



# The transformation of values into prices on the basis of random systems revisited

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## Abstract

The paper first defends the importance of the transformation of values into prices on the basis of textual analysis against those who believe that Marxian economics can dispense with prices of production and should be built on values and market prices only, or that the theory of the forms of value matters just for commodities, not for capital, or that one should only have prices of production, forgetting about values. It is shown that  $P=M$  is the lynchpin of the Marxian system, regarding the theory of the forms of value as well as the theories of exploitation and accumulation. Second, two proofs of the equality of the mass of profits  $P$  and total surplus value  $M$  are presented on the basis of random systems. This transformation is compared with Sraffa's use of the standard commodity for the transformation of values into prices. The third part concerns the interpretation of these results in the light of the fact that Marxian economics hinges on  $P=M$ , but that the assumption of random systems is more narrow than the conditions regarded as necessary by Marx or Engels and broader than modern critics, from Bortkiewicz to Sraffa, thought.

**Keywords** Marxian economics · Transformation problem · Labour values · Theory of distribution · Capital theory · Random matrix

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In this paper all quotes from Marx are in German, with short paraphrases where the assertion made in the quote is not clear from the context. Standard English translations have been added, where they were available. I should like to thank Kiichiro Yagi for constant encouragement. The paper was presented at the International Symposium “Marx in the twenty-first century” (on the occasion of the 200th anniversary of the birth of Karl Marx) at Hosei University, 22–23 December 2018, and at the Joint Seminar of the Institute of Business Research, Chuo University, and of the Nonlinear Research Division, Japan Association for Evolutionary Economics. At this seminar, convened by Professors Yuji Aruka and Kiichiro Yagi, a workshop was held on “Random System Solution of the Transformation Problem”, coordinated by Kiichiro Yagi, with discussion-papers by Yoshihiro Yamazaki, Kenji Mori and Masashi Morioka. I should like to thank for all comments received, in particular for the discussion-papers. My responses to them have been incorporated in the present version of my paper, with due acknowledgements.

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## 1 Introduction

I began to read Marx carefully a little more than 50 years ago. I have always remained convinced that the transformation of values into prices was of central importance in Marx' own eyes—the reasons are manifold. Let us call this the problem of *relevance*. It is more difficult to judge whether the transformation, in particular the use of labour values, can be circumvented as far as the theory of accumulation is concerned, for instance by translating the Marxian propositions about the production of absolute and relative surplus value into the language of the modern mathematical theory of prices of production (I think this can be done) or whether labour values can be dispensed with also with regards to other parts of the Marxian theory such as the theory of the forms of value (this is less clear). One speaks here of the question of the *redundancy* of the labour theory of value (Steedman 1977; Feess-Dörr 1989). Finally, my opinions have changed most dramatically with regard to the *feasibility* of the transformation, as I shall explain below (Sects. 5, 6).

The feasibility is intimately related to the interpretation of what, if anything, has to be transformed. There is the temptation to explain the transformation problem away. I once tinkered with Joan Robinson's idea of basing the transformation on a given money wage, but I gave that up even before it later became, in more sophisticated presentations, the so-called 'New Solution' (Schefold 1973).

I shall here return to what I now regard as the best possible—and, I think, quite satisfactory—solution, based on the traditional interpretation of the transformation problem, which is made solvable by means of random systems. It is an example of what historians of economic thought call an analytical reconstruction. Marx never claimed to have achieved more than an approximate solution, based on averages, and this is what the random approach allows to render more precise.

I shall begin with a simple description of the traditional framework of the transformation problem (Sects. 1, 2). I shall defend the traditional interpretation by emphasising the similarity of prices of production, natural prices and the normal prices of neoclassical theory in a history of economic thought perspective. Section 3 is a relatively extensive attempt to demonstrate the importance of a transformation which results in the equality of the rates of profit measured in values and in prices. This section concerns the relevance, but can be omitted by those who only want to see how I prove the feasibility of the random solution (Sects. 5, 6). I have added a new necessary and sufficient condition for the transformation to be possible in so-called one-industry systems (Sect. 6), and I have incorporated money (Sects. 2, 5). Section 4 touches the question of redundancy. Section 7 presents an evaluation of the results.<sup>1</sup>

<sup>1</sup> I apologise for having to repeat some derivations contained in my earlier article on the random solution (Schefold 2016). There are also a in part parallel texts in German (Schefold 2019a, b).

## 2 Prices of production in classical and neoclassical theory and in Marx

Our approach to Marxian theory begins with his opponents. Böhm-Bawerk was the first among the eminent neoclassicals to accept the Marxian challenge (von Böhm-Bawerk 1896). He was outstanding in his ability to isolate the logic of other thinkers by going deeply into their thought before disentangling it. He criticised the theories of interest based on abstinence and productivity, which Marx had characterised as vulgar economics, and he replaced them by time preference and roundabout production (von Böhm-Bawerk 1921).<sup>2</sup> It is little known that he shared with Marx the conviction that prices will tend in the long run to natural prices as in Adam Smith and David Ricardo (Garegnani 1960, Schefold 2018b). Böhm-Bawerk had taken over from Ricardo in particular the consideration that, if the distribution between wages and profits changes, relative prices change in general as well. If the rate of profit rises, the prices of commodities produced in labour-intensive industries will fall relatively to those in capital-intensive industries. For if the rate of profit rises, the wage rate will fall, given the technique, and a commodity produced in a capital-intensive industry will become more expensive insofar as now more profit will have to be contained in the price. That wage costs diminish simultaneously will be of lesser importance in capital-intensive industries; vice versa, if the industry is labour-intensive. Then, the fall of wage costs will predominate in comparison with the rise of the claim for profit corresponding to the general rate. Marx repeated this typically Ricardian exercise as it were upwards and downwards (with wages rising and falling) several times (MEGA II/4.2, p. 273, and earlier in the theories of surplus value, MEGA II/3, p. 841). Böhm-Bawerk followed Ricardo also in that he assumed commodities of an average capital composition and took their prices as the standard so that, relative to it, the prices of commodities produced in capital-intensive industries rose and those produced by labour-intensive industries fell. This implied that, compared to this average, always some prices increased and others fell, whenever distribution changed. Böhm-Bawerk used this insight to criticise Irving Fisher's proposition that prices would always fall with a rise of the rate of interest because of discounted expected future returns. Irving Fisher had forgotten, Böhm-Bawerk observed, that future returns would also be affected by the change in distribution.<sup>3</sup>

Böhm-Bawerk understood and emphasised that Ricardo had always a single technique (only one method of production in each industry) taken as given in these exercises. If the real wage is fixed by the subsistence need of the workers, the profit will remain as a residual in the surplus. Böhm-Bawerk therefore accepted that this profit was uniquely determined, once the technique and the real wage, therefore once the

<sup>2</sup> According to Böhm-Bawerk, the use of capital does not represent abstinence but a postponement of consumption, compensated by the hope for future interest.

<sup>3</sup> There is an analogy between Böhm-Bawerk's critique of Fisher and the Neoricardian critique of the Keynesian marginal efficiency of capital schedule. If future returns are regarded as expected and given, the value of capital that engenders those returns will fall as the rate of interest rises. But the returns themselves depend on the rate of profit in a full equilibrium, and hence the opposite movement of the value of capital is just as possible (Schefold 1997, p. 225).

surplus was given. But he now objected that also the claims of capital for a determinate earning of interest would have to be satisfied, if capital was to be advanced and if equilibrium should result. We assume to simplify the argument that the providers of capital demand a minimum rate of interest, in the same way as the workers demand a minimum wage. To make it possible that both claims could be fulfilled simultaneously and exhausted the product, Böhm-Bawerk now introduced the assumption that the technique was variable. According to neoclassical thought, the product per capita rises with the intensity of capital of the technique chosen, so that there is a technique for which the claims of both parts, expressed by the factor price ratio, can just be fulfilled. This does not yet determine the quantities of the factor that will be employed. If the factor prices are given, the factor supplies must be elastic to make equilibrium possible. Accordingly, Böhm-Bawerk further introduced corresponding hypotheses regarding the factor supplies and elastic demand on both sides of the markets, but his specific construction is of no concern to us, because the analogy between neoclassical, Marxian and classical thought ends here (Scheffold 2018a).

The accumulation of capital for Marx is a chase after surplus value. He does not try to determine in a formula how much capital will be supplied, given a certain expected rate of interest. For him, the supply depends on historical influences (Scheffold 2017b). This corresponds to the radical Keynesian position, according to which the future is uncertain and the demand for investment goods volatile and unpredictable. The idea that one could raise production only by increasing the rate of interest was ‘Blödsinn’ (‘nonsense’) for Marx. He pointed to the ‘Railway Mania’ in England: „Der Zinsfuß stieg nicht“ (‘The rate of interest did not rise’, MEGA II/4.2, p. 629). New techniques are introduced to use labour power profitably, and factor quantities are not in equilibrium. Given the minimum wage, there is no full employment of labour, instead there is a reserve army of workers which changes in magnitude and composition.

We are here only interested in the fact that the positions coincide insofar as the theory of long-run prices with a uniform rate of profit represents the main tool for the analysis of the process of accumulation both in the classical tradition according to Ricardo and Marx and in the neoclassical, represented by Böhm-Bawerk and his contemporary rivals. Each state of the system, which changes with the technique, is analysed as if it lasted indefinitely. The prices are used to assess the profitability of alternative processes of production—better processes lead to higher profits—and once the ‘socially necessary’ technique prevails, the rates of profit of individual processes can tend to uniformity. Long-run prices change slowly and fall in real terms as technical progress proceeds.

This classical and old neoclassical price system is to be distinguished from intertemporal systems of prices, where changes of prices from period to period are connected so as to describe the effects of changes in supply and demand and in the available techniques from period to period. Marx accepted the classical notion of price explicitly; we read in his manuscript of Volume III:

„Der Produktionspreis... ist in fact dasselbe was Adam Smith ‚natural price‘; Ricardo ‚price of production‘, ‚cost of production‘, die

Physiokraten, *prix nécessaire* ‘nennen... weil er in the long run [note the explicit use of the concept of the long run – BS] Bedingung der Reproduktion der Waaren jeder besonderen Productionssphäre ist“ (MEGA II/4.2, p. 272).

This has been rendered by Engels in his edition of *Das Kapital*: „Der Produktionspreis schließt den Durchschnittsprofit ein. Wir gaben ihm den Namen Produktionspreis; es ist tatsächlich dasselbe, was A. Smith natural price nennt, Ricardo price of production, cost of production, die Physiokraten *prix nécessaire* nennen - wobei keiner von ihnen den Unterschied des Produktionspreises vom Wert entwickelt hat -, weil er auf die Dauer Bedingung der Zufuhr, der Reproduktion der Ware jeder besondern Produktionssphäre ist.“ (MEGA II/15, p. 198)

Translation: “The price of production includes the average profit. We call it price of production. It is really what Adam Smith calls natural price, Ricardo calls price of production, or cost of production, and the physiocrats call *prix nécessaire*, because in the long run it is a prerequisite of supply, of the reproduction of commodities in every individual sphere.” (Capital Vol. III, p. 144)

Such prices of production are defined for a given technique and a given distribution. They are used in a changed framework and with a new representation of technology by the main representatives of the old neoclassical school, Walras, Marshall, Böhm-Bawerk and J. B. Clark. Neoclassical theory passes to intertemporal prices only in the twentieth century with Hicks. Von Weizsäcker and his pupil Wolfstetter intended to develop Marxian ideas by means of intertemporal price systems, first based on a model of planning (von Weizsäcker and Samuelson 1971). I regard this as a misleading interpretation of Marx: the unheard-of dynamic, which is inherent in the Marxian representation of the process of accumulation, cannot be represented by means of the soft and continuous connection of the periods in an intertemporal system with steady state growth. Older systems of production must be destroyed times and again, to give room for the emergence of a new one. This dynamic is historical, and this is the reason why the idea of a stable investment function cannot find a place in Marx. The system is thrown into a new state, for instance, because of the mechanisation of an important branch of industry: the market prices must adapt and tend to new prices of production. This adaptation is described in a more round about manner by Marx, since he first measures in value and hence has market values. These must be expressed in money terms and tend to value prices, these tend to prices of production expressed in money. The modern neoclassicals believe to catch this dynamic directly by assuming a steady growth of productivity, which shows in the shift of intertemporal prices over time. But this means to miss the character of innovations as events. It also means, from a Keynesian point of view, that abstraction is made from uncertainty.

It still surfaces in Böhm-Bawerk that long-run prices change as in Ricardo and Marx with changes of technique and in function of distribution and that the task is set to analyse these changes. Since Böhm-Bawerk interprets interest and normal profit as the result of an intertemporal exchange, he gets closer to the later theory of

intertemporal prices, but he sticks to the uniform rate of profit and to the classical method for the analysis of prices in the long run.

Marx started, as is well known, from labour values, as had been customary since Smith and Ricardo. The question then was how the deviation of prices of production from labour values could be represented. Ricardo had compared commodities produced by the same total amounts of labour expended at different dates. The profits made on earlier expenditures of labour had to be added to the wage costs, for repeated expenditure of labour meant repeated advances of capital for which profits could be expected, and if these profits were reinvested, profits made from profits would accumulate as in the case of compound interest. These profits depended on how much time it took to bring the commodity to market, as Ricardo would put it.<sup>4</sup> It is essential to see that Ricardo did not view these profits as a pure fruit of time in the way interest seems to depend on time only, but as a result of repeated investment. Böhm-Bawerk refined the terminology by introducing the idea of roundabout production and tried to provide a, as it were, two-dimensional measurement of capital. The subsistence fund represented the quantity of capital and the distribution of the labour inputs over time represented its structure or its quality. He differed from Ricardo in that he focused on interest rather than on profit. The concept of long-run price, however, remained always the same.

Marx based his analysis on labour values both in the First Volume, when he discussed accumulation, and in the Second, when he discussed circulation. Although he had hinted in several places in the First Volume that the proportionality of labour values and of long-run prices did not hold, if the capitals employed in different industries were of different organic composition, the transformation was revealed only in the Third Volume. Marx clearly hoped to surprise his readers in this manner. Engels turned this into a kind of prize competition, when he published the Second Volume and challenged the economists to solve the problem (on this and on the history of the transformation problem see Howard and King 1987). Having announced the riddle (MEGA II/13, p. 21), he praised the better solutions and criticised the bad ones in his introduction to the Third Volume (MEGA II/15, p. 11–23). Engels treated Marx's own solution as impeccable, but Böhm-Bawerk doubted it and von Bortkiewicz showed that it could not be generally true, and this assessment is on the whole still in vigour. One could try to translate Marxian results into a modern theory of prices of production, as I have done myself, and as Heinz Kurz resolutely continues to do (Kurz 2018), but there have also been attempts to reinterpret Marx, as if he had never had prices of production in the sense of the natural prices of Adam Smith or David Ricardo in view or as if the transformation problem did not exist, because Marx had always reckoned in terms of prices of production (Mohun and Veneziani 2017). But on what could Marx base his prices of production, if not on values, since the alternative of basing prices on the structure of production, as represented

<sup>4</sup> By 'value', Ricardo intended natural price or price of production, and he approximated natural prices by abstracting first from land and profit with embodied labour. Then he introduced profit as a modification of 'value'. Marx reserved the term 'value' for embodied labour and foisted the same distinction on Ricardo.

in an input–output system, seems not to have been known to him? He commented on the Corn Model, polemicizing against John Stuart Mill (Schefold 2017a), while he praised the *Tableau économique* of François Quesnay, without recognising the similarity of both conceptions.

To do justice to Marx, one should endeavour to understand his specific method, in particular his dialectic and the theory of the forms of value. It will be seen that the three volumes are organised according to a coherent conception, in which the theory of the forms of value is not only in the First Volume, as the foundation of the fetish character of the commodity and of the theory of money, but that it extends throughout to the Third Volume and reaches its culmination point precisely with the transformation from values into prices. Hence the significance of the transformation problem.

It has turned out recently that Marx's transformation remains correct after all, if the investigation is restricted to so-called random systems. The result shows that the transformation is essentially more general than critics of Marx had thought up to now—I must say that I did not expect this confirmation at all. However, the result is not fully general, and it is a question of interpretation, whether it is as general as Marx himself affirms, for he does not claim that his theory is completely exact; it holds only “on average” and as a tendency. The meaning of such formulations is open to debate. It may be that our result will disappoint both Marxists and critics of Marx: the former, because the sting of the critique only becomes looser, it is not withdrawn completely, the latter, because the critique of the logic of the transformation is unable to pull the floor away from under Marx's feet. Approval or not, it is necessary to stick to the truth, to work out the correct analysis and then to interpret the result. If it changes the vision with which one began, this has to be modified.

Hence we want to retrace the most important steps, which led Marx from the foundation of the theory of value to the execution of the transformation, to be conscious again of the reasons why it seems so important for Marx to represent profit as redistributed surplus of value. We shall then reconstruct the transformation in the case of random systems and turn eventually to the evaluation. It would be appropriate to have a section on the history of the transformation problem, but it has to be left out for reasons of space.<sup>5</sup>

### 3 From the theory of value to the problem of transformation

The contrast is old between the Marxian method to derive prices of production from labour values and the modern procedure to start from the structure of the production of values in use and to calculate prices of production directly. There are not only the authors of the twentieth century, who nowadays are mostly considered as predecessors of Sraffa, such as Potron (Bidard and Erreygers 2007),

<sup>5</sup> I apologise for this shortcoming. An important survey on the earlier debates was provided by Howard and King (1987). A survey of the more recent contributions is to be found in Mohun and Veneziani (2017).



Charasov (Gehrke 2015) or Remak (Kurz and Salvadori 2003, p. 64), but there was an attempt to derive prices of production from an early form of an input–output table already in the discussion of the problem of transformation and at the time of the publication of the Third Volume by Mühlpfordt. He made a remarkable progress, but remained little understood (Quaas 1994). A linear technique of production can be found in Walras. Count Buquoy (2005) composed an input–output table to reflect economic interconnectedness already at the time of Ricardo and dedicated his book to his venerated friend Goethe in 1818. Marx seems not to have known Buquoy’s work, which also contains an early mathematical formulation of Smith’s theory of natural prices. The most important early appeal to the structure of production of use values is to be found in the Corn Model by Ricardo himself, of course. Whether Sraffa was right to infer it from the Ricardo–Malthus exchange of letters can be left open; it is certain that several of his pupils used it in several contexts, as Skourtos showed (1991). It is to be found in John Stuart Mill and in R. Torrens, and Marx encountered it there and it irritated him (cf. Schefold 2017a). Where was the value? Marx remarked that the material inputs and outputs were not enumerated completely in this structure of values in use, but this was true also for his own system where only those goods enter as values which are traded as commodities, and Marx does not discuss which goods rise to the rank of circulating as commodities. Only neoclassical theory offers a general criterion: those goods which are scarce but in demand, will receive prices. The classical authors confined their attention to goods that could be reproduced, but did not pay much attention to joint products that were reproduced in excess and therefore had prices tending to zero or no prices at all.

Marx is virtually unique with his focus on labour values as the foundation of the explanation of prices. His theory of the genesis and the reproduction of money are clothed in the theory of the forms of value: the dark gateway to *Das Kapital* which helps to keep out the unbidden. Keynes confessed his utter lack of comprehension defiantly, when Sraffa imposed the lecture of the first volume on him. Sraffa reported on the result in a letter to Rajani Palme Dutt, a representative of the British Communist Party, leading in ideological matters, on 19 April 1932:

“You will be interested in this extract [...] from a letter of Keynes which I found here: ‘I made a good try at the Marx volumes, but I swear that it absolutely beats me what you find in them, or what you expected me to find! I did not discover a single sentence of any conceivable interest to a rational human being. For next vacation you must give me a marked copy’. What is interesting [...] is not K.’s individual case (in fact it is not at all individual, all economists, + most intellectuals in England must react in a similar way), nor even that of bourgeois intellectuals as a whole. The question I am puzzled by is this: the intellectual + literary food of the working class in England is entirely provided, in the form of school-teaching, newspapers, fiction, sermons, popular science, films, political and T. U. speeches etc. by people of Keynes’s mentality; and if



they start with such a ‘culture’ how can they pass on directly to Marx without some ‘mediation’?<sup>6</sup>

Sraffa had been worried for some time as to how Marx could be represented in a manner appropriate and understandable for the Anglo-Saxon world. According to a fragmentary note, which is to be found in the Sraffa Archive, he felt that this was a “metaphysical” problem. He wrote in 1927:

“I foresee that the ultimate result will be a restatement of Marx, by substituting to his Hegelian metaphysics and terminology our own modern metaphysics and terminology: by metaphysics here I mean, I suppose, the emotions that are associated with our terminology and frames (*schemi mentali*) - that is, what is absolutely necessary to make the theory living (*lebendig*), capable of assimilation and at all intelligible. If this is true, it is an exceptional example of how far a difference in metaphysics can make to us absolutely unintelligible an otherwise perfectly sound theory. This would be simply a translation of Marx into English, from the forms of Hegelian metaphysics to the forms of Hume’s metaphysics...”.<sup>7</sup>

Sraffa’s use of the concept of “metaphysics” is unconventional and ironic. He does not mean, like Heidegger (1976), the great philosophical question, why there is something at all and not nothing, but he denotes by metaphysics the feelings associated with our conceptualisations and the schemes of our thinking, which lend an inner life to our understanding of theories. We should rather speak of a form of representation and of a method, which Marx used and which Keynes, self-assured, reprimanded so severely.

We cannot discuss here where Marx took his mode of expression from, from Hegel, from Aristotle or from yet other authors, nor what was philosophical tradition and what his own contribution, but we can use an important hint which is to be found as a footnote to a significant passage in the First Volume of *das Kapital*, where Marx (1969, p. 392) refers to Darwin:

„Darwin hat das Interesse auf die Geschichte der natürlichen Technologie gelenkt, d.h. auf die Bildung der Pflanzen- und Tierorgane als Produktionsinstrumente für das Leben der Pflanzen und Tiere. Verdient die Bildungsgeschichte der produktiven Organe des Gesellschaftsmenschen, der materiellen Basis jeder besondern Gesellschaftsorganisation, nicht gleiche Aufmerksamkeit? Und wäre sie nicht leichter zu liefern, da, wie Vico sagt, die Menschengeschichte sich dadurch von der Naturgeschichte unterscheidet, daß wir die eine gemacht und die andre nicht gemacht haben? Die Technologie enthüllt das aktive Verhalten des Menschen zur Natur, den unmittelbaren Produktionsprozeß seines Lebens, damit auch seiner gesellschaftlichen Lebensverhältnisse und der ihnen entquellenden geistigen Vorstellungen. Selbst alle Religionsgeschichte, die von dieser materiellen Basis abstrahiert, ist - unkritisch.

<sup>6</sup> Quoted from Labour History Archive and Study Centre in Munari (2017, p. XVII).

<sup>7</sup> Sraffa Archive D3/12/04/15.

Es ist in der Tat viel leichter, durch Analyse den irdischen Kern der religiösen Nebelbildungen zu finden, als umgekehrt, aus den jedesmaligen wirklichen Lebensverhältnissen ihre verhimmelten Formen zu entwickeln. Die letztere ist die einzig materialistische und daher wissenschaftliche Methode.“

“Darwin has interested us in the history of Nature’s Technology, i.e., in the formation of the organs of plants and animals, which organs serve as instruments of production for sustaining life. Does not the history of the productive organs of man, of organs that are the material basis of all social organisation, deserve equal attention? And would not such a history be easier to compile, since, as Vico says, human history differs from natural history in this, that we have made the former, but not the latter? Technology discloses man’s mode of dealing with Nature, the process of production by which he sustains his life, and thereby also lays bare the mode of formation of his social relations, and of the mental conceptions that flow from them. Every history of religion, even, that fails to take account of this material basis, is uncritical. It is, in reality, much easier to discover by analysis the earthly core of the misty creations of religion, than, conversely, it is, to develop from the actual relations of life the corresponding celestialised forms of those relations. The latter method is the only materialistic, and therefore the only scientific one.” (Marx 1977, p.330)

The passage contains a short representation of the materialist method: We shall infer from the development of technology, which—we are referred to Darwin—follows a logic of selection, the conditions of life and from there the spiritual representations. The chosen example leads as far away from economics as far as possible. The point is not, Marx asserts, to denounce the superstitious character of religion (as the atheists perceive it), but to derive religion as a phenomenon, which is by necessity dependent on the state of development. This is difficult, and not even Marx really tried to construct materialist derivations of different religions. Rather, the note shall induce the reader to reflect on how economic relationships appear to people, for instance, profit as a result of the “abstinence” of capitalists or of the “productivity of capital”. Hence the economic process must appear as seemingly natural; it is not controlled by the action of fully conscious persons, but it conditions their actions and generates their perceptions and their thought. And certainly Marx wanted to *derive* the “vulgar” dynamic representations of *economic* relations *as if* they constituted a religion. To do this was, he thought, more than to reveal them as false and inconsistent. He wanted to denounce them as historically specific ideological expressions.

Prior to expressions are perceptions that are similarly conditioned. Scholars in the Humanities and Social Sciences follow Marx, when they speak of the “fetishism” of commodities as the opaque, hence mysterious property of use values, such as gold or even precious objects of consumption, of signalling a high value of exchange. To perceive commodities as precious precedes exchange; it is a passive feeling induced by exchange. Commodity fetishism according to Marx is not the appreciation of a preciousness stemming from rare beauty; what seems splendour arises from the fascination by abstract value.

But is the elementary act of exchange not an “act”? “Acting” in German means “handeln”, and “handeln” also means “marketing”. Hence is marketing and exchange not something which one does consciously and deliberately? The beginning of *Kapital* is so difficult because Marx sets the task to himself to represent the economic process as autonomous, as something determining man precisely where the economic subject feels most free, because one can choose with whom one wants or does not want to undertake an economic transaction. Gift-giving results from and leads to obligations. Buying and selling need not be tied to tradition. The traders are free from duties, once the commodity has been paid.

The discussion of the act of choice and of the motives underlying it is what Austrian theory begins with, to *find* the relation of exchange and the price. Marx, on the contrary, starts in his derivation of exchange and money from the *given equivalence* of commodities which are of equal value; this equivalence is prior and socially determined. In the end, each school of economics accepts such determination. It is always the case that, in competition, the price is given to the single trader, but the individual schools, the mercantilists more than the classicals, the Austrians more than the Marshallians, leave room for the description of what happens in the market so that, in the end, the competitive price is found: as a result of active trading.

Marx follows the converse method. Competition is what is to be *derived* (in the Third Volume); here, in the First Volume, the result of the competitive process is anticipated and is explained by means of the social character of labour. The process of work is on the one hand concrete and directed towards the manufacture of certain values in use, on the other, the productions have in common that they are based on labour, taken in its abstract meaning, and they result in the creation of value.

If a jacket, which has been tailored, is set as an equivalent of potatoes produced by means of the concrete labour of peasants, farm work and tailor work cannot explain the equality, for as concrete labours they are as different as the use values themselves, the jacket and the potatoes. Moreover, there is the difficulty that the jacket and the potatoes, which form the equivalent of the jacket in the market, will in general not have been produced in the same timespan as measured by the clock, because both tailor work and farm work presuppose specific gifts, experience and training. Does not one kind of concrete labour result in more value than another? How can the appeal to labour then be helpful? But the basic Marxian assumption is that abstract labour constitutes value, and labour itself has none. Marx abstracts from talents and experiences and presupposes that by considering the cost of training, both activities, tailor work and farm work, can be reduced to average labour. He was quite conscious of the difficulty of this reduction, but presupposed that the problem could be solved. The reader is mystified how the complex activity of programming shall result from the training by means of the simple labour of uneducated men and women or how stitching and shovelling sand shall in the end be equalised as abstract labours. An increased “Anspannung der Arbeitskraft” (“heightened tension of labour-power”) makes a “vergrößerte Arbeitsaufgabe in derselben Zeit” (“increased expenditure of labour in a given time”) and “Zusammenpressung einer größeren Masse Arbeit in eine gegebene Zeitperiode” possible; this “zählt ... als größtes Arbeitsquantum” (“counts... for a greater quantity of labour”) (Marx 1969, p. 432, 1977, p. 386). But if labour is not measured by means of time alone, how can

quantities of labour be compared and measured so that they define value? Ricardo was more cautious, for he did not interpret homogeneity of labour in this strict manner, but he aggregated different kinds of labour by means of the wage rates, which he regarded as given. I used to call this “weak homogeneity” of labour, in contrast to Marx’s postulate of “strong homogeneity” (Scheffold 1989, pp. 314–323).

The essential of what Marx wants to show by means of his theory of the forms of value is independent of the reduction of complicated labour to simple labour. If the jacket and the potatoes represent equal values, the relationship is nonetheless not only symmetric, insofar as the value of the jacket is equal to that of the potatoes and vice versa, but there is also an asymmetric relationship, insofar as the value of the jacket is represented by a certain amount of potatoes from the point of view of the tailor. This occurs on the subjective plane, where the reflection of the reader of the *Kapital* may have started, and Marx concedes this representation by means of small hints: the producer realises with pleasure or disappointed, depending on the expectations, how many potatoes in the end may be given for the jacket. But precisely the substance of this individual experience is socially determined: in how many potatoes the value of the jacket is reflected. The social result is that a determined amount of a value in use, of potatoes, becomes the expression of value, that is of the value of the jacket. A value in use (an amount of potatoes) becomes the “phenomenal form” of its “opposite” of the value (of the jacket). This is the decisive and difficult step. Use value and value of exchange are not thought of as “opposites” in ordinary logic. They are either taken as different dimensions like time and space (time is not the opposite of space) or, as in neoclassical theory, as being in the same dimension: if the use value of the jacket is subjective, it can be estimated. The peasant, who buys the jacket, may form an idea of how many of his potatoes he wants to forego to get it. But if value in use and value are thought of as opposites, one may use this Marxian “mental scheme” (Sraffa, see the preceding quote from him) to characterise, indeed to explain “commodity fetishism”. Marx offers a language game to his followers that promises explanatory power.

What is to be explained is clear: The reader will remember how Marx goes from here to the “equivalent form”, to the “general equivalent” and from there to “money”. Here that quasi-religious fetishism of value embodied in silver or gold finds its place. Precious metals become the mirrors, then the incarnation of all values. Exchange becomes sale against money, the relative form of value becomes the price in money terms. The laws of the capitalist mode of production therefore are not developed out of the consensual acting of individuals, prepared by discourses among them, but the process is always predetermined and the individuals perceive only aspects of it in a narrow perspective, for instance, when the tailor experiences in the market what he gets for his jacket.<sup>8</sup>

<sup>8</sup> Sraffa, as we have seen, spoke of a “metaphysics” of Marx that he regarded as different from modern “metaphysics”. Analysing Sraffa’s early notes, Gehrke and Kurz (2018) show that he hesitated to accept labour as a measure of value. He seems not to have recognized that the theory of the value forms constitutes an essential building block of Marxian theory. This was my impression from conversations with him around 1970. It was the same in either Cambridge. When I presented a paper on the theory of the forms of value in Samuelson’s seminar in the winter of 1973/4, he said a week later that it had been interesting, but that it was difficult to recall its contents. Joan Robinson avoided the subject. Only Maurice Dobb realised and admitted that he had missed something essential in Marx, when I spoke to him about

We shall come back to the theory of the value forms in Sect. 3, when we shall deal with the “fetishism of capital”. The reader will be familiar with the steps subsequent to Marx’s introduction of money. Commodities are exchanged at their values. More precisely, commodities are sold and the money is used to buy other commodities with a different use value. But there is a second form of circulation: money is used to buy commodities and commodities are sold against money. This series of transaction leads from money to money and is meaningless, if, since the use value is in the end the same as at the beginning, the quantity is not increased. Marx explains this profit on the basis of the labour theory of value by arguing that the buying of the commodity in the second form of circulation includes the buying of the commodity labour power, and that the commodity labour power is endowed with the property of being able to create more value than its reproduction costs. The worker labours during more hours for the capitalist, who has acquired his labour power for a determinate amount of time, than the reproduction of labour power, also measured in hours, costs. The difference is surplus value. The capitalist exploits labour power, given the cost of reproduction, therefore given the real wage, by letting it labour as long as possible, and this process of exploitation determines the character of the process of accumulation.

In Austrian theory the process of accumulation is initiated by an entrepreneur, who enters as an acting subject, pursuing the aim to obtain a profit above the cost of capital. Sombart had begun around 1900 to develop a colourful historical typology of entrepreneurs (Sombart 1923). The peculiar dynamic of capitalist development in Schumpeter’s theory is due to his sociology of entrepreneurship. By contrast, the entrepreneur is only a “character mask” in Marx. Accumulation is represented as a movement of capital. A money capitalist lends money to an industrial capitalist, who then buys labour power and other means of production, to produce a new commodity, the value of which is realised in its sale, provided there is demand for it, in the form of money. The surplus value in monetary form then is divided between the industrial capitalist and the monetary capitalist. The former retains the industrial profit, the latter receives interest. The process as a whole can be represented as changes in the form of a value, which is first present in the form of money, which is transformed into commodities and which again, together with the surplus value, takes the form of money. Money in this process is capital, that is, value destined to the production of surplus value, but this capital had to be formed first, for instance, because revenue accumulated in the hands of the money capitalist, therefore a hoard in monetary form. This becomes capital only as an advance destined to obtain surplus value. Hence the representation of accumulation through changes in the form of the substance, value, shall describe the process of accumulation in such a way that the two capitalists, the money capitalist and the industrial one, each appear only as representatives of social relationships, who possess only limited insight into the social and economic relations of which they are part. The Marxian entrepreneur never lifts his mask, to look out as a human being. Marx sticks to his materialist

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Footnote 8 (continued)

it in 1972 and pointed out that the theory of the forms of value is an essential connection between Volume One and Volume Three.

approach with iron consistency. He condemns, for instance, the manner of speaking of money as an “instrument”, used as a tool to acquire capital. This is doubly wrong for him: The transformation of revenue into capital precedes the acquisition of labour power and other means of production. Hence this money has been capital ever since it ceased to be revenue or part of a hoard. It is destined to change its form further; it will be transformed into labour power and means of production. Moreover, the term “instrument” is inadequate, because it seems to signify a subjective action where Marx wants to represent an objective process (MEGA II/15, p. 418).

Now Marx encountered a known difficulty. Malthus expressed it as follows (in Marx’s translation): “Der Capitalist erwartet gleichen Vortheil auf alle Theile des Capitals, die er vorstreckt” („Malthus, Principles of Political Economy, 2nd ed., London 1836, p. 268“, MEGA II/1.2, p. 687)—the capitalist expects the same advantage on all parts of the capital which he advances. Marx would quote this phrase in all manuscripts of *Das Kapital*, from the *Grundrisse* to the manuscript of the Third Volume. Marx did not grant arms and legs to capital, though it moves about a great deal in his account, but “organs”. It breaks down into variable capital, the value of labour power, creating surplus value depending on the degree of exploitation or of surplus value, and constant capital, the other means of production, bought at their values. The “organic” composition, the relation of constant to variable capital, is different in the several sectors of industry. Marx had hinted already in the First Volume that profit according to the uniform rate of profit ought to be proportional to the entire advance of capital, but that this was not possible according to the assumption of the labour theory of value, for then surplus value is proportional to variable capital with a uniform rate of surplus value. The solution to this conundrum consisted in the assumption that sectors with a low organic composition of capital, where surplus value is larger relative to total capital than in sectors, where, given the same amount of variable capital, there is more constant capital, the rate of profit is higher to begin with and that competition then causes capital to flow from sectors with a low rate of profit to sectors with a higher rate, so that market prices fall in sectors which are losing capital and rise in sectors towards which capital flows. This movement of capitals ends in theory, when the rates of profit have become equal and the surplus value has been redistributed, but this movement never ends, because there are new disturbances all the time. This redistribution means that, if one sector receives what the other loses, that the sum of profit  $P$  can be represented as a redistribution of surplus value  $M$ .

We formalise the transformation in an elementary manner to isolate the problem, then we shall discuss its meaning and eventually (Sect. 5) provide a deeper solution.

Two commodities  $i = 1, 2$  suffice at this stage. Let  $c_i$  be constant capital,  $l_i$  living labour,  $\alpha l_i$  necessary labour time and  $(1 - \alpha)l_i$  surplus labour—all measured in abstract labour time. We obtain  $(1 - \alpha)/\alpha$  as the rate of surplus value.

To transform value into prices of production, which, as prices, have for Marx to be expressed in money, we have first to go from values expressed in labour time to value expressed in money (gold). Gold is a produced commodity; the bar shall denote values expressed in gold prices

$$\bar{c}_i + \bar{v}_i + \bar{m}_i = \bar{u}_i$$

here  $\bar{c}_i = c_i/u_g$ , where  $u_g$  is the labour value of gold.  $\bar{v}_i = v_i/u_g$  is variable capital and equal to the value of subsistence goods in terms of gold. Similarly  $\bar{m}_i = m_i/u_g$  and  $\bar{u}_i = u_i/u_g$ , the latter is the value price of the commodities.

Marx introduces the cost prices of the commodities  $k_i$ ; they comprise the cost of the means of production to produce commodity  $i$ , that is the cost of the raw materials plus depreciation plus subsistence. Since  $k_i$  is a price, it is expressed in money. Marx quite explicitly distinguishes between cost prices estimated in value (and expressed in terms of money) and cost prices estimated in prices of production (and expressed in terms of money), but although he distinguishes these two concepts in words, he fails to provide two different names and two different formulas for them. We shall speak of a value cost price (from which the analysis starts) and a final cost price (the sum of the means of production, estimated in prices of production). The final cost price is supposed to be arrived at after the transformation.

Since this point is important and controversial, I here give two quotes from the Marxian manuscript of what became volume III to defend my interpretation:

„Wir hatten in Buch I) und II) nur mit den Werthen der Waaren zu tun. Einerseits hat sich jetzt abgesondert als ein Theil dieses Werths der *Kostenpreis*, andererseits entwickelt als eine verwandelte Form des Werths der *Produktionspreis* der Waare.“ (MEGA II/4.2, p. 239)

“Es ist durch die jetzt gegebene Entwicklung allerdings eine Modifikation eingetreten in respect to the determination of the *cost price* of commodities. Ursprünglich angenommen, daß der *Kostpreis* einer Waare = dem *Werth* der in ihrer Production *consummirten* Waaren. Da aber der *Produktionspreis* einer Waare als *Kostpreis* in die Preisbildung einer anderen Waare eingeht und da der *Produktionspreis* abweichen kann vom *Werth* der Waare, kann also auch der *Kostpreis* einer Waare, *über* oder *unter* dem *Theile ihres Gesamtwerts* stehen, der durch den *Werth* der in sie eingehenden Produktionsmittel gebildet wird.“ (MEGA II/4.2, p. 241–2)

This has been rendered by Engels: „Es ist durch die jetzt gegebene Entwicklung allerdings eine Modifikation eingetreten bezüglich der Bestimmung des *Kostpreises* der Waaren. Ursprünglich wurde angenommen, daß der *Kostpreis* einer Waare gleich sei dem *Werth* der in ihrer Produktion konsumirten Waaren. Der *Produktionspreis* einer Waare ist aber für den Käufer derselben ihr *Kostpreis* und kann somit als *Kostpreis* in die Preisbildung einer andren Waare eingehn. Da der *Produktionspreis* abweichen kann vom *Werth* der Waare, so kann auch der *Kostpreis* einer Waare, worin dieser *Produktionspreis* andrer Waare eingeschlossen, *über* oder *unter* dem *Theil ihres Gesamtwerts* stehn, der durch den *Werth* der in sie eingehenden Produktionsmittel gebildet wird.“ (MEGA II/15, p. 166)

Translation: “The foregoing statements have at any rate modified the original assumption concerning the determination of the cost-price of commodities. We had originally assumed that the cost-price of a commodity equalled the value of the commodities consumed in its production. But for the buyer



the price of production of a specific commodity is its cost-price, and may thus pass as cost-price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost-price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it.” (Capital Vol. III, p. 125)

In these two quotes, Marx looks back on the transformation, which we now have to explain.

Denote the value cost price by  $(\bar{c}_i + \bar{v}_i)$  so that as before

$$(\bar{c}_i + \bar{v}_i) + \bar{m}_i = \bar{u}_i \quad (1)$$

and let us use Marx’s  $k_i$  for the final cost price. With rate of profit  $r$  and prices of production, expressed in gold,  $\bar{p}_i$ , we have

$$(1 + r)k_i = \bar{p}_i. \quad (2)$$

The rate of profit is unknown. To effect the transformation, Marx assumes that the total sum of prices must be equal to total values, and this is often written as

$$K + W + P = C + V + M \quad (3)$$

where in our case  $C = \bar{c}_1 + \bar{c}_2$ ,  $V = \bar{v}_1 + \bar{v}_2$ ,  $M = \bar{m}_1 + \bar{m}_2$ .  $K + W = k_1 + k_2$  is means of production measured in prices and  $P = r(k_1 + k_2)$  total profits. The easy way to defend assumption (3)—as often found in the literature, some of my own writings included—is to say that (3) makes the system of prices commensurable with that of values by defining the numéraire for prices. But if we remember Marx’s monetary theory, we must say that the structure of commodity production, evaluated in values and in prices, is in each case expressed in a total (imaginary) mass of gold which can buy the commodities, the means of production and the surplus. This mass of gold must be the same in the system of values and in the system of prices, hence assumption (3) becomes assumption

$$\bar{u}_1 + \bar{u}_2 = \bar{p}_1 + \bar{p}_2 \quad (4)$$

Marx argues twice in his exposition of the transformation that the final total cost price of the means of production must be equal to the value cost price “on average”, so that

$$(\bar{c}_1 + \bar{v}_1) + (\bar{c}_2 + \bar{v}_2) = k_1 + k_2 \quad (5)$$

He argues that die “Abweichungen vom *Werthe*, die in den Kostenpreisen der Waaren stecken, sich gegeneinander aufheben” (MEGA II/4.2, p. 237). The deviations contained in the cost prices cancel each other, it indeed follows from (4), (5) and (2) that

$$M = m_1 + m_2 = r(k_1 + k_2) = P \quad (6)$$

and the rate of profit results, it is the same in value terms; we denote it now as  $\bar{r}$ :

$$\bar{r} = \frac{M}{C + V} = \frac{\bar{m}_1 + \bar{m}_2}{\bar{c}_1 + \bar{c}_2 + \bar{v}_1 + \bar{v}_2} = \frac{\bar{r}(k_1 + k_2)}{k_1 + k_2} = \frac{P}{K + W}. \quad (7)$$

This derivation is correct, if two properties of the system of production hold.

- I) To understand the first condition, note that (7) is independent of the numéraire, since it appears always as a linear factor in the nominator and the denominator. Prices of production  $\mathbf{p}$  for the system as a whole are given as a vector by

$$\mathbf{p} = (1 + r)(\mathbf{A}\mathbf{p} + \mathbf{w}\mathbf{l}), \quad (8)$$

where  $\mathbf{A}$  is an indecomposable productive input–output matrix,  $\mathbf{l}$  a positive labour vector, and  $r$  is the rate of profit to be determined. Prices in terms of labour commanded  $\mathbf{p}/w$  are equal to values  $\mathbf{u}$  at  $r = 0$ , where  $\mathbf{u} = \mathbf{A}\mathbf{u} + \mathbf{l}$ . The first condition then is: The prices must be such that the value of the means of production at  $r = 0$  are equal to the price of the means of production at  $r = \bar{r}$ , to fulfil (5), and moreover, prices  $\mathbf{p}$  as a function of the rate of profit must be such that (6) holds, therefore such that the value of the commodities constituting the surplus acquired out of profits or from surplus value must be the same at  $r = 0$  and at  $r = \bar{r}$ .

As we shall see, Sraffa conceded at one time that this condition would be fulfilled approximately but that it could be fulfilled exactly only if the means of production, the necessary wage, the physical surplus all were vectors proportional to total output (and proportional to Sraffa's standard commodity), and some interpreters of Marx have followed him in this.

But this rigid restriction is not necessary. Marx thought that the total value cost price would on average and approximately be equal to the final cost price. We shall show that this is true, and that the approximation is the better, the closer the system is to a random system, for then the labour theory of value holds on average. This is the essential assumption the system must have for the first property to hold.

- II) The second condition concerns the production of money. To justify  $K + W + P = C + V + M$  better than by definition of the numéraire, we assume that  $C + V + M$  and  $K + W + P$  are represented by the same amount of gold (the imaginary mass of gold mentioned above): the labour value and the price of the gold representing total production must be the same. Hence we must assume that the structure of gold production follows the same laws of average as total production, an assumption similar to that made by Ricardo, when he postulated in his search for an invariable standard of value that the capital composition in gold production corresponded to that of the average of the economy as a whole, where the average is that of the actual activities, not of an imaginary standard commodity.<sup>9</sup>

So this is the place also to mention the Ricardian exercise: if the rate of profit falls in an industry of average composition such as the gold industry, the

<sup>9</sup> Here I should like to thank Kenji Mori for his valuable comments provided at the Joint Seminar mentioned above and in particular for his insistence that the gold value and the gold price of total production should be represented in the transformation. It is true that gold value and gold price will not coincide, if gold is produced by an industry with an arbitrary composition of inputs, but the Ricardian assumption made here solves the problem, which he raised with good reason.

price of production in that industry according to (8) will not change, but the wage rate will rise. By how much? And does it remain true that the price of production of a capital-intensive industry (with a high organic composition of capital) will fall? These are matters to be discussed. It turns out that the wage curve is linear, if the random properties are fulfilled, and again the standard commodity is not needed.

Before we turn to the precise analysis of the conditions under which the transformation works after all, we want to investigate why Marx insisted so much on his transformation. The answer is, of course, that he wanted to interpret profits as resulting from a redistribution of surplus value.

#### 4 Profits as redistributed surplus value

The formula  $P = M$  is the lynchpin of the Marxian system, for three reasons.

1. The formula  $P = M$  is fundamental for the Marxian explanation of *class struggle*. He ironically speaks of a “communism of capital”, because the capitalists divide the surplus value among themselves, and he represents this as follows:

„Aus dem Gesagten ergibt sich, daß jeder einzelne Kapitalist, wie die Gesamtheit aller Kapitalisten jeder besondern Produktionssphäre, in der Exploitation der Gesamtarbeiterklasse durch das Gesamtkapital und in dem Grad dieser Exploitation nicht nur aus allgemeiner Klassensympathie, sondern direkt ökonomisch betheilig ist, weil, alle andern Umstände, darunter den Werth des vorgeschossenen konstanten Gesamtkapitals als gegeben vorausgesetzt, die Durchschnittsprofitrate abhängt von dem Exploitationsgrad der Gesamtarbeit durch das Gesamtkapital. Der Durchschnittsprofit fällt zusammen mit dem Durchschnittsmehrwerth... (MEGA II/15, S. 196).

Man hat also hier den mathematisch exakten Nachweis, warum die Kapitalisten, so sehr sie in ihrer Konkurrenz unter einander sich als falsche Brüder bewähren, doch einen wahren Freimaurerbund bilden gegenüber der Gesamtheit der Arbeiterklasse.“ (MEGA II/15, p. 198)

“It follows from the foregoing that in each particular sphere of production the individual capitalist, as well as the capitalists as a whole, take direct part in the exploitation of the total working-class by the totality of capital and in the degree of that exploitation, not only out of general class sympathy, but also for direct economic reasons. For, assuming all other conditions – among them the value of the total advanced constant capital – to be given, the average rate of profit depends on the intensity of exploitation of the sum total of labour by the sum total of capital.” (Capital Vol. III, p. 143)

“Here, then, we have a mathematically precise proof why capitalists form a veritable freemason society vis-a-vis the whole working-class, while there is little love lost between them in competition among themselves. (Capital Vol. III, p. 144)

On the other hand, the capitalists are not at all fully conscious of their position. According to Marx, they are prisoners of the illusion of being able to create profit themselves by charging a corresponding mark-up on their costs. Marx would therefore still have had to show whether and in how far the capitalists could act jointly in their class struggle against the labour class because of their joint dependence on the common surplus value, although they believe, that they “den Profit selbst schaffen” (“create the profit itself”—see full quote below). Marx probably planned to deal with this consideration, which might have crowned his system, in the last chapter of the Third Volume about the classes, which he left unfinished. The ideology that capitalists create profits is formulated as follows:

„Nur vergißt der Kapitalist – oder sieht vielmehr nicht, da die Konkurrenz ihm das nicht zeigt – daß alle diese, in der wechselseitigen Berechnung der Waarenpreise verschiedener Produktionszweige von den Kapitalisten gegen einander geltend gemachten Kompensationsgründe sich bloß darauf beziehen, daß sie alle, pro rata ihres Kapitals, gleich großen Anspruch haben auf die gemeinschaftliche Beute, den Total-Mehrwerth. Ihnen scheint vielmehr, da der von ihnen einkassirte Profit verschieden von dem von ihnen ausgepreßten Mehrwerth, daß seine Kompensationsgründe nicht die Betheiligung am Gesamtmehrwerth ausgleichen, sondern den Profit selbst schaffen, indem dieser einfach aus dem so oder so motivirten Aufschlag auf den Kostpreis der Waaren herstamme.“ (MEGA II/15, p. 208)

“The capitalist simply forgets – or rather fails to see, because competition does not point it out to him – that all these grounds for compensation mutually advanced by capitalists in calculating the prices of commodities of different lines of production merely come down to the fact that they all have an equal claim, pro rata to the magnitude of their respective capitals, to the common loot, the total surplus-value. It rather seems to them that since the profit pocketed by them differs from the surplus-value they appropriated, these grounds for compensation do not level out their participation in the total surplus-value, but create the profit itself, which seems to be derived from the additions made on one or another ground to the cost-price of their commodities.” (Capital Vol. III, p. 152)

2. Further, the equation  $P = M$  is essential for Marx’s *theory of accumulation*. This is expounded magnificently and artfully in the First Volume of *Das Kapital* in the sections on the production of absolute and surplus value. The level of the profit made and the amount of the capital advanced are not functionally related as in neoclassical theory—we remember Böhm-Bawerk’s contrary opinion—but Marx always assumes that the chase for surplus value takes place in specific historical conditions, and this influences how much surplus value is made. As we saw, the

process of investment is not analytically described in the form of a model as in Keynes, when Keynes (1936) derives his investment function using the concept of the marginal efficiency of capital, but it is represented in a descriptive and historical manner, which Keynes occasionally approximates, when he speaks of the uncertainty regarding the historical conditions in which future markets will take place.

Regarding the production of *absolute* surplus value, the question is how far capital will succeed, given the value of labour power and of constant capital, and given the system of production in existence, to expand the production of surplus value by lengthening the working day. It is not so much the lengthening of the working day which is controversial nowadays but the shortening of labour time per day, per week and per year (lengthening of holidays), which trade unions try to impose.

On the other hand, surplus value can be increased *relatively*, not by all forms of technical progress but by those which lead directly or indirectly to a cheapening of the subsistence of the workers, where the necessary wage is by Marx regarded as given. Marx now wants to show that all these forms of progress ultimately are based on innovations of processes such that the amount of labour time per unit of product is shortened at the expense of using more other means of production. This is discussed on the basis of historical arguments in chapters on cooperation, division of labour and the introduction of machinery, combining theoretical and historical arguments. These are chapters which realise the ideal of the historical school to develop history and theory in parallel, at a level of conceptual precision and with a wealth of visual illustration that the historical school itself never was able to reach.

The increased use of raw materials in the production of relative surplus value, which results in this form of technical progress regarded by Marx as dominant, seems to imply an increase of the organic composition of capital. Marx then affirms that the rate of profit will tend to fall, insofar as he isolates this increase in the organic composition of capital and assumes it as the predominant tendency. If the rate of surplus value cannot be increased indefinitely, in particular, if there are limits to the production of absolute surplus value, the rate of profit must fall, as the formula easily shows. Real wages then rise, of course, but not necessarily the value of labour power (Schefold 1997, p. 274).

It is possible to translate the debate about the falling rate of profit into more modern terms by expressing the analytical parts of Marx's account of the different forms of the production of relative surplus value by means of the modern theory of prices of production, following Sraffa, as I have shown earlier (Schefold 1997, p. 257–75). It turns out that the production of relative surplus value shows in the form of an increase of output per head and therefore of an increase of real wages at any given low rate of profit, accompanied, however, by a fall of the maximum rate of profit. Whether the actual rate of profit increases or whether the counter tendency predominates, based on a cheapening of machinery and raw materials, cannot be determined in this manner. What interests us here is not the outcome of this debate but the fact that Marx, to describe the production of relative surplus value, had no choice but to rely on a measurement in terms of labour values. A

measurement in terms of prices of production alone was not possible, since it could not be said how large these prices were without solving a system of linear equations (a possibility of which Marx was not even aware, let alone that he could have solved such a system). Since prices of production hung in the air, as it were, as long as they were not based on labour values, the necessary microeconomic foundation of Marxian macroeconomics (if this modernised form of expression is permissible) was lacking. But the measurement in terms of labour values led to a measurement of the rate of profits, and the question resulted whether this rate of profits in value terms was the same as the one which would be obtained from the transformation. This would be the case, as we saw, if profits were equal to redistributed surplus value. Hence we may conclude that  $P = M$  was essential as the analytical link between the First and the Third Volume of *Das Kapital*.

3. The equality  $P = M$  is the expression of a quantitative equality of value in different forms. Insofar as Marx is a monetary economist, the equation is dimensionally correct, if the value magnitudes on the right and the price magnitudes on the left are expressed in terms of gold, per period. But we can also express the equality in terms of labour time. This is the original meaning of exploitation. The aggregate of surplus labour extracted by the collective of the capitalists is redistributed among them so that it is proportional to the cost measured in labour time. The analysis of the quality brings us back to the theory of the forms of value. This is not just a matter of the First Volume, and it certainly was not only meant to provide a foundation for the fetish character of commodities. On the contrary, the *theory of the forms of value* reaches its culminating point in the formula  $P = M$ .

Profits now flow regularly, as long as the economy develops steadily. Regularity of the flow of profits implies that capital is not only value, it now also has a value in use, in that it empowers capitalists to extract surplus value and to make profits, and more capital means more possibilities for such extraction which appears, since the labour value relationships are below the surface, as the possibility of capital to yield interest. Hence the notion of a price of capital arises; one can get capital at the prevailing rate of interest:

„Im zinstragenden Kapital ist daher dieser Fetisch rein herausgearbeitet, der sich selbst verwerthende Werth, Geld heckendes Geld... Das gesellschaftliche Verhältniß ist vollendet als Verhältniß eines Dings, des Geldes, zu sich selbst“ (MEGA II/15, p. 381).

„In interest-bearing capital, therefore, this automatic fetish, self-expanding value, money generating money, are brought out in their pure state and in this form it no longer bears the birth-marks of its origin. The social relation is consummated in the relation of a thing, of money, to itself.” (Capital Vol. III, p. 266)

According to Marx, the mystification of the relationships in the capitalist mode of production here reach their culminating point because, what is due to labour, what results from the expenditure of labour power, surplus value, here appears as a profit on capital which may be purchased by means of money (MEGA II/15, p. 381–382). Hence there results a curious “Verdoppelung” of the price of capi-

tal, for on the one hand, capital now costs just as much as the sum of money of which it consists. On the other hand, interest now appears as a second price. Marx therefore speaks of an “irrationellen” form:

„Zins als Preis des Kapitals ist von vornherein ein durchaus irrationaler Ausdruck. Hier hat eine Waare einen doppelten Werth, einmal einen Werth, und dann einen von diesem Werth verschiedenen Preis...” (MEGA II/15, pp. 345–346).

“Interest, signifying the price of capital, is from the outset quite an irrational expression. The commodity in question has a double value, first a value, and then a price different from this value, while price represents the expression of value in money.” (Capital Vol. III, p. 239)

This reproach does not concern the later Austrian theory, for it does not interpret interest as a rent for the lending of capital in analogy to the renting of a house, but as the price for a promise of delivery according to an intertemporal exchange. To have money today or tomorrow is not the same thing, and if the money capitalist puts a sum at the disposal of the industrialist, the latter promises to pay an increased sum at a predetermined future date. But this dating of money or commodities does not exist in the Marxian theory of value, so that the paradoxical form must appear in Marx and the *fetish of capital* surpasses the commodity fetish. Capital is a fetish, insofar as it appears to be productive of profit and is valuable according to that potential—it costs interest—but the surplus value is created by labour:  $P = M$ .

It is rarely observed that this conception already played a significant role in the discussion of scholastic authors. When Thomas Aquinas deplores the vicious character of usury according to the tradition of the church, he uses not only biblical arguments and those of the fathers of the church, but he also tries to denounce usury as something illogical. First, he indicates, like Aristotle, that money seems to multiply as if it had children, when interest is paid, which is absurd in itself (Aquinas 1963). Further he argues that one has to distinguish between goods for immediate consumption and goods for durable use. Goods for durable use, like houses, can be rented. The rent is paid for the use, in the end the house is given back. Goods for consumption like bread or wine, by contrast, cannot be used without annihilating them. If bread is lent, the borrower will consume it. One cannot return the borrowed bread, only its value, and that is the same thing as buying it. The borrower buys by paying the same amount which the bread costs to the lender, with a delay, without a mark-up, for the value of the bread, in the case of Thomas the just price of the bread, has not changed. Now the conclusion: Interest as a second price for money is not logical, therefore not to be justified, therefore vicious, for money is according to Aristotle, Thomas says, a good to be used in consumption: it is there to be exchanged. Another function of money is not admitted—hoarding is not licit. In a licit act of selling, a commodity is given away against money, at a just price. In a licit act of lending, money is given away against money, and the price is just, if the sum given and the sum returned are equal.



Curiously, a mendicant friar found the most powerful counter arguments in the debate on usury. Pietro of Giovanni Olivi has become famous in the history of economic thought in the last 30 years, because he observed the circulation of money, trade and capital in the late Middle Ages and felt compelled, although he was against usury in principle, to advance somewhat different arguments from Thomas, and this most clearly in his *Dubia circa materiam contractuum* (Olivi 2012). Two citations may suffice to illustrate this:

*Quamvis... pecunia ex se non valeat plus seipsa, ex utentis tamen falcul-tate et industria acquirit aliquem valorem aut potest adquirere, et ideo ille usus ac facultas utendi potest ab eo, cuius ille usus est, vendi.* (Olivi 2012, p. 206).

Although... money is not worth more in itself than (it represents) itself, yet it acquires - or can acquire - a certain (additional) value through the ability and diligence of the one who uses it, and therefore that use and opportunity for use can be sold by the one whose use it is. (My transl.)

A second quotation:

*Illud quod in firmo proposito domini sui est ordinatum ad aliquod probabile lucrum, non solum habet rationem simplicis pecuniae seu rei, sed etiam ultra hoc quandam rationem seminalem lucri quam communiter capitale vocamus, (cf. the logos spermatikos of the Stoics), et ideo non solum debet reddi simplex valor ipsius, sed etiam valor superadiunctus.* (Olivi 2012, p. 232).

Whatever is determined in the firm intention of its owner for a certain probable profit has not only the character of simple money or a thing, but also a certain character fertile of profit, which we usually call capital, and therefore not only the simple value of it must be returned, but also an added (surplus) value. (My transl.)

If Olivi regards capital as fertile of profit (or as containing the semen of profit) he clearly feels compelled to concede a productive character to it. From the point of view of later neoclassical theory, Marx can be criticised for a backward mistake in the use of the dimensions, when he speaks of the irrational form of interest as a price of capital, ignoring that capital today and capital tomorrow are different commodities. But there is a positive result in conformity with his materialist explanation of the genesis of ideology: according to his derivation, it must *appear* as if capital was productive, although the production of surplus value originates from labour. This conception of a productivity of capital thus surfaces already in the Middle Ages in contradistinction to the teaching of Thomas Aquinas. Hence this seems to be the earliest explicit formulation of the central assertion of “vulgar” economics which we know. Marx would probably have regarded Olivi’s stance as progressive relative to scholastic thought as represented by Thomas, because Olivi asks where interest comes from, whereas Thomas only rejects it. The productivity of capital had become a “vulgar” idea after Ricardo. Thomas and

Olivi had in common the attempt to reduce legitimate profit to work and diligence (*labor, industria*), but to discuss this would lead to far astray.

## 5 Do prices of production result from redistribution or from new calculation?

We have dealt with three central domains in which the formula  $P = M$  plays an essential role for Marx (class struggle, accumulation and measurement of the rate of profit, fetish of capital). We now return to the problem of proving the formula. In the manuscript which later was edited as “Theories of Surplus Value”, Marx formulated the transformation in particularly simple form (MEGA II/3, p. 685):

„... da die Profitrate bestimmt ist durch das Verhältniß des Mehrwerths zum vorgeschossenen Capital, da dieß aber nach der Voraussetzung gleich in  $A, B, C$  u.s.w., so wären, wenn das vorgeschossene Capital, die verschiedenen Profitraten

$$= \frac{A)}{\frac{3M}{C}}, \quad \frac{B)}{\frac{2M}{C}}, \quad \frac{C)}{\frac{M}{C}}.$$

Die Concurrenz der Capitalien kann also nur die Profitraten ausgleichen, indem sie z. B. in dem angeführten Falle die Profitraten  $= \frac{2M}{C}, \frac{2M}{C}, \frac{2M}{C}$  setzt in den Sphären  $A, B, C$ .“

„Since the rate of profit is determined by the ratio of surplus value to capital advanced, and as on our assumption this is the same in  $A, B, C$ , etc., then if  $C$  is the capital advanced, the various rates of profit will be

$$= \frac{A)}{\frac{3M}{C}}, \quad \frac{B)}{\frac{2M}{C}}, \quad \frac{C)}{\frac{M}{C}}.$$

Competition of capitals can therefore only equalise the rates of profit, for instance in our example, by making the rates of profit equal to  $= \frac{2M}{C}, \frac{2M}{C}, \frac{2M}{C}$  in the spheres  $A, B, C$ .” (MECW 31, p. 263)

The equalisation of the rates of profit between sectors turns into a simple equalisation of the profits themselves by redistribution, if the capitals advanced in the sectors are of equal magnitude. Marx does not ask here how the transformation of values into prices affects the measurement of the capitals advanced in each sector. The simplicity of the redistribution lets value appear like a substance that changes only its form without ever changing its quantity except when labour, as the source of this substance, creates value. The profits in the three industries  $A, B, C$  above are like icicles of unequal length. They melt, the water flows into a pool and equal quantities of it are apportioned to the equal capitals  $A, B, C$ . But the metaphor is deceptive from the start. How much value is created depends on the intensity of labour, and only socially necessary labour time counts. If a clumsy bricklayer needs 2 days to build a wall, while his colleagues need only one on average, only 1 day is added to

the labour embodied in the house. The observer, who wants to predict values, must deduce the extra day in his estimate of value created.

Value also is destroyed. Machines not only depreciate, but they become obsolescent as well. How much value is destroyed—evaporates, as it were—and how much is transferred to the product depends on a calculation. And so the sum of direct and indirect labour embodied in a commodity must be calculated by having explicitly (as in Sraffa) or implicitly (as in Marx) recourse to the structure of use values. Insofar, the redundancy of labour values is not just the problem of the transformation into prices. It is deeply rooted in the foundation of the Marxian system. Since the “law of value” rules capitalist production, according to Marx, it must be possible for him to predict values, at least in principle, to demonstrate that capitalism could be understood and replaced. If labour values must be predicted from the structure of use values, prices of production can also be derived in this manner, and labour values seem necessary as an intermediate concept for prediction only as long as one does not know how to derive prices of production directly.

Calculation is necessary in particular if only one considers the redistribution of surplus value in the transformation as in the example with the three capitals *A*, *B*, *C*. Since the sectors are different, the structure of use values is going to be different in each sector, and if the amount of capital is the same, in value terms, the sum of prices of the capital goods in each sector will in general be different, so that the simple consideration, that only profits have to be redistributed, is invalidated. A redistribution must take place among the costs as well.

Marx dedicated himself to this challenge only in the manuscript to the Third Volume of *Das Kapital*. In the version published by Engels it is said<sup>10</sup>:

„Es ist durch die jetzt gegebne Entwicklung allerdings eine Modifikation eingetreten bezüglich der Bestimmung des Kostpreises der Waaren. Ursprünglich wurde angenommen, daß der Kostpreis eine Waare gleich sei dem Werth der in ihrer Produktion konsumirten Waaren. Der Produktionspreis einer Waare ist aber für den Käufer derselben ihr Kostpreis, und kann somit als Kostpreis in die Preisbildung einer andren Waare eingehn. Da der Produktionspreis abweichen kann vom Werth der Waare, so kann auch der Kostpreis einer Waare, worin dieser Produktionspreis andrer Waare eingeschlossen, über oder unter dem Theil ihres Gesamtwerths stehn, der durch den Werth der in sie eingehenden Produktionsmittel gebildet wird. Es ist nöthig sich an diese modificirte Bedeutung des Kostpreises zu erinnern und sich daher ||144| zu erinnern, daß wenn in einer besondern Produktionssphäre der Kostpreis der Waare dem Werth der in ihrer Produktion verbrauchten Produktionsmittel gleich gesetzt wird, stets ein Irrthum möglich ist. Für unsre gegenwärtige Untersuchung ist nicht nöthig, näher auf diesen Punkt einzugehn...“ (MEGA II/15, p. 166)

“The foregoing statements have at any rate modified the original assumption concerning the determination of the cost-price of commodities. We had

<sup>10</sup> The first part of this citation has already been quoted in Sect. 2 above.

originally assumed that the cost-price of a commodity equalled the value of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost-price, and may thus pass as cost-price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost-price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it. It is necessary to remember this modified significance of the cost-price, and to bear in mind that there is always the possibility of an error if the cost-price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point.” (Capital Vol. III, p. 125)

Similarly in another location:

„Was den konstanten Theil betrifft, so ist er selbst gleich Kostpreis plus Mehrwerth, also jetzt gleich Kostpreis plus Profit, und dieser Profit kann wieder größer oder kleiner sein als der Mehrwerth, an dessen Stelle er steht. Was das variable Capital angeht, so ist der durchschnittliche tägliche Arbeitslohn zwar stets gleich dem Werthprodukt der Stundenzahl, die der Arbeiter arbeiten muss, um die notwendigen Lebensmittel zu produzieren; aber diese Stundenzahl ist selbst wieder verfälscht durch die Abweichung der Produktionspreise der notwendigen Lebensmittel von ihren Werthen. Indeß löst sich die immer dahin auf, daß, was in der einen Waare zu viel, in der anderen zu wenig für Mehrwerth eingeht, und daß daher auch die Abweichungen vom Werth, die in den Produktionspreisen der Waaren stecken, sich gegeneinander aufheben.“ (MEGA II/15, p. 162)

“So far as the constant portion is concerned, it is itself equal to the cost-price plus the surplus-value, here therefore equal to cost-price plus profit, and this profit may again be greater or smaller than the surplus-value for which it stands. As for the variable capital, the average daily wage is indeed always equal to the value produced in the number of hours the labourer must work to produce the necessities of life. But this number of hours is in its turn obscured by the deviation of the prices of production of the necessities of life from their values. However, this always resolves itself to one commodity receiving too little of the surplus-value while another receives too much, so that the deviations from the value which are embodied in the prices of production compensate one another.” (Capital Vol. III, pp. 122–123)

The formula „sich gegeneinander aufheben“ (“compensate one another”) is a proof that Marx had a classical system of long-run prices in view and not intertemporal prices (on this see also in the manuscripts of *Das Kapital*, MEGA II/4.2, pp. 236–7, 250–3, 268, 283). It has been known since Bortkiewicz that the heuristics of regarding the capitals advanced as invariant is not correct in general. I was asked to write a “Einführung” for the new edition of the Third Volume of *Das Kapital* in

the context of the new *MEGA*. It did not have to be editorial like the introductions to the other volumes, for the editorial introduction to this volume had been written, and it was described elsewhere how Engels became the editor and in what his merits and faults as an editor consisted. I had been charged by the committee of editors of *MEGA* to discuss the material problems which Marx encountered in the Third Volume from the point of view of modern economic theory. This was to remain an exception in the context of the *MEGA*. Hence I could not avoid focusing on the specificities of the Marxian theory of value and hence on the difficulty of the transformation of values into prices, since this has been a focus of the academic discussion for a century (*MEGA* II/15, Apparatus, p. 871–910). I had to explain the deficiency of the Marxian derivation in accordance with the dominant literature of the time and hence to criticise Marx in this regard. This led to a polarisation of the reviewers who in part welcomed the deviation from the practice of writing only an editorial introduction as in other volumes and who in part condemned it. However, I have since been able to show that the sum of profit and value do coincide after all, if, as Marx seems to presuppose in the passages cited, that the structure of the economy has a random character. For the insight that deviations of prices from values cancel each other can be true only on average. The question is, however, what “im Durchschnitt” (“on average”) really means.

## 6 The new solution

I shall now try to give an intuitive account of what is meant by random systems. Those who regard the modelling as too complicated may read the text without the formulas. Those who would like to see the rigorous proofs should study the paper in the *Cambridge Journal* (Schefold 2016).

No transformation is needed in the case of the Corn Model with only one sector. If there are many sectors with different compositions of capital, the transformation becomes complex, but one can imagine an average industry, for which the organic composition corresponds to the average of the economy. Sraffa defined the average industry more specifically as that combination for which the product is a vector of the same composition as the composition of the means of production. If the workers get their wage also in the form of a vector of this same composition and if the surplus of the capitalists, too, consists in an amount of this “standard commodity”, the analogy with the one-product model is complete: the number of baskets of commodities, which workers or capitalists receive, expresses the relationship of wages to profits, and this ratio will be the same irrespective of whether one measures in terms of values or prices, so that  $P = M$  results, and the rate of profit in value terms equals the rate of profit in price terms. But this theoretical construction of a standard does not correspond to the distribution of goods in reality, for instance, because the gross product will contain investment goods, which are not part of wages.

The average is meant in a different sense in Marx. He takes the average of the organic compositions without changing the composition of output, but, as we shall see, his arguments amount to assuming a random structure of production such that deviations of prices from values in the sum of all the cost prices cancel. Such a

random structure is implied by his claim quoted above that cost prices in values and in prices of production are equal on average. A model can be constructed, for which this equality holds without being trivial because of a uniform composition of capital. What we need is a precise and modern expression for randomness.

We may begin by assuming that the methods of production in the several industries are proportional. We seem to be far removed from the real economy with such a supposition, but the point is that one can now further assume that this very even input–output structure is perturbed randomly. We could also say that the input–output system follows a slow random process. The reader who is not used to visualising production in this manner should consider that in the opposite case, if the evolution of production was determinate, it would have to be possible to predict individual future input–output coefficients, but prediction is successful only with regard to certain averages, e.g. the stylised facts predicted the constancy of the capital–output ratio. The perturbation means that the use of some inputs may be increased, some others are not used at all. The essential assumption now is that the input–output structure after the perturbation is such that the input-coefficients are independently and identically distributed, with means specific for each industry.<sup>11</sup> The main mathematical conclusion is that the non-dominant eigenvalues in such random systems are small, if the number of sectors is large.<sup>12</sup>

So far, we have only discussed assumptions about the structure of the input–output table. We also must make an assumption about the labour inputs. In principle, there is to each input–output matrix one hypothetical vector such that, if this vector was the labour vector, the labour theory of value would hold and the organic composition of capital would become uniform, hence such that prices of production would be proportional to labour values, and the rate of profit in value terms equal to that in price terms. For Marx’s transformation to hold it is not necessary that this vector is the labour vector, but the labour vector cannot be arbitrary either, but it must be independent of the numéraire in which we measure prices (in a sense which will be rendered precise below).

Relative prices will change in a non-linear fashion with every change of the rate of profit in regular systems<sup>13</sup>: if one considers the price vector as a function of the rate of profit, this function will never stay in a hyperplane of dimension lower than  $n$ . One may call this the fundamental neoricardian theorem. Random systems imply that prices tend to stay in a two-dimensional hyperplane as functions of the rate of profit (Scheffold 2013, p. 1177). The production of surplus value results in a revenue of the capitalists, which enables them to buy a physical surplus (we assume that they neither save nor dissave). If one varies the rate of profit of the system, determined by the Eq. (8), one finds, given this physical surplus, that there is exactly one rate of profit, at which this surplus can be bought by means of the profits made at the

<sup>11</sup> Farjoun and Machover (1987) also follow a probabilistic approach. It is quite different from the present because they abandon the uniform rate of profits.

<sup>12</sup> Yoshihiro Yamazaki gave a valuable introduction to the history of the concept of random matrix and its use in statistics and physics at the Joint Seminar mentioned above.

<sup>13</sup> I here refer to the concept of regular systems, which were introduced in my PhD-thesis in 1971 (enlarged edition, Scheffold 1989). The original thesis of 1971 will be found on my homepage.

corresponding prices of production. This rate of profit denotes an equilibrium: the physical surplus, evaluated at prices and bought by the capitalists, is equal to the total profit, which arises at this rate of profit.

Now we need a third assumption about the random character of the system. The physical surplus must be independent from the labour vector of the system. Then it can be concluded that the surplus valued at prices of production, i.e. profits, will be equal to the surplus evaluated by means of labour values, i.e. surplus value. And this then means that the transformation has been effected, as Marx wanted to have it.

We repeat this result, using formulas, but without giving the proofs, which are to be found in Schefold (2016).

The square semi-positive indecomposable input–output matrix is given by  $\mathbf{A}$ , the labour vector is  $\mathbf{l}$ ;  $\mu_1, \dots, \mu_n$ ;  $\mathbf{q}_1, \dots, \mathbf{q}_n$ ;  $\mathbf{x}^1, \dots, \mathbf{x}^n$  are the eigenvalues, assumed to be different from each other, the left-hand side and the right-hand side eigenvectors, respectively; with  $\mu_1 > 0$  being the dominant eigenvalue and with  $\mathbf{q}_1 > 0$ ,  $\mathbf{x}^1 > 0$  as Frobenius eigenvectors. We normalise the eigenvectors (which are linearly independent) in such a way that linear combinations result in the equations  $\mathbf{y} = \mathbf{q}_1 + \dots + \mathbf{q}_n$  and  $\mathbf{l} = \mathbf{x}^1 + \dots + \mathbf{x}^n$ , where  $\mathbf{y}$  represents the vector of activity levels which is at the same time the numéraire, or if one prefers, a vector of activity levels, to which the activity levels of the gold industry are proportional, as we discussed above. The vector  $\mathbf{q}_1$  (called Sraffa-vector) then is proportional to Sraffa's standard commodity. If and only if  $\mathbf{x}^1 = \mathbf{l}$ , prices  $\mathbf{p}$  defined by (8) are equal for all rates of profit  $r$  to the labour values  $\mathbf{u} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l}$ . In the former case,  $\mathbf{q}_2 = \dots = \mathbf{q}_n = 0$ , in the latter  $\mathbf{x}^2 = \dots = \mathbf{x}^n = 0$ .  $\mathbf{x}^1$  is called Marx-vector.

Now  $\mathbf{y}$  is also the vector of gross outputs and it is in general not proportional to the standard commodity. This gross output is composed in each sector of the capital goods used  $\mathbf{yA}$  (the commodities of which constant capital is composed), the wage goods  $\mathbf{b}$  (necessary wage) and the physical surplus  $\mathbf{s}$ :

$$\mathbf{y} = \mathbf{yA} + \mathbf{b} + \mathbf{s}. \quad (9)$$

Normalised prices and wage rates are denoted by  $\bar{\mathbf{p}}$  and  $\bar{w}$ . Because of the normalisation by means of the gold industry, we find that the gross product in values and in normalised prices is equal, and by convention equal to 1:

$$\mathbf{y}\bar{\mathbf{p}} = \mathbf{y}\mathbf{u} = 1. \quad (10)$$

The more money is advanced in the circuit, aided by credit creation (in Marx mostly bills), the higher the gross product  $K + W + P$ , if there is no overproduction.<sup>14</sup> Now we write for constant and variable capital (or wages) and for surplus value (profits) as usual

<sup>14</sup> Several authors, Moseley (2016) in particular, emphasise rightly that the circuit of capital in Marx always starts with an advance of capital in the form of money. The advances in (10) include those made in the sectors producing the commodities for the consumption of the capitalists and for their investments. At the level of the individual firm one would have to say after Keynes, that it is not the amount of money that determines the level of activity, but that, on the contrary, the investment plans of the entrepreneur, the level of planned production, determine the amount of money to be borrowed (if credit is needed) and to be advanced. The amount of money needed for circulation becomes endogenous and is a fraction of the total amount of money held.



$$C = \mathbf{y}\mathbf{A}\mathbf{u}, K = \mathbf{y}\mathbf{A}\bar{\mathbf{p}}, V = \mathbf{b}\mathbf{u}, W = \mathbf{b}\bar{\mathbf{p}}, M = \mathbf{s}\mathbf{u}, P = \mathbf{s}\bar{\mathbf{p}}$$

in values and prices. This corresponds to Eq. (3). One can show that there is exactly one equilibrium rate of profit  $\bar{r}$  for which we obtain with the prices according to (8) and the quantities according to (9)  $\bar{r} = P/(K + W)$ . One further needs the vectors  $\mathbf{m} = \mathbf{y} - \mathbf{q}_1$ ,  $\mathbf{v} = \mathbf{l} - \mathbf{x}^1$ . They represent the deviations of the activity levels vector from the standard proportions and of the labour vector from that vector which, if it was the labour vector, would cause the labour theory of value to hold (Marx-vector).

With this we can formulate the conditions which, taken together, are sufficient for obtaining asymptotically  $P = M$  and hence also  $M/(C + V) = P/(K + W) = \bar{r}$ , hence which are sufficient to guarantee that the rate of profit measured in values is equal to that measured in prices in any desired approximation.

- I. The eigenvalues  $\mu_2, \dots, \mu_n$  must be “small”, and for this is essentially sufficient that  $\mathbf{A}$  is a random matrix of sufficiently large dimension.
- II. We must have approximately  $\text{cov}(\mathbf{m}, \mathbf{v}) = 0$ : in this sense, the vectors of activity levels and of labour must be independent. To be more precise, the differences between the activity levels vector and the standard vector on the one hand and the differences between the labour vector and the vector which would yield the labour theory of value must not be correlated.
- III. We postulate that approximately  $\text{cov}(\mathbf{s}, \mathbf{v}) = 0$ ; this is essentially the independence of the surplus vector and the labour vectors.

The mathematical derivation yields for prices and for total profits, if conditions I and II hold:

$$\bar{\mathbf{p}} = \frac{1}{\mathbf{q}_1 \mathbf{x}^1} [\mathbf{x}^1 + (1 - (1 + r))\mathbf{v}] \text{ and } P = \frac{1}{\mathbf{q}_1 \mathbf{x}^1} [\mathbf{s} \mathbf{x}^1 + (1 - (1 + r))\mathbf{s} \mathbf{v}]. \quad (11)$$

If III holds as well,  $\mathbf{s} \mathbf{v}$  can be shown to vanish in (11), and the price of the surplus  $s$  will at all possible levels of the rate of profit, therefore in particular at  $r = \bar{r}$ , be equal to surplus value evaluated at  $r = 0$ ; we therefore have in particular  $\mathbf{s} \bar{\mathbf{p}}(\bar{r}) = \mathbf{s} \mathbf{u}$  and  $P = M$ . But this does not mean that prices are equal to values; they change linearly according to (11) with the rate of profit in such a manner that the vector of the differences between prices and values is for each  $r$  in a hyperplane orthogonal to  $\mathbf{s}$ .

In this way, one can show that  $P = M$  holds asymptotically in random systems, and the equality of the total product, valued in labour values and in prices, is assured by definition. The derivations allow to obtain further results. The results turn out to be robust. For instance, if one of the eigenvalues, say  $\mu_2$ , turns out not to be small after all, this violation of the randomness condition will be compensated, if  $\mathbf{q}_2 \mathbf{x}^2$  is small.

One finds, given the assumptions made above, that prices are not proportional to values, but that the deviation from prices and values vanishes on average. For prices will be given by (11) and one shows  $\mathbf{e} \mathbf{v} = 0$ , hence  $\mathbf{e} \bar{\mathbf{p}} = \mathbf{e} \mathbf{u}$  ( $\mathbf{e}$  summation vector,  $\mathbf{u}$  vector of values).

One can also show, using (8) in combination with (9), that a linear wage curve results, similar to that obtained by Sraffa using the standard commodity and similar

to that which obtains if the labour theory of value holds because of equal organic compositions. Define  $\tilde{w} = \rho\bar{w}$ , where  $\rho = 1 + r$ . This is the wage paid ex post, that is augmented by the rate of profit, which we need to make the derivation comparable with that of Sraffa. One finds

$$\tilde{w} = \rho\bar{w} = (1 - \rho\mu_1)/\mathbf{q}_1\mathbf{x}^1 \quad (12)$$

This relationship (12) facilitates the Ricardian exercise. The price of the average industry and of the gold industry is constant by definition. Now one can describe how prices change, if the rate of profit rises, because (12) tells us by how much the wage falls accordingly. This means that the price of a commodity produced in an industry with a high organic composition of capital will rise relative to that of a commodity produced in the average industry, while the price of a commodity produced in an industry with a low organic composition of capital will be reduced. Such a change of distribution implies that the capitalists can buy a different (more expensive) physical surplus. The equality  $P = M$  will continue to hold, if condition III continues to hold.<sup>15</sup>

Whether our assumptions I, II, III can be defended on empirical grounds will have to be discussed elsewhere, as we are here concerned with the analytical reconstruction of the Marxian system. Especially assumptions I and II are in the spirit of Marx, insofar as his theory of the production of relative surplus value (see Sect. 3, point 2 above) takes the saving of labour at the expense of using more materials and machines as the main form of technical progress. Hence the change of individual input–output coefficients is unpredictable, but one expects that the coefficients of the labour vector will fall, and they do that fairly uniformly at the level of aggregation of empirical input–output tables, while the individual input–output coefficients go up and down in a haphazard manner and the average does not move much; according to the stylised facts, the capital coefficient is constant. To assume that the input–output matrix is random thus seems compatible with the theory of production of relative surplus value, while the two covariance conditions II and III correspond to the absence of any postulate of some kind of correlation between the corresponding variables in Marx.

To regard an input–output matrix as random may still seem unusual, although random matrices get more and more used in many applications in the sciences. One tends to think of the technique as something rather static and determinate, but the coefficients do change, if only slowly, and a slow random process is still random—if it were determinate, one should be able to predict the evolution of individual coefficients, but that does not seem to be possible. Unlike labour coefficients, they do not go down uniformly because of saving of raw materials: on the contrary, they often go up to facilitate the saving of labour, in accordance with the Marxian hypotheses about the production of relative surplus value. Kenji Mori observed in his discussion of my paper in the Joint Seminar mentioned above that Marx regarded the “principle

<sup>15</sup> We here abstract from subtler effects considered by Sraffa (1960), who considers more complicated cases. For instance, the product of a more capital-intensive industry can fall with an increase in the rate of profit, according to Sraffa, if the means of production are produced by labour-intensive industries.

of multiples” as a characteristic feature of the machinery system and referred to the contemporary literature on technology by e.g. Charles Babbage; it results in a fixed proportion in the “instruments of labour”, referring to MEGA II/3.6, p. 1964 and to Marx (1969, p. 366). It is true that cars usually have four wheels and hence need four tires, but such links between inputs in the small do not necessarily exclude an at least approximately normal distribution in the large.

Anwar Shaikh and his school have undertaken empirical investigations, in particular Theodore Mariolis and Lefteris Tsoulfidis (see Shaikh 2016). Luis Daniel and Torres Gonz  les (2018) has examined hypotheses I and II in his thesis with satisfactory results. Torres finds a number of statistical regularities in the form of highly peaked empirical frequency distributions for several aspects of matrix  $\mathbf{A}$  (p. 68). Further research will show how far the simple concept of random matrices should be qualified.

## 7 Another derivation of $P = M$

We have provided only a sketchy derivation of  $P = M$  in the previous section, since the exact analysis is to be found in Schefold (2016). It is based on the theory of random matrices, introduced into the economic theory of capital in Schefold (2013), with its ultimate mathematically rigorous basis in the theorem of Goldberg and Neumann (Goldberg and Neumann 2003). They show the exact conditions under which the subdominant eigenvalues with tend to zero for large random matrices. If the subdominant eigenvalues are equal to zero, the matrix is of rank one, and each semipositive matrix  $\mathbf{A}$  of rank one is positive and can be written as  $\mathbf{A} = \mathbf{c}\mathbf{f}$  with  $\mathbf{c} > 0$ ,  $\mathbf{f} > 0$ .

The structure can be interpreted economically as a one-industry system (Schefold 2013, pp. 1176–1179); I call  $\mathbf{f}$  (row vector) the composition and  $\mathbf{c}$  (column vector) the distribution of capital. Random matrices thus are close to, and for large dimensions of the system they tend to, one-industry systems with a ‘flat’ structure  $\mathbf{A} = \mathbf{c}\mathbf{e}$ , where  $\mathbf{e} = (1, 1 \dots 1)$  is formally equal to the summation vector; it comes in here because the distribution of the coefficients on the rows of random matrices is i.i.d. Random matrices thus are close to matrices of the type  $\mathbf{A} = \mathbf{c}\mathbf{e}$  only on average; many components can actually be equal to zero. It follows from the continuity properties of eigenvalues that it ought to be possible to extend the Goldberg-Neumann theorem to matrices of the type  $\mathbf{A} = \mathbf{c}\mathbf{f}$ , generalising the random distribution of the coefficients on the rows, but at the expense of getting more complicated assumptions as sufficient conditions for convergence. As an economist, I took the liberty of introducing one-industry systems without having a precise mathematical theorem that would show how the properties of one-industry systems would tendentially be preserved in perturbations of  $\mathbf{A} = \mathbf{c}\mathbf{f}$  with perturbations similar to those that are admitted for random-matrices of the type  $\mathbf{A} = \mathbf{c}\mathbf{e}$ .

At any rate, the one-industry system is an economic model that deserves investigation, and we want to derive the conditions under which  $P = M$  will result in this case fully and explicitly.<sup>16</sup>

We write  $\rho$  for  $1 + r$ . Note that  $\mathbf{fA} = \mathbf{f}(\mathbf{cf}) = (\mathbf{fc})\mathbf{f}$ . We assume that  $\mathbf{A}$  is productive.

Hence the scalar  $\mathbf{fc} = \text{dom}\mathbf{A} < 1$  equals  $1/(1 + R)$ , where  $R > 0$  is the maximum rate of profit.

The matrix  $\mathbf{cf}/\mathbf{fc}$  is idempotent, for

$$(\mathbf{cf}/\mathbf{fc})^2 = [1/(\mathbf{fc})^2]\mathbf{cfcf} = \mathbf{cf}/\mathbf{fc}$$

and the following inverse results:

$$(\mathbf{I} - \rho\mathbf{cf})^{-1} = \sum_{t=0}^{\infty} (\rho\mathbf{cf})^t = \mathbf{I} + \rho\mathbf{cf}(1 + \rho\mathbf{fc} + \rho^2(\mathbf{fc})^2 + \dots) = \mathbf{I} + \rho\mathbf{cf}(1 - \rho\mathbf{fc})^{-1} \quad (13)$$

The vector  $\mathbf{f}$  is proportional to Sraffa's standard commodity. Prices  $\tilde{\mathbf{p}}$  in terms of this standard, with  $1 = \mathbf{f}\tilde{\mathbf{p}}$  can be calculated in a surprisingly simple manner, without using (13):

$$\tilde{\mathbf{p}} = \rho(\mathbf{cf}\tilde{\mathbf{p}} + \tilde{w}\mathbf{l}) = \rho(\mathbf{c} + \tilde{w}\mathbf{l})$$

hence from

$$1 = \mathbf{f}\tilde{\mathbf{p}} = \rho(\mathbf{fc} + \tilde{w}\mathbf{fl}),$$

we get a linear wage curve as in the case of Sraffa, if the wage is paid *ex post*, that is, for the *ex-post-wage*  $\rho\tilde{w}$ :

$$\rho\tilde{w} = (1/\mathbf{fl})(1 - \rho\mathbf{fc}) \quad (14)$$

The means that these 'standard' prices themselves get linear:

$$\tilde{\mathbf{p}} = \rho\mathbf{c} + [1/\mathbf{fl} - \rho(\mathbf{fc}/\mathbf{fl})]\mathbf{l} = \rho\left(\mathbf{c} - \frac{\mathbf{fc}}{\mathbf{fl}}\mathbf{l}\right) + \frac{1}{\mathbf{fl}}\mathbf{l} \quad (15)$$

We therefore get the extraordinary result that these 'standard'<sup>17</sup> prices are linear.

Each price is algebraically a linear function of  $r$ , and the price vector moves on a one-dimensional hyperplane (or a line) as a function of  $r$ , whereas prices are never confined to a hyperplane of dimension less than  $n$ , if the system is regular, according to the Fundamental Neoricardian Theorem.

<sup>16</sup> Masashi Morioka presented a comment at the Joint Seminar mentioned in the first Note. He derived conditions that were not only sufficient (as above in Sect. 5) but also necessary for  $P = M$  in one-industry systems. I am very grateful for this inspiration. My derivation and my conditions are somewhat different from his.

<sup>17</sup> These prices would be standard prices in the sense of Sraffa, if we normalised  $\mathbf{fl} = 1$ ; the *ex-post-wage* rate (14) would then be equal to the familiar  $1 - r/R$ .

Marx was not concerned with ‘standard’ prices, but with prices of production normalised by taking the vector of gross outputs  $\mathbf{y}$  as the numéraire. Prices in terms of  $\mathbf{y}$  are denoted by  $\bar{\mathbf{p}}$ , as in the previous section.

We get, using (13) and  $1 = \mathbf{y}\bar{\mathbf{p}} = \mathbf{y}(\bar{\mathbf{p}}/\bar{w})\bar{w}$ ,

$$\begin{aligned}\bar{\mathbf{p}} &= \rho\bar{w}(\mathbf{I} - \rho\mathbf{c}\mathbf{f})^{-1}\mathbf{l} = \rho\bar{w}\left(\mathbf{I} + \frac{\rho\mathbf{c}\mathbf{f}}{1 - \rho\mathbf{f}\mathbf{c}}\right)\mathbf{l} \\ \bar{w} &= \left[\rho\mathbf{y}\mathbf{l} + \rho^2 \frac{\mathbf{y}\mathbf{c}\mathbf{f}\mathbf{l}}{1 - \rho\mathbf{f}\mathbf{c}}\right]^{-1} \\ \bar{\mathbf{p}} &= \frac{\rho\mathbf{l} + \rho^2 \frac{\mathbf{c}\mathbf{f}\mathbf{l}}{1 - \rho\mathbf{f}\mathbf{c}}}{\rho\mathbf{y}\mathbf{l} + \rho^2 \frac{\mathbf{y}\mathbf{c}\mathbf{f}\mathbf{l}}{1 - \rho\mathbf{f}\mathbf{c}}} = \frac{\mathbf{l} + \rho((\mathbf{f}\mathbf{l})\mathbf{c} - (\mathbf{f}\mathbf{c})\mathbf{l})}{\mathbf{y}\mathbf{l} + \rho(\mathbf{y}\mathbf{c}\mathbf{f}\mathbf{l} - \mathbf{y}\mathbf{l}\mathbf{f}\mathbf{c})}\end{aligned}\quad (16)$$

Prices  $\bar{\mathbf{p}}$  are, like  $\tilde{\mathbf{p}}$ , not regular, but they move in a two-dimensional hyperplane and are not linear.

The conditions that are both necessary and sufficient for  $P = M$  to hold are now given by

$$P = \mathbf{s}\bar{\mathbf{p}}(\bar{\rho}) = \mathbf{s}\bar{\mathbf{p}}(1) = M, \quad (17)$$

where  $\bar{\rho} = 1 + \bar{r}$  corresponds to the rate of profit  $\bar{r}$ , at which the surplus appropriated by the capitalists,  $\mathbf{s}$ , can be bought by them at the prices pertaining to  $\bar{r}$ .

It has been shown (Schefold 2016, p. 175) that there is exactly one  $\bar{r}$  such that for  $\rho = 1 + \bar{r}$  and for  $\bar{w} = \bar{w}(\bar{r})$  one has

$$\mathbf{s}\bar{\mathbf{p}}(\bar{\rho}) = \mathbf{y}(\mathbf{I} - \mathbf{c}\mathbf{f})\bar{\mathbf{p}}(\bar{\rho}) - \bar{w}(\bar{\rho})\mathbf{y}\mathbf{l}. \quad (18)$$

While a unique solution  $\bar{\rho} = 1 + \bar{r}$  of (18) can be shown to exist, it cannot be given in explicit form and inserted in (17). We must therefore be content with a more modest solution and ask for the necessary and sufficient conditions under which (17) holds *identically* in  $\rho$ , that is, the conditions under which  $P = \mathbf{s}\bar{\mathbf{p}}(\rho)$  is constant.

These conditions are obtained by setting the derivative of  $\mathbf{s}\bar{\mathbf{p}}(\rho)$  equal to zero. The necessary and sufficient condition turns out to be simple<sup>18</sup>:

$$\frac{\mathbf{s}\mathbf{c}}{\mathbf{y}\mathbf{c}} = \frac{\mathbf{s}\mathbf{l}}{\mathbf{y}\mathbf{l}} \quad (19)$$

The labour value of the surplus stands in the same relation to the labour value of output as the corresponding compositions of capital. The condition is independent of the distribution of capital  $\mathbf{f}$ ;  $\mathbf{f} = \mathbf{e}$  is not excluded. Prices are not equal to labour values on average here, if  $\mathbf{f} \neq \mathbf{e}$ . The labour theory of value holds in one-industry systems, if and only if  $\mathbf{l}$  is proportional to  $\mathbf{c}$ ; (19) is then fulfilled. The analogy with the standard system is given, if  $\mathbf{s}$  and  $\mathbf{y}$  are proportional; (19) is then also fulfilled. Moreover, the conditions I–III of Sect. 5 are sufficient for (19) to hold, if  $\mathbf{f} = \mathbf{e}$ . For

<sup>18</sup> Using that  $(A + Bx)/(C + Dx)$  is constant, if and only if  $BC = DA$ .

$0 = \text{cov}(\mathbf{m}, \mathbf{v}) = \mathbf{m}\mathbf{v} - n\bar{m}\bar{v}$  implies  $\mathbf{m}\mathbf{v} = 0$ , since  $n\bar{v} = \mathbf{e}\mathbf{v} = 0$ , as we have seen, and similarly  $\mathbf{sv} = 0$ . Hence  $\mathbf{y}\mathbf{l} = \mathbf{q}_1\mathbf{x}^1$  and, with  $\mathbf{s} = \alpha_1\mathbf{q}_1 + \dots + \alpha_n\mathbf{q}_n$ ,  $\mathbf{sl} = \alpha_1\mathbf{q}_1\mathbf{x}^1$ , and  $\mathbf{yc} = \beta\mathbf{q}_1\mathbf{x}^1$  (with  $\mathbf{c} = \beta\mathbf{x}^1$ ),  $\mathbf{sc} = \alpha_1\mathbf{q}_1\beta\mathbf{x}^1$ . We get

$$\mathbf{ylsc} = \mathbf{q}_1\mathbf{x}^1\alpha_1\mathbf{q}_1\beta\mathbf{x}^1$$

$$\mathbf{yysl} = \beta\mathbf{q}_1\mathbf{x}^1\alpha_1\mathbf{q}_1\mathbf{x}^1$$

which proves (19). Conversely, if one assumes  $\mathbf{f} = \mathbf{e}$  and either  $\mathbf{mv} = 0$  or  $\mathbf{sv} = 0$ , one can deduce the covariance conditions II and III from (19), using

$$\mathbf{ylsc} - \mathbf{yysl} = \mathbf{mv} - \mathbf{sv}$$

The case  $\mathbf{mv} = \mathbf{sv} > 0$  represents a generalisation of the assumptions made in Sect. 5, but it is economically not as intuitive as the covariance conditions: We can easily imagine why  $\mathbf{m}$  and  $\mathbf{v}$  are not correlated and why therefore  $\mathbf{mv} = \mathbf{sv} = 0$ —there is no reason why a correlation should obtain—but it is not clear why these magnitudes should be equal, if there is a correlation. The assumptions made in Sect. 5 to explain an equality of total surplus value and of total profits therefore seem to be more natural than the ones made in Sect. 6, even if the latter are more general.

However that may be, we may again conclude that the requirements regarding the relationship between capital and labour can be less restrictive than the equal organic composition of capital condition, if the structure of production is more even by being close to a one-industry system. And an advantage of the new restrictions (that  $\mathbf{A}$  is, apart from perturbations, equal to  $\mathbf{cf}$  and that  $\mathbf{sc}/\mathbf{yc} = \mathbf{sl}/\mathbf{yc}$  holds approximately) exists. It consists in the fact that the number of restrictions is smaller and in the possibility of interpreting the condition  $\mathbf{f} \neq \mathbf{e}$  directly. We may say, for instance, that Marx, by asserting  $P = M$ , unknowingly made the hypothesis that all industries were somewhat similar. Such assumptions are, as so many others in economics, like that of the homogeneity of commodities and of prices, not exactly realistic but suggestive and, as a foundation for an analytic reconstruction of Marx's theory, taken together a touchstone for comparing the success of his explanations with those of others.

## 8 Evaluation

We have found systems for which  $P = M$  holds on average, but this in a wider and different sense than if one talks about Sraffa's standard system. According to Sraffa, one obtains  $P = M$ , if the activity vector, the vector of wage goods and that of the surplus are all in standard proportions, which is only theoretically possible, because workers do not consume investment goods. But here, the vector of activity levels can be arbitrary; only the randomness and the independence conditions (covariance conditions) must hold. In these systems the rates of profit in values and in prices coincide, as we have shown, so that the condition is fulfilled, by which the analysis of the First and the Third Volume are held together. Our result holds only asymptotically,

but Marx did not affirm more. Engels expressed the result in the Third Volume of *Das Kapital* as follows:

“Es ist überhaupt bei der ganzen capitalistischen Production immer nur in einer sehr verwickelten und annähernden Weise, als nie festzustellender Durchschnitt ewiger Schwankungen, daß sich das allgemeine Gesetz als die beherrschende Tendenz durchsetzt.” (MEGA II/15, p. 162)

“Under capitalist production, the general law acts as the prevailing tendency only in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations.” (Capital Vol. III, p. 123)

Engels here identifies the formation of the average with tendencies of temporal sequences. Böhm-Bawerk (1896, in Weiß 1968) criticised this in his critique of Marx, presumably because, trained in Ricardian economics, he thought of a formation of averages for a given long period, in the sense of Ricardo’s invariable standard of value, which Sraffa developed further, turning it into the standard commodity, which refers to a given technique. In Sraffa’s case, the averages thus formed in a statical system. Marx himself, however, wrote, using what in English is a four-letter word and mixing the languages:

“Es ist überhaupt bei dieser ganzen bürgerlichen Scheisse immer nur in a very complicated, and very rough way, daß sich das allgemeine Gesetz als die beherrschende Tendenz durchsetzt.” (MEGA II/4.2, p. 237)

This has been rendered by Engels: „Es ist überhaupt bei der ganzen kapitalistischen Produktion immer nur in einer sehr verwickelten und annähernden Weise, als nie festzustellender Durchschnitt ewiger Schwankungen, daß sich das allgemeine Gesetz als die beherrschende Tendenz durchsetzt.“ (MEGA II/15, p. 162)

Translation: “Under capitalist production, the general law acts as the prevailing tendency only in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations.” (Capital Vol. III, p.123)

This original quote expresses very clearly that Marx did not expect to find a general law that would be fulfilled in each particular case, but that he was looking for an approximation, and he liked to situate it in a temporal sequence, in a process, as the following quote shows (it is not concerned with  $P = M$  directly, but with the equalisation of the rates of profit as an expression of competition, which is in Marx not any kind of rivalry but the pursuit of profit):

„Bei den vielen verschiedenen Gründen, welche nach ch.I die Profitrate steigen oder fallen machen, sollte man glauben, daß die allgemeine Profitrate, so zu sagen jeden Tag changiren müßte. Aber die Bewegung in einer Productions-sphäre wird in der andern aufgehoben, die Einflüsse kreuzen und paralysiren sich. Man wird später weiter entwickelt finden, nach welcher Seite die changes tendiren; aber sie sind langsam; die Plötzlichkeit, Vielseitigkeit und Beständigkeit der changes in den einzelnen Productionssphären macht daß sie sich zum Theil in ihrer Reihenfolge aufheben (in der Zeit, heute up, morgen down),

daß sie local bleiben (ich verstehe hier unter local die Beschränkung innerhalb der besondern Productionssphäre) und daß andererseits die verschiedenen localen changes sich wechselseitig aufheben.“ (MEGA II/4.2, p. 245)

This has been rendered by Engels: „Bei den vielen verschiedenen Ursachen, welche nach dem Entwickelten die Profitrate steigen oder fallen machen, sollte man glauben, daß die allgemeine Profitrate jeden Tag wechseln müßte. Aber die Bewegung in einer Produktionssphäre wird die in der andern aufheben, die Einflüsse kreuzen und paralysieren sich. Wir werden später untersuchen, nach welcher Seite die Schwankungen in letzter Instanz hinstreben; aber sie sind langsam; die Plötzlichkeit, Vielseitigkeit und verschiedene Dauer der Schwankungen in den einzelnen Produktionssphären macht, daß sie sich zum Teil in ihrer Reihenfolge in der Zeit kompensieren, so daß Preisfall auf Preissteigerung folgt und umgekehrt, daß sie also lokal, d.h. auf die besondere Produktionssphäre beschränkt bleiben; endlich, daß die verschiedenen lokalen Schwankungen sich wechselseitig neutralisieren.“ (MEGA II/15, p. 170)

Translation: “In view of the many different causes which make the rate of profit rise or fall one would think, after everything that has been said and done, that the general rate of profit must change every day. But a trend in one sphere of production compensates for that in another, their effects cross and paralyse one another. We shall later examine to which side these fluctuations ultimately gravitate. But they are slow. The suddenness, multiplicity, and different duration of the fluctuations in the individual spheres of production make them compensate for one another in the order of their succession in time, a fall in prices following a rise, and vice versa, so that they remain limited to local, i.e., individual, spheres. Finally, the various local fluctuations neutralise one another.” (Capital Vol. III, p. 127)

It cannot be doubted that Marx liked to analyse the economy sequentially, partly, because this reflected his dynamical view of capitalist accumulation, partly for the mundane reason that solving equations in successive steps as in his tables for the transformation into prices or in his analysis of circulation in the two-sector model of volume II is easier than a simultaneous solution.<sup>19</sup> But he remained faithful to the classical method of using prices of production as centres of gravitation, and this is no contradiction, since intertemporal systems such as the simple

$$\mathbf{p}_{t+1} = (1 + r)(\mathbf{A}\mathbf{p}_t + \mathbf{l}) \quad (20)$$

converge to (8) with  $w = 1$ .<sup>20</sup> Often, the process will seem more important than where it converges to, but  $P = M$  was a result that Marx and Engels announced

<sup>19</sup> It has been observed [MEGA (1958 sq) I/1, p. 1204] that Marx showed more proficiency in the solution of geometrical than of algebraical problems, when he had to pass the final school examinations in Trier in 1835 [see MEGA (1958 sq) I/1, p. 460–464]. There seems to be a mistake in the diagram of p. 463 due to the editors. Marx made a mistake in the solution of a very simple system of linear equations.

<sup>20</sup> A differentiated answer thus must be given to the authors who interpret Marx in terms of temporal systems like Andrew Kliman (2007).



triumphantly, because they thought that profit ultimately was nothing but surplus value.

Sraffa studied the Marxian approach intensely, but he in effect dissociated himself from it.<sup>21</sup> His “metaphysics” were somewhat different after all. We find the following note in the archive (here quoted following Schefold 2016, p. 197):

„The propositions of M. are based on the assumption that the *comp. of any large aggr.* of commodities (wages, profits, const cap.) consists of a *random selection* {my emphasis – BS}, so that the ratio between their aggr. (rate of s.v., rate of p.) is approx. the same whether measured at ‘values’ or at the p. of prod. corresp. to any rate of s.v.

This is obviously true, and one would leave it at that, if it were not for the tiresome objector {= Sraffa?! – BS}, who relies on hypothetical deviations: suppose, he says, that...the caps switched part of their consumption from comms of lower to higher org. comp., while the workers switched to the same extent theirs from higher to lower, the aggr. price of each remaining unchanged....“

Sraffa thus anticipated our result, but portrayed himself as the toilsome pedant, who rendered the calculation in Marxian aggregates more complicated by asking what would happen in the case of deviations from the average. Such disturbances of statistic ensembles are not licit in thermodynamics. Imagine the room, in which you sit, divided in two by a wall passing through the observer. Imagine you could open the wall each time, when a fast molecule of air arrives from the right and also each time when a slow molecule arrives from the left. After a while, all the fast molecules will be found on the left, all the slow ones on the right, and one could, thanks to the difference in temperature so obtained, produce electricity by means of a steam engine. By opening and closing the wall with a negligible expense of energy, one could thus violate the entropy theorem. The Maxwellian demon, as the opener and closer of the wall is called in this thought experiment, can therefore not exist.

Sraffa’s tiresome objector reminds us of the Maxwellian demon, and one has to concede to Sraffa, of course, that his thought experiment, in which people demanding goods of higher and lower capital composition randomly are divided into two groups as capitalists and labourers, is possible. If one likes: the Maxwellian demon is licit in economics. But we may express our interpretation of the transformation problem by saying that Marx presupposed consciously, but without rendering precise, a random structure of the capitalist system—no actual

<sup>21</sup> How much Sraffa was intent to master the transformation problem can be inferred from a letter recently published of 1 August 1957 (therefore 3 years prior to the publication of Sraffa’s „Production of Commodities“) to his close friend Raffaele Mattioli, in which he writes (printed in Munari 2017, p. 105, my translation from the Italian): „Today the, Review of Economic Studies’ has arrived with the paper by a certain Seton (I do not know who that is), who anticipates an important part of my work. This has happened before several times, but this is perhaps worse... it seems to me as if a part of myself had vanished.“Sraffa confesses to his friend that he lost tears—it was the paper, now well known, about the transformation problem by Seton (1957). Seton’s price system bears a similarity to a Sraffa system at the maximum rate of profit. The second footnote mentions the original Frobenius theorem with reference to the original publication (hence Sraffa knew that reference after all!), and even something like the standard system is described in a rudimentary form.

Maxwellian demon takes away the property of the system of having an identical and independent distribution of coefficients—and therefore thought to be entitled to abstract from deviations such as the ones here constructed by Sraffa, because he thought that capitalism had the property which we know from thermodynamics: in the long run, the average prevails.

This was admitted by Sraffa, who continued in the citation introduced above:

„It is clear that M’s pros are not intended to deal with such deviations. They are based on the assumption (justified in general) that the aggregates *are* of some average composition. This is in general justified in fact, and since it is not intended to be applied to detailed minute differences it is all right.

This should be good enough till the tiresome objector arises. If then one must define which is the average to which the comp. should conform for the *result to be exact* {my emphasis – BS} and not only approximate, it is the St. Comm....

But what does this average ‘approximate’ to? i.e. what would it have to be composed of (what weights shd the average have) to be exactly the St. Com.?“

Sraffa arrives at his decisive conclusion:

„i.e. {my emphasis, the subsequent emphases by Sraffa – BS} Marx *assumes* that wages and profits consist *approximately* of quantities of st. com.“

But this conclusion is not compelling according to the result which we have found. We do not need to form a special average, that of the standard commodity, across industries of different organic compositions, if the system itself is a random system. For each physical composition of the product, not only for the standard commodity,  $P = M$  will hold approximately, if only the random properties of the system are approximately fulfilled. And the solutions of random systems are mathematically exact, even if only with asymptotic precision. And so our solution, using random systems is, I want to affirm, closer to Marx and more general than that by means of the standard commodity, which exists, of course, as well and with traces of the idea even in Marx. Sraffa renounced in his book “Production of Commodities by Means of Commodities” to derive the result  $P = M$ , which meant that he touched the Marxian approach but severed the link between the First and the Third Volume.

It also has become clear that, contrary to suppositions to be found in the literature and contrary to what I myself thought on occasions, that the average industry, which Marx uses in his transformation of values into prices times and again, is not the average in the sense of the standard commodity (the vector of activities, for which the vector of inputs is proportional to the vector of outputs), but the ordinary arithmetical average, given an arbitrary level of activities. The organic compositions of capital are different. Yet, the average of values corresponds to the average of prices.  $P = M$  presupposes in particular that the condition (III) is fulfilled. If that constraint on the covariance is violated, if one introduces for s another vector in (11), the value of capital will turn out to be dependent on

distribution. Moreover, even if (11) is fulfilled, only total capital, not the mass of constant capital or of variable capital individually, will be independent from distribution.

What is the general result? That labour values are not necessary to derive or predict prices of production, that they are not even sufficient for the derivation, if one does not know the temporal distribution of past labour inputs over time, has remained true. If one is only concerned with prices of production in their function for the explanation of accumulation, the theory of surplus value remains redundant. Redundance does not exclude the use of values for the theory of employment or with regard to considerations of distributional justice (Bortis 1997). Who uses the theory of surplus value and the Marxian conceptual apparatus now has got the confirmation that, to the extent that capitalist production may be understood as a random system, it is possible to produce the essential propositions of Marx in *Kapital*. That had been possible only under more restrictive assumptions, since the objections by Bortkiewicz became known.

How does this theoretical insight change our vision? The argument that profit is unpaid labour has been strengthened. But one can accept the theory of value and exploitation as an interpretation of reality and yet think that it is advisable to stick to capitalism. Hyman Minsky used to say that it is necessary to force a surplus so that the system may grow, also in the interest of the workers, for they are consumers and get richer as the economy grows. This justification of capital is old. It is not cynical but the fruit of long experience and sceptical analysis. There is no ideal economic system, but there are improvements. In fact: If wages rise with productivity, each worker gains from the exploitation of the others and profit appears as a compensation for entrepreneurial action. Such an acceptance of exploitation becomes bearable only, if the resulting distribution is regarded as fair after all, if taxation helps to correct imbalances and if extreme forms of exploitation and its consequences like child labour and unemployment are avoided.

## Compliance with ethical standards

**Conflict of interest** There is no conflict of interest regarding this paper.

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