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PRICES vs. QUANTITIES

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1. Introduction

A classical problem of economics is how to design a system which best controls economic activity for the benefit of society. So phrased, this is a very broad topic which is difficult to analyze as such. From the beginning it will be useful to pose the problem somewhat more narrowly and precisely so it is easier to isolate the basic issues that are involved.

The setting for the problem under consideration is a large economic organization or system which in many cases is best thought of as the entire economy. Within this large economic organization resources are allocated by some combination of commands and prices (the exact mixture is inessential) or even by some other unspecified mechanism. The following question arises. For one particular isolated economic variable that needs to be regulated, what is the best way to implement control for the benefit of the organization as a whole? Is it better to directly administer the activity under scrutiny or to fix transfer prices and rely on self-interested profit maximization to achieve the same ends in decentralized fashion? This issue is taken as the prototype problem of central control which is studied in the present paper. There are a great many specific examples which fit nicely into such a framework. One of current interest is the question of whether it would be better to control certain forms of pollution by setting emission standards or by charging appropriate

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pollution taxes.

When quantities are employed as planning instruments, the basic operating rules from the center take the form of quotas, targets, or commands to produce a certain level of output. With prices as instruments, the rules specify either explicitly or implicitly that profits are to be maximized at the given parametric prices. Now a basic theme of resource allocation theory emphasizes the close connection between these two modes of control. No matter how one type of planning instrument is fixed, there is always a corresponding way to set the other which achieves the same result when implemented.[1] From a strictly theoretical point of view there is really nothing to recommend one mode of control over the other. This notwithstanding, I think it is a fair generalization to say that the average economist in the Western marginalist tradition has at least a vague preference toward indirect control by prices, just as the typical non-economist leans toward the direct regulation of quantities.

That a person not versed in economics should think primarily in terms of direct controls is probably due to the fact that he does not comprehend the full subtlety and strength of the invisible hand argument. The economist's attitude is somewhat more puzzling. Understanding that prices can be used as a powerful and flexible instrument for rationally allocating resources and that in fact a market economy automatically regulates itself in this manner is very different from being under the impression that indirect controls are generally preferable for the kind of problem considered in this paper. Certainly a careful reading of economic theory yields little to support such a universal proposition.

Many economists point with favor to the fact that if prices are the planning

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[1]Given the usual convexity assumptions.

instrument then profit maximization automatically guarantees total output will be efficiently produced, as if this result were of any more than secondary interest unless the prices (and hence total output) are optimal to begin with. Sometimes it is maintained that prices are desirable planning instruments because the stimulus to obtain a profit maximizing output is built right in if producers are rewarded in proportion to profits. There is, of course, just as much motivation to minimize costs at specified output levels so long as at least some fraction of production expenditures are borne by producers. With both modes of control there is clearly an incentive for self-interested producers to systematically distort information about hypothetical output and cost possibilities in the pre-implementation planning phase. Conversely, there is no real way to disguise the true facts in the implementation stage so long as actual outputs (in the case of price instruments) and true operating costs (in the case of quantity instruments) can be accurately monitored. For the one case the center must ascertain <u>ceteris paribus</u> output changes as prices are varied, for the other price changes as outputs are altered.

A reason often cited for the theoretical superiority of prices as planning instruments is that their use allegedly economizes on information. The main thing to note here is that generally speaking it is neither easier nor harder to name the right prices than the right quantities because in principle exactly the <u>same</u> information is needed to correctly specify either. It is true that in a situation with many independent producers of an identical commodity, only a single uniform output price has to be named by the center, whereas in a command mode separate quantities must be specified for each producer. If such an observation has meaningful implications, it can only be within the artificial milieu of an iterative

<u>tâtonnement</u> type of "planning game" which is played over and over again approaching an optimal solution in the limit as the number of steps becomes large. Even in this context the fact that there are less "message units" involved in each communication from the center is a pretty thin reed on which to hang claims for the informational superiority of the price system. It seems to me that a careful examination of the mechanics of successive approximation planning shows that there is no principal informational difference between iteratively finding an optimum by having the center name prices while the firms respond with quantities, or by having the center assign quantities while the firm reveals costs or marginal costs.[1]

If there were really some basic intrinsic advantage to a system which employed prices as planning instruments, we would expect to observe many organizations operating with this mode of control, especially among multi-divisional business firms in a competitive environment. Yet the allocation of resources within private companies (not to mention governmental or non-profit organizations)

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[1] The "message unit" case for the informational superiority of the price system is analogous to the blanket statement that it is better to use dual algorithms for solving a programming problem whenever the number of primal variables exceeds the number of dual multipliers. Certainly for the superior large step decomposition type algorithms which on every iteration go right after what are presently believed to be the best instrument values on the basis of all currently available information, such a general statement has no basis. With myopic gradient methods it is true that on each round the center infinitesimally and effortlessly adjusts exactly the number of instruments it controls, be they prices or quantities. But who can say <u>how many</u> infinitesimally small adjustments will be needed? Gradient algorithms are known to be a bad description of iterative planning procedures, among other reasons because they have inadmissably poor convergence properties. If the step size is chosen too small convergence takes forever. If it is chosen too large, there is no convergence. As soon as a finite step size is selected on a given iteration to reflect a desire for quick convergence, the "message unit" case for prices evaporates. Calculating the <u>correct</u> price change puts the center right back into the large step decomposition framework where on each round the problem of finding the best iterative prices is formally identical to the problem of finding the best iterative quantities. For discussion of these and various other aspects of iterative planning, see the articles of Heal [4], Malinvaud [5], Marglin [7], Weitzman [9].

is almost never controlled by setting administered transfer prices on commodities and letting self-interested profit maximization do the rest.[1] The price system as an allocator of internal resources does not itself pass the market test!

Of course all this is not to deny that in any particular setting there may be important practical reasons for favoring either prices or quantities as planning instruments. These reasons might involve ideological, political, legal, social, historical, administrative, motivational, informational, monitoring, enforcing, or other considerations.[2] But there is little of what might be called a system-free character.

In studying such a controversial subject, the only fair way to begin must be with the tenet that there is no basic or universal rationale for having a general predisposition toward one control mode or the other. If this principle is accepted, it becomes an issue of some interest to develop strictly economic criteria by which the comparative performance of price and quantity planning instruments might be objectively evaluated. Even on an abstract level, it would be useful to know how to identify a situation where employing one mode is relatively advantageous, other things being equal.

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[1]About a decade ago, Ford and G.M. performed a few administrative trials of a limited sort with some decentralization schemes based on internal transfer prices. The experiments were subsequently discontinued in favor of a return to more traditional planning methods. See Whinston [10].

[2]For example, there is no sense considering quantities in a legal-political milieu which effectively prohibits direct regulation. Alternatively, if it happens to be the case that it is difficult or expensive to monitor output on a continuous scale but relatively cheap to perform a pass-fail litmus type test on whether a given output level has been attained or not, the price mode may be greatly disadvantaged from the start.

2. The Model

We start with a highly simplified prototype planning problem. Amount q of a certain commodity can be produced at cost $C(q)$, yielding benefits $B(q)$. The word "commodity" is used in an abstract sense and really could pertain to just about any kind of good from pure water to military aircraft. Solely for the sake of preserving a unified notation, we follow the standard convention that goods are desirable. This means that rather than talking about air pollution, for example, we instead deal with its negative--clean air.

Later we treat more complicated cases, but for the time being it is assumed that in effect there is just one producer of the commodity and no ambiguity in the notion of a cost curve. Benefits are measured in terms of money equivalents so that the benefit function can be viewed as the reflection of an indifference curve showing the trade-off between amounts of uncommitted extra funds and output levels of the given commodity. It is assumed that $B''(q) < 0$, $C''(q) > 0$, $B'(0) > C'(0)$, and $B'(M) < C'(M)$ for M sufficiently large.

The planning problem is to find that value q^* of q which maximizes

$$B(q) - C(q).$$

The solution must satisfy

$$B'(q^*) = C'(q^*).$$

With

$$p^* \equiv B'(q^*) = C'(q^*),$$

it makes no difference whether the planners announce the optimal price p^* and have the producers maximize profits

$$p^*q - C(q)$$

or whether the center merely orders the production of q* at least cost. In an environment of complete knowledge and perfect certainty there is a formal identity between the use of prices and quantities as planning instruments.

If there is any advantage to employing price or quantity control modes, therefore, it must be due to inadequate information or uncertainty. Of course it is natural enough for planners to be unsure about the precise specification of cost and benefit functions since even those most likely to know can hardly possess an exact account.

Suppose, then, that the center perceives the cost function only as an estimate or approximation. The stochastic relation linking q to C is taken to be of the form

$$C(q,\theta)$$

where θ is a disturbance term or random variable, unobserved and unknown at the present time. While the determination of θ could involve elements of genuine randomness[1], it is probably more appropriate to think primarily in terms of an information gap.

Even the engineers most closely associated with production would be unable to say beforehand precisely what is the cheapest way of generating various hypothetical output levels. How much murkier still must be the center's _ex ante_ conception of costs, especially in a fast moving world where knowledge of particular circumstances of time and place may be required. True, the degree of fuzziness could be reduced by research and experimentation. But it could never be truly eliminated because new sources of uncertainty are arising all the time.[2]

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[1]Like day-to-day fluctuations.

[2]For an amplification of some of these points, see Hayek [3].

Were a particular output level really ordered in all seriousness, a cost minimizing firm could eventually grope its way toward the cheapest way of producing it by actually testing out the relevant technological alternatives. Or, if an output price were in fact named, a profit maximizing production level could ultimately be found by trial and error. But this is far from having the cost function as a whole knowable a priori.

While the planners may be somewhat better acquainted with the benefit function, it too is presumably discernable only tolerably well, say as

$$B(q,\eta)$$

with η a random variable. The connection between q and B is stochastic either because benefits may be imperfectly known at the present time or because authentic randomness may play a role. Since the unknown factors connecting q with B are likely to be quite different from those linking q to C, it is assumed that the random variables θ and η are independently distributed.

As a possible specific example of the present formulation, consider the problem of air pollution. The variable q could be the cleanliness of air being emitted by a certain type of source. Costs as a function of q might not be known beyond doubt because the technology, quantified by θ, is uncertain. At a given level of q the benefits may be unsure since they depend among other things on the weather, measured by η.

Now an ideal instrument of central control would be a contingency message whose instructions depend on which state of the world is revealed by θ and η. The ideal ex ante quantity signal $q^*(\theta,\eta)$ and price signal $p^*(\theta,\eta)$ are in the form of an entire schedule, functions of θ and η satisfying

$$B_1(q^*(\theta,\eta),\eta) = C_1(q^*(\theta,\eta),\theta) = p^*(\theta,\eta).$$

By employing either ideal signal, the <u>ex ante</u> uncertainty has in effect been eliminated <u>ex post</u> and we are right back to the case where there is no theoretical difference between price and quantity control modes.

It should be readily apparent that it is infeasible for the center to transmit an entire schedule of ideal prices or quantities. A contingency message is a complicated, specialized contract which is expensive to draw up and hard to understand. The random variables are difficult to quantify. A problem of differentiated information or even of moral hazard may be involved since the exact value of θ will frequently be known only by the producer.[1] Even for the simplest case of just <u>one</u> firm, information from different sources must be processed, combined, and evaluated. By the time an ideal schedule was completed, another would be needed because meanwhile changes would have occurred.

In this paper, the realistic issue of central control under uncertainty is considered to be the "second best" problem of finding for each producer the <u>single</u> price or quantity message which optimally regulates his actions. This is also the best way to focus sharply on the essential difference between prices and quantities as planning instruments.

The optimal quantity instrument under uncertainty is that target output $\hat{q}$ which maximizes expected benefits minus expected costs, so that

$$E[B(\hat{q},\eta) - C(\hat{q},\theta)] = \max_{q} E[B(q,\eta) - C(q,\theta)]$$

where $E[\cdot]$ is the expected value operator. The solution $\hat{q}$ must satisfy the first order condition

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[1] So that it may be inappropriate, for example, to tell him to produce less if costs are high unless a very sophisticated incentive scheme goes along with such a message. For an elaboration of some of these points see Arrow [1], pp. 321-322.

$$E[B_1(\hat{q},\eta)] = E[C_1(\hat{q},\theta)]. \tag{1}$$

When a price instrument p is announced, production will eventually be adjusted to the output level

$$q = h(p,\theta)$$

which maximizes profits given p and θ. Such a condition is expressed as

$$ph(p,\theta) - C(h(p,\theta),\theta) = \max_q \quad pq - C(q,\theta),$$

implying

$$C_1(h(p,\theta),\theta) = p. \tag{2}$$

If the planners are rational, they will choose that price instrument $\tilde{p}$ which maximizes the expected difference between benefits and costs given the reaction function $h(p,\theta)$:

$$E[B(h(\tilde{p},\theta),\eta) - C(h(\tilde{p},\theta),\theta)] = \max_p E[B(h(p,\theta),\eta) - C(h(p,\theta),\theta)].$$

The solution $\tilde{p}$ must obey the first order equation

$$E[B_1(h(\tilde{p},\theta),\eta)\cdot h_1(\tilde{p},\theta)] = E[C_1(h(\tilde{p},\theta),\theta)\cdot h_1(\tilde{p},\theta)],$$

which can be rewritten as

$$\tilde{p} = \frac{E[B_1(h(\tilde{p},\theta),\eta)\cdot h_1(\tilde{p},\theta)]}{E[h_1(\tilde{p},\theta)]}. \tag{3}$$

Corresponding to the optimal <u>ex ante</u> price $\tilde{p}$ is the <u>ex post</u> profit maximizing output $\tilde{q}$ expressed as a function of θ,

$$\tilde{q}(\theta) \equiv h(\tilde{p},\theta). \tag{4}$$

In the presence of uncertainty, price and quantity instruments transmit central control in quite different ways. It is important to note that by choosing a specific mode for implementing an intended policy, the planners are at least temporarily locking themselves into certain consequences. The values of η and θ are at first unknown and only gradually, if at all, become recognized through their effects. After the quantity $\hat{q}$ is prescribed, producers will continue to generate that assigned level of output for some time even though in all likelihood

$$B_1(\hat{q},\eta) \neq C_1(\hat{q},\theta).$$

In the price mode on the other hand, $\tilde{q}(\theta)$ will be produced where except with negligible probability

$$B_1(\tilde{q}(\theta),\eta) \neq C_1(\tilde{q}(\theta),\theta).$$

Thus neither instrument yields an optimum _ex post_. The relevant question is which one comes closer under what circumstances.[1]

In an infinitely flexible control environment where the planners can continually adjust instruments to reflect current understanding of a fluid situation and producers instantaneously respond, the above considerations are irrelevant and the choice of control mode should be made to depend on other factors. Similar comments apply to a timeless _tâtonnement_ milieu where iterations are costless, recontracting takes place after each round, and in effect nothing real is presumed to happen until all the uncertainty has been eliminated and an equilibrium is approached. In any less hypothetical world the consequences

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[1] We remark in passing that the issue of whether it is better to stabilize uncertain demand and supply functions by pegging prices or quantities can also be put in the form of the problem analyzed in this paper if benefits are associated with the consumers' surplus area under the demand curve and costs with the producers' surplus area under the supply curve.

of an order given in a particular control mode have to be lived with for at least the time until revisions are made, and real losses will be incurred by selecting the wrong communication medium.

Note that the question usually asked whether it is better to control prices or quantities for <u>finding</u> a plan is conceptually distinct from the issue treated in this paper of which mode is superior for <u>implementing</u> a plan. The latter way of posing the problem strikes me as more relevant for most actual planning contexts - either because there is no significant informational difference between the two modes in the first place, or because a step in the <u>tâtonnement</u> planning game cannot meaningfully occur unless it is really implemented, or because no matter how many iterations have been carried out over time there are always spontaneously arising changes which damp out the significance of knowing past history. In the framework adopted here, the planners are at the decision node where as much information as is feasible to gather has already been obtained by one means or another and an operational plan must be decided on the basis of the available current knowledge.

3. Prices vs. Quantities

It is natural to define the comparative advantage of prices over quantities as

$$\Delta \equiv E[(B(\tilde{q}(\theta),\eta) - C(\tilde{q}(\theta),\theta)) - (B(\hat{q},\eta) - C(\hat{q},\theta))]. \tag{5}$$

The loss function implicit in the definition of Δ is the expected difference in gains obtained under the two modes of control. Naturally there is no real distinction between working with Δ or with $-\Delta$ (the comparative advantage of quantities over prices).

The coefficient Δ is intended to be a measure of comparative or relative advantage only. It goes without saying that making a decision to use price or quantity control modes in a specific instance is more complicated than just consulting Δ. There are also going to be a host of practical considerations formally outside the scope of the present model. Although such external factors render Δ of limited value when isolated by itself, they do not necessarily diminish its conceptual significance. On the contrary, having an objective criterion of the *ceteris paribus* advantage of a control mode is very important because conceptually it can serve as a benchmark against which the cost of "non-economic" ingredients might be measured in reaching a final judgement about whether it would be better to employ prices or quantities as planning instruments in a given situation.

As it stands, the formulation of cost and benefit functions is so general that it hinders us from cleanly dissecting equation (5). To see clearly what Δ depends on we have to put more structure on the problem. It is possible to be somewhat less restrictive than we are going to be, but only at the great expense of clarity.

In what follows, the amount of uncertainty in marginal cost is taken as sufficiently small to justify a second order approximation of cost and benefit functions within the range of $\tilde{q}(\theta)$ as it varies around $\hat{q}$.[1] Let the symbol "$\overset{\circ}{=}$" denote an "accurate local approximation" in the sense of deriving from the assumption that cost and benefit functions are of the following quadratic form within an appropriate neighborhood of $q=\hat{q}$:

$$C(q,\theta) \overset{\circ}{=} a(\theta) + (C'+\alpha(\theta))(q-\hat{q}) + \frac{C''}{2}(q-\hat{q})^2 \qquad (6)$$

$$B(q,\eta) \overset{\circ}{=} b(\eta) + (B'+\beta(\eta))(q-\hat{q}) + \frac{B''}{2}(q-\hat{q})^2 \qquad (7)$$

In the above equations $a(\theta)$, $\alpha(\theta)$, $b(\eta)$, $\beta(\eta)$ are stochastic functions and C', C'', B', B'' are fixed coefficients.

Without loss of generality, $\alpha(\theta)$ and $\beta(\eta)$ are standardized in (6), (7) so that their expected values are zero:

$$E[\alpha(\theta)] = E[\beta(\eta)] = 0. \qquad (8)$$

Since θ and η are independently distributed,

$$E[\alpha(\theta)\cdot\beta(\eta)] = 0. \qquad (9)$$

Note that the stochastic functions

$$a(\theta) \overset{\circ}{=} C(\hat{q},\theta)$$

$$b(\eta) \overset{\circ}{=} B(\hat{q},\eta)$$

translate different values of θ and η into pure vertical shifts of the cost and benefit curves.

Differentiating (6) and (7) with respect to q,

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[1]Such an approximation can be rigorously defended along the lines developed by Samuelson [8].

$$C_1(q,\theta) \overset{\circ}{=} (C'+\alpha(\theta)) + C''\cdot(q-\hat{q}) \tag{10}$$

$$B_1(q,\eta) \overset{\circ}{=} (B'+\beta(\eta)) + B''\cdot(q-\hat{q}). \tag{11}$$

Employing the above equations and (8), the following interpretations are available for the fixed coefficients of (6), (7):

$$C' \overset{\circ}{=} E[C_1(\hat{q},\theta)]$$
$$B' \overset{\circ}{=} E[B_1(\hat{q},\eta)]$$
$$C'' \overset{\circ}{=} C_{11}(q,\theta)$$
$$B'' \overset{\circ}{=} B_{11}(q,\eta).$$

From (1),

$$B' = C'. \tag{12}$$

It is apparent from (8) and (10) that stochastic changes in $\alpha(\theta)$ represent pure unbiased shifts of the marginal cost function. The variance of $\alpha(\theta)$ is precisely the mean square error in marginal cost

$$\sigma^2 \equiv E[(C_1(q,\theta) - E[C_1(q,\theta)])^2] \overset{\circ}{=} E[\alpha(\theta)^2]. \tag{13}$$

Analogous comments hold for the marginal benefit function (11) where we have

$$E[(B_1(q,\eta) - E[B_1(q,\eta)])^2] = E[\beta(\eta)^2].$$

From (10) and (2),

$$h(p,\theta) = \hat{q} + \frac{p-C'-\alpha(\theta)}{C''}, \tag{14}$$

implying

$$h_1(p,\theta) \overset{\circ}{=} \frac{1}{C''}. \tag{15}$$

Substituting from (15) into (3) and cancelling out C" yields

$$\tilde{p} \overset{\circ}{=} E[B_1(h(\tilde{p},\theta),n)]. \tag{16}$$

Replacing q in (11) by the expression for $h(\tilde{p},\theta)$ from (14) and plugging into (16), the following equation is obtained after using (8)

$$\tilde{p} \overset{\circ}{=} B' + \frac{B''}{C''}(\tilde{p}-C'). \tag{17}$$

From (12) and the condition $B'' < 0 < C''$, (17) implies

$$\tilde{p} \overset{\circ}{=} C'. \tag{18}$$

Combining (4), (14), and (18),

$$\tilde{q}(\theta) \overset{\circ}{=} \hat{q} - \frac{\alpha(\theta)}{C''}. \tag{19}$$

Now alternately substitute $q = \hat{q}$ and $q = \tilde{q}(\theta)$ from (19) into (6) and (7). Then plugging the resulting values of (6), (7) into (5), using (8), (9), and collecting terms,

$$\Delta \overset{\circ}{=} \frac{\sigma^2 B''}{2C''^2} + \frac{\sigma^2}{2C''}. \tag{20}$$

Expression (20) is the fundamental result of this paper.[1] The next section is devoted to examining it in detail.

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[1] In the stabilization context B" is the slope of the (linear) demand curve, C" is the slope of the (linear) supply curve, and σ^2 is the variance of vertical shifts in the supply curve.

4. Analyzing the Coefficient of Comparative Advantage

Note that the uncertainty in benefits does not appear in (20). To a second order approximation it affects price and quantity modes equally adversely. On the other hand Δ depends linearly on the mean square error in marginal cost. The *ceteris paribus* effect of increasing σ^2 is to magnify the expected loss from employing the planning instrument with comparative disadvantage. Conversely, as σ^2 shrinks to zero we move closer to the perfect certainty case where in theory the two control modes perform equally satisfactorily.

Clearly Δ depends critically on the curvature of cost and benefit functions around the optimal output level. The first thing to note is that the sign of Δ simply equals the sign of $C''+B''$. When the sum of the "other" considerations nets out to a zero bias toward either control mode, quantities are the preferred planning instrument if and only if benefits have more curvature than costs.

Normally we would want to know the magnitude of Δ and what it depends on, as well as the sign. To strengthen our intuitive feeling for the meaning of formula (20), we turn first to some extreme cases where there is a strong comparative advantage to one control mode over the other. In this connection it is important to bear in mind that when we talk about "large" or "small" values of B'', C'', or σ^2, we are only speaking in a relative sense. The absolute measure of any variable appearing in (20) does not really mean much alone since it is arbitrarily pegged by selecting the units in which output is reckoned.

The coefficient Δ is negative and large as either the benefit function is more sharply curved or the cost function is closer to being linear. Using a price control mode in such situations could have detrimental consequences. When marginal costs are nearly flat, the smallest miscalculation or change results in either much more or much less than the desired quantity. On the other hand, if benefits are almost kinked at the optimum level of output, there is a high degree of risk aversion and the center cannot afford being even slightly off the mark. In both cases the quantity mode scores a lot of points because a high premium is put on the rigid output controllability which only it can provide under uncertainty.

From (20), the price mode looks relatively more attractive when the benefit function is closer to being linear. In such a situation it would be foolish to name quantities. Since the marginal social benefit is approximately constant in some range, a superior policy is to name it as a price and let the producers find the optimal output level themselves, after eliminating the uncertainty from costs.

At a point where the cost function is highly curved, Δ becomes nearly zero. If marginal costs are very steeply rising around the optimum, there is not much difference between controlling by price or quantity instruments because the resulting output will be almost the same with either mode. In such a situation, as with the case $\sigma^2=0$, "non-economic" factors should play the decisive role in determining which system of control to impose.

It is difficult to refrain from noticing that although there are plenty of instances where the price mode has a good solid comparative advantage (because $-B''$ is small), in some sense it looks as if prices can be a <u>disastrous</u> choice of instrument far more often than quantities can. Using (20), $\Delta\to-\infty$ if either $B''\to-\infty$ or $C''\to 0$ (or both). The only way $\Delta\to+\infty$ is under the thin set of circumstances where simultaneously $C''\to 0$, $B''\to 0$, and $C''>-B''$. In a world where C'' and B'' are themselves imperfectly known it seems hard to avoid the impression that there will be many circumstances where the more conservative quantity mode will be preferred by planners because it is better for avoiding very bad planning mistakes. This idea could be formalized as follows.

Consider two generalizations of formulae (6) and (7):

$$C(q,\theta) \overset{\circ}{=} a(\theta) + (C'+\alpha(\theta))(q-\hat{q}) + \frac{C''}{2f(\theta)}(q-\hat{q})^2 \tag{21}$$

$$B(q,\eta) \overset{\circ}{=} b(\eta) + (B'+\beta(\eta))(q-\hat{q}) + \frac{B''g(\eta)}{2}(q-\hat{q})^2 \tag{22}$$

The only difference between (6), (7) and (21), (22) is that in the latter approximation $\frac{1}{C_{11}(q,\theta)}$ and $B_{11}(q,\eta)$ are allowed to be uncertain. The change in the profit maximizing output response per unit price change is now stochastic, $h_1(p,\theta) \overset{\circ}{=} \frac{f(\theta)}{C''}$. Without loss of generality we set

$$E[f(\theta)] = E[g(\eta)] = 1.$$

Note that increasing the variance of f (or g) is a mean preserving spread of $\frac{1}{C_{11}}$ (B_{11}). Suppose for simplicity that f and α are independent of each other, and so are g and β. Then we can derive the appropriate generalization of (20) as

$$\Delta \overset{\circ}{=} \frac{B''\sigma^2(1+\delta^2)}{2C''^2} + \frac{\sigma^2}{2C''}, \tag{23}$$

where $\delta^2 \equiv E[(f(\theta)-E[f(\theta)])^2]$ is the variance of $f(\theta)$. Formula (23) can be interpreted as saying that other things being equal, greater uncertainty in $\frac{1}{C_{11}(q,\theta)}$ increases the comparative advantage of the quantity mode.

Having seen how C" and B" play an essential role in determining Δ, it may be useful to check out a few of the principal situations where we might expect to encounter cost and benefit functions of one curvature or another. We start with costs.

Contemporary economic theory has tended to blur the distinction between the traditional marginalist way of treating production theory with smoothly differentiable production functions and the activity analysis approach with its limited number of alternative production processes. For many theoretical purposes convexity of the underlying technology is really the fundamental property.

However, there are very different implications for the efficacy of price and quantity control modes between a situation described by classically smooth Marshallian cost curves and one characterized by piecewise linear cost functions with a limited number of kinks. In the latter case, the quantity mode tends to have a relative advantage since $\Delta=-\infty$ on the flats and $\Delta=0$ at the elbows. Note that it is impossible to use a price to control an output at all unless some hidden fixed factors take the flatness out of the average cost curve. Even then, Δ will be positive only if there are enough alternative techniques available to make the cost function have more (finite difference) curvature than the benefit function in the neighborhood of an optimal policy.

What determines the benefit function for a commodity is contingent in the first place on whether the commodity is a final or intermediate good. The benefit of a final good is essentially the utility which arises out of consuming the good. It could be highly curved at the optimum output level if tastes happen to be kinked at certain critical points. The amount of pollution which makes a river just unfit for swimming could be a point where the marginal benefits of an extra unit of output change very rapidly. Another might be the level of defence which just neutralizes an opponent's offence or the level of offence which just overcomes a given defence. There are many examples which arise in emergencies or natural calamities. Our intuitive feeling, which is confirmed by the formal analysis, is that it doesn't pay to "fool around" with prices in such situations.

For intermediate goods, the shape of the benefit function will depend among other things on the degree of substitutibility in use of this commodity with other resources available in the production organization and upon the possibilities for importing this commodity from outside the organization. These things in turn are very much dependent on the planning time horizon. In the long run the benefit function probably becomes flatter because more possibilities for substitution are available, including perhaps importing. Take for example the most extreme degree of complete "openness" where any amount of the commodity can be instantaneously and effortlessly bought (and sold) outside the production organization at a fixed price. The relevant benefit function is of course just a straight line whose slope is the outside price.

There is, it seems to me, a rather fundamental reason to believe that quantities are better signals for situations demanding a high degree of coordination. A classical example would be the short run production planning of intermediate industrial materials. Within a large production organization, be it the General Motors Corporation or the Soviet industrial sector as a whole, the need for balancing the output of any intermediate commodity whose production is relatively specialized to this organization and which cannot be effortlessly and instantaneously imported from or exported to a perfectly competitive outside world puts a kink in the benefit function. If it turns out that production of ball bearings of a certain specialized kind (plus reserves) falls short of anticipated internal consumption, far more than the value of the unproduced bearings can be lost. Factors of production and materials that were destined to be combined with the ball bearings and with commodities containing them in higher stages of production must stand idle and are prevented from adding value all along the line. If on the other hand more bearings are produced than were contemplated being consumed, the excess cannot be used immediately and will only go into storage to lose implicit interest over time. Such short run rigidity is essentially due to the limited substitutability, fixed coefficients nature of a technology based on machinery.[1] Other things being equal, the asymmetry between the effects of overproducing and underproducing are more pronounced the further removed from final use is the commodity and the more difficult it is to substitute alternative slack resources or to quickly replenish supplies by emergency imports. The resulting strong curvature in benefits around the planned consumption levels of intermediate materials tends to create a very high comparative advantage for quantity instruments. If this is combined with a cost function that is nearly linear in the relevant range, the advantage of the quantity mode is doubly compounded.[2]

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[1]The existence of buffer stocks changes the point at which the kink occurs, but does not remove it. For a more detailed treatment of this entire topic, see Manove [6].

[2]Note that in the context of an autarchic planned economy, such pessimistic conclusions about the feasibility of using Lange-Lerner price signals to control short run output do not carry over to, say, agriculture. The argument just given for a kinked benefit function would not at all pertain to a food crop, which goes more or less directly into final demand. In addition, the cost function for producing a given agricultural commodity ought to be much closer to the classical smooth variety than to the linear programming type with just a few kinks.

5. Many Production Units

Consider the same model previously developed except that now instead of being a single good, $q=(q_1,..,q_n)$ is an n-vector of commodities. The various components of q might represent physically distinct commodities or they could denote amounts of the same commodity produced by different production units. Benefits are $B(q,\eta)$ and the cost of producing the i^{th} good is $c^i(q_i,\theta_i)$. As before, for each i the two random variables η and θ_i are distributed independently of each other.

Suppose the issue of control is phrased as choosing either the quantities $\{\hat{q}_i\}$ which maximize

$$E[B(q,\eta) - \sum_1^n c^i(q_i,\theta_i)],$$

or the prices $\{\tilde{p}_i\}$ which maximize

$$E[B(h(p,\theta),\eta) - \Sigma c^i(h_i(p_i,\theta_i),\theta_i)],$$

where $\{h_i(p_i,\theta_i)\}$ are defined analogously to (2).

Naturally the coefficient of comparative advantage is now defined as

$$\Delta_n \equiv E[(B(\tilde{q}(\theta),\eta) - \sum_1^n c^i(\tilde{q}_i(\theta_i),\theta_i)) - (B(\hat{q},\eta) - \sum_1^n c^i(\hat{q}_i,\theta_i))].$$

Assuming locally quadratic costs and benefits, it is a straightforward generalization of what was done in section 3 to derive the analogue of expression (20),

$$\Delta_n \doteq \sum_{i=1}^n \sum_{j=1}^n \frac{B_{ij}\sigma_{ij}^2}{2c_{11}^i c_{11}^j} + \sum_{i=1}^n \frac{\sigma_{ii}^2}{2c_{11}^i}, \qquad (24)$$

where

$$\sigma_{ij}^2 \overset{\circ}{=} E[(c_1^i(q_i,\theta_i) - E[c_1^i(q_i,\theta_i)])(c_1^j(q_j,\theta_j) - E[c_1^j(q_j,\theta_j)])]. \tag{25}$$

To correct for the pure effect of n on Δ_n, it is more suitable to work with the transformed cost functions

$$C^i(x_i,\theta_i) \equiv nc^i(x_i/n,\theta_i). \tag{26}$$

The meaning of C^i is most readily interpreted for the situation where n different units are producing the same commodity or a close substitute with similar cost functions. Then C^i is what total costs would be as a function of total output if each production unit were an identical replica of the ith unit. When "other things being equal" n is changed, it is more appropriate to think of C^i being held constant rather than c^i.

With C^i defined by (26), we have

$$C_1^i = c_1^i$$

$$C_{12}^i = c_{12}^i \tag{27}$$

$$C_{11}^i = \frac{c_{11}^i}{n} \tag{28}$$

Relation (27) means that in the quadratic case the coefficients of the marginal cost variance-covariance matrix for the $\{C_1^i\}$ are the same as those given by (25) for the $\{c_1^i\}$. Substituting (28) into (24),

$$\Delta_n \overset{\circ}{=} \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{B_{ij}\sigma_{ij}^2}{2C_{11}^i C_{11}^j} + \frac{1}{n}\sum_{i=1}^{n}\frac{\sigma_{ii}^2}{2C_{11}^i}\,. \tag{29}$$

The above formula shows that in effect the original expression for Δ holds <u>on the average</u> for Δ_n when there is more than one producer. Naturally the generalization (29) is more complicated, but the interpretation of it is basically similar to the diagnosis of (20) which was just given in the previous section.

There is, however, a fundamental distinction between having one and many producers which is concealed in formula (29). With some degree of independence among the distributions of individual marginal costs, less weight will be put on the first summation term of (29). Other things being equal, in situations with more rather than fewer independent units producing outputs which substitute for each other in yielding benefits, there is a correspondingly greater relative advantage to the price mode of control. Although this point has general validity, it can be most transparently seen in the special regularized case of one good being produced by many micro-units with symmetrical cost functions. In such a case

$$\begin{aligned} &\text{(i)} \quad B_{ij} = B'' \\ &\text{(ii)} \quad C^{i}_{11} = C'' \\ &\text{(iii)} \quad \sigma^2_i = \sigma^2 \\ &\text{(iv)} \quad \sigma^2_{ij} = \rho\sigma^2, \quad i \neq j, \quad -1 \leq \rho \leq 1. \end{aligned} \tag{30}$$

The coefficient ρ is a measure of the correlation between marginal costs of separate production units. If all units are pretty much alike and are using a similar technology, ρ is likely to be close to unity. If the cost functions of different units are more or less independent of each other, ρ should be nearly zero. While in theory the correlation coefficient can vary between plus and minus unity, for most situations of practical interest the marginal costs of two different production units will have a non-negative cross correlation.

Using (30), (29) can be rewritten as

$$\Delta_n \overset{\circ}{=} \rho\left(\frac{B''\sigma^2}{2C''^2} + \frac{\sigma^2}{2C''}\right) + (1-\rho)\left(\frac{1}{n} \; \frac{B''\sigma^2}{2C''^2} + \frac{\sigma^2}{2C''}\right). \tag{31}$$

If the marginal costs of each identical micro-unit are perfectly correlated with each other so that $\rho=1$, it is as if there is but a single producer and we are exactly back to the original formula (20). With $n>1$, as ρ decreases, Δ_n goes up. A ceteris paribus move from dependent toward independent costs increases the comparative advantage of prices, an effect which is more pronounced as the number of production units is larger. If there are three distinctly different types of sulfur dioxide emitters with independent technologies instead of one large pollution source yielding the same aggregate effect, a relatively stronger case exists for using prices to regulate output.

When it is desired to control different units producing an identical commodity by setting prices, only a single price need be named as an instrument. The price mode therefore possesses the ceteris paribus advantage that output is being produced efficiently ex post. With prices as instruments

$$c_1^i(\tilde{q}_i,\theta_i) = c_1^j(\tilde{q}_j,\theta_j) = \tilde{p},$$

whereas with quantities

$$c_1^i(\hat{q}_i,\theta_i) \neq c_1^j(\hat{q}_j,\theta_j)$$

except on a set of negligible probability.

Using prices thus enables the center to automatically screen out the high cost producers, encouraging them to produce less and the low cost units more. This predominance in efficiency makes the comparative advantage of the price mode go up as the number of independent production units becomes larger, other things being equal. The precise statement of such a proposition would depend on exactly what was held equal as n was increased - the variance of _individual_ costs or the overall variance of _total_ costs. For simplicity consider the case of completely independent marginal costs, $\rho=0$. Then (31) becomes

$$\Delta_n \doteq \frac{1}{n}\frac{B''\sigma^2(n)}{2C''^2} + \frac{\sigma^2(n)}{2C''}, \tag{32}$$

where $\sigma^2(n)$ is implicitly some (given) function of n. If the "other thing" being equal is the constant variance of marginal costs for each individual producing unit, then $\sigma^2(n) \equiv \sigma^2$. If the variance of total costs is held constant as n varies, $\sigma^2(n) \equiv n\sigma^2$. Either way Δ_n in (32) increases monotonically with n and eventually becomes positive.

It is important to note that such _ceteris paribus_ efficiency advantages of the price mode as we have been considering for large n are by no means enough to guarantee that Δ_n will be positive in a particular situation for any _given_ n. True, what aggregate output is forthcoming under the price mode will be produced at least total cost. But it might be the wrong overall output level to start with. If the $\{-B_{ij}\}$ are sufficiently large or the $\{C^i_{11}\}$ sufficiently small, it may be advantageous to enjoy greater control over total output by setting individual quotas, even after taking account (as our formula for Δ_n does) of the losses incurred by the _ex post_ productive inefficiency of such a procedure.[1]

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[1] An even better procedure from a theoretical point of view in the case when an identical output is produced by many firms would be to fix _total_ output by command and subdivide it by a price mechanism. This kind of solution is proposed by Dales [2] who would set up a market in "pollution rights," the fixed supply of which is regulated by the government. In effect, such an approach aggregates the individual cost functions, and we are right back to a single cost function.

Returning to the general case with which this section began, we note that the basic difference between benefits and costs becomes somewhat more transparent in the n commodity vector formulation. Only the center knows benefits. Even if it could be done it would not help to transmit $B(\cdot)$ to individual production units because benefits are typically a non-separable function of all the units' outputs, whereas a particular unit has control only over its own output. In any well formulated mode of decentralized control, the objective function to be maximized by a given unit must depend in some well defined way on its decisions alone. For the purposes of our formulation B need not be a benefit and the $\{c^i\}$ need not be costs in the usual sense, although in many contexts this is the most natural interpretation. The crucial distinction is that B is in principle knowable only by the center, whereas c^i is best known by firm i.

When uncertainties in individual costs are unrelated so that the random variables θ_i and θ_j are independently distributed, the decision to use a price or quantity instrument to control q_i alone is decentralizable. Suppose it has already been resolved by one means or another whether to use price or quantity instruments to control q_j for each $j \neq i$. To a quadratic approximation, the comparative advantage of prices over quantities for commodity i is

$$\Delta^i \doteq \frac{\sigma_i^2 B_{ii}}{2 c_{11}^{i\,2}} + \frac{\sigma_i^2}{2 c_{11}^i}, \tag{33}$$

which is exactly the formula (20) for this particular case.

In some situations, "mixed" price-quantity modes may give the best results. As a specific example, suppose that q_1 is the catch of a certain fish from a large lake and q_2 from a small but prolific pond. Let q_1 be produced with relatively flat average costs but q_2 have a cost function which is curved at the optimum somewhat more than the benefit function. The optimal policy according to (33) will be to name a quota for q_1 and a price for q_2.

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