labor value system approximates the production price system. In retrospect, one finds that this statement has been repeated since Ricardo's time. The labor value system is much simpler than the production price system. We could even say that, according to this result, labor value system is "fundamental" to the production price system. This logic may suit Marx's operable and pragmatic use of mathematics.

Compliance with ethical standards

References

Dyson FJ (1962) A Brownian-motion model for the eigenvalues of a random matrix. J Math Phys 3:1191–1198

Leibnitz GW (1684) Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas nec irrationals quantitates moratur, et singular pro illis calculi genus. Acta Eruditorum 3:467–473

Schefold B (2016) Profits equal surplus value on average and the significance of this result for the Marxian theory of accumulation. Camb J Econ 40:165–199

Sraffa P (1960) Production of commodities by means of commodities: prelude to critique of economic theory. Cambridge University Press, Cambridge

Wigner EP (1956) Results and theory of resonance absorption. In: Conference on neutron physics by time-of-flight held at Gatlinburg, Tennessee, November 1 and 2, pp 59–70