

OBSERVATIONS
ON
REVERSIONARY PAYMENTS;
ON
SCHEMES for providing ANNUITIES
for WIDOWS, and for Persons in OLD AGE;
ON
The METHOD of Calculating the VALUES
of ASSURANCES ON LIVES;
AND ON
THE NATIONAL DEBT.

To which are added,

FOUR ESSAYS

On different Subjects in the Doctrine of LIFE
ANNUITIES and POLITICAL ARITHMETICK.

ALSO,

AN APPENDIX,

Containing a complete Set of TABLES; particularly,
Four New Tables, shewing the Probabilities of Life in
LONDON, NORWICH, and NORTHAMPTON; and the
Values of *joint* Lives.

BY RICHARD PRICE, D.D. F.R.S.

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Printed for T. CADELL, in the Strand.



INTRODUCTION.

BEFORE the Reader enters upon this Work, it will not be improper to give him the following information concerning it.

A few years ago, many gentlemen; of the first eminence in the law, formed themselves into a *Society*, for providing annuities for the widows of all such persons in judicial offices, barristers, civilians, and solicitors, as should chuse to become members. A plan was agreed upon and printed; but, some doubts happening to arise with respect to it, the directors resolved to ask the opinion and advice of three gentlemen; well known for their skill in calculation. This occasioned a further reference to me; and the issue was, that the plan being found to be insufficient; the whole design was laid aside.

About the same time, several other societies were formed with the same views; but all on plans alike improper and insufficient.

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Finding, therefore, that the public wanted information on this subject, I was led to undertake this work; imagining, that it might be soon finished, and that all I could say might be brought into a very narrow compass. But in this I have been much mistaken. A design, which I at first thought would give little trouble, has carried me far into a very wide field of enquiry; and engaged me in many calculations that have taken up much time and labour. I shall, however, be sufficiently rewarded for my labour, should it prove the means of preventing any part of that distress, which is likely to be hereafter produced by the societies now subsisting for the benefit of widows.—I have proved the inadequateness of their plans, by undeniable facts and mathematical demonstration (a).—I have, further, given an account of some of the best plans, that are consistent with a sufficient probability of permanency and success.—Should, therefore, any of these so-

(a) In the note p. 68, I take notice of the five guineas *fine* required by some of these societies at admission; and mention some reasons for not making any particular allowance for it. But I have since learnt, that it is indeed no more, than the first of the yearly payments, which I have always supposed to begin immediately. No allowance, therefore, was necessary to be made for it.

cieties determine to reform themselves; or should any institutions of the same kind be hereafter established, they will here find direction and assistance (*b*).

In

(*b*) I have lately learnt, that Mr. *Cadell*, the publisher of this work, and also Mr. *Becket*, Bookseller in the *Strand*, are commissioned to deliver in *London*, printed accounts of the scheme of a society, established five years ago at *Amsterdam*, for granting annuities on survivorship.---I cannot satisfy my own mind without introducing here, though an improper place, the following remarks on this scheme.

From the solution of Questions I and IV. in the First Chapter of the following Work, it may be gathered, that, (reckoning interest at $3\frac{1}{2}$ per cent. and the probabilities of life as they are in Tables III. IV. and V. in the *Appendix*) the value of an annuity of *1l.* for life, to be enjoyed by a person aged 20, provided he survives another person aged 60, is *8l. 16s. 6d.* in one present payment; and *18s. 6d.* in annual payments, during the two joint lives: the first payment to be made immediately. A single payment, therefore, of 130 florins, entitles to an annuity of 15 florins; and an annual payment of 110 florins, to an annuity of 119 florins; and both together, to an annuity of 134 florins. If the annual payments are to be made, not during the joint lives, but during the whole continuance of the oldest single life, they will, together with the single payment, entitle to an annuity of 144 florins. But this society promises, for these payments, an annuity of 100 florins, if the oldest life fails in the first year after admission; 200 florins, if it fails in the 2d year; 300 florins, if it fails in the third; 400 florins, if it fails in the 4th; and 500 florins, if it fails in the fifth year, or at any time afterwards. It is, therefore, evident that the scheme of this society is, in this instance, grossly defective. There are other instances in which it is even more defective; and the whole of it, like the schemes of most of the *London*

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In Question VI. Chap. I. a general method is described of finding the values, in *single* and *annual* payments, of all life-annuities which are to begin after a given term of years; and, in the 4th Section of the 2d Chapter, the plans of the societies for granting such annuities are particularly considered; and proved to be extremely deficient.—
Indeed, the general disposition which has lately shewn itself to encourage these societies; is a matter of the most serious concern; and ought, I think, to be taken under the notice of the Legislature. The leading persons among the *present* members, will be the *first*

don societies, appears to have been contrived by persons who had no principles to go upon. And yet it has been much encouraged. Many have entered themselves into it from different parts of *Europe*; and the printed plan acquaints us, that it is now in possession of an annual income of 200,000 *florins*. What disappointment then must it in time produce?—It is provided by its rules, that the terms of admission shall become less and less advantageous, the longer it has subsisted; just as if the value of the annuities it promises depended, not on the probabilities of life, and the improvement to be made of money, but on the age of the society.—I have taken notice of a similar absurdity in the rules of our own societies. But it is easy to see what is meant by it.

Mr. *Cadell* can procure from his correspondents in *Holland*, any information for those who may want to know more of this society. But indeed I should be sorry to find it much enquired after in *LONDON*.

annui-

annuitants; and they are sure of being gainers: and the more insufficient the scheme is, on which a society is formed, the greater will be the gains of the first annuitants. The same principle, therefore, that has produced and kept up other *bubbles*, has a tendency to preserve and promote these; and, for this reason, it is to be feared, that, in the present case, no arguments will be attended with any effect (c). The consideration, that “the gain made by some in these societies, will be so much plunder taken from others,” ought immediately to engage all to withdraw from them, who have any regard to justice and humanity; but experience proves, that this argument, when opposed to

(c) This apprehension has been verified by fact.—At the beginning of last winter a letter was published to the *Provident Society*, containing a clear proof of the insufficiency of the plans of all these societies. It was at least to be expected, that such a publication would prevent the rise of new societies, formed on *more* inadequate plans. But this was so far from being the effect, that, soon afterwards, a society sprung up which calls itself the *Rational Annuity Society*; and which, though it does not take *half* the values of the annuities it promises, has had the shamelessness to *assure* the public, that it is formed on a plan *incontestably durable*. The *Consolidated*, the *Public Annuitant*, and the *Westminster Union Societies*, are yet *worse* institutions, which have been since formed; and there may, for ought I know, be many more: for, indeed, all LONDON seems to be now entering into associations of this kind.

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private interest, is apt to be too feeble in its influence.

It cannot be said with precision, how long these societies may continue their payments to annuitants, after beginning them. A continued increase, and a great proportion of young members, may support them for a longer time than I can foresee. But the longer they are supported by such means, the more mischief they must occasion.—So, a tradesman, who sells cheaper than he buys, may be kept up many years by increasing business and credit; but he will be all the while *accumulating* distress; and the longer he goes on, the more extensive ruin he will produce at last.

In the latter end of the first chapter, I have stated very particularly, the method of computing the values of *assurances* on lives and survivorships, in all cases where no more than two lives are concerned: and, in the 3d Essay, I have pointed out a considerable error, into which there is danger of falling in computing some of these values. The societies and offices for transacting business in this way, are very useful; and it is necessary that they should go upon the best principles,
and

and possess all the information that can be given them.

But there is no part of this work in which the public is so much concerned, as the 3^d Chapter. It will be there proved, that had the sums raised for public services since the REVOLUTION, been much greater than they have been, the increase of the public debts to their present state might have been prevented in the easiest manner, and at a trifling expence. A method, likewise, of reducing within due bounds these debts, heavy as they now are, will be proposed.—All competent judges will, I believe, see, that this method, being founded on the most perfect improvement that can be made of money, is the most expeditious and effectual that the natures of things admit of. Nor, in my opinion, if the nation is not yet too near the *limit* of its resources, can there be any *good* reason against carrying it into execution.—It is well known, to what prodigious sums, money, improved for some time at *compound interest*, will increase (a). A state, if there is no mis-

(a) A penny, put out to 5 per cent. compound interest at our Saviour's birth, would, by this time, have increased to more money than would be contained in 150 millions of globes, each equal to the earth in magnitude, and all solid gold.

application of money, must necessarily make this improvement of any savings, which can be applied to the payment of its debts. It need never, therefore, be under any difficulties; for, with the *smallest* savings, it may, in as little time as its interest can require, pay off the *largest* debts.——Several of the observations I have made on this subject, have not, perhaps, been duly considered. Could they engage the attention of the managers of our public affairs, they might, I think, be of some service. But this, I am sensible, I cannot expect. I have, however, in some degree, satisfied my own mind; and I shall always reflect with pleasure, that, in this part of the following treatise, I have endeavoured to convey to the public, an information which is of particular importance to it.

In the *first* Essay I have made many observations on the expectations of lives, the pernicious influence of great towns on health, and manners, and population; the increase of mankind; and other subjects in the doctrine of Annuities and Political Arithmetick.——In the Last Essay I have stated carefully the proper method of forming tables of the probabilities of human life, from given observations:

tions: And, in the *Appendix*, besides several new Tables, I have thought it necessary to give Mr. *Simpson's* Tables of the values and expectations of LONDON lives; and all the other Tables which can be wanted in the perusal of this work.—I have also, in the *Appendix*, given the Demonstrations of the Answers to the *Questions* in Chap. I. These Demonstrations I have chosen to keep out of sight in the body of the work, in order to avoid discouraging such readers as may be unacquainted with mathematics.

Upon the whole. A great part of this work is, I believe, new; and I am in hopes also, that it will be found to contain some improvements in those branches of philosophical enquiry, which are the subjects of it.

The Reader is desired to correct the following Error.

In page 148, line 11. *instead of 11, read 9 years purchase; and in the next line, instead of $6\frac{1}{2}$, read $15\frac{1}{2}$ years purchase.*



C H A P. I.

Questions relating to Schemes for granting Reversionary Annuities, and the Values of Assurances on Lives.

QUESTION I.

 Set of married men enter into a
“ A “ society for securing annuities to
“ their widows. What sum of
“ money, in a single present pay-
“ ment, ought every member to
“ contribute, in order to entitle his widow
“ to an annuity of 30*l.* per annum for her life,
“ estimating interest at 4 per cent ?”

A N S W E R.

It is evident, that the value of such an expectation is different, according to the different ages of the purchasers, and the proportion of the age of the wife to that of the husband. Let us then suppose, that every person in such a society is of the same age with his wife, and that one with another all the members when they enter may be reck-

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oned 40 years of age, as many entering above this age as below it. It has been demonstrated by Mr. *De Moivre* and Mr. *Simpson*, that “the value of an annuity on the joint continuance of any two lives, subtracted from the value of an annuity on the life in expectation,” gives the true present value of an annuity on what may happen to remain of the latter of the two lives after the other.

In the present case, the value of an annuity to be enjoyed during the *joint continuance* of two lives, each (*a*) 40, (*b*) is 9.826, according

(*a*) See Table VII. Appendix.

(*b*) The values of *joint* lives and reversions, as deduced from the *Breslaw* observations, are not given in any part of this work from Mr. *De Moivre*'s rules in his treatise on annuities on lives. For these rules are approximations, which give results so far from the truth, as to be, not only useless, but dangerous. In the second essay in the Appendix, a particular account of this will be given, and also of the method in which these values have been calculated.

Mr. *De Moivre* has calculated the values of *single* lives, on the supposition of an *equal decrement of life* thro' all its stages till the age of 86, which he considered as the utmost probable extent of life. Thus; let there be 56 persons alive at 30 years of age. It is supposed that one will die every year till, in 56 years, they will be all dead. The same will happen to 46 at 40, in 46 years. To 36 at 50, in 36 years, and so on for all other ages. The number of years which a given life wants of 86, he calls the *complement* of that life. Fifty-six, therefore, is the *complement* of 30; 46 of 40, and 36 of 50.

This hypothesis eases very much the labour of calculating the values of lives; and it is so conformable to Dr. *Halley*'s table of observations, that there is little or no reason

ing to the probabilities of life in the table of observations formed by Dr. *Halley*, from the bills of mortality of *Breslaw* in *Silesia*. The value of a single life 40 years of age, as given by Mr. *De Moivre*, agreeably to the same table, is 13.20 (a); and the former subtracted from the latter, leaves 3.37, or the true number of years purchase, which ought to be paid for any given annuity, to be enjoyed by a

son for distinguishing between the values of lives as deduced from this Table, and the same values deduced from the hypothesis.

In order to avoid putting the reader to trouble, I have given this table at the end of this work. And I have also given two other tables which I have formed from the bills of mortality at *Northampton* and *Norwich*. These last tables answer more nearly to Mr. *De Moivre*'s hypothesis than even Dr. *Halley*'s table; and the difference between the values of *single* and *joint* lives by the *hypothesis*, and the same values computed strictly from the tables, is generally less in these tables than in Dr. *Halley*'s, as will be shewn in the last Essay. When, therefore, in the course of this work the values of *single* and *joint* lives are mentioned, as given agreeably to Dr. *Halley*'s table, it must be understood, that they are taken from Tables VI. and VII. in the Appendix, and given in strict agreement only to the *hypothesis*; and that for this reason, they are in reality still more conformable to the *Northampton* and *Norwich* tables.

The inhabitants of *London*, as is well known, not living so long as the rest of mankind, the values of *single* and *joint* lives there, are considerably less than in any other place where observations have been kept. Whenever, therefore, I have had *London* lives in view, I have given particular notice of it, and taken their values from Mr. *Simpson*, who has calculated them with much accuracy from the *London* tables of observation. See Tables X. and XI.

(a) See Table VI. Appendix.

person 40 years of age, *provided* he survives another person of the same age, interest being reckoned at 4 *per cent. per annum*. The annuity, therefore, proposed in this question being 30*l.* the present value of it is 30 multiplied by 3.37, or 101*l.* 2*s.*

By calculating from Mr. *Simpson's* tables (a), formed from the bills of mortality of *London*, this value comes out 102*l.*

The difference in the value of the reversion will be inconsiderable, whether the common age is taken a few years more or less than 40. Thus married men of 30 ought not, according to Dr. *Halley's* table, to give two fifths of a year's purchase more, for any given reversionary annuity for their wives, than married men of 50, provided they are of the same ages with their wives; and one quarter more, according to Mr. *Simpson's* table. If the wives are younger (as is generally the case) there will indeed be a considerable difference; for the value now determined would be 120*l.* according to the *Breslaw* observations, supposing the two lives to be 40 and 33, or that wives are one with another seven years younger than their husbands; and 118*l.* 10*s.* according to the *London* observations.

(a) See Table X. and XI. Appendix.

QUESTION II.

“ Supposing such a society as that describ-
 “ ed in the preceding question, to be limit-
 “ ed to a certain number of members, and
 “ constantly kept up to that number, by the
 “ admission of new members as old ones are
 “ lost, in consequence of their own deaths,
 “ and the deaths of their wives: What is the
 “ number of annuitants which, in some time
 “ after its establishment, will come to be
 “ constantly upon it?”

ANSWER.

Since every marriage produces either a widow or widower; and since all marriages taken together would produce as many widows as widowers, were every man and his wife of the same age, and the chance equal which shall die first; it is evident, that the number of widows that have ever existed in the world would, in this case, be equal to *half* the number of marriages. And what would take place in the world must also, on the same suppositions, take place in this society. — In other words; every *other* person in such a society leaving a widow, there must arise from it a number of widows equal to half its own number. — But this does not determine what number, all living at one and the same time, the society may expect will

come to be constantly upon it. For if every widow lived no more than a year, the society would never have more annuitants upon it than came on in a year. And on the contrary, if none ever died, the number of annuitants would go on increasing for ever.—'Tis, therefore, necessary, in order to answer the present enquiry, to determine how long the *duration of survivorship* between persons of equal ages will be, compared with the *duration of marriage*. And the truth is, that, supposing the probabilities of life to decrease uniformly (*a*), the former is equal to the latter; and consequently, that the number of *survivors*, or (which is the same, supposing no second marriages) of *widows* and *widowers* alive together, which will arise from any given set of such marriages constantly kept up, will be equal to the whole number of marriages; or *half* of them (the number of widows in particular) equal to *half*.

(*a*) That is, supposing that out of any given number alive at any age the same number will die every year 'till all are dead. See the preceding note. That on this hypothesis, the duration of survivorship is equal to the duration of marriage, when the ages are equal; or, in other words, that the *expectation* of two joint lives, the ages being equal, is the same with the *expectation* of survivorship, may be learnt from the 18th and 20th problems of Mr. *De Moivre's* treatise on annuities; and a demonstration of it, together with a particular explanation of this subject, may be found at the beginning of the first Essay to which I must beg the reader to turn, if he is at any loss about the full meaning of what is here said.

the

the number of marriages.—Now, it appears that the decrease in the probabilities of life, is in fact nearly uniform. According to the *Breslaw*, the *Northampton* and *Norwich* tables of observation, almost the same numbers die every year from 20 years of age to 77 (a). After this, indeed, fewer die, and the rate of decrease in the probabilities of life is retarded. But this deviation from the hypothesis is inconsiderable; and its effect, in the present case, is to render the duration of survivorship *longer* than it would otherwise be. According to the *London* table of observations, the numbers dying every year begin to grow less at 50 years of age; and from hence to extreme old age, there is a constant retardation in the decrease of the probabilities of life (b). Upon the whole, therefore, it appears in answer to the present question, that “ according to the *three former tables* of observations, and supposing no widows to marry, the number enquired after is *somewhat greater* than half the number of the society; but, according to the *London table*, a *good deal greater*.”

It must be carefully remembered, that this has been determined on the supposition, that

(a) See Tables III. IV. and V. Appendix.

(b) The reason of this difference between the *London* and other tables, will be given at the end of the fourth Essay.

husbands and their wives are of equal ages, and that in this case it becomes an equal chance which shall die first. In reality neither of these suppositions is just. Husbands in general are older than their wives; and in equal ages the mortality of males has been found to be greater than the mortality of females. For both these reasons, it is much more than an equal chance that the husband will die before his wife, or that the woman shall be the survivor of a marriage, and not the man. This will increase considerably the duration of survivorship on the part of the woman, and consequently the number enquired after in this question. The marriage of widows will also diminish this number, and the operation of these causes will be different in different situations. But it is by no means to be expected (in the situation of the societies I have in view) that the diminution from the latter cause will be considerable enough, to overbalance the operation of all the other causes which have been mentioned, and reduce the number under consideration so low, as half the number of marriages (a).

SCHOLIUM.

In *London* it appears, that there is a retardation of the decrease in the probabilities

(a) It will be observed hereafter, that this observation has been found to be true in fact.

of

of life, which renders the duration of survivorship between two lives of equal ages, considerably longer than their joint continuance. It seems worth observing, that this is the reason why, though the probabilities of life, and therefore the values of single and joint lives, are less in *London* than in other places, yet the values of reversions depending on survivorships, are in some cases greater there. It is proper to add, that this likewise is the reason why, in calculating the values of joint lives and reversions, the present value of an annuity payable yearly to the survivor of two equal lives, may come out equal to, or even greater than, the present value of a like annuity for the joint lives. As an annuity, during such survivorship, will probably not become payable for some years, and therefore the money given for it will have time to accumulate, it is manifest, that the value of it could never be equal to the value of an annuity on the joint lives, the payment of which begins immediately, were not the observation now made true.

QUESTION III.

“ Such a society as that described in the preceding questions being supposed, in what time will the number of annuitants upon it come to a *maximum* ? ”

ANS-

ANSWER.

In order to be more clear in answering this question, I will first suppose the society to comprehend in it from its first establishment, *all* the married persons of *all* ages in any town or country, where the number of people continue constantly the same. In this case, the whole collective body of members will be at their greatest age, at the time of the establishment of the society; and the number of members, together with the number of widows left every year, will, taking one year with another, admit of no increase or diminution. The number of widows in life together, derived from any given number coming on a society every year, will increase continually, till as many die off as are added every year; that is, till they come to die off as fast as possible. But they cannot die off as fast as possible, till the whole collective body of widows are at their greatest age; or, till there is among them the greatest number possible of the oldest widows; and, therefore, not till there has been time for an accession to the oldest widows, from the youngest part of the widows that come on annually.

Let us, for the sake of greater precision, divide the whole medium of widows that come on every year, into different classes according to their different ages, and suppose some to be left at 56 years of age, some at 46,
some

some at 36, and some at 26. The widows, constantly in life together, derived from the first class, will come to their greatest age, and to a *maximum*, in 30 years, supposing with Mr. *De Moivre*, 86 to be the utmost extent of life. The same will happen to the second class in 40 years, and to the third in 50 years (*a*). But the whole body, composed of these classes, will not come to a *maximum*, till the same happens to the fourth or youngest class; that is, not till the end of 60 years. After this, the affairs of the society will become *stationary*, and the number of annuitants upon it of all ages will keep always nearly the same.

Such is the answer to this question, supposing a society to begin with its complete number of members, consisting of married persons of all ages, in the same proportions to one another, with the proportions in which they exist in the world.—If it begins with its complete number of members, but at the same time admits none above a particular age: If, for instance, it begins with 200 members all under 50, and afterwards limits itself to this number, and keeps it up by admitting every year, at all ages between 26 and 50, new members as old ones drop off;

(*a*) In the *Appendix*, note (*a*), a rule is given, by which the numbers alive at the end of any particular number of years may be very easily determined.

in this case, the period necessary to bring on the *maximum* of annuitants will be just doubled. For, in the first place, the whole collective body of members will be 60 years in getting to their greatest age, as may easily appear from what has been just said. The annual medium of widows, therefore, that will come on the society will increase continually for 60 years, it being evident, that the older any set of married men are, taken one with another, the faster they will leave widows. And after this annual medium is increased to a *maximum*, 60 years more will be necessary to bring to a *maximum* the number in life together, derived from such a *fixed* annual medium constantly coming on.—If such a society is any number of years in gaining its *maximum* of members, the time necessary to bring on the *maximum* of annuitants will be still further prolonged, and will be equal to twice 60 years with that number of years added.—Most of the societies for granting annuities to widows are of this kind; and, therefore, supposing them to gain their complete number of members in ten years, and for ever afterwards to preserve it, the number of annuitants upon them will go on increasing for 130 years.—It is proper, however, to be remembered, that the increase will be quicker at first, and afterwards slower; and that, within 20 or 30 years of the end

of this term, it will be so slow as scarcely to be sensible, though still real.

All who will bestow due attention on this subject must see these decisions to be just; and a demonstration of them might be given, in a form more strictly mathematical, were it necessary.

QUESTION IV.

“ Suppose the members of such a society
 “ as that described in the preceding questi-
 “ ons, to chuse making *annual payments during*
 “ *the continuance of marriage*, in lieu of the
 “ sum which the reversionary annuity for
 “ their widows is worth in *present money* :
 “ What ought these *annual payments* to be,
 “ estimating interest at 4 per cent ?”

ANSWER.

This will be easily determined, by finding what annual payments, during two joint lives of given ages, are equivalent to the value of the reversionary annuity in *present money*. — Suppose, as in Question I. the two joint lives to be each 40, and the reversionary annuity 30*l. per annum*. An annual payment during the continuance of two such lives is worth, according to Dr. *Halley's* table of observations, 9.82 (*a*) years purchase. The annual

(*a*) See Table VII.

payment

payment then ought to be such as being multiplied by 9.82, will produce (a) *l.* 101.1, the present value of the annuity in one payment by Question I. Divide then *l.* 101.1 by 9.82, and the *quotient*, or *l.* 10.3 will be the answer.—This is very nearly the annual payment of all the members at an average, supposing equal numbers to offer themselves for admission of every age between 30 and 50. As much as some give less, others ought to give more, according to their excess of age. Thus, the annual payment of a married person, 30 years of age, ought to be *l.* 9.39; and of a person 50 years of age *l.* 11.33.—If the values of joint lives and the reversionary annuity are taken agreeably to the *London* table of observations, these annual payments will be, for 30 years of age (b), *l.* 10.9,—for 40, *l.* 12.5,—for 50, *l.* 14.5.

If

(a) Particular notice should be taken of the method of notation here used, because it will be carried through the whole of this work.—The figures on the right hand of the full-point, signify the decimal parts of *1l.* Thus; *l.* 101.1, is *l.* 101 and the 10th of *1l.* or *l.* 101 and 2*s.*—*l.* 9.39, is *l.* 9, and 39 hundredths of *1l.* or *l.* 9 : 7*s.* : 10*d.*—*l.* 11.33, is *l.* 11, and 33 hundredths of *1l.* or *l.* 11 : 6*s.* : 7*d.*—In general; it should be remembered, that 2 shillings allowed for every unit in the first place of decimals, and two-pence half-penny for every unit in the second place of decimals, will give, nearly enough, the value of the decimal part of every such expression.

(b) The value of two joint lives of 30, taken from Table XI. is 9.6. This subtracted from the value of the life in expectation, or from 13.1, by Table X. gives 3.5,
the

If either the rate of interest is supposed lower, or wives are supposed younger than their husbands, the annual payments will be increased. But there is no occasion for pointing out particularly the difference. It may be easily found in any cases by the directions now given. There is, however, one observation which ought to be here carefully attended to.—This method of calculation supposes, that the first annual payment is not to be made 'till the end of a year. If it is to be made *immediately*, the value of the joint lives will be increased one year's purchase; and, therefore, in order to find in this case the annual payments required; the value in present money found by Quest. I. must be divided by the value of the joint lives increased by unity, and, in this way, the preceding values at 4 *per cent.* according to the *Breslaw* observations, will be found to be *l.8.62—l.9.35—l.10.07.*—According to the *London* observations, *l.10,—l.11.2,—l.12.7.*

the number of years purchase which an annuity for a life of 30 years of age; *after* another life of the same age, is worth. This remainder, multiplied by 30, gives 105*l.* the value in a single payment, supposing the reversionary annuity to be 30*l.* And 105*l.* divided by 9.6, gives *l.10.9*, the value of the same annuity in annual payments, during the joint continuance of the two lives, according to the *London* observations.—By similar operations all the other values above given have been found.

QUES-

QUESTION V.

“ A society may chuse to make abate-
 “ ments in these annual payments, and to re-
 “ quire the remainder of the value of the
 “ reversionary annuity to be given, in fines
 “ or premiums at the time of admision; it
 “ may, for instance, chuse to fix the annual
 “ payments of all the members to 5 guineas.
 “ What, in this case, would be the premium
 “ due at admision, the annuity being sup-
 “ posed 30*l. per annum*, and interest being
 “ at 4 *per cent* ?”

ANSWER.

From the whole present value of the annuity in one payment, subtract the value of 5 guineas *per annum*, during the joint lives; and the remainder will be the answer.

Supposing the joint lives both 40, the whole present value of the annuity in one payment is, according to the *Breslaw* observations, *l. 101.1*, by Quest. I.—The value of 5 guineas *per annum*, or of *l. 5.25 per annum*, during two such joint lives, is 5.25, multiplied by the value of the joint lives; that is, 5.25, multiplied by 9.82, or *l. 51.55*; and this subtracted from *l. 101.1*, gives *l. 49.5*, the answer required for two lives at the age of 40.—The answer found in the same way for two lives whose common age is 30, is *l. 46.5*,—and for two lives at 50, *50l.*

Accord-

According to the *London* observations, these values are, for two lives at 30, *l.*54.6.—At 40, *l.*59. 4.—At 50, *l.*63. 3.

If the first of the annual payments is to be made immediately, the true answer will, in every instance, be the values found in the manner now directed, diminished by the annual payment; or, in the present case, 5 guineas less than the values specified.

The values, in *premiums* and *annual payments*, of any other reversionary annuity, will be as much greater or less than these, as the annuity itself is greater or less.

QUESTION VI.

“ A person 35 years of age wants to buy
 “ an annuity, for what may happen to re-
 “ main of his life after 50 years of age.
 “ What is the value of such an annuity in
 “ *ready money*; and also in *annual payments*,
 “ till he attains to the said age; that is, in
 “ annual payments for 15 years, subject in
 “ the mean time to failure, should his life
 “ fail?”

ANSWER.

The present value of such an annuity is the *present* value of a life at 50, in money to be received 15 years hence, and the payment of which depends on the contingency of the continuance of the given life 15 years. That is; it is equal to the value of a life at 50,

multiplied by the present value of 1*l.* to be received at the end of 15 years, and also by the probability that the given life will continue so long.—A life at 50, according to Mr. *De Moivre's* valuation of lives, and reckoning interest at 4 *per cent.* is worth 11, 34 year's purchase. The present value of 1*l.* to be received at the end of 15 years, is, by Table I, 0.5553. And the probability that a life at 35, will continue 15 years, is, according to the *Breslaw* observations $\frac{346}{490}$ (a). And these three values, multiplied by one another, give 1.4.44, or the number of years purchase that ought to be given for the annuity.—The annuity then being supposed 50*l.* its value in present money is 222*l.*

(a) The probability that a given life shall continue any number of years, or attain to a *given age*, is (as is well known) the fraction, whose *numerator* is the number of the living in any table of observations opposite to the *given age*, and *denominator*, the number opposite to the present age of the given life.—Thus, in the present instance; 346 is the number in Dr. *Halley's* table opposite to 50, and 490 the number opposite to 35.— $\frac{346}{490}$, (or the odds of 17 to 7) is, therefore, the probability that a person whose age is 35 shall attain to 50, or live 15 years. In the same manner it will appear, that, according to the same table, the probability that a person at this age shall live 25 years, is $\frac{242}{490}$; or nearly an even chance.

At *Northampton* and *Norwich* a person at the same age, has an even chance of living 26 years; but in *London*, scarcely 20 years. See Tables III, IV, V, and VIII. Appendix. I will add, though foreign to my present purpose, that a person at the same age has in these towns a better chance of living one year, than in *London*, in the proportion of 3 to 2.

In

In order to find this value in *annual payments*, while the given life is attaining to 50, it is necessary to find the value of an annuity for 15 years, subject to failure on the extinction of the given life. And the value of such an annuity is, evidently, the last value subtracted from the value of the given life; or, in the present instance, *l.* 4.44, subtracted from *l.* 13.97. (See Table VI, Appendix) that is, *l.* 9.53,—222*l.* then, being the present value of an annuity of 50*l.* for the remainder of a life now 35, after attaining to 50; and 9.53 being the number of years purchase, which ought to be given for an annual payment to last 15 years, if a life now 35 lasts so long, it follows, that the value of the same annuity in annual payments till this life attains to 50, is 222*l.* divided by 9.53; or *l.* 23.3:

This calculation supposes; that the first of the annual payments is not to be made till the end of a year. If the first payment is made immediately, the value will be, the *single payment* divided by the value of the life for the given term increased by unity; that is, in the present case, 222*l.* divided by 10.53; or *l.* 21.08.

If the value of the annuity is required in a single payment, over and above any given annual payment; deduct the value of the annual payment from the whole value in a single present payment, and the remainder will

be the answer.—Thus; let 5 guineas, in the present instance, be the given annual payment for the assigned term; and let the enquiry be, how much more in present money the supposed annuity is worth. By what has been just said, 9.53, multiplied by 5 guineas, that is, 50*l.* is the value of the annual payment; and this sum deducted from 222*l.* leaves 172*l.* the answer.

If the annual payment begins immediately, its value is 10.53, multiplied by 5 guineas, and the answer comes out *l.* 166.75.

In this way may be found the value, in single and annual payments, of any other annuity, payable to an assigned life, after a given term of years, taking any valuation of lives or interest of money. But care must be taken to remember, that it is the title to the annuity that will commence at the end of the given term, and that the first payment is not to be made 'till a year afterwards; that is, in the case here specified, not 'till the end of 16 years.

SCHOLIUM.

The value of the *remainder* of two joint lives, after a given term of years, is likewise the value of 1*l.* due at the end of the given term, multiplied by the value of two joint lives, each older by the given term than the given lives; and this product, multiplied by the probability, that the given joint lives shall
not

not fail in the given term; or (which is the same) by the product of the two probabilities, that the single lives shall each continue the given term. And the value of an annuity, on any given joint lives for a term of years beginning now, is this last value, subtracted from the whole present value of the joint lives. Thus; the value of two joint lives, one 40 years of age, and the other 50, (see Table VII.) is 8.91; which, multiplied by 0.6755, the value of 1*l.* due 10 years hence, and by $\frac{44}{111}$, (the probability that a life at 30 shall continue 10 years) and also by $\frac{345}{447}$, (the probability that a life at 40 shall continue 10 years) gives 3.92, the present value of the remainder of two joint lives, aged 30 and 40, after 10 years; and this value, subtracted from 10.43 (the value in Table VII. of two joint lives, aged 30 and 40) leaves 6.51, their value for 10 years.

As the value of the longest of two lives is always the value of the *joint* lives, subtracted from the sum of the values of the two *single* lives; their value also for any *given term*, is the value of the *joint* lives for the given term, subtracted from the sum of the values of the *single* lives for the given term.

The truth of these rules may easily appear without particular proof. I have, however, pointed out the method of demonstrating them in a note (a) at the end of this work.

(a) See note (B) in the Appendix.

By similar operations, may be found the values of 3 or more *joint* lives, or the longest of *three* or more lives, for a given term of years, or of what shall remain of them after a given term of years.

QUESTION VII.

“ The present value is required of an annuity to be enjoyed by one life, for what may happen to remain of it beyond another life, after a given term; that is, provided *both* lives continue, from the present time, to the end of a given term of years?”

ANSWER.

Find the value of the annuity for two lives greater, by the given term of years, than the given lives. Discount this value for the given term; and then, multiply by the probability, that the two given lives shall *both* continue the given term; and the product will be the answer.

EXAMPLE.

Let the two lives be each 30. The term seven years. The annuity, 10%. Interest, 4 *per cent.* — The given lives, increased by 7 years, become each 37. The value of two joint lives each 37, is (by Table VII) 10.25.
The

The value of a single life at 37, is (by Table VI) 13.67. The former, subtracted from the latter, is 3.42, or the value of an annuity for the life of a person 37 years of age, after another of the same age, by *Quest. I.*—3.42 discounted for 7 years, (that is, multiplied by 0.76, the value of 1*l.* due at the end of seven years, by Table I.) is 2.6.—The probability that a single life at 30 shall continue 7 years, is (by the hypothesis explained page 2.) $\frac{49}{56}$ (a). The probability, therefore, that two such

(a) In this case, it is on some accounts best, as well as easiest, to take the probabilities of life from the hypothesis, rather than immediately from the Tables.—Fifty-six persons being supposed alive at 30, one will die every year, according to the hypothesis. At the end of seven years then, the number of the living will be 49, and $\frac{49}{56}$, or the odds of 7 to 1, is, by note p. 18. the probability, that a life, aged 30, will continue 7 years; and this fraction, multiplied by itself, is the probability, that two lives of this age, shall *both* continue 7 years. In general, it must be remembered, that the probability, that any two or more events shall *all* happen, is the product arising from multiplying by one another, the probabilities of all the events taken separately. The probability, therefore, that any number of persons will *all* live any given time, is rightly found by multiplying into one another the probabilities that each of them will live that time.—It may further be of use to some, that I should observe here, that the difference between unity and the fraction expressing the probability, that an event will happen, gives the probability that it will *not* happen. Thus; the probability, that a person 40 years of age will live 11 years, is by the *Breslaw* Table $\frac{335}{445}$. The probability, therefore, that he will *not* live 11 years, is $\frac{110}{445}$, subtracted from

such lives shall both continue 7 years, is $\frac{7}{11\frac{1}{2}}$, or, in decimals 0.765. And 2.6 multiplied by 0.765, is 1.989, the number of years purchase which ought to be given for an annuity, to be enjoyed by a life now 30 years of age, after a life of the same age, provided both continue 7 years. The annuity then being 10*l.* its present value is *l.* 19.89.

By similar operations, it may be found, that supposing the term one year, and the ages and the rate of interest the same, the present value of the same reversionary annuity is *l.* 32.4; and that if the term is 15 years, the value is *l.* 9.7.

For two lives each 40, these values are *l.* 30.33.—*l.* 17.44.—*l.* 7.3. the term being 1, 7, or 15 years.

For two lives each 50, the same values for the same terms, are *l.* 28.2,—*l.* 13.86,—*l.* 4.34 (a).

These values, according to the *London* observations and Mr. *Simpson's* Tables of the values of single and joint lives, are,

from unity or $\frac{110}{445}$.—In like manner: The probability that two persons aged 30, shall *both* live 7 years, being 0.765, the probability that they will *not* both live so long, or that *one or other* of them will die in 7 years, is 0.765 subtracted from unity, or .235.

If any reader is unwilling to take these assertions for granted, he should consult the beginning of Mr. *De Moivre's*, or Mr. *Simpson's* Treatises on the Doctrine of Chances, where he will find them demonstrated.

(a) See Note (C) Appendix.

For

For 2 lives at 30—*l.* 32.05—*l.* 18.62—*l.* 7.66,
 at 40—*l.* 30.7 —*l.* 15.6 —*l.* 5.45.
 at 50—*l.* 29.36—*l.* 12.33—*l.* 3.24.

QUESTION VIII.

“ Let the scheme of a society for granting
 “ annuities to widows, be, that, if a member
 “ lives *a year* after admission, his widow shall
 “ be entitled to a life annuity of 20*l.* If
 “ *seven years*, to 10*l.* more, or 30*l.* in the
 “ whole. If *fifteen years*; to another addi-
 “ tional 10*l.* or 40*l.* in the whole. What
 “ ought to be the annual payments of the
 “ members for the ages of 30, 40, and 50,
 “ supposing them of the same ages with their
 “ wives, and allowing compound interest at
 “ 4 per cent ?”

ANSWER.

According to the *hypothesis*, explained p.
 2; and, therefore, very nearly, according to
 the Tables of observation for *Breslaw, Nor-*
wich, and Northampton,

l. 8.44—*l.* 8.69—*l.* 9.05.

According to the *London* observations,

l. 9.41—*l.* 10.17—*l.* 10.92.

These

These values are easily deduced from the values in the last question. For example, The value of 10*l.* *per annum* for life to 40 after 40, provided the joint lives do not fail in *one* year, is, according to the *hypothesis*, *l.* 30.33. The value of 20*l.* *per annum*, in the same circumstances, is, therefore, *l.* 60.66.— In like manner, the value of 10*l.* after *seven* years, is *l.* 17.44. And of 10*l.* after 15 years *l.* 7.3.—These values together make *l.* 85.4, or the value of the expectation, described in this question, in a *single present payment*; which, divided by 9.82, (the value by Table VII. of two joint lives at 40) gives *l.* 8.69, the value of the same expectation in *annual payments*, during the joint lives.—In the same manner may be found the answer in all cases to any questions of this kind.

These calculations suppose, that the annual payments do not begin till the end of a year. If they are to begin *immediately*, the true *annual payments* will be, as was before observed, the *single payments*, divided by the value of the joint lives increased by unity; and in the present case they will be, by the *hypothesis*,

$$l. 7.75—l. 7.9—l. 8.07.$$

By the *London* observations,

$$l. 8.52—l. 9.06—l. 9.51.$$

By

By the method of calculation now explained, may be easily found in all cases, supposing the annual payments previously settled, what the reversionary annuities are corresponding to them in value.—Thus, the annuities being the same with those mentioned in this question, the *mean* annual payments for all ages between 30 and 50, are nearly 8*l.* according to the *highest* probabilities of life; 9*l.* according to the *lowest*; and 8 guineas the *medium* (a); interest being at 4 *per cent.* and the first payment to be made immediately.

If the mean annual payments, beginning immediately, are fixed to five guineas, the corresponding life annuities will be nearly (by the *hypothesis*) 12*l.* if the contributor lives a year, and 24*l.* if he lives seven years; or (by the *London* observations) 12*l.* if he lives a year, and 20*l.* if he lives seven years (b).

It

(a) The value of this expectation, supposing married men 40 years of age, and their wives 30, is, in a *single* payment, 113*l.* In annual payments beginning immediately 1.9.88, by the *hypothesis*. And 107*l.*—and 1.10.93, by the *London* observations.

(b) If the annuities in expectation are 14*l.* provided a member lives a year, and 20*l.* provided he lives seven years, the proper *mean single* payments for all ages, taken one with another, under 50 or 52, is 50 guineas nearly, according to all the Tables of observation, supposing equality of age between men and their wives. And the addition which ought to be made, on account of excess of age on the man's side is, taking the nearest and the easiest

It is observable, that the difference in the values of the annuities, arising from difference of ages, and the difference in the probabilities of life, is less in this question than in question 4th; and that, consequently, the plan proposed in it, is the safest, as well as the most equitable and encouraging, that a society can adopt.

It is necessary to remark here further, that *yearly* payments which begin immediately, are more advantageous than *half-yearly* payments which begin immediately. Mr. Simpson (in his Treatise on *The Doctrine of Annuities and Reversions*, pag. 78, and also in his *Select Exercises*, p. 283) has shewn, that, in the case of life annuities, *half-yearly* payments, which begin at the end of half a year, are $\frac{1}{4}$ of a year's purchase better than *yearly* payments, which begin at the end of a year. And it is manifest, that *half-yearly* payments, which begin immediately, are no

easiest round sums, about a guinea and $\frac{1}{2}$ for every year as far as 17 years; or, in the annual payments, (supposed 5 guineas) $\frac{1}{2}$ a guinea *per annum* for five years excess, and $\frac{1}{2}$ a guinea more for every four years excess beyond five years, till the excess comes to be 17 years. And, I believe, that 60 guineas in *single payments*, and six guineas in *annual payments* beginning immediately, may very well be stated as the *lowest common* payments proper to be required, supposing all married men under 52, taken into a society, without enquiring into the difference of age between them and their wives, the annuities being all along supposed to be *life annuities*, and interest reckoned at 4 *per cent.*

more than half a year's purchase better than those which begin at the end of half a year. But *yearly* payments, which begin immediately, are a *whole year's* purchase better than the same payments to begin at the end of a year. The difference of value, therefore, between *yearly* and *half-yearly* payments, supposing both to begin immediately, is a quarter of a year's purchase in favour of the former.

QUESTION IX.

“ The value is required of an annuity to
 “ be enjoyed for what may happen to re-
 “ main of one life after another, provided
 “ the life in expectation continues a given
 “ time ? ”

ANSWER.

Find by Question VI. the present value of the annuity for the remainder of the life in expectation, after the given time, and multiply this value by the probability, that the other life shall fail within that time. Find also, by Question VII. the value of the reversion, provided *both* lives continue the given time. Add these values to one another, and the *sum* will be the answer in a single present payment. /

EXAM-

EXAMPLE.

An annuity of 10*l.* for the life of a person now 30, is to commence at the end of 11 years (*a*), if another person now 40, should be then dead; or, if this should not happen, at the end of any year beyond 11 years in which the former shall happen to survive the latter. What is the present value of such an annuity, reckoning interest at 4 *per cent.* and taking the probabilities of life as they are in *Dr. Halley's Table*?

The value of 10*l. per annum*, for the remainder of the life of a person now 30, after 11 years, found by *Quest. VI.* is *l.* 69.43.—The probability that a person 40 years of age shall live 11 years, is, by *Dr. Halley's Table*, $\frac{335}{445}$. The probability, therefore, that he will die in 11 years, is $\frac{335}{445}$ subtracted from unity (*b*), or $\frac{110}{445}$; which multiplied by *l.* 69.43; gives *l.* 17.16.—The value of the reversion; provided *both* live 11 years, found by *Quest. VII.* is 17*l.* And this value added to the

(*a*) That is, the title to the annuity is to commence at the end of 11 years, and the first payment to be made a year afterwards, in case the life in expectation should continue so long, and the other fail. But if *both* lives should continue the given term, the first payment is always to be made at the end of the year, in which the former life shall happen to survive the latter. See *Quest. VI.*

(*b*) See the Note, p. 23.

former,

former, makes $l. 34.16$, the value required in a *single present payment*; which payment divided by $l. 11.43$, (the value by Table VII. of two joint lives, aged 30 and 40, with unity added) gives $3l$; (a) or the value required in annual payments during the joint lives, the first payment to be made immediately.—If, every thing else being the same, the assigned term is 15 years, the value required will be $29l.$ in a *single payment*, and $l. 2.55$ in *annual payments*.

QUESTION X.

“ What money in hand, and also in annual payments during life, ought a person of an assigned age to give for a sum of money, payable at his death to his heirs (b)?— In other words, what money in hand, and in annual payments during life, ought a person of a given age to pay for an *assurance* of any given sum on his life?”

ANSWER.

Subtract the value of the life from the *perpetuity*. Multiply the remainder by the

(a) See the demonstration of this rule in Note (D) Appendix.

(b) This question is the same with Problem 16th, in Mr. *De Moivre's* Treatise on Annuities, and Problem 26th, in Mr. *Simpson's* Select Exercises; but the answers there given are right only when applied to reversionary *estates*, and therefore must be materially wrong, when applied to reversionary *sums*, as will appear from the *Scholium* to this Question, and from note (E) in the Appendix.

product

product of the given sum into the interest of 100*l.* for a year: and this last product, divided by 100*l.* increased by its interest for a year, will give the answer in a *single present* payment. And this payment, divided by the value of the life, will give the answer in *annual* payments, during the continuance of the life.

Example. Let the life be 30. The sum 100*l.* The rate of interest 4 *per cent.* And the valuation of lives, that in Table VI. The perpetuity, therefore (a), is 25. The interest of 100*l.* for a year, is 4*l.* 100*l.* increased by its interest for a year is 104*l.* And the value of the life 14.68.—The value of the life, subtracted from the perpetuity, gives 10.32, which, multiplied by the product of 100*l.* into 4, or by 400, gives 4128. And this, divided by 104, gives *l.* 39.7, the value of 100*l.* payable at the death of a person aged 30, in a single present payment.—And this payment, divided by 14.68, is *l.* 2.7, the same value in annual payments during the continuance of the life.

These values found in the same way, agreeably to the valuation of lives for *London*, in Table X, are *l.* 45.76, and *l.* 3.49.—If the life is 36, and interest 4 *per cent.* these values are 43*l.* and *l.* 3.1, by Table VI, and *l.* 49.6,

(a) That is; the value of the *fee simple* of an estate found by dividing 100*l.* by the rate of interest.

and

and *l.* 4.1, by Table X.—If interest is reckoned at 3 *per cent.* the same values are, by Table VI, for 30 years of age, *l.* 48.14.—2.86.—For 36 years of age, *l.* 51.43, and *l.* 3.28.

It appears here, that difference of interest makes no considerable difference in the answers to questions of this kind, except when the values are required in a single payment.

If the first of the annual payments is to be made immediately, the single payment is to be divided by the value of the life, with unity added to it, agreeably to what has been already observed; and the annual payments in this case (interest supposed at 4 *per cent.*) will be by Table VI, for a life at 30, *l.* 2.53—At 36, *l.* 2.9.

If the payments are half yearly payments beginning immediately, the single payment must be divided by the value of the life increased by $\frac{1}{2}$, or .75, (see Quest. VIII.) And the half yearly payments, for the age of 36, will be half 2.9, or 1.45. And half 1.45, or .725, is likewise nearly the proper quarterly payments.

Again; if an annual payment, beginning immediately, of *l.* 2.9, ought (reckoning interest at 4 *per cent.*) to purchase 100*l.* payable at the failure of a life now 36; 5*l.* by the rule of proportion, ought to purchase 172*l.* And in like manner, it may be found, that the same annual contribution, in half-

D

yearly

yearly or quarterly payments, beginning immediately, ought to purchase 170*l.*— These sums, according to the *London* observations, are 132*l.* and 130*l.* nearly.

The reason of mentioning these particulars will be seen in the next chapter.

SCHOLIUM.

If the reversion is not a *sum*, but an annuity for ever, or an *estate in fee simple*, to be entered upon after a given life, its present value, *in a single payment*, will be “ the value
“ of the life subtracted from the perpetuity,
“ and the remainder multiplied by the an-
“ nuity, or the annual rent of the estate.”—
And the value, in *annual payments*, will be, as before, the single payment divided by the value of the life.—Universally. It ought to be remembered, that a reversionary *estate*, after any given life or lives, is worth as much more than a corresponding reversionary *sum*, as 100*l.* increased by its interest for a year, is greater than 100*l.*—Thus, the present values, in single and annual payments, of 4*l.* *per annum* for ever, and of 100*l.* in money after any assigned life, are to one another, (interest being at 4 *per cent.*) as 104 to 100, or 1.04 to 1.—The reason of this difference is, that the calculations suppose, that the reversionary *sum*, and the first yearly rent of the *estate*, or first payment of the annuity,
are

are to be received at the same time, after the extinction of the lives in possession. It is easy to see, that this is a circumstance which must make the latter of most value. But to prevent any doubts about it, I shall explain it more particularly in a note in the Appendix (a).

QUESTION XI.

“ A person of a given age, having an year-
 “ ly income which will fail with his life,
 “ wants to make provision for another per-
 “ son of a given age, in case the latter should
 “ happen to survive. What ought the for-
 “ mer to give in a single payment, and also
 “ in annual payments during their joint lives,
 “ for a given sum, payable at his death to
 “ the latter ?”

It is manifest, that the value of the given sum in this case, must be less than in the case stated in the last Question ; because, here the payment of it is suspended on the contingency, that one life shall survive another, whereas in the other case, it is *certainly* to be paid at the failure of a given life.

ANSWER.

Find, by the solution of problem 32d, p. 297, Mr. *Simpson's* Select Exercises, the

(a) Vid. Appendix note (E).

value of an estate, corresponding to the given sum, and depending on the given survivorship. Divide this value by 1% increased by its interest for a year, and the quotient will be the value of the given sum in a single present payment. And the single payment, divided by the value of the given joint lives, will be the answer in annual payments during the joint lives.

The solution I have referred to is as follows.

“ Find the value of an annuity on two equal joint lives, whereof the common age is equal to the age of the older of the two proposed lives; which value, subtract from the perpetuity, and take half the remainder. Then say, as the *expectation* of the duration of the younger of the two lives is to that of the elder, so is the said half remainder to a 4th proportional, which will be the number of years purchase to be given for the estate when the life in expectation is the oldest of the two. But if this life is the youngest, then add the number of years purchase just found to the value of the joint lives, and let the sum be subtracted from the perpetuity, and you will also have the answer in this case (a).”

Let

(a) Mr. *Simpson* has given the following examples of this solution, adapted to *London* lives.—Example I.
 “ Suppose the age of the *expectant* to be 40; of the possessor 30. The rate of interest 4 per cent. and the
 “ given

Let the life in expectation be 30; and the other life 40: The sum, 100*l.* Interest, 4 per cent. The valuation of lives, that in Table VI.

The *expectation* of the first life, is 28; of the second life 23, by Mr. *De Moivre's hypothesis*. The value of the joint lives is 10.43,

“ given legacy 5000*l.* or 200*l.* per annum. Then the value of two equal joint lives of 40, being 8.1, by Table XI, and the perpetuity 25, the remainder or difference will be here 16.9; whereof the half is 8.45. Therefore, it will be as 23.6 to 19.6, so 8.45 to 7.02 years purchase, or *l.* 1404, the required value.”

Example II. “ Let the age of the *expectant* be 30, of the *possessor* 40, and the rest as in the preceding example. Here the value of the joint lives 30 and 40, will be 8.8; which added to 7.02, (found above) the sum will be 15.82; whence the answer, in this case, is 9.18 years purchase, or 1836.”

I have shewn, that the values of reversionary *estates*, and reversionary *sums*, are not the same as is here supposed.—The rule gives the true value when applied to the former; but, when applied to the latter, the values given by it must be divided by 1*l.* increased by its interest for a year, as above directed.—The same observation is to be applied to Mr. *Simpson's* next Problem, or the 33^d.

In these Examples, 23.6 and 19.6, are the expectations, in Table IX, of 30 and 40, according to the *London Tables of Observation*; and the method of finding them for any age, and from any Tables of observation, is explained at the beginning of the first Essay.

In Mr. *De Moivre's hypothesis*, the expectation of a life, is always *half* the complement. See note p. 2.—Sometimes the *complement* of a life is mentioned without any view to Mr. *De Moivre's hypothesis*, and it then means double the *expectation* of the life whatever that may be according to any Table of observations.

by Table VII. The value of two joint lives, both 40, is 9.82, by the same Table. The estate corresponding to 100*l.* is 4*l.* *per ann.* and the present value of such an estate to be entered upon by a person 30 years of age, provided he survives a person 40 years of age is, by the rule just quoted, 1.33.32. And this value, divided by 1*l.* increased by its interest for a year, or by 1.04, is 1.32.03. the value in a *single present payment* of the sum of 100*l.* dependent on the given survivorship. And this single payment, divided by 10.43, is 1.3.07, the required value in *annual payments*, during the joint lives, if the first payment is not to be made till the end of a year. But if the first payment is to be made immediately, the required value in *annual payments* will be 1.32.03, divided by 11.43, or 1.2.8.—These values, according to the *London* observations, or Mr. *Simpson's* Tables founded upon them, are 1.35.30, in a *single payment*, and 1.3.6, in *annual payments*, beginning immediately.

Mr. *Simpson*, in the Problems following that here quoted, has given solutions of most other Questions, concerning the values of reversions depending on survivorships, where the whole duration of two or three lives is concerned. And I am acquainted with no other solutions of these Questions, which are applicable to all Tables of observations, and which at the same time (proper regard being

paid to the correction explained in the last Question) may be considered as sufficiently correct (a).

QUESTION XII.

“ Suppose an institution for the relief of
 “ widows to extend its assistance likewise
 “ to the families of married men, provided
 “ they leave no widows. Suppose, for in-
 “ stance, that in this case children are to be
 “ entitled to 100*l.* What is such an expect-
 “ tation worth, in present payment, accord-
 “ ing to Dr. *Halley's* Table, interest being at
 “ 4 per cent ?”

ANSWER.

If 40 is the mean age at which members are admitted on such an institution, and 32 the mean age of their wives, the answer (supposing no subsequent marriages) is, by the 33d Problem in Mr. *Simpson's* Select Exercises, p. 298, and the correction already explained, *l.* 13.80 (b).

But

(a) See the third Essay.

(b) This Problem and its solution are given by Mr. *Simpson* in the following words. “ A and his heirs are entitled to an estate of a given value, upon the decease of B, provided B survives A ; to find the value of their expectation in *present* money.”—Solution. “ Find the value of an annuity on the longest of two equal
 D 4 “ lives,

But there is a reduction necessary, on account of the chance there is, that a widower may marry again. Suppose, therefore, one half of all widowers to marry a second and third time, and that two fifths of such widowers survive these subsequent marriages. In this case, $\frac{1}{2}$ added to $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{7}{5}$ of all who become widowers, will die without leaving widows, and therefore $\frac{3}{5}$ of $l. 13.8$, or $l. 9.66$, will be the answer. If only one *fourth* of all who become widowers marry again, and two fifths of these survive, the answer will be $l. 11.73$.

“ lives, whereof the common age is that of the older of
 “ the lives A and B; which value subtract from the
 “ perpetuity, and take half the remainder; then it will
 “ be as the expectation of duration of the younger of
 “ the lives A and B, is to that of the older, so is the
 “ said half remainder to the number of years purchase
 “ required, when the life B is the older of the two. But
 “ if B be the younger; then to the number thus found,
 “ add the value of an annuity on the longest of the lives
 “ A and B, and subtract the sum from the perpetuity, for
 “ the answer in this case.”

If the estate is $4l.$ *per annum*, the age of B 40, and of A 32, interest 4 *per cent.* the answer by this rule comes out $l. 14.35$, which divided (as in the preceding question) by 104, gives $l. 13.80$, the value, as above, of 100*l.* in money. If B is 30 and A 40, the same value is 20*l.*

N. B. The value of the longest of two lives is always the *difference* between the value of the *joint* lives, and the *sum* of the values of the two given *single* lives. Thus; the value of a life at 40, is, by Table VI, 13.2. The *sum* of the values of two such lives, is 26.4. The value of two *joint* lives, whose common age is 40, is, by Table VII, 9.82; and the difference is 16.58, or the value of the *longest* of two lives at 40.

This

This calculation supposes all marriages to leave children who survive their parents. If this is considered as uncertain, the values now determined must be diminished in the proportion of this uncertainty.—Thus; if one marriage in seven fails of leaving children (*a*) that survive their parents; these values will be reduced a *seventh* part, or to *l. 8.28*, if *half*, and *l. 10.05*, if a *quarter* of all widowers marry.

In this way may any other questions of the same kind be answered on any suppositions that may be thought most reasonable.

QUESTION XIII.

“ Let an establishment be supposed which
 “ takes in at once all the marriages in a
 “ country, or all marriages among persons
 “ of a particular profession within a given
 “ district, and subjects them for perpetuity
 “ to a certain equal and common tax, or an-
 “ nual payments, in order to provide life an-
 “ nuities for such widows as shall result from
 “ these marriages. What ought the tax to
 “ be, supposing the annuity *20l.* and calcu-
 “ lating at *4 per cent.* from Mr. *De Moivre’s*
 “ valuation of lives; or, which is nearly the
 “ same, from the probabilities of life in Dr.
 “ *Halley’s Table of observations?*”

(*a*) This for many years has been nearly the fact among the ministers and professors in *Scotland*.

ANSWER.

ANSWER.

Since at the commencement of such an establishment, all the oldest, as well as the youngest marriages, are to be entitled equally to the proposed benefit, a much greater number of annuitants will come immediately upon it, than would come upon any similar establishment, which limited itself in the admission of members to persons not exceeding a given age. This will check that accumulation of money, which should take place at first, in order to produce an income equal to the disbursements at the time when the number of annuitants comes to a *maximum*; and, therefore, will be a particular burden upon the establishment in its infancy. For this, some compensation must be provided; and the equitable method of providing it, is, by levying *finés* at the beginning of the establishment, on every member *exceeding* a given age, proportioned to the number of years which he has lived beyond that age. But in the present question, it is supposed, that such *finés* cannot be conveniently levied, or that every payment must be equal and common, whatever disparity there may be in the value of the expectations of different members. The *finés*, therefore, must be reduced to one common one, answering as nearly as possible to the disadvantage I have mentioned, and payable

payable by every member at the time when the establishment begins. After this, the establishment will be the same with one that takes upon it all at the time they marry; and the tax or annual payment of every member adequate to its support, will be the annual payment during marriage, due from persons who marry at the mean age at which, upon an average, all marriages may be considered as commencing.—There are then two points to be here determined. The *fines* necessary to be paid at first, according to the account I have just given; and the *constant annual payment*, necessary to be made by every member, as an equivalent for the expectation provided by the establishment.—The *fines* to be paid at first are, for every particular member, the same with the difference between the value of the expectation to him at his present age, and what would have been its value to him had the scheme begun at the time he married? Or, they are, for the whole body of members, the difference between the value of the common expectation, to persons at the mean age of all married persons taken together as they exist in the world, and to persons at that age, which is to be deemed their mean age when they marry.

Thus; let 33 for the man, and 25 for the woman, be the mean ages of all that marry annually. Let also 48 be the mean age of all the married men in the world, and 40 of married

married women (a).—Now, he that will calculate for these ages, in the manner directed in Quest. IV. will find, that the value in *annual payments* during marriage, and beginning immediately, of the expectation of an annuity of 20*l. per annum*, by a person 25 years of age, after a life whose age is 33, is *l. 6.64*.—And that *l. 8.04*, is the value of the same expectation, the ages being 48 and 40.

The former, therefore, is the payment for perpetuity from every member of the establishment; and the value of the *difference* between it and the latter, or of *l. 1.4 per ann.* payable during two joint lives, whose ages are 40 and 48, that is, *l. 14.2*, is the fine necessary to be levied on every married member at the beginning of the establishment (b).

It would be easy to extend the benefit of such an establishment, so far as to provide 100*l.* for the children of members, provided

(a) I must beg leave to refer to note (F) in the Appendix, for an explanation of what I mean by the mean ages of married men and women, and also for a confirmation of the answer I have given to this question.

(b) An annuity for ever, the first payment of which is to be made immediately, is worth 26 years purchase, interest being at 4 *per cent.* *l. 14.2* therefore, is equivalent in value to 0.55*l.* or 11*s. per annum*, for ever. Add this to *l. 6.64*, and it will appear, that *l. 7.19 per annum*, beginning immediately, is the answer to this question, supposing the value of the *fine* to be provided for in the perpetual annual payments.

they

they leave no widows; and the necessary addition on this account to the perpetual annual payments, can scarcely, in the circumstances this question supposes, be much more than about 15*s.* payable during life, and excluding from all benefit such as happen to be widowers at the commencement of the establishment, and do not afterwards marry.

If, in such an establishment, all persons of a particular denomination, whether married men, widowers, or batchelors, are subjected alike to the taxes and fines; they ought to be as much *less*, as the whole number of persons subjected to them, is *greater* than the number of marriages constantly existing.

In carrying these schemes into execution, there cannot be a more easy, or equitable way of raising the necessary fines, than by providing, that none shall be entitled to any expectation for a few of the first years. Thus; an establishment, entitling widows to 20*l.* *per annum* for life, and consisting of 667 married members, and 344 unmarried, always kept up at an average, ought to begin with a capital of *l.* 14.2 multiplied by 667, or 9471*l.* besides one payment in hand of the constant annual payments. That is, (the proper annual payment of every member being in this case $\frac{20}{1000}$, multiplied by *l.* 6.64, or *l.* 4.38) it ought to begin with a capital
of

of 13,899*l.* over and above the payment of 4.38, at the *end* of every year for ever afterwards (a). — The exclusion of all the first members from any benefit, unless they survive the first *two* years, or live to make *three* payments, would raise this capital nearly. And such an exclusion for *three* or *four* years, would be an advantage so considerable, that it would probably give security and stability to the scheme for all subsequent time.

In these observations, I have had in view, several schemes of the kind described in it, which are now actually established in this kingdom; but more particularly, one begun among the *London* and *Middlesex* clergy, and another which is established by act of parliament, among the clergy in *Scotland*, of both which, I shall have occasion in the next chapter to take further notice.

I have chosen to calculate here only from Dr. *Halley's* Table, or Mr. *De Moivre's hypothesis* grounded upon it, because the *London* Table is, by no means, adapted to the cases in view.

The difference of eight years between the ages of men and their wives, as here taken, is probably too little; and for this reason,

(a) Or, supposing the value of 9471 *l.* (the fine) provided for in the annual payments, it ought to receive every year, at the *beginning* of the year, a contribution from each member of 4.74.

and also on account of the greater mortality of males, the values I have given should be considered as the lowest that any scheme ought to provide.

It should be further remembered, that when the mean ages, at which marriages commence, are supposed to be 33 and 25, all second and third marriages are included; and that it is to be expected, that almost all these marriages will begin after these ages; and likewise, that a considerable proportion of the first marriages will begin a much longer time *after* these mean ages, than any of the other first marriages will begin before them.—Probably, therefore, these mean ages should not be taken younger. One or two years, however, more or less, in every supposition I have made, will make no difference of any consequence.

QUESTION XIV.

“ A person of a given age has an estate depending on the continuance of his life for a given term. What ought he to give for having it *assured* to him for that term ? ”

ANSWER.

From the value of an annuity certain for the given term, found by Table II, subtract the value of the life for the given term,
found

found by Quest. VI. and *reserve* the remainder.—Multiply the value of 1*l.* due at the end of the given term, (found by Table I.) by the *perpetuity*, and also by the *probability*, that the given life shall fail in the given term. The *product* added to the *reserved* remainder, and the *sum* multiplied by the given annuity, will be the required value of the assurance in one present payment (a).

E X A M P L E.

An estate or annuity of 10*l.* *for ever*, will be lost to the heirs of a person now 34, should his life fail in 11 years. What ought he to give for the *assurance* of it for this term?—That is; what is the present value of such an annuity to be entered upon at the failure of such a life, should that happen in 11 years?

The value of the life of a person whose age is 34 for 11 years, is, by Quest. VI. (reckoning interest at 4 *per cent.* and calculating from Dr. *Halley's* Table of observations) 7.76; which, subtracted from 8.760, (the value of an annuity certain for 11 years) leaves 1*l.* the *remainder* to be reserved.

The value of 1*l.* to be received at the end of 11 years, is, 0.6496, by Table I. The probability that the life of a person, aged 34,

(a) See the demonstration in note (G) Appendix.

shall fail in 11 years, is, by Dr. *Halley's* Table, $\frac{1}{4} \frac{03}{99}$; and the perpetuity is 25. These numbers, multiplied by one another, and 1 added to the product, make 4.34, which, multiplied by 10, (the given annuity) gives *l.* 43.4, the required value in a single present payment.

l. 43.4, divided by 1.04, gives *l.* 41.7, the true value, by Scholium to Quest. X. of the assurance of an *equivalent* sum, or of 25*l.* for 11 years on the given life.

Again. 41.7, divided by 8.76, (the value of the given life for the given time with unity added to it) gives 4.76, the same value in annual payments beginning immediately, for 11 years (*a*), subject to failure should the life fail.

SCHOLIUM.

In a similar way may the price of assurances on any two joint lives; or the *longest* of two lives for any given terms, be calculated; the rule being as follows:

“ From the value of an annuity certain
 “ for the *given term*, subtract the value of
 “ the joint lives, or the longest of the two
 “ lives for the *given term*, found by Scholium to Quest. VI. and *reserve* the remainder.—Multiply the value of 1*l.* to be re-

(*a*) The last payment to be made at the end of the 11th year; or 12 payments in all.

“ ceived at the end of the given term by the
 “ perpetuity, and also by the probability
 “ that the *joint lives*, or the *longest of the two*
 “ *lives*, shall fail within the given term. This
 “ product added to the reserved *remainder*,
 “ and the *sum* multiplied by the annuity to be
 “ assured, will be the value of the assurance
 “ in a single present payment.”

EXAMPLE.

“ What is the value of 10*l.* *per annum*, to
 “ be entered upon, should *either* of two
 “ persons, one 40 and the other 30 years of
 “ age, die in ten years, reckoning interest
 “ at 4 *per cent.* and calculating from Dr.
 “ *Halley's Table.*”

The value of two joint lives at these ages, for 10 years, (found by *Scholium* to *Quest. VI.*) is, 6.51; which, subtracted from 8.111, (the value of an annuity certain for 10 years, at 4 *per cent.*) leaves 1.60, the remainder to be reserved.

The value of 1*l.* to be received at the end of 10 years, is, .6755, by Table I.

The probability, that the lives of one or other of two persons, aged 30 and 40, shall fail in 10 years, is, $\frac{4}{5}\frac{8}{3}\frac{5}{1}$, by Table III (a).

(a) The probability taken from the Table, that a person aged 30, shall live 10 years, is, $\frac{4}{5}\frac{4}{3}\frac{5}{1}$. That a person, aged 40, shall live 10 years, is, $\frac{3}{4}\frac{4}{1}\frac{6}{5}$. That they shall both live 10 years, is, $\frac{3}{4}\frac{4}{1}\frac{6}{5}$, multiplied by $\frac{4}{5}\frac{4}{3}\frac{5}{1}$, or $\frac{3}{5}\frac{4}{1}\frac{6}{1}$. That they shall *not both* live 10 years, or that *one or other* of them shall die in this time, is, $\frac{3}{5}\frac{4}{3}\frac{6}{1}$, subtracted from unity, or, $\frac{4}{5}\frac{8}{3}\frac{5}{1}$. See note p. 23.

And

And the perpetuity 25. These numbers, multiplied by one another, and 1.60 added to the product, make 7.48, which, multiplied by 10, (the given annuity) gives $l. 74.8$, the answer in a single present payment.

$l. 74.8$, divided by 1.04, gives $l. 71.92$, the value of the assurance of an *equivalent sum*; or of 250*l.*— $l. 71.92$, divided by 7.51, (the value of the two joint lives for 10 years with unity added) gives 9.57, the value of the same sum in annual payments beginning immediately, for 10 years, subject to failure should the joint lives fail.

EXAMPLE II.

“What is the value of 10*l. per annum*, to be entered upon, should two persons one 30, and the other 40, *both* die; that is, should the *longest* of the two lives fail in 10 years, reckoning interest at 4 *per cent.* and calculating from Dr. *Halley's* Table?”

The value of the *longest* of the two lives for 10 years, (that is, the value of the joint lives for 10 years, subtracted from the sum of the (a) values of the single lives for 10 years) is, 7.91; which, subtracted from 8.111, the value of an annuity certain for 10 years, leaves .20 the remainder to be reserved.—The value of 1*l.* to be received at the end

(a) See Scholium to Quest. VI.

of 10 years, is, .6755. The probability that the lives of two persons, aged 30 and 40, shall fail in 10 years, is, by Table III, $\frac{86}{531}$, multiplied by $\frac{99}{443}$, or $\frac{8514}{236295}$; and the perpetuity 25. These numbers, multiplied by one another, and .20 added to the product, make .740, which, multiplied by 10, (the given annuity) gives 7.4, the answer in a single payment.

7.4, divided by 1.04, gives 7.115, the value of the assurance of 250*l*.

R E M A R K I.

The values of single lives for given terms, when these terms are less than 10 years, must, in answering these Questions, and also in answering the following Questions, be found true to at least 2 or 3 places of decimals. But they cannot be found to this exactness by any Tables that are extant; and, therefore, they must be calculated in the following manner:

“ Multiply the probability, taken out of
 “ the Table of observations, that the life
 “ shall exist 1, 2, 3, &c. years, by the value
 “ of 1*l*. due at the end of 1, 2, 3, &c. years;
 “ and the sum of the products will be the
 “ value of the life for 1, 2, 3, &c. years.”

For Example. The probability, that a person whose age is 34, shall live a year, is,
 by

by Dr. Halley's Table, $\frac{490}{499}$. The probability, at the same age, of living 2 years, is, $\frac{481}{499}$; 3 years, $\frac{472}{499}$.— $\frac{490}{499}$ multiplied by .9615, (the value, by Table I, of 1*l.* due at the end of a year, interest being at 4 *per cent.*) is, .942; or the value of the life for *one* year.— $\frac{481}{499}$, multiplied by .9245, (the value of 1*l.* due at the end of 2 years) is, .891. And this added to the former product, gives 1.833; or the value of the life for 2 years.— $\frac{472}{499}$, multiplied by .8890, (the value of 1*l.* due at the end of 3 years) is, .841; and this product, added to 1.833, makes 2.674, or the value of the given life for 3 years.

When the term exceeds 10 years, the rule in Quest. VI. will give these values with sufficient exactness; and it would do the same in all cases, were the values of lives given true to 3 or 4 places of decimals, and in strict agreement to the Tables of observation used.

The remark now made is to be extended to the values of *joint* lives for given terms. For these values, like those of *single* lives, cannot be found in solving these Questions with sufficient accuracy, when the terms are small, by any method, except the tedious one, of multiplying the probability that the 2 lives shall *both* continue 1, 2, 3, &c. years, by the value of 1*l.* due at the end of 1, 2, 3, &c. years, and taking the sum of the products in the manner just described.

REMARK II.

If the annuity is to be entered upon, in case of the failure within a given time of any life or lives, *at the end of that time*; and not *at the end of the year in which the failure may happen*; its present value will be the product arising from the continual multiplication by one another of the perpetuity increased by unity; the value of 1*l.* due at the end of the given time; the annuity; and the probability that the life, or lives, shall fail within the given time. And care should be taken not to confound these two sorts of Questions with one another.—Thus; the value in one payment of 10*l. per ann.* to be entered upon eleven years hence, in case a person aged 34 should not live so long, is 26, (the perpetuity increased by unity, interest being at 4 *per cent.*) multiplied by .6496, and by 10*l.* and also by $\frac{1.03}{1.04}$; or 34.8.—This value, divided by 1.04, is, 33.5; the value of an equivalent sum, or of 250*l.* to be obtained on the same conditions.

The value of the *assurance* of any annuity on the whole continuance of any single life is, by Quest. X. the *excess* of the perpetuity above the value of the life, multiplied by the annuity. And in like manner; the value of the *assurance* of any annuity on the whole continuance of any two *joint* lives, or the *longest* of two lives, is the excess of the per-
petuity

petuity above the value of the joint lives, or of the longest of two lives, multiplied by the annuity. This is very obvious; but no general method has been yet explained of finding the values of *assurances* on lives and survivorships for terms of years less than the whole continuance of the lives. For this reason, I have been here more explicit than I should otherwise have been; and, as such assurances are now much practised, and may be very useful if their values are rightly determined, I have thought proper to add the two following Questions, which, when joined to Question XI. and Mr. *Simpson's* 33d *Problem* given in the note p. 39, will, I believe, exhaust this subject as far as two lives can be concerned.

QUESTION XV.

“ B, expectant, will lose a given sum,
 “ should he survive A, *within a given time*.
 “ What ought he to pay for the *assurance* of
 “ it?—In other words: “ What ought he to
 “ pay for a given sum to be received at the
 “ death of A, should he happen to survive
 “ him within a given time?”

ANSWER.

Divide the *sum* of the decrements of life in the Table of observations from the age of A, for the given time, by the given time; and, by the *quotient*, divide the number of

the living in the Table at the age of A ; and again, by this *second* quotient (*a*), divide the given sum reserving the *third* quotient.

Find the value of an annuity on the life of B, for the given time. To this value add the *quotient*, that will arise from dividing the value of an annuity certain, for the given time, by twice the *complement* of the life of B ; and the *sum*, multiplied by the *reserved quotient*, will be the required value in a single present (*b*) payment.

EXAMPLE.

Let the Table of observations be Mr. *Simpson's* for *London*, or Table VIII. Let the rate of interest be 3 *per cent*. A, seven years of age. B, 30. The given time 14 years. The given sum 100*l.*—The sum of the *decrements*, in Table VIII. for 14 years from the age of seven, is 73, which, divided by 14, gives 5.2. The number of the living at seven is 430, which, divided by 5.2, and 100*l.* divided by the quotient, gives *l.* 1.21, the *quotient* to be *reserved*.

(*a*) When the age of A is under 60, and the term so large as to exceed the difference between it and 70, it will be best, when the *London* Table is used, to divide the given sum, not by the second quotient here mentioned, but by the *complement* of the life of A, taken out of Table IX.

(*b*) See the demonstration of this rule, and also of the rule that will be given for solving the next Question, in the Appendix, note (H).

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The value of an annuity for 14 years on the life of B, is, by Quest. VI. 9.5.—The value of an annuity certain for 14 years, is, by Table II. 11.296, which, divided by 94.4, (twice the *complement* of the life of B, by Table IX) (*a*), gives .12, which, added to 9.5, gives 9.62; and this again multiplied by 1.21, the *reserved quotient*, gives 11.64, the *present* value in *one* payment of 100*l.* payable at the death of A aged 37, to B aged 30, should A die and leave B the survivor within 14 years.

The present value for 14 years of two joint lives, one 7 and the other 30 years of age, may be found, by the help of Table XI, and the rule in the *Scholium* to Quest. VI. to be nearly 9 years purchase; and, 1.11.64 divided by this value with unity added, or by 10, gives 1.164, the foregoing value in *annual payments* during the joint lives for 14 years, the *first* payment to be made immediately, and the *last* payment at the end of 14 years, should the joint lives not fail.

SCHOLIUM.

It deserves particularly to be remembered, that in this method likewise may be calculated, what sums ought to be paid on any survivorship, within a given time, of one life

(*a*) This Table gives the *expectations* only, but it should be remembered, that twice the *expectation* is always the *complement* of a life. See note, p. 37.

beyond

beyond another, in consideration of any given sum now advanced.—The following Example of this is a case which has offered itself in practice.

“ A person, aged 30, has in expectation
 “ an estate which is to come to him, pro-
 “ vided he survives a *minor*, aged 7, before
 “ he is out of his minority; that is, pro-
 “ vided he should be himself living at the
 “ time of the *minor*'s death, should that hap-
 “ pen before he is 21.—In these circum-
 “ stances, he wants to borrow 1000*l.* on his
 “ *expectation*. What *reversion* out of the
 “ estate depending on such a survivorship, is
 “ a proper equivalent for this sum now ad-
 “ vanced, interest being reckoned at 3 *per*
 “ *cent.* and the probabilities of life being
 “ supposed the same with those in Mr. *Simp-*
 “ *son*'s Table of *London* observations?”

A N S W E R.

It appears from what has been just determined, that for *l.* 11.64 now advanced, the proper equivalent in such circumstances, is, 100*l.* to be paid, in case the survivorship should take place; or, by the *correction* in page 34, as much of the estate as 100*l.* will buy at 3 *per cent.* supposing the first rent to be received immediately; (that is, supposing the estate worth 34.33 years purchase.) or *l.* 2.912 *per annum.* — By the rule of proportion, therefore, for 1000*l.* the proper equi-

equivalent will be 859*l.* in money, or 250*l.* per annum out of the estate.

QUESTION XVI.

“ 100*l.* will be lost to B’s heirs, should he
 “ happen to die after A, *within a given time*,
 “ What is the price of the *assurance* of it?—
 “ That is: What is the present value of
 “ 100*l.* payable at the death of B, provided
 “ his death should happen *after A’s death*,
 “ *within a given time*?”

ANSWER.

Divide the sum of the decrements of life in the Table of observations from the age of B, for the given time, by the given time; and by the *quotient* divide the number of the living at the age of B; and again, by this *second quotient* (a), divide the given sum, reserving the *third quotient*.

Find the value of an annuity on the life A for a number of years, less by *one* year than the given time, which subtract from the value of an annuity certain for the same number of years. Multiply the *remainder* by the *reserved quotient*, and divide the *product* by the amount of *1*l.** for one year, and let this be a *second reserved quotient*.

(a) Or rather, if the *London Table* is used, by the *complement* of the life of B, when his age is under 60, and the term exceeds the *difference* between it and 70.

Again.

Again. Multiply into one another the *first* reserved quotient, and the value of an annuity certain for the given time; and divide the product by twice the *complement* of A's life. This *last* quotient, added to the *second* reserved quotient, will be the *answer* in a present single payment.

EXAMPLE.

Let the age of B be 40. Of A 30. The sum 100*l.* Rate of interest 4 *per cent.* The given time 20 years. The Table of observations, Mr. *Simpson's*, or Table VIII.—The sum of the decrements of life, in this Table, from the age of 40 for 20 years, is 127, which, divided by 20, (the given time) gives 6.38.—The number of the living at 40 is 229, which, divided by 6.38, gives 35.8; and 100*l.* (the given sum) divided by 35.8, gives 2.79, the *first* quotient to be reserved.

The value of an annuity for 19 years on a life at 30 years of age, is 10.3; which, subtracted from 13.134, (the value of an annuity *certain* for 19 years, by Table II) and the remainder multiplied by 2.79, gives 7.89. This product divided by 1.04, (the amount of 1*l.* in one year) gives 7.60; the *second* reserved quotient.

2.79 multiplied by 13.59, (the value of an annuity certain for 20 years) gives 37.916; and this *product* divided by 94.4, (twice the
com-

complement of A's life by Table IX.) gives .401, which, added to 7.60, gives 8*l.* the *Answer*; or, the value of 100*l.* payable at the death of B, on the contingency of his surviving A aged 30, and *both* dying in 20 years.

It is plain, that this is likewise the sum that ought to be lent to B now, on the expectation of 100*l.* at his death, provided it should happen after A's death in 20 years.

This rule gives the just solution in all cases, except when B, the expectant, is the *youngest* of the two lives, and at the same time the term of years *greater* than the complement of A's life. In this particular case the following rule must be used.

Find, by the preceding rule, the value of the assurance of the given sum for a term of years, equal to the complement of A's life, and let this value be *reserved*. Multiply by one another the given *sum*; the *value* of 1*l.* to be received at the end of a number of years equal to the complement of A's life; and the value of an *annuity certain* for as many years as the given term exceeds this complement. And the *product*, divided by the complement of B's life, and the *quotient* added to the *value reserved*, will be the true value sought.

EXAMPLE.

Let the age of B be 30; of A 40. The term 47 years; and every thing else as in the

the last Example. The complement of A's life, is, by Table IX, 39.2. The value of 100*l.* to be received at the death of B, if he survives A within 39 years, may be found by the preceding rule to be *l.* 16.15; the value to be reserved.—The value of 1*l.* to be received at the end of 39 years is, by Table II, .2166. The value of an annuity *certain* for 8 years, (the excess of the given term above the complement of the life of B by Table IX.) is, 6.733.

And these two values multiplied by one another, and by 100*l.* give 145.83; which, divided by 47.2, (the complement of the life of B) and 16.15, added to the quotient, make *l.* 19.23, the value sought.

R E M A R K.

As after finding the present value of an estate, or annuity, it is necessary to *divide* that value by the amount of 1*l.* in one year, in order to find the present value of a *sum* equivalent to the *annuity*; so, after finding the value of a *sum*, it is necessary to *multiply* that value by the said amount, in order to find from it the value of an equivalent annuity.

In the first Example, therefore, the value of an estate of 4*l. per annum*, would be *l.* 8.32. In the second Example, 20*l.* And this is, as it ought to be, the value for the whole duration of the lives, agreeably to the Problem in the note page 37.

In solving this Question, care also must be taken not to forget the *first* Remark under the foregoing Question.

In the same way with that in which the rules in the three last Questions have been discovered, it is possible to find rules for calculating the values of *assurances, for given terms*, on lives and survivorships, where three or more lives are concerned. But this is of less importance; and I chuse to leave to others the further prosecution of this subject.

C H A P. II.

Containing an Application of the Questions in the foregoing Chapter to the Schemes of the Societies in Great Britain, for making Assurances on Lives and Survivorships, and for granting Annuities to Widows, and to Persons in old Age.

S E C T. I.

Of the London Annuity, and the Laudable Societies for the Benefit of Widows.

THE scheme mentioned in Quest. VIII. is nearly that of the *London Annuity Society*. The *Laudable Society* is also formed on a similar plan. In both, the *annual contribution* of every member is five guineas, payable half-yearly; and for this a title is given to an annuity of 20*l.* to every widow during widowhood, if the husband, after admission, lives *one* year according to the *first* scheme; or *three* years according to the (a)

(a) In this society a member who lives but *one* year, is entitled to no more than an annuity of 10*l.* for his widow; if he lives two years, 15*l.* If he lives three years, 20*l.* four years, 25*l.* seven years, 30*l.* ten years, 35*l.* thirteen years, 40*l.*

second;

second; of 30*l.* if the husband lives *seven* years, according to both schemes; and 40*l.* according to the *first* scheme, if he lives 15 years, or 13 years, according to the *second*. — In both schemes also, there is no other premium or fine required, than five guineas extraordinary, at admission, from every member whose age does not exceed 45. The *Laudable Society* admits none above 45, and the *London Annuity Society* obliges every person between 45 and 55 to pay, at admission, five guineas extraordinary, for every year that he is turned of 45.

These are the main particulars in these schemes; and, therefore, both of them, were the annuities to be enjoyed for life, would receive (supposing the members all under 46 at admission, and of the same ages with their wives, and money at 4 *per cent.*) but little more than three fifths of the true value of the annuities; or about one half, supposing wives, one with another, 10 years younger than their husbands; as appears from Question VIII.

It appears further in that Question, that, supposing the annuities to be *life* annuities, and men and their wives of equal ages, the expectation to which an annual payment of five guineas beginning immediately, entitles, is nearly 14*l.* if the contributor lives a year, and 20*l.* if he lives seven years (a), taking

(a) The same annual payment will, on the same suppositions, entitle to 14*l.* if a member lives a year, and 18*l.* if he lives *three* years.

the medium between the *London* and the other Tables of Observation.

It is likely, that many persons will be very unwilling to believe, that these schemes are so deficient as they have been now represented. I will, therefore, endeavour to prove this in a way which, tho' less strict, is sufficiently decisive, and may be more likely to be intelligible to persons unskilled in mathematical calculation.—I shall here confine myself to the scheme of the *London Annuity Society*. The differences between it and the scheme of the *Laudable Society* are inconsiderable, and what shall be said of the one will be fully applicable to the other.

According to this scheme, as it has been just described, all that live 15 years in the society will be entitled to annuities of 40*l.* per annum for their widows. Suppose the whole society, at admission, to be men of 40 years of age, taken one with another. A person of this age has an even chance of living 23 years; and he has an even chance of continuing with a wife of the same age, (that is, of continuing in the society) 13 years and $\frac{1}{2}$ (a). Not much less, therefore, than
half

(a) This is the exact truth according to Mr. *De Moivre's* Hypothesis, and the *Norwich Table*. But according to Dr. *Halley's* and the *Northampton Table*, a man 40 years of age has an even chance of living no more than 22 years, and of joint continuance with a wife of the same
same

half the members will continue in the society 15 years; and, consequently, not much less than half the widows that will come upon the society will be annuitants of 40*l.* *per annum*. These widows, however, being older than the rest when they commence annuitants, will continue on the society a shorter time; and, therefore, the number constantly in life together, to which they will in a course of years increase, will be proportionably smaller. Putting every thing as favourably as possible, let us suppose, that out of 20 annuitants constantly on the society, *five* will be annuitants of 40*l.* *six* of 30*l.* and *nine* of 20*l.* To 20 annuitants then the society will pay 560*l.* *per annum*, or the 20th part of this sum, that is 28*l.* to *every* annuitant at an average. But such an annuity for a life at 40, after another equal life, provided both survive one year, is worth (by *Quest. VII.* p. 24.) in a single present payment, 85*l.* nearly, according to the *London*, and all the *Tables of Observations*, interest being all along supposed at 4 *per cent.*

It cannot appear improbable to any one, that this should be the true value of such a reversion. It is not credible, that there is

same age, 13 years.—Forty must be more than the mean age of the members of the society at admission, and on this account the number of annuitants of 40*l.* must be proportionably greater. The mean age, therefore, has been taken very moderately.

any situation in which the decrements of life are such as can make it a tenth part more or less.—85*l.* in present payment is the same with 3*l.* 8*s.* *per annum* for ever.—But is an annual payment of five guineas, which must cease as soon as either of two lives each 40, fails, equal in value to such a perpetuity? Every one must see, that there is a great difference.—A set of marriages between persons all 40, will, according to the probabilities of life in Dr. *Halley's* Table, last, one with another, 15 years (*a*); and an annual payment beginning immediately, during the joint continuance of two persons of this age, is worth 10 years purchase (*b*). The comparison then, in the present case, is between 3*l.* 8*s.* *per annum* for ever, and five guineas *per annum* for 15 years; or between an annuity of 3*l.* 8*s.* worth 25 years purchase, and an annuity of five guineas worth only 10 years purchase.

But to throw this subject into another light.

(*a*) See the beginning of Essay I.

(*b*) The value of such an annual payment, by Table XI, or the *London Observations*, is 9.1; and 10.8, by Mr. *De Moivre's* Hypothesis.—I have not taken into this account the five guineas *fine* paid at admission, because it is obviously of too little consequence to make any considerable difference. The allowances I have made in favour of these schemes are more than equivalent to it. In particular; it should be remembered, that the payments required by these schemes, are *half-yearly payments* beginning immediately; and that these, by Quest. VIII. are less advantageous than the payments I all along suppose them to require, or, “*yearly payments* beginning immediately.”

Let the number to which the society is kept up be supposed to be 200. It has been demonstrated in *Quest. II*, that at least half this number of widows will in time come to be constantly on the society; and it has also been just now shewn, that the medium of annuities, payable to them, will be at least 28*l.* After a course of years, then, the society will have a constant expence to bear of 2800*l. per annum.*—But what will be its income?—In order to determine this, we must consider, that there are two sources from whence its income will be derived. First, the annual payments of the members. And, secondly, the money accumulated, or the *capital* raised during the time the number of annuitants is coming to a *maximum.*—The first of these sources affords 1000 guineas, or 1050*l. per annum.* This wants 1750*l.* of the annual expence just mentioned; and, therefore, in order to have the income of the society equal to the burden upon it, when the annuitants come to a *maximum*, there must be a fund raised in the mean time equal to 43,750*l.* or to an estate in perpetuity of 1750*l. per annum.*—But 1050*l. per annum* beginning immediately, and forborn 24 years, and improved, without loss or delay, all that time at 4 *per cent.* compound interest, will but just raise such a capital (*a*). There is, therefore,

(*a*) Every Question of this kind may be easily solved in the following manner. In Table I, find the value

fore, the fullest proof, that the scheme I am considering is extremely deficient. The truth is, that scarcely a *third* of such a capital could be raised, as will appear from the following observations.

Out of 200 persons, all 40 years of age, *more* than five, according to the *London Table of Observations*, and not so many by Dr. *Halley's Table*, may be expected to die in a year. Suppose then five to be the real number of members that will die the first year of the society. In subsequent years the collective body of members will be continually growing older; and, therefore, the proportion of them that will die every year, will be continually increasing, till it gets to a *maximum*. I will, however, suppose, that during the first 20 years no more than the

of 1*l.* payable at the end of any number of years; and any given annuity divided by that value, will be the annuity to which the given annuity will in that number of years increase. — Thus; the present value of 1*l.* payable at the end of 25 years, is .3751, reckoning interest at 4 *per cent.* and 1050*l. per annum* divided by .3751, gives 2,800*l. per annum*, the increased annuity arising from 1050*l. per ann.* In the same manner; it may be found, that the same annuity, forborn 11 years, will increase to 1610*l. per annum.* This supposes the first payment of the annuity to be made a year hence. If the first payment is to be made immediately, which is the present case, the annuity will increase to the same sums in one year less time.—But a more particular account of this will be given in the rules annexed to the Tables at the end of this work.

.number

number just specified will die every year; and that, consequently, no more than *five widows* will come every year on the society. The ages of all these widows, when they commence widowhood, will, it is evident, be between 40 and 60. One with another then, they may be considered as having commenced widowhood at 50 years of age. Now, five widows left every year at this age, will, in 10 years, increase to 43 constantly in life together, according to the expectations of life in Tables III, IV, and V; and, in 20 years, to 70 (a). Suppose the true number alive together at the end of 20 years to be only 62. the greater part of these will be annuitants of 30*l.* and 40*l. per ann.* and the rest 20*l.* Were the former only equal to the latter, the medium of annuities payable to them would exceed 25*l.* Suppose then this medium to be no more than 26*l.* and it will

(a) Every calculation of this kind is easily made by the rule in note (A) in the Appendix.—I have put the number living together at the end of 20 years at 62, not only that the reader may be better satisfied that I have kept low enough, but also to make an allowance for such widows as will be left by those members who die within a year after admission, and who, therefore, according to these schemes, will be entitled to no annuities. This allowance is too large: For, after the first year of the scheme, it will not happen above once in 4 or 5 years, that the death of a member will be so circumstanced, supposing the probability that a man at 40 will live a year, to be, as all but the *London Tables* make it, 50 to 1.

follow, that, at the end of 20 years, the society will have an annual rent to pay of 26*l.* multiplied by 62 or 1612*l.* and, if then able to bear such an expence, it must, in the intermediate time, have acquired an increase of income equal to the difference between 1050*l.* and 1612*l. per ann.* That is; it must, with its savings, have accumulated a stock equal to 562*l. per ann.* and worth 14,050*l.* But, as during this time, there will be a number of annuitants constantly increasing, to whom yearly payments must be made, the savings of the society cannot certainly be one half of what they would have been had it been all the time free from all burdens. Suppose then the stock produced by these savings, to be equal to the stock that would arise from an income of 1050*l. per ann.* beginning immediately, and improved perfectly at 4 *per cent.* compound interest, for half the time I have mentioned, or for 10 years, without being subject to any checks or deductions. Such an income thus improved, would, in 10 years, produce an additional income of 560*l. per annum,* or a capital of 14,000*l.*—According to these observations, therefore, the annual income of the society at the end of 20 years, and before a third part of the highest annuitants could come upon it, would begin to fall short of its expences. About that time then it would necessarily run aground; and long before the number of annuitants could
rise

rise to a 100, it would spend its whole stock, and find itself under a necessity of either doubling the annual payments of its members, or of reducing the annuities one half.

All I have now said is meant on the supposition, that the society begins with 200 members at 40 years of age, and is afterwards limited to that number, by admitting no more new members than will just supply the vacancies occasioned by the loss of old members. If it is allowed to increase, it may continue a longer time. And, for this reason, a society that wants half the income necessary to render it permanent, may very well subsist, and even prosper for 30 or 40 years.—Thus, the *Laudable Society*, was it to keep to its present number of members, might possibly feel no deficiencies for 20 or 30 years to come; but if it should continue to increase at the rate of 70 or 80 every year, it would, at the end of that time, possess a balance so much in its favour, as might enable it to support itself for 20 or 30 years more (a). But bankruptcy would
come

(a) What has been before demonstrated in Quest. III. should be here recollected, that the number of annuitants on such a society as this, must go on to increase for more than 100 years, after acquiring its greatest number of members.

The *Laudable Society*, I am informed, took its rise from a calculation contained in a pamphlet entitled, *The Possibility and Probability of a SCHEME intended for the Benefit of Widows being able to support itself*. The scheme here referred to, is the same with that which this Society has
since

come at last, and with the more terrible weight the longer it had been deferred.

The rule in the *London Annuity Society*, which obliges every person between the ages of 45 and 55, to pay at admission 5 guineas extraordinary, for every year that he exceeds 45, is an advantage to it, but it is a very inadequate, and also a very unequitable advantage. For at the same time, that it obliges a person 55 years of age, to give *more* than the value of his expectation, it takes *above* two fifths *less* than the value from a person who is 45 years of age.

Should any persons remain still doubtful about what I have said, I would beg them to attend to one further argument.

It must be reckoned upon that every other member of these societies, supposing them to consist of persons all of the same ages with their wives, will leave widows to whom one with another, (as already shewn) at least 28/.

since followed; and I am afraid I shall not be credited, when I say, that the calculation to prove its capacity of supporting itself, is founded on the supposition, that a hundred married men whose common age is 36, will leave but *one* widow every year, tho' at the same time it is supposed that two of them will die every year.

This mistake has made the whole calculation one half wrong.—Nothing can be plainer than that, if the death of a married man does not leave a widow at the end of the year, the reason must be, that both himself and his wife have happened to die in the year. But it is always very improbable this should happen.

per ann. must be allowed, for as many years as there have been payments from each member. For every 10 guineas then received they must some time or other hereafter pay 28*l.* But let it be well considered what can enable them to do this. Did money bear no interest, for any given sum now received, they could not afford at any time hereafter to pay more than an *equal* sum. That is; (since the duration of *survivorship* is in the present case, by *Quest. II*, equal to the duration of *marriage*) the proper consideration for any given reversionary annuity, to be allowed to *all* the survivors of a set of marriages, would be, supposing no interest of money, an equal annuity payable by each marriage during its existence; and just *half* the reversionary annuity, if it is to be allowed only to half the survivors, or to widows exclusive of widowers. The annual payment then of *five* guineas, during marriage, can entitle widows to no more than an annuity of *ten* guineas, supposing money to bear no interest. But if it does bear interest, the same payment will entitle them to more, in proportion to the degree in which it is capable of being improved, during the time between that in which the annual payments begin, and the commencement of widowhood. Now, it is easy to see, that unless money bears very high interest, this improvement cannot be likely in any circumstances to produce a capital, the
interest

interest of which shall be equal to the annual payment itself. Any given annual payment perfectly improved at 4 *per cent.* compound interest, requires 17 years to double itself, supposing the first payment made immediately; or, near 18 years (*a*), if the first payment is not made till the end of a year. But no marriages are *likely* to last so long as this, except those among persons who are very young. A marriage between two persons, both 40, will not *probably* last longer than 13 years, according to the probabilities of life in Dr. *Halley's* Table. A marriage between two persons, both 50, will not probably, by the same Table, last longer than *eleven* years; nor a marriage between two persons, both 30, longer than 16 years. Such marriages, it is true, may possibly last 30 or 40 years. But this circumstance is more than balanced by the fact, that no less possibly they may not last *one* year. The annual payments, then, being incapable of such an improvement as shall produce an additional income equal to themselves; it is obvious, that no society ought to go so far as to allow to widows annuities twice as great as those which might be allowed, supposing no

(*a*) At 3 *per cent.* the period of doubling money by compound interest, is nearly 23 years. At 5 *per cent.* 14 years.

interest of money (a); so far, for instance, as to allow, instead of 10 guineas, 20 guineas for an annual payment of five guineas. In the circumstances of most of these societies three fifths addition may be the full allowance. That is; supposing the annual payment of each member to be five guineas, time may be expected for gaining from hence a capital of 75 guineas, or that shall produce three guineas *per annum* interest; and the proper reversionary annuity will be 16 guineas; or six guineas more than the proper reversionary annuity, did money admit of no improvement.

The preceding observations have gone on the supposition, that the reversionary annuities are to be *for life*. What difference in favour of these societies arises from the circumstance, that the annuities are to be paid only *for widowhood*, cannot be exactly determined. Some judgment, however, may be formed of it from what has been said at the conclusion of Quest. II. Were even one half

(a) The money accumulated will not be exactly the same with that to which the annual payment would increase, if improved at compound interest for a number of years, equal to that which the joint lives have an *equal chance* of existing. Much less will the increase be the same with that which would arise from the annual payment forborn, and improved for a number of years equal to the *expectations* of the joint lives. It will be less than either of these, for a reason explained in note (L) Appendix.

of the widows to marry, still the schemes I have been considering would probably be insufficient. But in the circumstances of these societies it cannot be expected, that above one in 10, or perhaps one in 20, will marry. The persons most likely to enter into them, are such as have not the prospect or ability of making competent provisions for their widows in other ways. The widows left, therefore, will in general be unprovided for, and being also left with families of children, it is quite unreasonable to expect, that any considerable proportion should marry. This is true of such as may happen to be left young; but when a society has subsisted some time, the *greater* part will not be young when left, and these, at the same time that no advantage can be expected from their marrying, will be in general the *highest* annuitants, and, therefore, the *heaviest burdens*.—Moreover, the prospect of the loss of their annuities will have a particular tendency to check marriage among them.—For all these reasons it seems to me likely, that the benefit, which these societies will derive from marriage among their annuitants, will not be very considerable; or at least not *so* considerable as to be equal to the advantages I have allowed them, by calculating on the suppositions, that the money they receive will be *always improved perfectly, without loss or delay, at the rate of 4 per cent. compound interest*; that the probabilities
of

of life among males and females are the same, and all husbands likewise of the same ages with their wives, and that consequently the *maximum* of widows on such societies can amount to no more than half the number of marriages (a). — With respect to the last of these suppositions, it deserves to be particularly observed, that by an enquiry made some years ago in *Scotland*, it was found, that the widows of *ministers* and *professors* there, (b) notwithstanding the diminution occasioned by their marrying, did exceed half the number of marriages. And certainly it would be unreasonable in these societies not to reckon that the same will happen among them. — Indeed it seems certain that, notwithstanding

(a) Care should be taken in these societies, not to judge of the proportion of widows that will marry, from the proportion that may happen to marry during their first years. For most of the widows that will be left at first will be young; whereas the greater part will not be young when they commence widowhood, after a society has subsisted 30 or 40 years; and, therefore, tho' one in 3 or 4 should marry at first, it will not be reasonable to expect, that half so many should marry after the affairs of the society become stationary.

(b) 364 widows, all living at one time, were counted; and the number of married ministers and professors for many years past has been, at an average, 667. — Twenty widows likewise are left one year with another; and, for 10 years, ending in the year 1767, but nine of these had married. — Of the annuitants likewise (about 160 in number) on the fund established among the Dissenters in *London*, for relieving the widows of indigent ministers, it is found that few ever marry. See the latter end of the 4th Essay; and note (A) in the Appendix.

the

the hazards that attend child-bearing, the probability, that the woman shall survive in marriage, and not the man, is much greater (a) than is commonly imagined. It will be shewn in the last Essay, that it is not less than the odds of 3 to 2; and had I calculated agreeably to this fact, the values of annuities for widows, would have been given near a quarter greater than they have been given on the supposition, that the chance of survivorship is equal between men and their wives.—It must be added, that I have made no account of any expences attending the execution and management of the schemes of these societies. Some such expences there must be, and some advantages should be always provided in order to compensate them.

There are in this kingdom several institutions for the benefit of widows, besides the two on which I have now remarked; and in general, as far as I have had any information concerning them, they are founded on plans

(a) Partly, as observed in page 8, on account of the greater mortality of males, but chiefly on account of the excess of age on the man's side.—According to the printed articles of agreement, the *Laudable Society* pays no regard to this excess of age; and the allowance required on this account by the *London Annuity Society* is so trifling that it deserves no notice.

In March 1770, thirty-two husbands had died in the *Laudable Society*, and 27 wives. They seem, therefore, to be already beginning to experience, that the chances of survivorship in marriage are in favour of the wife.

equally

equally inadequate. The motives which influence the contrivers of these institutions are, without doubt, *laudable*; but they ought, I think, to have informed themselves better. This appears sufficiently from what has been said; but I will just mention one further proof of it.

The *London Annuity Society* promises that, if in 21 years; and the *Laudable Society* that, if in 25 years, it shall appear that there has been all along an annual surplus in favour of the societies, it shall be employed in either raising the *annuities*, or in sinking the *annual payments*. Now, they may be assured, that, if at the end of these periods, they should not be possessed of a considerable surplus, the true reason will be, their having granted much higher annuities than the annual contributions are able permanently to support: For it has been demonstrated, that the number of annuitants, and consequently the amount of the annual expences, will go increasing for a long course of years beyond these periods. The effect, therefore, of carrying into execution this regulation will be, precipitating that bankruptcy which would have come too soon had there been no such regulation.

It has been said in defence of these Societies, that the deficiencies in their plans cannot be of much consequence, because their rules oblige them to preserve a constant equality between their income and expences, by reducing the annuities as there shall be occasion.

And from hence it is inferred, that they can never be in any danger of a bankruptcy.—In answer to this, it has appeared, that the time when they will begin to feel deficiencies is so distant, that it will be too late to remedy past errors, without sinking the annuities so much, as to render them inconsiderable and trifling: All that is given too much to *present* annuitants is so much taken away from *future* annuitants. And if a scheme is *very* deficient, the first annuitants may, for 30 or 40 years, receive so much more than they ought to receive, as to leave little or nothing for any who come after them. Deficient schemes, therefore, are attended with particular injustice; and this injustice will be the same, if, instead of *reducing* the annuities, the annual payments should be increased; for all the difference this can make will be, to cause the injustice to fall on *future contributors*, instead of *future annuitants*.

But what requires most to be considered here is, that, after either the annuities have been for some time in a state of reduction, or the contributions in a state of increase, it will be seen that these Societies have gone upon wrong plans, and, therefore, they will be deserted and avoided; the consequence of which will prove still greater deficiencies in their annual income, and a more rapid desertion and decline, 'till a total dissolution and bankruptcy take place.—This will be the

death of most of the present societies for providing for widows, if they continue to be encouraged, and do not soon alter their plans: And at that period the number of *annuitants* will be greater than ever; whose annuities, having no other support than the poor remains of a stock always insufficient, will be soon left, without the possibility of relief, to lament that ignorance and credulity which gave rise to these societies, and which had so long supported them.

In the *London Annuity Society*, there is an encouragement to *batchelors* and *widowers* to join them, arising from the additional annuities to which they will be *immediately* entitled, when they marry, in consequence of having made their payments a greater number of years; and it is imagined, that particular advantages will be derived from such members. But even these will in general pay much less than the value of their expectations.—A person who begins an annual contribution of five guineas at the age of 24 will, should he live 11 years, and marry a woman of the same age at the end of that time, entitle her immediately to 35*l.* *per ann.* during survivorship, and to 41*l.* *per annum* should he live four years after marrying. (interest being at 4 *per cent.*) (a). In this
par-

(a) The value of five guineas *per annum* (first payment made immediately) for 11 years, subject to failure should a life now 24 fail; and, after 11 years, for the joint lives

particular case, therefore, a person will pay nearly the true value of his expectation. But *all* at all ages who *marry*; and *most* of those who *die*, in less time than 11 years after admission, will pay less than the value of their expectations.

S E C T. II.

*Of the Association among the London Clergy,
and the Ministers in Scotland, for providing
Annuities for their Widows.*

IN April, 1765, the clergy within the bills of mortality, and the county of *Middlesex*, at a general meeting in *Sion-College*, agreed to form themselves into a society for the support of their widows and orphans. Many in this respectable body may be capable of doing, in a better manner, what I have attempted in this Treatise; and they are, perhaps, already sensible of the deficiencies in the plan which they have established. I shall not, however, I hope, do wrong, in taking the liberty to recite briefly this plan, in order to introduce a few observations upon it.

of two persons both 35, is, by the Table of *London Observations*, l. 69.3.—By Dr. *Halley's* Table l. 76.44.—The present value of 35*l.* per annum for life to the widow of a person now 24, should he live 11 years, and marry a woman of the same age with himself at the end of that time; and also of 6*l.* more, or 41*l.* per annum in all, should he live after marriage four years; is, by the Table of *London Observations*, l. 69.36,—By Dr. *Halley's* Table, l. 76.03.

According to the printed articles, every clergyman possessed of any benefice, lectureship, or licensed curacy, within the bills of mortality, and the county of *Middlesex*, who subscribes annually one guinea, or two guineas, or more, shall entitle his widow to an annuity; or, if he leaves no widow, he shall entitle any such children as he shall leave, to the same annuity for seven years as his widow would have had. And, in case a widow possessed of an annuity, should either *die* or *marry* before the lapse of 10 years, from the commencement of her annuity, such children of her former husband, as shall be then alive, are to be entitled to as many of the ten years payments of the annuities as she shall not have received.—The annuity is fixed to no particular sum, but instead of this, it is ordered, that a fourth part of the annual subscriptions and interest shall be divided the first three years after the establishment of the society; half only the next four years; and 3-4ths the next 5 years; provided, however, that in no one of these 12 years the dividend shall exceed 20*l.* to the widows and orphans of the clergy subscribing two guineas or more; and 10*l.* to the widows and orphans of the subscribers of one guinea. And, after the expiration of 12 years, the whole amount of the subscriptions, and of the interest of the capital stock, is to be divided proportionably for ever.—It is further provided, that every clergyman, who shall be married, or have chil-

dren, at the time of his subscription, shall pay a fine of two guineas towards a capital stock, if a subscriber of two guineas or more, and 40 years of age or upwards. If 50 years of age or upwards, he shall pay a fine of three guineas; if 60 or upwards, five guineas. But, if not married at the time of his subscribing, and shall afterwards marry, he shall pay a fine according to the age he shall be of at the time of his marrying. The obligation laid upon all, whether married or unmarried, to become subscribers, is, an incapacity of being admitted members without the consent of a general court, unless, within two years after becoming possessed of any ecclesiastical employment, they subscribe.

Every one who has attended to the observations in this and the preceding chapter, must know what judgment to form of these regulations.

Let us suppose, that all the clergy in *London* and *Middlesex* came into this association from the first; and that one with another they are subscribers of two guineas annually; and that there are among them as many unmarried persons as married.

In this case, it may be learnt from *Quest. XIII*, that the annuity to which widows should be entitled, (supposing no allowance to the children of any that die) ought not to exceed 10 or 11 guineas at most, and that, besides the annual subscriptions, there ought

to

to have been a fine paid at the commencement of the scheme, by every married person, of six guineas at least, or, by the whole number of subscribers, three guineas. If the number of married members is double the unmarried, the annuity ought not to exceed eight guineas; and the fine from every member should be about four guineas.—The order, that only a fourth part of the annual subscriptions and interest shall be divided the first three years, half the next four years, and three quarters the next five, is without reason; because the number of claimants, for the first 12 years of the scheme, will be so few, that it will not be possible, during that time, that there should be occasion for dividing any proportions so large of the annual subscriptions and interest, unless they are indeed beyond all bounds too little.—After 12 years, the number of annuitants will go on increasing for near 50 years, as appears from *Quest. III.* The consequence, therefore, of dividing, after this time, the whole amount of the annual subscriptions and interest, will be a constant yearly diminution in the dividends for near 50 years; and making the payments to the first claimants much more considerable than they ought to be, at the expence of all subsequent claimants.—For these reasons; it appears to me out of all doubt, that this scheme is by no means likely to answer the good ends proposed by it; and that, therefore, it will be best to lay it aside.

At the time it was settled it was, I find, further agreed, that the annual subscriptions of the *laity*, together with the interest of their benefactions, unless otherwise directed by the donors; and the annual subscriptions of such of the clergy as shall so direct, shall make a *charitable fund* to be applied to the relief of the distressed widows or children of all the clergy within the limits I have mentioned, whether subscribers or not, provided that in no one year of the first twelve more than 20*l.* be given out of the fund to any one family.— This is an excellent design; and if the money arising from all the subscriptions is thrown into this fund, an important means of relief may be provided, for such of the more indigent widows and families as will accept the help of charity.

There is one more association of particular consequence, which it is necessary I should take notice of. I mean, the *association* among the ministers and professors in *Scotland*, for making provisions for their widows and orphans. The last-mentioned association, and also several others of the same kind (a) in this kingdom, have been formed on the

(a) There is one among the Dissenting Ministers in the counties of *Chester* and *Lancashire*, and another among the Dissenting Ministers in *Cumberland*, *Northumberland*, *Westmoreland*, and *Durham*. — Even the *London Annuity Society*, tho' its plan is totally different, professes to form itself on the principles of the *Scotch* establishment, and to derive encouragement from it.

model of this establishment; and the success with which it has been hitherto attended, has been the principal cause to which they owe their rise.—I am afraid of being too tedious, and therefore I will not attempt to recite all the particulars contained in the plan of this establishment. It may be sufficient to observe, that for “an annual
“ payment, which begun immediately, of
“ five guineas from 1011 contributors, 667
“ of whom are married persons; besides a
“ tax on weddings producing about 142*l.*
“ *per annum*; it entitles every widow to an
“ annuity of 20*l.* during widowhood; and al-
“ so, every family of children that shall be
“ left by such members as die without leav-
“ ing widows, to 200*l.*” Now, by particu-
lar enquiry at the commencement of the scheme, it had been found, that there was reason to believe that, for many years back, 20 widows had been left annually by the whole body of ministers and professors; and that, also, six had died annually and left children without widows; and these facts have been since confirmed by the experience of 25 years.—Subtract, therefore, from 5,450*l.* (the whole annual income) 1,200*l.* payable every year to six families of children; and 4,250*l. per ann.* (the first payment of which was made (a) immediately) or 1.4.2 *per annum*

(a) The truth is, that a double payment was made at the beginning of the second year. This is of less value than
two

annum from each member, will remain as the standing provision for bearing the burden of the annuitants. — This provision, according to the calculation in *Quest. XIII*, and note (F) *Appendix*, ought to be at least 4,745*l. per annum*, from each member; from whence it seems to follow, that this establishment has not a sufficient income to afford it a *permanent* support. — But I do not by any means design to assert this. The difference between the real and calculated incomes, as it has been now stated, is not considerable enough to give sufficient reason for such an assertion. This establishment may have some advantages that I know nothing of, and that are not mentioned in the printed accounts; or, in consequence of the increase of luxury, and the higher price of all the means of subsistence, marriage may decline among the ministers; or, possibly, the probabilities of life among them, tho' much higher 'till towards 50, than is common among mankind in towns, may yet afterwards decrease much faster. The income, therefore, of this establishment, properly improved, may continue to be adequate to all its expences and burdens. — One observation, however, ought to be carefully attended to. The success it has hitherto met with, is no good reason for en-

two payments, one of which is made immediately, and the other a year hence; but, the difference not being considerable, I have reckoned them the same.

tertaining

tertaining this expectation, with any degree of confidence. It appears from Quest. III, that the number of annuitants, on such an establishment, must go on increasing for 60 years, from the time of its commencement; and it is obvious, that the *continuance* of a success which has not lasted *half* this period, cannot be absolutely depended on. I know, indeed, that, according to the calculations which were made when this establishment begun (*a*), the number of widows upon it will not increase sensibly after the year 1780, or for more than 10 years to come; and, were this true, all diffidence about it would, perhaps, be unreasonable. But these calculations cannot, in this instance, be right; for they imply, that none are left widows under 52 years of age. 'Tis certain, on the contrary, that many are left widows under 32; and that, consequently, the whole body in life together must go on to increase for 25 years, beyond the period assigned in these calculations; or till the year 1805 (*b*). It is necessary

(*a*) See Table III, in a book printed at Edinburgh in 1748, entitled, *Calculations, with the Principles and Data on which they are instituted, relative to a late act of parliament, entitled, An act for raising and establishing a fund for a provision for the widows and children of the ministers of the church, and of the heads, principals and masters of the universities of Scotland; shewing the rise and progress of the fund.*

(*b*) This is said on the supposition, that all the ministers and professors acceded to the scheme from the first.

cessary I should add, that the whole number alive, when the increase stops, will, most probably, be greater than the number provided for in these calculations. They are made on the supposition, that 52 is the mean age at which women commence widows. If this supposition is right, it is impossible that, according to the probabilities of life in Dr. Halley's Table, the number of widows living together at one time, derived from 20 left annually, should increase to more than 323, if none marry; or about 300, if one marries every year (a): Nor does it appear likely, that
 this

135 did not accede; but all their successors have been obliged to accede. This circumstance must add 30 or 35 years to the period of increase which I have mentioned; that is, as many years as are necessary to cause all the non-contributors to die off. See Quest. III.

(a) The *expectation* of a person at the age of 52 is, when taken exactly from Dr. Halley's Table, 16.16; and this number multiplied by 20, gives the *maximum*, to which 20 widows left annually, at 52, will increase in 34 years, supposing 86 the utmost limit of life. Vid, Essay I.—In the calculation to which I have referred, there is no account taken of those that die in the year in which they are left widows; and, for this reason, they are made to increase to a greater number than is consistent with the supposition, that 52 is their mean age when they commence widowhood.—It should be remembered here, that supposing this mean age, as explained in note (F) Appendix, rightly taken, the *maximum* of widows will be rightly found in the manner just specified. But the period, in which they would attain to that *maximum*, would be as many years greater than the difference between the mean age and the utmost limit of life, as the mean age is greater than
 than

this establishment will be able to bear the expence of above 30 or 40 more than the last-

than the *least* age at which widowhood ever commences. The calculation, therefore, which I have in view must be very wrong. It supposes not only, that all the widows left at all ages, will increase to a *maximum* in the same time with those left at the mean age; but that all left in the course of every year will certainly live to the beginning of the next year. It supposes, likewise, that no widows will marry; and those concerned will understand me when I add, that it supposes further, that *acciders*, tho' mostly young ministers unmarried, will leave widows as fast as the *noncontributors* whom they succeed. In consequence of these omissions it has all along given the numbers in life much higher than they ought to have been given; but yet the event has been, that these numbers have in fact corresponded nearly to the calculation: From whence it follows, that the widows in life have hitherto increased at a much greater rate than they could have done, according to Dr. *Halley's* Table, had their mean age, when left, been 52. Either, then, their mean age has been considerably less than 52; or, their probabilities of life must be considerably greater than those in Dr. *Halley's* Table; and, it ought, therefore, to be expected, that they will at last increase to higher numbers than those assigned in this calculation.

I cannot help here mentioning one more reason for entertaining this expectation.—Were the decrements of life *uniform*, and the chances of survivorship between men and their wives *equal*, the number of widows and widowers in the world, if none married, would also be equal; and both together equal to the whole number of marriages. See *Quest. II*, and note (F) Appendix.

If the chances for the survivorship of the wife are greatest, the number of widows in the world will be also greatest, and the whole number of widows and widowers greater than the whole number of marriages. In the present case experience proves, that the chances of survivorship in marriage are as 5 to 3 at least, in favour of the

last-mentioned number. But it was found, by enquiry, that the number of widows living

the wife; or that there are 20 widows left to 12 widowers. (See note (F) Appendix, and the end of the last Essay). The number of widows and widowers then would certainly, if none married, exceed 667, the whole number of marriages. Suppose, however, that they would be only equal; which is the same with supposing, that *widows* would not increase to more in life together, in proportion to the number left, than *widowers*; or that one with another they are of equal ages. 667 then being the number of marriages, this will likewise be the number of widows and widowers, 416 of whom must be widows and 250 widowers; that is, 5 to 3. Now as *widows* are certainly, one with another, younger than *widowers*; and likewise, very probably, more long-lived at equal ages; and as also, in the present case, but *one* marries of the *twenty* left annually; it follows, with demonstrative evidence, that if the annuitants on this establishment should not increase to 400, the reason must be, that the decrease in the probabilities of life, instead of continuing always uniform, is slow in the first stages of mature life, and accelerated afterwards, to a degree of which there is no example in any Tables. And this, possibly, may be indeed the case; for it is uncertain whether, in this particular situation, life may not waste according to a law not yet observed. This uncertainty it is in the power of the conductors of this scheme to remove, by keeping an account of the ages at which all the ministers and professors enter upon their offices and die; and also of the ages of their wives when they marry and die. From such an account kept for a course of years, Tables of Observation, adapted to the best sort of lives of both sexes, might be formed, which would contribute much to the improvement of this part of knowledge.

I have said nothing above of the advantage which this establishment derives from the marriages among widows. This advantage, it has appeared, were it enjoyed without abatement, would not be considerable; but it is in some measure given up by the order which makes a part of this
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ing in 1748, was *at least* 364; (a) and it may be learnt from notes (A and F) in the *Appendix*, that, according to Dr. *Halley's* Table, 20 widows left annually, must in time increase to near 400, tho' one of the youngest married every year. And it may be further learnt from note (A) Appendix, that the widows on this establishment have not hitherto increased more slowly, than is consistent with their actually increasing to 400: Nor, indeed, (as the probabilities of life in this case are higher than those given by Dr. *Halley's* Table) should I much doubt of their increasing to *more* than this number, were it not that 364 has been given as the number found upon enquiry. With respect to this, however, it ought to be mentioned, that another account had been taken which made the number of widows only 321. As, therefore, a more careful enquiry discovered 43 new widows; perhaps, an enquiry yet more careful would have made yet further discoveries. In taking such accounts, none can well be added;

establishment, that such children of an annuitant as are under 16, shall be entitled, if she marries, to as much as shall happen to be then unpaid of ten years purchase of her annuity. The same provision is made for the children of annuitants that die. There are other *burdens* on this establishment, and it has also *advantages* of which I have taken no notice; but, as far as I am acquainted with them, they are of no particular consequence, and they also nearly balance one another.

(a) See Calculations, with their Principles and Data, &c. Introduction.

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but in the strictest search it can hardly be possible to avoid omitting some.

The result of the last enquiry, in particular, is said to have been; not that it was found that there were *no more* than 364 widows, but that they did *exceed* this number. (a) However, let 364 be the true number living in 1748. Before that time, there had probably been more marriages among them, than there have been since; and this may have rendered their number less than it would otherwise have been, and less than it will be hereafter: For it seems to me, that this establishment, at the same time that it encourages marriage among the ministers, has a tendency to check it among their widows, by making the consequence of marrying to be the loss of their annuities.

I hope the venerable ministers and professors concerned in these remarks, and at present so eminent in all the departments of science, will excuse what has been now said. It may, perhaps, be of service, if not to them, yet at least to some in this part of the united kingdoms, by shewing them, that this establishment has been copied in it much too rashly; and that, however successful it may in the end prove, it is yet too soon to make it a model and an authority for similar establishments.

(a) See Calculations with their Principles, &c. p. 44.

S E C T. III.

Of the best Schemes for providing Annuities for Widows.

INstitutions for providing widows with annuities would, without doubt, be extremely useful, could such be contrived as would be *durable*, and at the same time *easy* and *encouraging*. The natures of things do not admit of this in the degree that is commonly imagined. The calculations and rules, in the preceding chapter, will enable any one to determine in all cases to what reversionary annuities any given payments entitle, according to any given valuation of lives or rate of interest. From Quest. VII and VIII, in particular, it may be inferred that (interest being at 4 *per cent.* and the probabilities of life as in Mr. *De Moivre's* Hypothesis, or the *Breslaw, Norwich, and Northampton* Tables) for an annual payment beginning immediately of *four guineas* during marriage; and also for a guinea and half in hand, on account of each year that the age of the husband exceeds the age of the wife, every married man, under 40, might be entitled to an annuity, during life, for his widow of 5*l.* if he lives a year, 10*l.* if he lives *three* years, and 20*l.* if he lives *seven* years. Money can scarcely now in this kingdom be improved

at so high a rate as 4 *per cent.* But, perhaps, it might be reasonably expected, that an advantage, sufficient to compensate this disadvantage, would be derived, from changing the annuities I have mentioned into annuities during widowhood. One may, at least, venture to pronounce, that nothing much worse could befall a society that went on this plan, than the necessity of some time or other adding half a guinea to the annual payments.

If such a society chuses, that those who shall happen to continue members the longest time, shall be intitled to still greater annuities, six guineas, additional to all the other payments at admission, would be the full payment for an annuity of 25*l.* and 12 guineas for an annuity of 30*l.* if a member should live 15 years.

All batchelors and widowers might be encouraged to join such a society, by admitting them on the following terms.—*Four guineas* to be paid on admission, and *three guineas* every year afterwards, during celibacy; and, on marriage, the same payments with those made by persons admitted after marriage; in consideration of which, *1*l.* per annum*, for every single payment before marriage, might be added to the annuities, to which such members would have been otherwise entitled.

For

For example. If they have been members four years, or made five payments before marriage, instead of being entitled to life-annuities for their widows of only 5*l.* 10*l.* 20*l.* 25*l.* and 30*l.* on the conditions I have specified, they might be entitled to annuities of 10*l.* 15*l.* 25*l.* 30*l.* and 35*l.* Or, if they have been members nine years, and made 10 payments, they might, instead of the same annuities, be entitled to annuities of 15*l.* 20*l.* 30*l.* 35*l.* and 40*l.*—In this case, the contributions of such members as should happen to desert, or die in celibacy, would be so much profit to the society, tending to give it more strength and security.

This is one of the best schemes that I am able to think of, or would chuse to recommend. There are, however, others no less safe and encouraging which some may prefer, and which therefore, I will just propose.

Let the probabilities of life be the same with those in the Tables just mentioned. Let money be supposed to be improved at no higher interest than 3 *per cent.* Let the reversionary annuities promised to widows be 10*l.* *for life*, if a member lives five years after admission, and 15*l.* more, or 25*l.* in all, if he lives 11 years. The proper payments for such an expectation, from married men not exceeding 50 years of age, will, in the

nearest and most convenient round sums, be four guineas *in annual payments* beginning immediately, and two guineas in hand for every year that his age exceeds his wife's, not admitting any greater excess than 15 years : Or, if the whole value is given in *one present payment*, 40*l.* added to a guinea, for every year that his age falls short of 50, besides the payment just mentioned on account of disparity of age.—For example. Four guineas in annual payments, besides 10 or 20 guineas in hand, according as the age of the husband exceeds the wife's 5 or 10 years. Or, if the whole value of the expectation is given in one payment, 10 guineas added to 40*l.* (that is 50*l.* 10*s.*) from a man whose age is 40; and, in like manner, 20 guineas added to 40*l.* (that is 60*l.*) from a man whose age is 30; besides the payment just mentioned on account of disparity of age.

If money is improved at 4 *per cent.* or, on account of any advantages attending a scheme, may be justly considered as so improved, the full payments for the expectation I have mentioned will be about one eighth, or half a guinea, less in the *annual payments* during marriage; and a quarter less in all the other payments. That is : A married man, *at or under* 50, would, besides three guineas and half in annual payments during marriage, be bound to add a guinea and half for every year he is older than his wife : Or, if he

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chuses

chuses to give the value of his expectation in one payment; besides the common contribution of 30*l.* and a guinea and half for every year his age exceeds his wife's; he would be bound to pay three quarters of a guinea, for every year he is less than 50 years of age; that is, 53*l.* 12*s.* 6*d.* in all, supposing him 40 years of age, and 10 years older than his wife. — All these payments doubled would entitle to double annuities.

There is one particular advantage which societies formed on a plan of this kind would enjoy (*a*) — Persons who know themselves subject to disorders, which are likely to render them short-lived, will have no great temptations to endeavour to gain admission into such societies; and, if admitted, the danger from them will be less than on any other plan. Were it not for this danger, one might recommend the following plan, as one of the most inviting.

In the plans hitherto mentioned it is implied, that, if either a member or his wife dies within any of the periods specified, the additional annuities, that would otherwise have become due, will be lost. But it would be much more agreeable to a purchaser, that they should be made certain to his wife, provided she lives to the end of these periods,

(*a*) See another advantage mentioned under Quest. VIII, p. 28.

tho' in the mean time his own life should fail. The value of such annuities may be computed by the rule in Quest. IX.

Suppose, for instance, the *scheme* to be “that a wife shall be entitled certainly to a life-annuity of 20*l.* the first payment of which shall be made at the end of 12 years, provided she should be then alive, and her husband dead; or at the end of any year beyond this term in which she may happen to be left a widow.” Suppose it also stipulated, “that she shall be entitled to 10*l.* more, or 30*l.* in all, on the same terms, provided she should live 16 years.” —The value of such an expectation (interest being at 3 *per cent.* and the probabilities of life as in Mr. *De Moivre's* Hypothesis) will be, in the most convenient round sums, supposing none admitted above 50 years of age, seven guineas in annual payments to be continued during marriage, and to begin immediately; besides four guineas in present money for every year, as far as 15 years, that the husband's age exceeds the wife's, if he is between 40 and 50, and three guineas on the same account if he is under 40: Or, if the whole value of the expectation is given in one present payment, 70*l.* added to a guinea and half, for every year that the husband's age falls short of 50, besides the payment just mentioned on account of disparity of age,

If the annuities are made to be annuities during *widowhood*, and not during *life*, and the advantage arising from hence, is supposed equivalent to the difference between the improvement of money at 4 *per cent.* and its real improvement; the value of the expectation just mentioned, (that is, its value at 4 *per cent.*) will be six guineas in annual payments; besides three guineas in present money, for every year that the husband's age exceeds the wife's, if he is between 40 and 50; and 2 guineas, if he is under 40: Or, if the whole value of the expectation is given in one present payment, 56*l.* added to 1*l.* 5*s.* for every year that his age falls short of 50, besides the payment last mentioned on account of inequality of age. (a)

(a) Supposing 16 years the only term, the annuity 20*l.* and interest at 4 *per cent.* the proper payments will be nearly, in the case of equal ages and *single* payments, 46*l.*—40*l.*—29*l.* as the age of the man is 30, 40, or 50. Or, in *annual* payments, 1.3.80.—1.3.66.—1.3.13.—Supposing the woman's age 10 years less than the man's, the same values will be, in *single* payments, 1.58.92.—1.56.56.—1.53.66.—In *annual* payments 1.4.63.—1.5.—1.5.41.—It appears, therefore, that a society, supposing money improved at the rate of 4 *per cent.* might entitle all married men *indiscriminately*, who are under 50 years of age, to such an expectation as this for their wives, for either 60*l.* in *one* payment, or five guineas in *annual* payments. — But equity requires, that different payments should be made, according to the different comparative ages of men and their wives; and Tables might be formed for shewing, at one view, what these different payments ought to be in all cases. If such Tables are wanting, recourse must be had to some such easy rules as those I have stated above.

He that will give himself the trouble to calculate, agreeably to the directions in the Questions to which I have referred, will find that, taking all particular cases together, the rules now given come as near the truth as there is reason to desire in an affair of this nature, the *defects* in some cases being nearly compensated by the *excesses* in others.

I have calculated here, as well as in most other places, from Mr. *De Moivre's Hypothesis*, because its conformity to the three Tables which I have so often mentioned, convinces me, that it gives a proper *medium* between the different values of *town* and *country* lives. In the country the probabilities of life are much higher; but in *London*, and probably in all *great* towns and some *smaller* ones, they are much lower.

It is proper to add, that, according to the values of lives and survivorships deduced both from the *London* and Dr. *Halley's* Table, and taking interest as low as 3 *per cent.* all women whose husbands are under 50 years of age, might be entitled to an annuity of 24*l.* during *life* (the first payment to be made at the end of the year in which they shall be left widows) for the sum of 100*l.* supposing 3*l.* additional given on account of every year that they are younger than their husbands.— At 4 *per cent.* an annuity of 30*l.* might be granted on the same terms.

In the year 1690, the company of *Mercers* in *London*, adopted such a scheme as that last mentioned. For 100*l.* in one present payment, they entitled every subscriber to a life annuity for his widow of 30*l.*; and this, at that time, (when money bore 6 per cent. interest) was considerably less than the value of the money advanced, supposing men and their wives of equal ages. As the interest of money sunk, they sunk also the annuity, first to 25*l.* and then to 20*l.* and 15*l.* But, at last, after carrying on the scheme for above 50 years, finding the burden of the annuitants too heavy, and likely to go on increasing, they were obliged to drop the scheme and to stop payment. In a little time, however, by a parliamentary aid of 3000*l. per ann.* which they are now enjoying, they were restored to a capacity of making good all their engagements, and of paying their arrears.— Their failure, is, indeed, much to be lamented; for, in consequence of it, the public has lost the benefit of an institution, that for many years promised the happiest effects, by encouraging marriage, and affording relief to indigence. The rapid fall of the interest of money; their admitting purchasers at too advanced ages; and, particularly, their paying no regard to the difference of age between husbands and their wives, must have contributed much to hurt them. Some of the principal causes, therefore, which have rendered them

them unsuccessful, may be now avoided; and for this reason I should be glad to see some similar scheme, providing, as this did, annuities for *life*, and not for *widowhood*, undertaken. If well planned it would, I think, be a proper object of *parliamentary* encouragement.

It must, however, be remembered, that the issue of the best schemes of this kind must be in some degree uncertain. For want of proper observations, it is not possible to determine what allowances ought to be made, on account of the higher probabilities of life among females than males. No prudence can prevent all losses in the improvement of money; nor can any care guard against the inconveniencies to such schemes, which must arise from those persons being most ready to fly to them who, by reason of concealed disorders, feel themselves most likely to want the benefit of them.

The societies, therefore, on which I have remarked in the first Section of this chapter, would have reason to take warning from what has happened to the *Mercer's Company*, were the schemes on which they are formed perfectly unexceptionable. But I have demonstrated that these schemes are very defective; and that the longer they are carried on, the more mischief they must produce. 'Tis vain (as appears from *Quest. III.*) to form such establishments with the expectation of see-
ing

ing their fate determined soon by experience. If not more extravagant than any ignorance can well make them, they *will* go on prosperously for 20 or 30 years; and, if at all tolerable, they *may* support themselves for 50 or 60 years; and at last end in distress and ruin. No experiments, therefore, of this sort should be tried hastily. An unsuccessful experiment must be productive of very pernicious effects. All inadequate schemes lay the foundation of *present* relief on *future* calamity, and afford assistance to a *few* by disappointing and oppressing *multitudes*.

As the persons who conduct these schemes can mean nothing but the advantage of the public, they ought to listen to these observations. At present their plans are capable of being reformed; but they cannot continue so always; for the greater number of exorbitant payments they now make to annuitants, the more they consume the property of future annuitants, and the less practicable a retreat is rendered to a rational and equitable and permanent plan (a). They should, therefore, *immediately* (b) either *reduce*

(a) See p. 82, 83. Sect. I.

(b) Thus; was the *London Annuity Society* to make their lowest annuity 10*l.* the next 20*l.* and the highest 30*l.* they would probably be safe. But, after proceeding on their present plan some years longer, such a reduction would by no means be sufficient.

their schemes, or change them into one of those which I have proposed. But, I am afraid, this is not to be expected. The neglect with which they have received some remonstrances that have been already made to them, gives reason to fear, that what has been now said will be in vain; and that those who are to come after them, must be left to *rue* the consequences of their mistakes.

S E C T. IV.

Of Schemes for providing Annuities for Old Age.

A General disposition has lately shewn itself, to encourage schemes for granting *annuities* to persons in the latter stages of life; and this has occasioned the 6th Question in the former Chapter; and, as a further and more particular direction in cases of this kind, I have thought it necessary here to give the following Table.

Valuo

Values of 1, per ann. for life, after 50, to persons whose ages are	Values in one present payment, interest 4 per cent.	Interest 3 per cent.	Values in annual payments, till 50, to begin at the end of a year, interest 4 pr ct.	Interest 3 per cent.
10	1.235	2.015	.0789	.113
15	1.583	2.444	.106	.146
20	2.028	2.989	.146	.193
25	2.594	3.644	.203	.259
30	3.369	4.508	.297	.366
35	4.446	5.667	.466	.559
40	5.953	7.232	.822	.950

Values of the same annuity, after 55, to ages				
30	2.114	2.937	.167	.211
35	2.722	3.632	.241	.297
40	3.732	4.708	.394	.464
45	5.088	6.115	.703	.803

Values of the same annuity after 60, to ages				
35	1.667	2.290	.135	.168
40	2.234	2.923	.203	.245
45	3.043	3.811	.327	.384
50	4.255	5.061	.600	.679

The numbers in the 2d and 3d columns of this Table, multiplied by any annuity, will give the value of that annuity in a *single* pay-

payment, to be enjoyed for life, by the ages corresponding to those numbers in the first column, *after* the age mentioned at the head of that column.—And in the same manner; the numbers in the 4th and 5th columns will give the values in *annual* payments.—Thus. The value of 44*l.* *per annum*, to be enjoyed for life, after 50, by a person now 40, (interest at 4 *per cent.*) is 5.95, multiplied by 44, or *l.* 261.9, in a *single* payment; and .822, multiplied by 44, or *l.* 36.16, in *annual* payments till 50, the first payment to be made at the end of a year.

In order to find the same values, partly in *annual payments*, and partly in any given *entrance* or *admission-money*; say; “As the value of the given *annuity* in a *single* payment, (found in the way just mentioned) is to the given *entrance-money*; so is its value in *annual payments*, to a fourth proportional; which, subtracted from the value in *annual payments*, the *remainder* will be the annual payment due, over and above the given *entrance-money*.”

EXAMPLE.

Suppose a person now 40, to be willing to pay 200*l.* entrance-money, *besides* such an annual payment for 10 years as shall, together with his entrance-money, be sufficient
to

to entitle him to a life annuity of 44*l.* after 50. What ought the annual payment to be?

ANSWER.

L.8.55. — For, *l.*261.9, is to 200*l.* as *l.*36.16, to *l.*27.61; which, subtracted from *l.*36.16, the remainder is *l.*8.55.

This Table has been calculated from the *probabilities* and *values* of lives in Tables III. and VI. The probabilities of life among the inhabitants of *London*, are (as I have often had occasion to observe) much lower than among the generality of mankind; and the values in the preceding Table, had they been given agreeably to the *London* Observations, would have been less. But, certainly, an office or society, that means to be a permanent advantage to the public, ought always to take higher rather than lower values, for the sake of rendering itself more secure, and gaining some *profits* to balance *losses* and *expences*.

There have lately been established, in *London*, several societies for granting such annuities as those now mentioned; and he that will compare their true values, as they may be learnt from the preceding Table, with the *terms* of admission into these societies, as given in their printed *Abstracts* and
Tables,

Tables, must be surpris'd and shock'd. They are all impositions on the public, proceeding from ignorance, and encouraged by credulity and folly.

It has been shewn; that the proper payment, (allowing compound interest at 4 *per cent.*) for an annuity of 44*l.* to be enjoyed by a person now 40, for what may happen to remain of his life after 50, is 200*l.* in *admission-money*; besides 1.8.55, or 8*l.* 11*s.*, in annual payments till he attains to 50, the first of these payments to be made at the end of a year. — The conditions of obtaining this annuity, according to the *Tables of the Laudable Society of Annuity-takers for the benefit of age*, are 76*l.* 17*s.* in *admission-money*; and 6*l.* 14*s.* in *annual payments*. — According to the *Tables of the society of London Annuity-takers for the benefit of age*, the conditions of obtaining the same annuity are 30*l.* in *admission-money*, and 10*l.* in *annual payments*. — The *Equitable Society of Annuity-takers* requires for the same annuity 38*l.* 10*s.* in *admission-money*, and 13*l.* in *annual payments*. The true value is, over and above the *admission-money* just mentioned, an *annual payment* of 30*l.* 17*s.* (interest reckoned at 4 *per cent.*) or an *annual payment* of 36*l.* 15*s.* interest reckoned at 3 *per cent.* — The *London Union Society for the comfortable support of aged members* promises an annuity of no less than 50 guineas for life, after 50,

to a person now 40 for 40*l.* 10*s.* in admission-money, and 7*l.* in annual payments.

The *Amicable Society of Annuitants for the benefit of age*, promises an annuity of 26*l.* per annum, for life, to a person now 40, after attaining to 50, for 28*l.* 16*s.* in admission-money, and 6*l.* in annual payments. — The true value of this annuity is 28*l.* 16*s.* in admission-money, and 17*l.* 8*s.* in annual payments; (interest supposed at 4 per cent.); or the same sum in admission-money, and 20*l.* 18*s.* in annual payments, interest supposed at 3 per cent.

The *Provident Society for the benefit of age* promises an annuity of 25*l.* to a person now 40, after attaining to 50, for 34 guineas in admission-money, and eight guineas in annual payments. The true value is, 34 guineas in admission-money; and 15*l.* 12*s.* in annual payments, interest at 4 per cent.; or, the same sum in admission-money, and 19*l.* in annual payments, interest being at 3 per cent. (a).

But I will not tire the reader, by going, in this manner, thro' the schemes of all these societies. The contrivers of them, it is certain, can know nothing of the principles on which the rule in *Quest. VI*, and the demonstration of it in the *Appendix* is founded; and, therefore, if unwilling to be guided by the autho-

(a) The account here given of the terms on which a person whose age is 40, is admitted into these societies, I have taken from their printed Tables as they stood at the end of the year 1770. — In the younger ages the deficiencies are greater.

rity of mathematicians, it may not be possible to convince them of their mistakes. I will, however, offer to them the following demonstration, which will be understood, without difficulty, by every one who knows how to compute (a) the increase of money at compound interest.

The value of a life at 50, (interest being at 4 *per cent.*) is $11\frac{1}{3}$ years purchase by Table VI. For an annuity, therefore, of 44*l. per annum* for life, to be enjoyed by a person at this age, 498*l.* ought to be given. *Two in three* of a number of persons at the age of 32 will, (by Tables III, IV, and V,) live to 50; and therefore, in order to be able to pay an annuity to them of 44*l.* for life, after 50, the money now advanced by every *three*, ought to be such as will, in consequence of being laid up to be improved, increase in 18 years to double 498*l.* or to 996*l.*—From the preceding Table it may be learnt, that the money which ought to be advanced by every single person is 165*l.* or by *three* persons 495*l.* and this, in 18 years, will double itself, or increase to just the sum that will then be the value of the annuities to be paid.—But the money required in this case by the *Laudable Society*, is 14*l.* 11*s.* 9*d.* from each member at admission, besides an *annual* payment of 4*l.* The admission-money, therefore, of two members, being 29*l.* 3*s.* 6*d.*

(a) The easiest method of doing this, is taught in the rules annexed to the Tables in the APPENDIX.

may

may be increased to twice this sum, or to 58*l.* 7*s.* An annual payment of 4*l.* for 18 years will, if perfectly improved at 4 *per cent.* compound interest, increase to 102*l.*; and two such annual payments will increase to 204*l.*

The whole pay, therefore, of *two* members will produce at the end of 18 years 262*l.* 7*s.* — A third part, I have said, will die without attaining to 50, and these will live one with another 10 years. An annuity of 4*l.*, for this time, will produce a capital of 48*l.* and this capital improved for eight years more will increase to 66*l.* The whole profit, therefore, from the member who will die is, his admission-money doubled and added to 66*l.* or 95*l.* 3*s.* 6*d.* And this sum added to 262*l.* 7*s.* makes 357*l.* 10*s.* 6*d.* the *whole* money with which the society can be provided, at the end of 18 years, to bear the expence of *two* life-annuities, worth together 996*l.*

By a similar computation it may be found, that the improvement of money at only 3 *per cent.* will *sink* the *former* sum to 329*l.* at the same time that the value of the *annuities* will be *raised* to 1100*l.*

The deficiencies in the schemes of all the other societies, except the *Provident Society*, are no less considerable (a). — What confusion

(a) Some of these societies tell us, that the payments on admission shall increase, as the number of members increases; and they have practised on this rule just as if

sion then must they produce some time or other? How barbarous is it thus to draw money

the value of an annuity was nothing determinate in itself, but depended on the number of persons who have been purchasers. But the true design may perhaps be, to quicken the public in their applications.

Should any of these societies, sensible of their mistakes, resolve to reform themselves, they ought to consider, that this cannot be done by only obliging *future* members to pay the just values of the annuities promised them. All the *present* members must likewise, besides raising their payments, make compensation for what they have hitherto paid too little; and this compensation is to be calculated in the following manner. — “ Find the whole amount to the present time of the payments which have been made. Subtract this from the whole amount of the payments which *should* have been made; and the remainder will be the compensation required.”

EXAMPLE. In the *Laudable Society of Annuityants*, the condition of a title to 44*l.* per annum for life, after 50, to a person at the age of 40, was, 4 years ago, 34*l.* 17*s.* in admission-money, besides an annual payment of 6*l.* 14*s.* till he attained to 50.—The admission-money will, (reckoning compound interest at 3 per cent.) amount in four years to 39*l.* 4*s.* and the annual payment to 28*l.* The whole amount, therefore, of the payments of a member admitted 4 years ago, is 67*l.* 4*s.*—But the value of the annuity was 37*l.* 4*s.* in annual payments, besides 34*l.* 17*s.* in admission-money; and these payments, during the 4 years, would have amounted to 195*l.* The difference, therefore, between these two amounts, or 127*l.* 16*s.* is the *compensation* which such member ought to pay; and if he continues a member without paying it, (besides raising his annual contribution to 37*l.* 4*s.*) he must either lose his annuity, or owe it to injustice.

I have taken interest here at 3 per cent. because I think these societies cannot reasonably depend on always improving the money they receive at a higher rate.

Since I writ the above, I have found, that the admission-money required by this society has lately received another

ney from the public by promises of advantages that *cannot* be obtained? Have we not already suffered too much by *bubbles*; and, if nothing else can check the frenzy that encourages them, ought not the legislature to interpose its authority?

I do not, however, mean to condemn all institutions of this kind. They may be very useful, if the full values are taken, and proper care is used in the *improvement* of money. Interest, in these cases, ought not to be reckoned higher than 3 *per cent.* and, supposing money improved at this rate, a person, for a single payment of 50*l.* before he is 40, might be entitled to a life-annuity of 10 guineas *after* 55; or, if he chuses it, to a life-annuity of 17*l.* *after* 60. But if he pays the same sum before he is 34, he might be entitled to a life annuity of 14*l.* *after* 55, or 22*l.* *after* 60. 25*l.* might purchase for him *half* these annuities; and 100*l.* *double.*

A society or office that would go on this plan, might do great service. Persons in

another advance. At the age of 40, in particular, it is advanced to 108*l.* 7*s.*—when they have further either advanced the admission-money to *double* this sum, or *tripled* the annual payments, they will be *almost* right with respect to this particular age, provided the *compensation-money*, just mentioned, has been paid.

These societies, tho' their plans are so insufficient, may, after beginning their payments to annuitants, continue them 15, or, perhaps, 20 years; but it will be by robbing all the younger members.

the lower stations of life might be brought to a habit of industry, in the beginning of life, by striving to get 25*l.* or 50*l.* beforehand in order to purchase such annuities, and thus to make provisions for themselves in the more advanced parts of life, when they will be incapable of labour.

There are now established in *Holland* some institutions of this kind.—Any poor persons there, I am informed, who can, before they attain to a particular age, lay up 50*l.* may make use of it in buying for themselves a right to be admitted, when 50, or at any time afterwards, to houses prepared on purpose, for providing them with all the conveniencies of lodging and board. This is an excellent institution; and I wish there was some imitation of it in this kingdom.

Considerable profits would, in this, case be received, from the payments of *some* who would chuse to *delay* going into such houses; and of others who would grow rich enough to be above them.

It is proper to observe here, that institutions of this kind would furnish one of the *safest* ways of providing for widows.—A married man might, by paying 100*l.* before his wife attained to 40, entitle her, after 55, or 60, to a life annuity of 2*l.* or 3*l.* Or, by paying the same sum before she attained to 34, he might entitle her, after the same ages,

to a life annuity of 28*l.* or 44*l.* (a); and in this case he would have a chance of sharing himself in the benefit of the annuity.

I have called this the *safest* way of providing for widows, because attended with none of the dangers arising from disproportion of age between men and their wives, and from the admission of persons labouring under concealed distempers.

I cannot conclude this Section, without mentioning the following plan of a provision for Old Age.

Let 13 guineas be given as *entrance-money*; and let besides 1*l.* 2*l.* 3*l.* 4*l.* &c. be given at the beginning of the 1st, 2d, 3d, 4th, &c. years, as the payments for these years respectively; and let the last payment be 16*l.* at the beginning of the 16th year. All these payments put together will, according to the probabilities of life in the 3d, 4th and 5th Tables, (interest being at 4 *per cent.*) entitle a person, whose age was 40 when he begun them, to an annuity, after 15 years, beginning with 15*l.* and increasing at the rate of 1*l.* every year, 'till, at the end of 15 years more, or (b) when he has attained to 70, it

(a) The same payment before 30, would entitle to an annuity of 22*l.* after 50.

(b) According to the probabilities of life in the *London* Table, this annuity should be greater.—A *Theorem* for finding what the annuity ought to be in these cases, is given in the Appendix, Note (I).

becomes a standing annuity of 30*l.* for the remainder of his life.

If the addition of three guineas is made to the *entrance-money*, for every year that any life between 30 and 40 falls short of 40, the value will be obtained nearly, of the same annuity to be enjoyed by that life, after the same number of years, and increasing in the same manner, 'till, in 30 years, it becomes *stationary* and *double*.—This plan is particularly inviting, as it makes the *largest* payments become due, when the *near* approach of the annuity renders the encouragement to them *greatest*; and as, likewise, the annuity is to increase continually with age, till it comes to be highest (*a*), when life is most in the decline,

(*a*) The lower part of mankind are objects of particular compassion, when rendered incapable, by accident, sickness, or age, of earning their subsistence. This has given rise to many very useful societies among them, for granting relief to one another, out of little funds supplied by *weekly* contributions. A society of this kind, formed on the following plan, would probably thrive, and might, on some accounts, be even more useful than the institutions in *Holland*, mentioned in p. 118.

Let the society, at its first establishment, consist of 100 persons, all between 30 and 40; and whose mean age may therefore be reckoned 36; and let it be supposed to be always kept up to this number, by the admission of new members, between the ages of 30 and 40, as old members die off. Let the contribution of each member be four-pence *per* week, making, from the whole body, an annual contribution of 85*l.* 17*s.*—Let it be further supposed, that seven of them will fall every year into disorders, that shall incapacitate them for seven weeks.—

cline, and when therefore it will be most useful. — It is further a recommendation of this plan, that less depends in it on the *improvement* of money than in most other plans. — But I must leave these hints to be pursued by others.

S E C T. V.

Of the Amicable Society for a perpetual Assurance Office: And the Society for equitable Assurances on Lives and Survivorships.

THE 10th Problem has been given, with a particular view to the corporation of the *Amicable Society*, for a perpetual Assu-

30*l.* 12*s.* of the annual contribution will be just sufficient, to enable the society to grant to each of these 12*s.* *per week*, during their illnesses. And the remaining 55*l.* *per annum*, laid up and *carefully* improved, at 3½ *per cent.* will increase to a capital that shall be sufficient, according to the chances of life in Tables III, IV, and V, to enable the society to pay to every member, *after* attaining to 67 years of age, or *upon* entering his 68th year, an annuity, beginning with 5*l.* and increasing at the rate of 1*l.* every year for seven years, 'till, at the age of 75, it came to be a standing annuity of 12*l.* for the remainder of life.

Were such a society to make its contribution *sevenpence per week*, an allowance of 15*s.* might be made, on the same suppositions, to every member during sickness; besides the payment of an annuity beginning with 5*l.* when a member entered his 64th year, and increasing for 15 years, 'till, at 79, it became fixed for the remainder of life at 20*l.*

If the probabilities of life are lower among the labouring poor, than among the generality of mankind, this plan will be so much the more sure of succeeding.

rance-

rance-office on single lives, kept in *Serjeant's-Inn*. This society was established in 1706, and is the only one I am acquainted with, which has stood any considerable trial from time and experience. The annual payment of each member used to be 6*l.* 4*s.* payable quarterly; but it has been lately reduced to 5*l.* The whole annual income, hence arising, is equally divided among the *nominees*, or heirs of such members as die every year; and this renders the dividends among the *nominees* in *different* years, more or less, according to the number of members who have happened to die in those years. But the society now engages, that the dividends shall not be *less* than 150*l.* to each claimant, tho' they may be *more*. — None are admitted whose ages are *greater* than 45, or *less* than 12; nor is there any difference of contribution allowed on account of difference of age.

This society has, I doubt not, been very useful to the public; and its plan is such, that it cannot well fail to *continue* to be so. It might, however, certainly have been much more useful, had it gone from the first on a different plan. It is obvious, that regulating the dividends among the *nominees* by the number of members who die every year, is not *equitable*; because it makes the benefit which a member is to receive to depend, not on the value of his contribution, but on a *contingency*; that is, the number of members that shall happen to die the same year

year with him. This regulation must also have been disadvantageous to the society; as will appear from the following account of the natural progress of the affairs of such a society, when established on a right plan.

Suppose a *thousand* persons, whose common age is 36, to form themselves into a society for the purpose of *assuring* a particular sum at their deaths, to such persons as they shall name, in consideration of a particular annual-contribution to be continued during their lives. Suppose the annual contribution to be 5*l.* and the first payment (*a*) to be made immediately. Suppose, likewise, the original number of the society to be constantly kept up by the admission of new members, at 36 years of age, in the room of such as die.—In Quest. X. p. 33, it appears, that an annual payment, beginning immediately, of 5*l.* during a life at the age of 36, should entitle, at the failure of such a life, to 172*l.* reckoning interest at 4 *per cent.* and taking Mr. *De Moivre's* valuation of lives.—A *thousand* persons, all 36 years of age, will die off at the rate of 20 every year. The disbursements, therefore, of such a society will be, the first year, 20 times 172*l.* or 3,440*l.* and its income will be 5000*l.* It will, therefore, at the end of the year, have a surplus

(*a*) Such payments, it has been shewn, Quest. VIII. p. 28, are better than any *half* yearly or *quarterly* payments, and at the same time they save some trouble.

of 1560*l.* to put to interest. — In consequence of the yearly accessions to supply vacancies, the number dying annually will be always increasing after the first year. In 50 years it will attain to a *maximum*; and then, the affairs of the society will become *stationary*, and the number dying annually will be 40, and its annual expence will be 6,880*l.* exceeding the annual contribution 1,880*l.* But, in the mean time, by improving its surplus moneys, it will have raised a capital equal to this excess, and, consequently, its affairs will be fixed on a firm basis for all subsequent times.

Suppose now, that such a society, at its establishment, should resolve to divide its whole yearly income among the *nominees* of deceased members. The effect of this would be, that no capital could be raised; that the dividends payable to *nominees* would diminish continually, till, at the time that the greatest number of members came to die *annually*, or at the end of 50 years, they would be reduced to half; and all claimants, after this period, receive too little, because the first claimants had received too much (a).

At

(a) The reverse of this will take place, if such a society *begins* with admitting all at all ages, and afterwards changes its plan, and *limits* the age of admission. In this case, the number of *yearly deaths* will be *greatest* at first, and the *dividends smallest*. In consequence of altering its plan, the *yearly deaths* will lessen gradually, and the *dividends* rise; but in time *both* would return again to their original state.

The

At the time of the institution of the *Amicable Corporation*, the interest of money was at 6 *per cent.* and, as they admit all between 12 and 45, the mean age of admission cannot probably be so great as 36. It appears, therefore, that had they avoided the error now mentioned, and gone from the first on the plan I have described; they might have all along paid to each *nominee* 172*l.* besides raising a capital much greater, in proportion to the number of members, than that I have specified; by the help of the excess of their annual payments above 5*l.* and some other advantages which they have enjoyed (a). Indeed, I cannot doubt but that, with these advantages, they might, before this time, have found themselves able to pay at least 200*l.* to each *nominee*; and at the same time

The following facts incline me to suspect, that this remark may be applicable to the *Amicable Corporation*.

First. In their *original charter*, as it is given in their printed abstracts, there is no limitation of age mentioned; but 31 years afterwards, I find a bye-law made against admitting any person who should be above the age of 45, or under 12. — Secondly. In their printed advertisements in 1770, it is said, that in 59 years they had paid, among 3643 claimants, 378,184*l.* from whence it follows, that tho' the average of their dividends, for the last 17 years, has been 154*l.* the same average, for 59 years, is only 104*l.*

(a) A surplus from a *thousand* members of only *five shillings per annum*, duly improved, at 4 *per cent.* would, in 41 years, produce a capital of 25,000*l.*

restricted

restricted themselves, as they now do, to an annual payment of 5*l.* (a).

I have already mentioned one instance in which the plan of this society is not equitable. Another instance of this is, their requiring the same payments from all persons under 45, without regarding the differences of their ages; whereas, the annual payments of a person admitted at 45, ought to be double the annual payment of a person admitted at 12.

Further. The plan of this society is so narrow, as to confine its usefulness too much. It can be of no service to any person whose age exceeds 45. It is, likewise, far from being properly adapted to the circumstances of persons, who want to make assurances on their lives, for only short terms of years.—Thus; the true value of the assurance of 150*l.* for 10 years, on the life of a person whose age is 30, is, by *Quest. XIV.* (interest being at 3 *per cent.*) 2*l.* 13*s.* in annual payments for 10 years, to begin at the end of the first year; and subject to failure when the life fails. But such an assurance could not be made, in this society, without an annual payment of 5*l.*—Neither is the plan of this society at all

(a) It should be remembered, that all this is said on the supposition, that proper care has been taken to keep out unhealthy persons; and that the probabilities of life among the members of this society, are the same with those in the 3d, 4th, and 5th Tables, in the *Appendix.*

adapted to the circumstances of persons, who want to make assurances on particular survivorships. — For example. A person possessed of an estate, or salary, which must be lost with his life, has a person dependent upon him, for whom he desires to secure a sum of money, payable at his death. But, he desires this only as a provision against the danger of his dying *first*, and leaving a wife, or a parent, without support. In these circumstances, he enters himself into this society; and by an annual payment of 5*l.* entitles his *nominee* to 150*l.* In a few years, perhaps, his *nominee* happens to die; and, having then lost the benefit he had in view, he determines to forfeit his former payments, and to withdraw from the society. In this way, probably, this society must have gained some advantages. But the right method would have been, to have taken from such a person the true value of the sum assured, “on the supposition of non-payment, provided he should survive.” In this way he would have chosen to contract with the society; and had he done this, he would have paid for the *assurance*, (supposing interest at 3 *per cent.* his age 30, the age of his *nominee* 30, and the probabilities of life as in the 3d, 4th, and 5th Tables) 3*l.* 8*s.* (a) in annual payments, to begin immediately, and
to

(a) The value of 150*l.* payable at the death of a person, aged 30, provided he survives another person of the same

to be continued during the *joint* continuance of his own life, and the life of his *nominee*.

All these objections are removed by the plan of the society kept in *Nicholas-Lane, Lombard-Street*, which has justly stiled itself the Society for *Equitable Assurances on Lives and Survivorships*. This Society, if due care is taken, may prove a very great public benefit. It was founded, in consequence of proposals which had been made, and lectures, recommending such a design, which had been read by Mr. *Dodson*, the author of the *Mathematical Repository*. It assures any sums, or reversionary annuities, on any lives, for any number of years, as well as for the whole continuance of the lives, at rates settled by particular calculation, and in any manner that may be best adapted to the views of the persons assured. That is; either by making the assured sums payable *certainly* at the failure of any given lives; or on *condition* of survivorship; and also, either by taking the price of the assurance in *one present payment*; or in *annual payments*, during any single or

same age, is, by *Quest. XI. Chap. I. l. 45.65*; and this value divided by 13.43, (the value increased by unity, of two joint lives both 30) gives *l. 3.4*, or *3l. 8s.* — The value of the same reversion, according to the probabilities of life in *London*, is, *l. 49.19*, in *one payment*; and *4.16*, in *annual payments*, during the joint lives, the first payment to be made immediately.

joint

joint lives, or any terms less than the whole continuance of the lives.—In short; the plan of this society is so extensive, and so important, that I cannot satisfy my own mind, without offering to the gentlemen concerned in the direction of it, the following observations, hoping they will not think them impertinent or improper.

First. They should consider what distress would arise from the failure of such a scheme in any future time; and what dangers there are, which ought to be carefully guarded against in order to secure success. I have already more than once observed, that those persons will be most for flying to these establishments, who have feeble constitutions, or are subject to distempers, which they know render their lives particularly precarious; and it is to be feared, that no caution will be sufficient to prevent all danger from hence.

Again. In matters of chance, it is impossible to say, that an unfavourable run of events will not come, which may hurt the best contrived scheme. The calculations only determine probabilities; and, agreeably to these, it may be depended on, that events will happen on the whole. But at particular periods, and in particular instances, great deviations will often happen; and these deviations, at the commencement of a scheme, must prove either very favourable, or very unfavourable.

But further. The calculations suppose, that all the monies received are put out immediately to accumulate at compound interest. They make no allowance for losses, or for any of the expences attending management. On these accounts, the payments to a society of this kind, ought to be somewhat more than the calculations will warrant. The interest of money ought to be reckoned low; and such Tables of Observations used as give the highest values. Mr. *Dodson*, I find, has paid due attention to all this, by reckoning interest, in his calculations for this society, at 3 *per cent.* and taking the lowest of all the known probabilities of life, or those deduced from the *London* bills of mortality (*a*). There is, besides, a liberty provided of making a call on all the members, in case of any particular emergency. It is, therefore, highly probable, that this society is secure. The last expedient, however, would be a very disagreeable one, should there be ever any oc-

(*a*) It ought, however, to be remembered here, that in selling life-annuities to commence either immediately, or after given terms; and also in some other cases, the values come out *less* in consequence of *lower* probabilities of life. Would it, in *such* instances, be taking an unfair advantage, to estimate the values by the 3^d, 4th, or 5th Table in the Appendix, rather than the *London* Table?— Thus; was the society to sell 20*l.* *per annum*, for life, to a person now 30, after attaining to 50, the value, according to Dr. *Halley's* Table, would, reckoning interest at 3 *per cent.* be 90*l.* in a single payment; but, according to the *London* Table, the value would be only 70*l.*

caſion

caſion for having recourſe to it; and, in order to guard ſtill more effectually againſt danger, it would not, I think, be amiſs to charge a profit of 3 or 4 *per cent.* on all the payments. —Should the conſequence of this prove, that in ſome future period the ſociety ſhall find itſelf poſſeſſed of too large a capital, the harm will be trifling, and future members will reap the advantage. But this leads me to repeat an obſervation of particular conſequence.

As this ſociety is guided in every inſtance, by ſtrict calculation, it is not to be expected that it can meet with any difficulties for many years; becauſe, not till the end of many years after it has acquired its *maximum* of members, will the *maximum* of yearly claimants and annuitants come upon it? Should it, therefore, thro' inattention to this remark, and the encouragement ariſing from the poſſeſſion of a large ſurplus, be led to check or ſtop the increaſe of its ſtock by enlarging its dividends too ſoon, the conſequences might prove pernicious.

Again; I would obſerve, that it is of great importance to the ſafety of ſuch a ſociety, that its affairs ſhould be under the inſpection of able mathematicians. Melancholy experience ſhews, that none but mathematicians are qualified for forming and conducting ſchemes of this kind. —In ſhort; dangerous miſtakes may ſometimes be committed if the

affairs of such a society are not managed carefully and prudently. One instance of this I cannot avoid mentioning.

A person, who desires to assure a particular sum, to be paid at the failure of his life, on condition of the survivorship of another life, may chuse to pay the value in annual contributions during the continuance of his own single life, rather than during the continuance of the joint lives, because the annual contributions, in this case, ought to be much less. But a society that would practise such a method of *assurance* would hurt itself; for, as soon as the life, on whose survivorship the assurance depends, is extinct, the person assured, if then living, would have no longer any benefit in view; and, therefore, would make his payments with reluctance, and, in time, perhaps, entirely withdraw them; the consequence of which would be, that the society would suffer a loss by being deprived of the just value of the expectation it had granted. The plan of a society ought always to be such, as that the losses arising from discontinuance of payment, should fall on the purchaser, and never on the society.

I must not forget to add, that it is necessary, that such a society should be furnished with as complete a set of Tables as possible. This will render the business of the society much more easy, and also much more capable of being conducted by persons unskilled
in

in mathematics. It will also contribute much to its *safety*. For in all cases to which Tables can be extended, there would be no occasion for employing any calculators; and, consequently, a danger would be prevented to which, tho' it is not *now*, it may *hereafter* be exposed; I mean, the danger of happening to trust unskilful, or careless calculators.—Mr. *Dodson*, I find, has furnished this society with some important Tables; and his skill was such, that there is no reason to doubt, but they may be depended on. They have also others which, I believe, are safe and accurate. But there are some still wanting which should be supplied; and all should be subjected to the examination of the best judges, and afterwards published; together with a minute account of the principles assumed, and the method taken in composing them. Such a publication would be a valuable addition to this part of science; and it would also be the means of increasing and establishing the credit of the society.

In Questions 4th, 6th, 10th, 11th, 14th, 15th, and 16th, I have, with a particular view to this society, given rules, by which may be formed every Table it can want, for shewing the values of assurances on the *whole duration*, or any *terms*, of any *one* or *two* lives, in all possible cases; and nothing but care and attention can be necessary to enable any good arithmetician to calculate from them.

Perhaps, this may be as much business as any one society should undertake. Rules, however, for finding the values of *assurances*, in most cases, where the whole duration of any *three* lives is concerned, may be found in Mr. *Simpson's* Select Exercises, from pag. 299 to p. 307; and it is not possible they should follow a better guide.

C H A P. III.

Of PUBLIC CREDIT, and the NATIONAL DEBT.

THE *National Debt* is a subject in which the public is deeply interested. Some observations have occurred to me upon it, which I think important; and for this reason, though foreign to my chief purpose in this work, I cannot help here begging leave to offer them to the reader's attention.

The practice of raising the necessary supplies for every public service, by borrowing money on interest, to be continued 'till the principal is discharged, must be in the highest degree detrimental to a kingdom, unless a plan is settled, for putting its debts into a regular and certain course of payment. When this is not done, a kingdom, by such a practice, obliges itself to return for every sum it borrows infinitely greater sums; and, for the sake of a present advantage, subjects itself to a burden which must be always growing heavier and heavier, 'till it becomes insupportable.

This seems to be now the very state of this nation. At the REVOLUTION, an æra

in other respects truly glorious, the practice I have mentioned begun. Ever since, the public debt has been increasing fast, and every new war has added much more to it, than was taken from it, during the preceding period of peace. In the year 1700, it was 16 millions. In 1715, it was 55 millions. A peace, which continued 'till 1740, sunk it to 47 millions; but the succeeding war increased it to 78 millions; and the next peace sunk it no lower than 72 millions. In the *last* war it rose to 148 millions; and, at a few millions less than this sum it now stands, and probably will stand, 'till another war raises it perhaps to 200 millions.—One cannot reflect on this without terror.—No resources can be sufficient to support a kingdom long in such a course. 'Tis obvious, that the consequence of accumulating debts so rapidly; and of mortgaging posterity, and funding for eternity, in order to pay the interest of them; must in the end prove destructive. Rather than go on in this way, it is absolutely necessary, that no money should be borrowed, except on annuities, which are to terminate within a given period. Were this practised, there would be a LIMIT beyond which the national debts could not increase; and time would do that *necessarily* for the public, which, if trusted to the oeconomy of the conductors of its affairs, might possibly never be done.

This, therefore, is one of the proposals to which, on this occasion, I wish I could engage attention. — I am sensible, indeed, that the *present* burdens of the state would, in this case, be increased, in consequence of the greater present interest, that would be necessary to be given for money. But I do not consider this as an objection of any weight. For let the annuity be an annuity for a 100 years. Such an annuity is, to the present views of men, nearly the same with an annuity for ever; and it is also nearly the same in calculation, its value at 4 *per cent.* being $24\frac{1}{2}$ years purchase, and therefore only half a year's purchase less than the value of a *perpetuity*. Supposing, therefore, the public able to borrow money at 4 *per cent.* on annuities for ever, it ought not to give above 1*s.* 7*d.* *per cent.* more for money borrowed on annuities for 100 years: But should it be obliged to give a *quarter*, or even an *half per cent.* more (a), the additional burdens derived from hence,

(a) These annuities might be kept 18 years without being much diminished in value; for, supposing interest at 4 *per cent.* an annuity for 82 years, is within a 49th part, or 2*l.* in 98*l.* worth as much as an annuity for a 100 years.

Perhaps, in this way of raising money, it might be best to offer a higher interest at first, which should fall to a lower, at the end of given intervals. Thus, tho' $4\frac{1}{2}$ for 100 years is equal in value to 5 *per cent.* for 17 years, and after that 4 *per cent.* for 83 years, yet the latter might appear more inviting.

would

would not be such as could be very sensibly felt; and the advantages, arising from the necessary annihilation of the public debts by time, would abundantly overbalance them.

These advantages would be, indeed, unspeakably great. By such a method of raising money, the expence of one war would, in time, come to be always discharged, before a new war commenced; and it would be impossible, that a state should ever have upon it, at any one time, the expence of many wars; or any larger debts than could be contracted, within the limited period of the annuities: and, consequently, it would enjoy the invaluable privilege of being rendered, in some degree, independent of the management of its finances by ignorant or unfaithful servants.

I must add, that it is by no means necessary, that the limited period of the annuities should be so long as I have mentioned, or 100 years: And that, at any time before the expiration of this period, the public might employ any surplus monies, in extinguishing part of the annuities, by purchasing them for itself at the market price; and thus it might aid the operations of time, and keep its debts within any bounds, that its interest rendered necessary.—Our government has, I know, in some instances adopted the plan now proposed; but it is to be wished that, instead of retracting

retracting (a) it, as was once done, it had been carried much further.

I am, however, far from intending to commend this plan as the best a state can pursue. There is another method of gaining the same end, which is, on many accounts, preferable to it. I mean, “ by providing an
“ annual saving, to be applied invariably,
“ together with the interest of all the sums
“ redeemed by it, to the purpose of discharg-
“ ing the public debts : Or, in other words,
“ by the establishment of a permanent SINK-
“ ING FUND.”

It is well known, that this plan has been also adopted by our government ; but, tho' capable of producing the *greatest* effects in the *easiest* and *surest* manner, it has never been carried into execution. It will abundantly appear from what follows that this observation is just.

Suppose the annual saving to be 100,000*l.* This sum, applied now to discharge an equal debt, bearing interest at 4 *per cent.* will transfer to the public, from its creditors, an annuity of 4,000*l.* At the end of a year, then, there would be a saving of 104,000*l.* which would transfer to the public another annuity of 4,160*l.* and make the saving, at

(a) In the year 1720, the nation was put to the expence of three millions, in order to reduce several long and short annuities then subsisting, to redeemable *perpetuities.*

the end of two years, to be 108,160*l.* — Thus, the original fund would go on increasing, at the same rate with money improved at 4 *per cent.* compound interest. — At the end of three years it would be 112,486*l.* At the end of 18 years, 202,587*l.* Of 36 years, 410,393*l.* and of 95 years (a), 4,151,138*l.* — At the end of 93 years, then, the nation might be eased of above 4 millions *per annum* in taxes; and above 100 millions of its debts would be discharged, gradually and insensibly, at no greater expence than 100,000*l.* *per annum*; and, without interfering with any of the resources of government; or making any other difference, than causing *funds* to be engaged for a course of time to the *public*, that would have been otherwise necessarily engaged to its *creditors*, and which, therefore, must have been entirely useless to it.

It is an observation that deserves particular attention here, that, on this plan, it is of little importance what interest a state is obliged to give for money: For the higher the interest, the sooner will such a fund pay off the principal. Thus; a 100 millions borrowed at 8 *per cent.* and bearing an annual interest of eight millions, would be paid off by a fund, producing annually 100,000*l.* in 56 years; that is, in 39 years less time, than if the same money had been borrowed at 4 *per cent.*

(a) See the Questions annexed to the Tables in the Appendix.

It follows from hence, that reductions of interest would, on this plan, be no great advantage to a state. They would, indeed, lighten its *present* burdens; but this advantage would be balanced, by the addition that would be made to its *future* burdens, in consequence of the longer time, during which it would be necessary to bear them.—I mean this on the supposition, that the savings produced by reductions of interest, are immediately applied to the relief of the state, by annihilating taxes equivalent to them. But if this is not the case; and if, likewise, there is either no plan established for putting the public debts into a certain course of payment, or it is not faithfully carried into execution; in these circumstances, reductions of interest may prove hurtful. For, first, They would only furnish with more money for supplying the deficiencies arising from bad management. And, secondly, As, in such circumstances, they would only *retard*, and not *prevent* the increase of the burdens occasioned by the public debts, a period would come when the affairs of the state would get to a *crisis*; and, at such a period, its danger would be increased, in proportion to the reductions of interest that had been made.

In order to understand this; let us suppose, that a debt, bearing an annual interest of five millions, is the whole debt, which a state can bear without being so much oppressed as
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to be near sinking. Let it, however, be supposed to have still some last resources left, which may enable it to bear, for 23 years to come, this load, together with every additional load, which, during this time, may be necessary to be thrown upon it.—Let it further be supposed, that at this time, the state, urged by the fear of an approaching bankruptcy, resolves upon entering into some effectual measures for preserving itself.—Certain it is, that in this case, no measure *so* effectual can be pursued, as the establishment of a *sinking fund*, and such a faithful application of it as I have explained. Let this then be the measure entered upon; and let the state be supposed capable of providing a fund, producing a million annually. If all the debts bear interest at *6 per cent.* this fund would pay off three fifths of them, within the time I have mentioned; or, in 23 years; and the state might be saved. But if, in consequence of reductions, they bear interest at no more than *3 per cent.* the same fund would not give the same relief, in less than *double* that time; and, therefore, a bankruptcy might prove unavoidable (a).

(a) In some other kingdoms a sponge might be applied in such circumstances, or the funds reduced one half by an act of despotism, without occasioning any convulsions; but this is not possible in this free country; and, it is to be hoped, never will be possible.

I wish

I wish I could think, that there is nothing in this representation, that can be applied to the present state of this nation. The interest of the public debts has been reduced, at different periods, from 6 to 5, and from 5 to 4, and 3 *per cent.*; but still they have grown with rapidity; and we now see ourselves overloaded, and in no way of gaining relief. Had there been no reductions of interest, we should, indeed, have been in the same condition sooner; but, we might have been relieved also sooner, and with less difficulty and danger.

What I have now said implies, that a state always discharges its debts, whatever interest they bear, by paying the original sum borrowed. It may, perhaps, be imagined, that when a loan is under *par*, it may be discharged at a less expence. But this is by no means so practicable as it may seem; for it should be considered, that a public loan, now under *par*, would not long keep so, after being put into a course of payment: And, for this reason, as a state can never be obliged, in redeeming its debts, to pay *more* than the original sum borrowed, so neither ought it to expect, in general, to be able to redeem them by paying *less*. I have said, *in general*; for I am sensible, that at the beginning of the operations of a fund, when its produce is small; and also, in a time of war, a state might derive great advantages from the low price

price of its debts. And I am sensible also, that considerable advantages might be derived from *lotteries* (a), in paying the public debts: But *lotteries* do great mischief in a state, by fostering the destructive spirit of gaming. It is wretched policy to make them familiar, by recurring to them in the ordinary course of government. There are great occasions on which they may be necessary, and for such occasions they should be reserved.

But to return to the subject I have principally in view.

The advantages of putting the public debts into such a course of payment, as I have described, are scarcely to be imagined. It would give a vigour to public credit, which would enable a state always to borrow money easily, and on the best terms. And the encouragement to lenders might be always improved, without any inconvenience, by making every loan irredeemable, during the first 20 or 30 years; for, there could seldom be any occasion, for beginning to discharge any *one* loan sooner.

It might be easily shewn, that the faithful application, from the beginning of the year

(a) Thus; 800,000*l.* of the 3 *per cents.* at 87; or 1,000,000, at 70, might be redeemed with half a million of money, consisting of 50,000 lottery tickets at 10*l.* each, real value; but capable of being sold at 14*l.*, as was done in some of the last lotteries.

1700, of only 200,000*l.* annually, would long before this time, notwithstanding the reductions of interest, and every waste that has been made of the public money, have caused above half the public funds to revert to the public, and paid off above 80 millions of its debts. The nation might therefore, some years ago, have been eased of the greatest part of the taxes with which it is loaded. The most important relief might have been given to its trade and manufactures; and it might now have been in much better circumstances, than at the beginning of the last war; its credit firm; respected by foreign nations; dreaded by its enemies; and ready to punish any insult that could be offered to it. The near view, likewise, of such a period, during the course of the last war, would have given higher spirits to the nation, and encouraged it to bear the expence occasioned by the war with more chearfulness, and to continue it with vigour for two or three years longer; the consequence of which would, probably, have been, gaining a full indemnification from our enemies, and weakening them to such a degree, as would have given us effectual security against them for many years to come.—A new account might also now have been begun; and another fund, not much more considerable, applied in the same way, would, in 60 or 70 years more, have paid, not only all that would

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have been now unpaid, but also, probably, a great proportion of such further debts as must be contracted within this time (a). And thus, without any expence that could be sensibly felt, its debts, as soon as they began to grow heavy, might have been constantly reduced to a *half*, or a *third*; and not only all danger, but all considerable *inconvenience* from them prevented.

All I have now said, supposes a *single* fund with a *general* appropriation to the payment of the public debts. The same ends might be answered by *particular* funds, with small surplusses, appropriated to *particular* debts. In the wars of King *William* and Queen *Anne*, 6 *per cent.* interest was given for all loans. It would have been easy to have annexed to each loan a *fund* producing a *surplus* of 1 *l.* *per cent.* after paying the interest; and such a *surplus* would have been sufficient to annihilate the principal of every loan in 33 years. Had this plan been followed, the disengagement of the public funds, and the relief attending it, would have begun 50 years ago; and the debts contracted, during the reigns

(a) One of the properest objects of taxation in a state is *celibacy*. I doubt not, but that by a fund supplied only from hence, the end I have in view might have been easily accomplished; and, consequently, the very means of paying off the debts of the nation, rendered at the same time the means of increasing its chief strength, by promoting population in it.

of King *William* and Queen *Anne*, would have been all cancelled near 20 years ago, without any of that trouble, tumult and distress, which have been occasioned by reductions of interest, and by the various schemes which have been tried for lessening the debts (*a*). — A fund, yielding 1*l.* per cent. surplus, annexed to a loan at 5 per cent., would discharge the principal in 37 years (*b*). At 4 per cent., in 41 years. At 3 per cent., in 47 years.

These observations relate only to what *might* have been the state of the nation with respect to its debts, had a right plan been pursued from the first. But it will be asked, What can be done with them as they *are*? — I wish I was able to give a more satisfactory answer to this enquiry. Every one must see our prospect to be discouraging, and our state hazardous. Some have thought, that a good

(*a*) The sums to be laid out would, in this case, be so small at first, that it would be proper to employ them in purchasing part of the loan to be annihilated at the prices in the public market; and this, as far as it can be carried, is the most easy and quiet and silent way possible of extinguishing the public debts.

(*b*) I have all along supposed the produce of the public funds to come in yearly. The truth is, that it comes in *half-yearly*; but this gives no advantage in the payment of the public debts worth taking into account. 1*l.* per annum, together with its growing interest, at 4 per cent. taken yearly out of 100*l.* will reduce it to nothing in 41 years; if taken *half-yearly*, it will annihilate the same capital only four months and 12 days sooner. See the questions annexed to the Tables in the *Appendix*.

method might be found out of discharging the national debt, by life annuities. The following observations will shew how vain an imagination this is.

Let us suppose, that 33,333,000*l.* is to be paid off, by offering to the public creditors life-annuities, in lieu of their 3 *per cents.* A life at 60, supposing interest at $3\frac{1}{2}$ *per cent.*, and the probabilities of life as in the *Breslaw, Norwich, and Northampton Tables of Observation*, is worth 11 years purchase. A life at 30 is worth $6\frac{1}{2}$ years purchase. Certainly, therefore, no scheme of this kind would be sufficiently inviting, which did not offer 8 *per cent.* at an average, to all subscribers. Let us, however, suppose, that no more than $7\frac{1}{2}$ is given; and that there are 33333 subscribers, at 1000*l.* stock each, for which a life-annuity is to be granted of 75*l.* or, for the whole stock subscribed, two millions and a half. A million and a half extraordinary, therefore, must be provided towards paying these annuities.

Let us further suppose, that the subscribers are persons between the ages of 30 and 60; and that the numbers of them, at all the intermediate ages, are in the same proportions to one another, with the proportions of the living at these ages, as they exist in the world, or, as they are given in *Tables of Observation*. Let us again suppose, that as these *annuitants* die off, they are immediately

replaced by others, who are continually offering themselves at the same ages, and in the same proportional numbers at these ages, with those of the original subscribers at the time they subscribed; in consequence of which, the whole number of annuitants will be kept always the same. In these circumstances, it will be 30 years, at least, before a number will die off, (a) equal to the whole number; that is, before 33 millions of debts will be annihilated. But had the extraordinary million and half provided for paying these annuities, been employed during this time, in paying off so much of the debt at *par* every year, extinguishing at the same time every year an equivalent tax, 45 millions would have been paid. But had the savings, also, instead of being sunk as they arose, been employed in the same manner, 71 millions would have been paid.

The nation, therefore, must, without doubt, lose greatly by all schemes of this kind; and yet they have been often much talked of; and, indeed, I shall not wonder, should I hereafter see an attempt made to pay off the national debt in this way.

I must beg leave to detain the reader here some time longer. A more particular explanation of this subject, will lead to some

(a) A demonstration of this will be given in the Appendix, note (K).

observations on the best methods of raising money which, I think, deserve to be carefully considered.

When any sum is said to be the *value* of a life-annuity, the meaning is, that, in consequence of being improved at interest, and allowing for the chances of mortality, it will bear the whole expence of the annuity. If, therefore, instead of being *laid* up for improvement, it is either immediately applied to particular uses, or has been long since spent; there will be a loss, equal to the sum which would have been added to the purchase-money, had it been *improved*.—This is the reason of the loss which, I have shewn, the public would suffer by offering *life-annuities*, in lieu of *stock*, in order to extinguish its debts. And for the same reason, it must always lose considerably by *raising* money on life-annuities.

Suppose a million raised by *annuities* on a set of lives, all at 30 years of age. Persons at this age have, (according to Tables III, IV, and V,) an *expectation* of 28 years. That is; the duration of their lives, taking them one with another, will be 28 years; (see the beginning of the first Essay) and they will be entitled, supposing interest at 4 *per cent.* to 7*l. per annum*, for every 100*l.* advanced. For a million then, the public would make 28 payments of 70,000*l.*—Let us suppose next, that
a fund

a fund producing this sum annually, instead of being engaged to pay these life-annuities, is engaged for 28 years, to pay the principal and interest of a million, borrowed on *redeemable* perpetuities, at 4 per cent. There will, at the end of the first year, be a surplus of 30,000*l.*— In consequence of applying this to the extinction of the principal, it will be reduced to 970,000*l.* on which, at the end of the second year, the interest due will be 38,800*l.* There will, therefore, be a saving of 1200*l.* Instead of employing this saving in further sinking the *principal*, which would cause the fund to accumulate in the same manner with money at compound interest, let it be taken and employed in any other way: And let the same be done with all the subsequent savings, reserving only 30,000*l.* annually, for the purpose of sinking the principal. At the end of the second year, the principal will be 940,000*l.*; and the saving of interest upon it, at the end of the third year, 2400*l.* At the end of the 28th year, the principal will be reduced to 160,000*l.* The saving of interest that year will be, 1200*l.* multiplied by 27, or 32,400; and the sum of all the savings will be 453,600*l.*—Deduct from hence 160,000*l.* remaining then undischarged of the principal; and 293,600*l.* will be the loss the public would sustain, in the circumstances I have supposed, by raising money on life-annuities. But if we suppose the savings, as they arise, as well as the con-

stant sum of 30,000*l.* to be applied to the discharge of the principal, instead of being spent on current services; the whole million will be annihilated in 21 years and half; and the loss to the public by life-annuities, will be 6½ years purchase of the annuities; or 455,000*l.*—By similar deductions it may be easily found, that the loss, in *younger* lives, is greater; in *older* lives less; but never inconsiderable, except in the *oldest* lives.

It appears, therefore, that, in consequence of such a way of raising money, the public must always pay much more in interest than there is any occasion for; and *waste* a sum nearly equal to half the principal borrowed (*a*).

This,

(*a*) It is obvious, that the observations here made, may be applied to the common methods of raising money, on life-annuities, for building churches, paving streets, making navigations, &c. &c. And, in general, to all cases where the money received, is not laid up to be improved.—For, to view this subject in another light, let us suppose 10,000*l.* borrowed for any public work, on perpetuities, at 4 *per cent.* And, if that will afford more encouragement, let them be made irredeemable for any number of years less than seventeen. Let us further suppose, such rates, or tolls, established for the payment of the interest and principal, as shall produce *double* the interest of the sum borrowed; or 800*l.* *per annum*, instead of 400*l.* *per annum.* Let the *surplus*, as it comes in *half-yearly*, be laid up to accumulate in the public funds. In 17 years and half, reckoning interest at 4 *per cent.* a capital will be raised, equal to the whole sum borrowed; and, therefore; at the end of that time, the whole debt may be discharged, and the whole transaction finished.—But if the same sum had been borrowed on annuities, for

This, however, tho' so wasteful, is a more frugal way of procuring money than by borrowing on *perpetuities*, without putting them into a course of redemption; for in this case, (if a sponge is not applied) the loss must be *infinite*.

I must add, that these observations are particularly applicable to all the ways of raising money by the sale of reversions.—The public, for instance, might procure a million, by offering for it a fund, that will be disengaged at the end of 18 years; and then produce 80,000*l.* *per annum* for ever. This, supposing interest at 4 *per cent.*, would be the very same with offering *two* millions, 18 years hence, for *one* million now: And a private man, or an *office* for the sale of reversions, might gain by such a transaction; because, the money advanced, in consequence of being improved, might, in 18 years, be more than

the lives of a set of persons 50 years of age, at 8 *per cent.* which is 1*l.* *per cent.* less than the true value of such annuities: Had this, I say, been done, *half* the annuitants would have been alive at the end of the term I have mentioned; (see Tables III, IV, and V,) and the whole transaction, together with the expences and trouble attending the management of it, could not have been finally closed 'till the extinction of all the lives; that is, not in less time, most probably, than 35, or, perhaps, 40 years.—It is a necessary observation here, that, if public credit maintains its ground, much will not depend, in the plan now proposed, on the rise and fall of Stocks. If a *war* sinks them, the money laid out, while the war lasts, will accumulate faster. If a peace raises them, the money that had been previously laid out will be proportionably increased.

doubled.

doubled. But, as the *public* always borrows for immediate services, and never lays up money, it would necessarily lose a sum equal to the whole sum borrowed: And the same money might have been borrowed on a fund, producing 50,000*l.* *per annum*; which would not only pay the interest, but discharge the whole principal in 41 years (a).

By raising money on life-annuities, the *present members* of a state take a heavier load on themselves, in order to exempt *posterity*; and there would be a laudable generosity in this, were it not for the *folly* of it; the same exemption being equally practicable at *half* the expence.—On the other hand. By borrowing on *reversionary grants*, the *present members* of a state exempt themselves *entirely*, by throwing the load *doubled* on posterity; and there is a cruelty and injustice in this that nothing can excuse.

It is well known, that both these methods of raising money have been practised among us. This, however, is, by no means, the worst that has been done. It has been common to borrow money to pay the interest of money borrowed, and thus to give *compound interest* for money; and our *parliaments* have, sometimes, expressly provided, that this shall be done for a succession of years.

(a) The smallness of the sums, which I have here and elsewhere sometimes supposed to be employed in discharging the public debts, can create no difficulties, because there is no sum which may not be applied to this use by purchasing stock.

But

But to return to the main point I had in view.

The enquiry which has occasioned this digression, must be highly interesting to every person who wishes well to his country.—All schemes for discharging the public debts, by life-annuities, have been shewn to be absurd and extravagant.—In general; it may be observed, that it is far from probable, that any money which the nation can spare, if applied so as to bear only *simple* interest, can be capable of reducing its debts within due bounds; or of doing us, in our present circumstances, any essential service. A fund, producing a surplus of even two millions annually, would, when thus applied, pay no more than 40 millions in 20 years; and, in that time, a war might probably come, which would interrupt the application of it; and increase our debts much more than such a fund had lessened them.

Certain it is, therefore, that if our affairs are to be retrieved, it must be by a *fund* increasing itself in the manner I have explained. The smallest *fund* of this kind is, indeed, *omnipotent*, if it is allowed time to operate. But we are, I fear, got so near to the limits of the resources of the nation, that it cannot be allowed much time: And, in order to make amends for this, it is necessary that it should be *large*.—Let us then suppose,
3 that

that the nation is still strong enough to enable it to provide a fund, that shall yield a *million and half annually*, for 20 years to come: And also, that, together with all its *present* burdens, it is capable of bearing every *additional* burden that 20 years more can bring upon it. If this is not true, we have, I think, nothing to do but to wait the issue, and tremble.

A fund, producing annually a million and a half, would increase to three millions *per ann.* in 20 years (*a.*) At the end of this term, the nation might be eased of the most oppressive taxes, to the amount of a million and a half; and the consequence would prove, that, if there should have been a war, either the whole, or much the greatest part of the addition occasioned by it to the public burdens, would be taken off, and the nation reinstated nearly in its present circumstances. But, if there should have been no war, the national debt and the taxes charged with it, would be reduced a third below the sums at which they now stand; and the nation would be so much relieved as to be prepared for a war.—The remaining million and half would,

(*a*) It should be remembered, that in the year 1779, 11. *per cent.* on the consolidated 4. *per cents.* will be annihilated, and that I suppose the savings derived from hence to be taken at that time as a part of the fund.—Methods might be easily contrived for getting this saving immediately, which would be some advantage.

in 23 years, increase again to three millions *per annum*; and then, so much more of the public taxes would be set free; 50 millions more, or 93 millions in all, of the public debts would be discharged, and the difficulties of the nation would be, in a great measure, conquered.—During this whole course of time, there may possibly be but one war; and should that happen, the appropriation at the end of it, of about 400,000*l.* *per annum*, might be enough to answer all purposes.

In these observations, I suppose the 3 *per cents* to be paid off at *par*; and no advantage taken at any time of their low price. By taking this advantage, and with the help of a little management, a fund, producing annually a million and half, might be made to increase to another million and half, in less time than I have assigned. Should there be a war in a few years, the 3 *per cents.* would probably fall below 75; and then the proprietors of them must be glad to part with them at this price; the consequence of which, supposing the war to last eight years, would be, that the fund would double itself, and the nation be relieved in the manner I have mentioned, in 18, instead of 20 years.—The advantage will be the same, supposing the government at such a time to go on in paying off the 3 *per cents* at *par*. For the effect of this would be, that money might be borrowed for the public service on proportionably

ably better terms. Suppose, for instance, that four millions must be borrowed for the service of the year; and let the produce of the fund be then increased to two millions; and the interest of money in the *stocks*, above 4 *per cent*. In these circumstances, it would be the interest of the lenders of money, to take $3\frac{1}{2}$ *per cent*. for the sums they advanced, in consideration of having their 3 *per cents* paid off at *par*, to the amount of half these sums.—War, therefore, would accelerate the redemption of the public debts; and it would do this the more, the longer it lasted, and the higher it raised the interest of money. Or if, in consequence of paying always at *par*, this could not happen; an equivalent effect would be produced in the way just mentioned. The *stocks* would be always kept up by the operations of the fund; and, in proportion to the sums yielded by it, the public would be able to borrow money more advantageously, and less would be added to its burdens.—This seems to me an observation of particular consequence. It demonstrates, that the invariable application, in *war* as well as *peace*, of the produce of the fund I am supposing, to the payment of the national debts, rather than to any current services, would, independently of its effect in (a) redeeming these debts, be attended

(a) So true is this, that a war, were we now engaged in it, would only render the *present* time so much the more

ed with great advantages to the public. But this is a subject on which I shall have occasion to say more presently.

more proper for entering into measures for paying the public debts. And the following observations will put this out of doubt.

As it is now become the practice to have recourse to *lotteries* in *peace*, we may be sure, that no year will pass without them in *war*. I would, therefore, propose, that, instead of making use of them in raising the annual supplies in war, they should be then applied as an aid in discharging the public debts.—Suppose the war to last 10 years, and the 3 *per cents* at 70. Suppose also, each *lottery* to consist of 750,000*l.* in *tickets*, which, when disposed of to subscribers, will bring in 1,050,000*l.* On these suppositions, the whole *loss* to the public, from applying the lotteries to the payment of the public debts, rather than to the current supplies, will be 1,050,000*l.* annually, or 10 millions and $\frac{1}{2}$ in all.—The *gain* will be as follows. 750,000*l.* of the produce of the sinking fund, formed into tickets, will be the same with 1,050,000*l.*; and this sum will pay off a million and a half of the 3 *per cents*, every year, or 15 millions in all; and the growing savings arising from these payments will, at the end of 10 years, have paid, at least, two millions more. The nation, therefore, having paid off 17 millions of its debts, and added to them only 10 millions and $\frac{1}{2}$, will gain six millions and $\frac{1}{4}$. But this will be the smallest part of its gains. All the produce of the *sinking fund*, over and above 750,000*l.* might be charged with the payment of the interest of such new debts as would be necessary to be contracted during the war; and, at the end of it, the nation, with the help of 200,000*l.* to be disengaged in 1779, by the reduction of the 4 *per cents*, would find itself possessed of a *fund*, producing 1,450,000*l.* annually; which, faithfully employed, might probably be sufficient to extricate it from all its difficulties.—Besides this; such a scheme would not only *preserve*, but *raise* and *establish* the credit of the public: And he only can be duly sensible of the importance of this, who will consider, what danger there would be in another

The *sinking fund*, in its present state, and, after supplying the deficiencies of the peace establishment, yields, I suppose, a considerable part of the million and a half I have mentioned. An annual lottery might easily raise 200,000*l.* more. But this is a measure which I cannot wish to see carried into execution, unless absolutely necessary. Were the managers of our affairs sufficiently in earnest in this business, I cannot doubt but that such savings might be made in the collection and expenditure of the national revenue, as would cause the sinking fund to yield, for 18 or 20 years to come, the *whole* of this sum, without imposing any new burdens on the public. But, were there, indeed, no way of providing any part of it, but by creating new funds, or imposing new taxes; it *ought* to be done, because it *must* be done, or the nation be ruined.

The evils and dangers, attending an *exorbitant* public debt in this country, are so great, that they cannot be exaggerated.—Without repeating, what has been so often said, of its increasing the dependence on the crown, by jobs and places without number; occasioning
 ther war, should it continue long, of either *overwhelming* public credit; or of being terrified, by the apprehension of such a calamity, into an ignominious and fatal peace: The establishment, therefore, of some such plan as that now proposed, would, at the beginning of a war, be the most important of all works.

execrable

execrable practices of the Alley; rendering us tributary to foreigners; and raising the price of provisions and labour; and, consequently, checking population, and loading our trade and manufactures; I will only take notice of the two following evils which attend it.

In the first place. It must check the exertions of the spirit of liberty in the kingdom. The tendency of every government is to despotism; and in this it must end, if the people are not constantly jealous and watchful. Opposition, therefore, and resistance are often necessary. But they may throw things into confusion, and occasion the ruin of the public funds. The apprehension of this must influence all who have their interest connected with the preservation of the funds, and incline them always to acquiescence and servility.

But further. It exposes us to particular danger from *foreign* as well as *domestic* enemies, by making us fearful of war, and incapable of engaging in it, however necessary, without the hazard of bringing on terrible convulsions by overwhelming public credit.

All these are evils which must increase with every increase of the national debt; and there is a point at which, when they arrive, the consequences must be fatal (a). — I am

(a) “Either the nation (Mr. *Hume* says, *Essays* Vol. II. p. 145,) must destroy public credit; or public “credit will destroy the nation.”—A dreadful alternative! surely.

now writing under a conviction, that I am doing the little in my power to preserve my country from this danger. I have shewn, that an annual supply of a million and a half for 18, or at most 20 years, may be made the means of restoring and saving us. This, therefore, is our remedy; and it ought to be applied *immediately*, least it should not be applied time enough.

But to proceed to some further observations.

What has been said, has all along supposed a *sacred* and *inviolable* application of the fund I have described, and of all its earnings, to the purpose of sinking the national debt. The whole effect of it depends on its being allowed to operate, WITHOUT INTERRUPTION, a proper time. But it may be asked, how this can be secured? Or, by what method an object, that must be continually growing more and more tempting, can be defended against invasion and rapine?—I might here mention the superintendency and care of the representatives of the kingdom, the faithful guardians of the state, to whom ministers are responsible for the use they make of the public money. But experience has shewn, that we cannot rely on this security.—The difficulty, therefore, now mentioned, is the very greatest difficulty the nation
tion

tion has to struggle with in the payment of its debts.

The sinking fund was established in the year 1716, or soon after the accession of the present family, at a time when the public debts, tho' not much more than a third of what they are now, were thought to be so considerable as to be alarming and dangerous. It was intended as a SACRED DEPOSIT never to be touched; the law which established it declaring, that it was to be applied to the payment of the principal and interest of such national debts and incumbrances, as had been incurred before the 25th of *December 1716; and to no other use, intent or purpose whatever.* — The faith of *parliament*, therefore, as well as the security of the kingdom, seemed to require, that it should be preserved carefully and rigorously from alienation. But, notwithstanding this, it has been *generally* alienated; and the produce of it employed, in helping to defray such current expences as the exigencies of the state rendered necessary.

In order to justify this, it has been usual to plead, that when money is wanted, it makes no difference, whether it is taken from hence, or procured by making a new loan. There cannot be a worse sophism than this. The difference between these two methods of procuring money is no less than *infinite.*—For, let us suppose, a *million* wanted for any public service. If it is borrowed at

4 *per cent.* the public will lose by the payment of interest 40,000*l.* the first year, and the same the second year, and the same for ever afterwards. But if it is taken out of the *sinking fund*, the public will lose 40,000*l.* the *first* year; 4160*l.* the *second* year; 80,000*l.* the 18th year; a *million* the 85th year: For these are the sums that would, at these times, have otherwise necessarily reverted to the public. It loses, therefore, the advantage of paying in 85 years, with money of which otherwise no use could have been made, *twenty-five* millions of debt.—In other words; by employing the SINKING FUND, in bearing current expences, rather than borrowing *new* money; the state, in order to avoid giving *simple interest* for money, is made to alienate money, that *must* have otherwise been improved at *compound interest*; and that, in time, would have *necessarily* increased to *any* sum (a).--Had a faithful use been made from the first, of only one THIRD of the produce of this fund, near *three fourths* of our present debts might now have been discharged; and, in a few years more, the *whole* of them might have been discharged (b).—Can it be possible then

(a) The principal observations in this Chapter, I have given just as they occurred to my thoughts, without knowing that any of them had been made by other writers. Some proposals of a similar nature, but very differently represented, I have since found in Mr. *Postlethwayt's* Dictionary, under the articles *Public Credit, Debts, Funds, &c.*

(b) See a particular explanation and proof of this in the Questions following the Tables in the *Appendix.*

to think, without regret and indignation, of that misapplication of this fund, which, with the consent of parliaments always complying, our ministers have practised?—I find it difficult here to speak with calmness.—But I must restrain myself. *Calculation*, and not *censure*, is my business in this work.—I must believe, that the grievance I have mentioned, has proceeded more from inattention and mistake, than from any design to injure the public.



E S S A Y I. *

Containing Observations on the Expectations of Lives; the Increase of Mankind; the Number of Inhabitants in LONDON; and the Influence of great Towns, on Health and Population.

In a LETTER to BENJAMIN FRANKLIN, Esq; L. L. D. and F. R. S.

DEAR SIR,

I Beg leave to submit to your perusal the following observations. If you think them of any importance, I shall be obliged to you for communicating them to the Royal Society. You will find, that the chief subject of them is the present state of the city of *London*, with respect to healthfulness and number of inhabitants, as far as it can be collected from the bills of mortality. This is a subject that has been considered by others; but the proper method of calculating

* This Essay was read to the ROYAL SOCIETY, April 27th, 1769, and has been published in the *Philosophical Transactions*, Vol. 59. It is here republished with several additions; particularly, the *Postscript*.

from the bills has not, I think, been sufficiently explained.

No competent judgment can be formed of the following observations, without a clear notion of what the writers on *Life-Annuities* and *Reversions* have called the *Expectation of Life*. Perhaps this is not in common properly understood; and Mr. *De Moivre's* manner of expressing himself about it is very liable to be mistaken.

The most obvious sense of the *expectation* of a given life is, "That particular number " of years which a life of a given age has an " equal chance of enjoying." This is properly the time that a person may reasonably *expect* to live; for the chances *against* his living longer are greater than those *for* it; and, therefore, he cannot entertain an *expectation* of living longer, consistently with probability. This period does not coincide with what the writers on Annuities call the *expectation of life*, except on the supposition of an uniform decrease in the probabilities of life, as Mr. *Simpson* has observed in his *Select Exercises*, p. 273.—It is necessary to add, that, even on this supposition, it does not coincide with what is called the *expectation of life*, in any case of joint lives. Thus, two lives of 40 have an even chance, according to Mr. *De Moivre's* hypothesis (a), of continuing together only $13\frac{1}{2}$ years. But the *expectation*

(a) See the Notes in pag. 2, and 23.

of two equal joint lives being (according to the same hypothesis) always a *third* of the *common complement*; it is in this case $15\frac{1}{3}$ years. It is necessary, therefore, to observe, that there is another sense of this phrase, which ought to be carefully distinguished from that now mentioned. It may signify, “The *mean continuance* of any given *single, joint,* or *surviving* lives, according to any given *Table of observations:*” that is, the number of years which, taking them one with another, they actually enjoy, and may be considered as sure of enjoying; those who live or survive *beyond* that period, enjoying as much *more* time in proportion to their number, as those who *fall short* of it enjoy *less*. Thus; Supposing 46 persons alive, all 40 years of age; and that, according to Mr. *De Moivre's hypothesis*, one will die every year till they are all dead in 46 years; half 46, or 23, will be their *expectation of life*: That is; The number of years enjoyed by them all, will be just the same as if every one of them had lived 23 years, and then died; so that, supposing no interest of money, there would be no difference in value between annuities payable for life to every single person in such a set, and equal annuities payable to another equal set of persons of the same common age, supposed to be all sure of living just 23 years and no more.

In

In like manner; the *third* of 46 years, or 15 years and 4 months (a), is the *expectation* of two joint lives both 40; and this is also the *expectation* of the survivor. That is; supposing a set of marriages between persons all 40, they will, one with another, last just this time; and the survivors will last the same time. And annuities payable during the continuance of such marriages would, supposing no interest of money, be of exactly the same value with annuities to begin at the extinction of such marriages, and to be paid, during life, to the survivors. In adding together the years which any great number of such marriages, and their survivorships have lasted, the sums would be found to be equal.

One is naturally led to understand the *expectation* of life in the first of the senses now explained, when, by Mr. Simpson and Mr. De Moivre, it is called, *the number of years which, upon an equality of chance, a person may expect to enjoy*; or, *the time which a person of a given age may justly expect to continue in being*; and, in the last sense, when it is called, *the share of life due to a person*. But, as in reality it is always used in the last of these senses, the former language should not be applied to it: And it is in this last sense, that it coincides with the *sums* of the *present* probabilities, that any given single or joint lives shall attain to the end of the

(a) See Note (L) Appendix.

1st, 2d, 3d, &c. *moments*, from this time to the end of their possible existence; or, (in the case of survivorships) with the sum of the probabilities, that there shall be a survivor at the end of the 1st, 2d, 3d, &c. moments, from the present time to the end of the possible existence of survivorship. This coincidence every one conversant in these subjects must see, upon reflecting, that both these senses give the true present value of a life-annuity, secured by land, without interest of money (a).

This period in joint lives, I have observed is never the same with the period which they have an equal chance of enjoying; and in single lives, I have observed, they are the same only on the supposition of an uniform decrease in the probabilities of life. If this decrease, instead of being always uniform, is *accelerated* in the last stages of life; the former period, in single lives, will be *less* than the latter; if *retarded*, it will be *greater*.

It is necessary to add, that the number expressing the former period, multiplied by the number of single or joint lives whose expectation it is, added annually to a society or town, gives the whole number living together, to which such an annual addition would in time grow. Thus; since 19, or the third of 57, is the *expectation* of two

(a) See Note (L) in the Appendix.

joint lives whose common age is 29, or common *complement* 57; twenty marriages every year between persons of this age would, in 57 years, grow to 20 times 19, or 380 marriages always existing together. The number of *survivors* also arising from these marriages, and always living together, would, in twice 57 years, increase to the same number. And, since the *expectation* of a single life is always half its *complement*; in 57 years likewise, 20 single persons aged 29, added annually to a town, would increase to 20 times 28.5 or 570; and, when arrived at this number, the deaths every year will just equal the accessions, and no further increase be possible.

It appears from hence, that the particular proportion that becomes extinct every year, out of the whole number constantly existing together of single or joint lives, must, wherever this number undergoes no variation, be exactly the same with the *expectation* of those lives, at the time when their existence commenced. Thus; was it found that a 19th part of all the marriages among any body of men, whose numbers do not vary, are dissolved every year by the deaths of either the husband or wife, it would appear that 19 was, at the time they were contracted, the *expectation* of these marriages. In like manner; was it found in a society, limited to a fixed number of members,

members, that a 28th part dies annually out of the whole number of members, it would appear that 28 was their common expectation of life at the time they entered. So likewise; were it found in any town or district, where the number of births and burials are equal, that a 20th or 30th part of the inhabitants die annually, it would appear, that 20 or 30 was the *expectation* of a child just born in that town or district. These *expectations*, therefore, for all *single* lives, are easily found by a *Table of Observations*, shewing the number that die annually at all ages, out of a given number alive at those ages; and the general rule for this purpose, is “to divide
“ the sum of all the living in the Table, at
“ the age whose expectation is required, and
“ at all greater ages, by the sum of all that
“ die annually at that age, and above it; or,
“ which is the same, by the number in the
“ Table of the living at that age; and half
“ unity subtracted from the quotient will be
“ the required *expectation* (a).” Thus, in Dr. *Halley’s* Table, the sum of all the living at 20 and upwards is, 20,724. The number living at that age is 598; and the former number divided by the latter, and half unity

(a) This rule, and also rules for finding in all cases the expectations of joint lives and survivorships, may be deduced with great ease, by having recourse to the doctrine of fluxions. In this method, Mr. *De Moivre* says, he discovered them. See Appendix, Note (L), where an account will be given of these deductions, omitted by Mr. *De Moivre*.

(a) subtracted from the quotient, gives 34.15 for the *expectation* of 20. The *expectation* of the same life by Mr. *Simpson's* Table, formed from the bills of mortality of London, is 28.9 (b).

These

(a) If we conceive the *recruit* necessary to supply the *waste* of every year to be made always at the *end* of the year, the *dividend* ought to be the *medium* between the numbers living at the *beginning* and the *end* of the year. That is, it ought to be taken *less* than the sum of the living in the Table at and above the given age, by *half* the number that die in the year; the effect of which *diminution* will be the same with the *subtraction* here directed.—The reason of this subtraction will be further explained, in the beginning of the last Essay.

(b) It appears in p. 169 and 170, that the *expectations* of *single* and *joint* lives are the same with the values of *annuities* on these lives, supposing no interest or improvement of money.—In considering this subject, it will, probably, occur to some, that, allowing interest for money, the values of lives must be the same with the values of annuities *certain* for a number of years equal to the *expectations* of the lives. But care must be taken not to fall into this mistake. The latter values are always greater than the former: And the reason is, that, tho' a number of *single* or *joint* lives of given ages will, among them, enjoy a *given* number of years, yet some of them will enjoy a much *greater*, and some a much *less* number of years. Thus; 100 marriages among persons, all 29, would, as I have said, one with another, exist 19 years; and an office bound to pay annuities to such marriages during their continuance, might reckon upon making 19 payments for each marriage. But then, many of these payments would not be made 'till the end of 30, and some not 'till the end of 40 years. And it is apparent, that on account of the greater value of *quick* than *late* payments, when money *bears* interest, 19 payments so made cannot be worth as much, as the same number of payments made
regularly

These observations bring me to the principal point which I have had all along in view. They suggest to us an easy method of finding the number of inhabitants in a place, from a *Table of Observations*, or the *bills of mortality* for that place, supposing the yearly births and burials equal. “Find by “the Table, in the way just described, the “*expectation* of an infant just born, and this, “multiplied by the number of yearly births, “will be the number of inhabitants.” At *Breslaw*, according to Dr. *Halley's* Table, though half die under 16, and therefore an infant just born has an *equal chance* of living only 16 years; yet his *expectation*, found by the rule I have given, is near 28 years; and this, multiplied by 1238 the number born annually, gives 34,664, the number of inhabitants. In like manner, it appears from

regularly at the end of every year, 'till in 19 years they are all made.

This observation might be employed, to demonstrate further, the error of those who have maintained, that the value of a given life is the same, with the value of an annuity certain, for as many years as the life has an equal chance of existing. Were this true, an annuity on a life, supposed to be exposed to such danger in a particular year, as to create an equal chance, whether it will not fail that year, would, at the beginning of the year, be worth *nothing*, though supposed to be sure of continuing for ever, if it escaped that danger.--But there can be no occasion for taking notice of an opinion, which has been embraced only by persons ignorant of mathematics, and plainly unacquainted with the genuine principles of calculation on this subject.—See a Pamphlet on Life-Annuities by *Weyman Lee*, Esq; of the *Inner Temple*.

Mr.

Mr. *Simpson's* Table, that, though an infant just born in *London* has not an *equal chance* of living 3 years, his *expectation* is 20 years; and this number, multiplied by the yearly births, would give the number of inhabitants in *London*, were the births and burials equal.-- The medium of the yearly births, for 10 years, from 1759 to 1768, was 15,710. This number multiplied by 20, is 314,200; which is the number of inhabitants that there would be in *London*, according to the bills, were the yearly burials no more than equal to the births: that is, were it to support itself in its number of inhabitants, without any supply from the country. But for the period I have mentioned, the burials were, at an average, 22,956, and exceeded the christenings 7,246. This is, therefore, at present, the yearly addition of people to *London* from other parts of the kingdom, by whom it is kept up. Suppose them to be all, one with another, persons who have, when they remove to *London*, an *expectation* of life equal to 30 years. That is; suppose them to be all of the age of 18 or 20, a supposition certainly far beyond the truth. From hence will arise, according to what has been before observed, an addition of 30 multiplied by 7.246; that is, 217,380 inhabitants. This number, added to the former, makes 531,580; and this, I think, at most, would be the number of inhabitants in *London* were the bills perfect.

But

But it is certain, that they give the number of births and burials too little. There are many burying-places that are never brought into the bills. Many also emigrate to the navy and army and country; and these ought to be added to the number of deaths. What the deficiencies arising from hence are, cannot be determined. Suppose them equivalent to 6000 every year in the births, and 6000 in the burials. This would make an addition of 20 times 6000, or 120,000, to the last number; and the whole number of inhabitants would be 651,580. If the burials are deficient only two-thirds of this number, or 4000; and the births, the whole of it; 20 multiplied by 6000, must be added to 314,290, on account of the defects in the births: And, since the excess of the burials above the births will then be only 5,246; 30 multiplied by 5,246 or 157,380, will be the number to be added on this account; and the sum, or number of inhabitants, will be 591,580.—But if, on the contrary, the burials are deficient 6000, and the births only 4000; 80,000 must be added to 314,290, on account of the deficiencies in the births; and 30 multiplied by 9,246, or 277,380, on account of the excess of the burials above the births; and the whole number of inhabitants will be 671,580.

Every supposition in these calculations is too high. *Emigrants from London* are, in

particular, allowed the same *expectation* of continuance in *London* with those who are born in it, or who come to it in the firmest part of life, and never afterwards leave it; whereas it is not credible that the former *expectation* should be so much as half the latter. But I have a further reason for thinking that this calculation gives too high numbers, which has with me irresistible weight. It has been seen, that the number of inhabitants comes out less on the supposition, that the defects in the christenings are greater than those in the burials. Now it seems evident that this is really the case; and, as it is a fact not attended to, I will here endeavour to explain distinctly the reason which proves it.

The proportion of the number of births in *London*, to the number who live to be 10 years of age, is, by the bills, 16 to 5. Any one may find this to be true, by subtracting the *annual medium* of those who have died under 10, for some years past, from the *annual medium* of births for the same number of years.—Now, tho', without doubt, *London* is very fatal to children, yet it seems incredible that it should be so fatal as this implies. The *bills*, therefore, probably, give the number of those who die under 10 too great in proportion to the number of births; and there can be no other cause of this, than a greater deficiency in the *births* than in the *burials*.

burials. Were the deficiencies in both equal; that is, were the *burials*, in proportion to their number, just as deficient as the *births* are in proportion to *their* number, the proportion of those who reach 10 years of age to the number born, would be right in the *bills*, let the deficiencies themselves be ever so considerable. On the contrary; were the deficiencies in the *burials* greater than in the *births*, this proportion would be given too great; and it is only when the former are least, that this proportion can be given too little.—Thus; let the number of annual *burials* be 23,000; of *births* 15,700; and the number dying annually under 10, 10,800. Then 4,900 will reach 10, of 15,700 born annually; that is, 5 out of 16.—Were there no deficiencies in the *burials*, and were it fact that only *half* the number born die under 10; it would follow, that there was an annual deficiency equal to 4,900 subtracted from 10,800, or 5,900, in the *births*.—Were the *births* a third part too little, and the *burials* also a third part too little, the true number of *births*, *burials*, and of *children dying under 10*, would be 20,933--30,666, and 14,400; and, therefore, the number that would live to 10 years of age, would be 6,533 out of 20,933, or 5 of 16 as before.—Were the *births* a third part, and the *burials* so much as two-fifths wrong, the number of *births*, *burials*, and children dying under 10 would

be 20,933—32,200—and 15,120. And, therefore, the number that would live to 10 would be 5;813 out of 20,933, or five out of 18.—Were the *births* a 3d part wrong, and the *burials* but a 6th, the foregoing numbers would be 20,933—26,833—12,600; and, therefore, the number that would live to 10 would be 8,333 out of 20,933, or 5 out of 12.56: And this proportion seems as low as is consistent with probability. It is somewhat less than the proportion in Mr. *Simpson's* Table of *London Observations*; and much less than the proportion in the Table of *Observations* for *Breslaw*. The deficiencies, therefore, in the *births* must be greater than those in the *burials* (a); and the least number I have given, or 591,580 is nearest to the true number of inhabitants. However, should any one, after all, think that it is not improbable that only 5 of 16 should live in *London* to be 10 years of age; or that above *two-thirds* die under this age; the consequence will still be, that the foregoing cal-

(a) One obvious reason of this fact is, that *none* of the *births* among *Jews*, *Quakers*, *Papists*, and the *three denominations of Dissenters* are included in the bills, whereas *many* of their *burials* are. It is further to be attended to, that the abortive and still-born, amounting to about 600 annually, are included in the burials, but never in the births. If we add these to the christenings, preserving the burials the same, the proportion of the born, according to the bills, who have reached ten for the last sixteen years, will be very nearly one *third* instead of *five sixths*.

ulation

ulation has been carried too high. For it will from hence follow, that the *expectation* of a child just born in *London* cannot be so much as I have taken it. This *expectation* is 20, on the supposition that half die under 3 years of age, and that 5 of 16 live to be 29 years of age, agreeably to Mr. *Simpson's* Table. But if it is indeed true, that *half* die under 2 years of age, and 5 of 16 under 10, agreeably to the *bills*, this expectation cannot be so much as 17 (*a*); and all the numbers before given will be considerably reduced.

Upon the whole: I am forced to conclude from these observations, that the second number I have given, or 651,580, though short of the number of inhabitants commonly supposed in *London*, is, very probably, much *greater*, but cannot be *less*, than the true number. Indeed, it is in general evident, that in cases of this kind numbers are very much over-rated. The ingenious Dr. *Brakenridge*, 14 years ago, when the bills were lower than they are now, from the number of houses, and allowing six to a house, made the number of inhabitants 751,800. But his method of determining the (*b*) number of houses is too precarious;

(*a*) This may be deduced from the observations in the last Essay; and it will be there proved, that, in reality, this expectation does not exceed 18.

(*b*) Vid. Phil. Transactions, Vol. XLVIII, p. 788. In a paper subsequent to this, Dr. *Brakenridge* tells us, that

carious; and, besides, six to a house is too large an allowance.—Many families now have two houses to live in.—The magistrates of *Norwich*, in 1752, took an exact account of both the number of houses and individuals in that city. (a) The number of houses

that in a late survey it appeared, that in all *Middlesex*, *London*, *Westminster*, and *Southwark*, there were 87,614 houses, of which 19,324 were cottages, and 4810 empty. And he acknowledges, that this, if right, proves *London* to be much less populous than he had made it. See *Phil. Transf.* Vol. 50, p. 471. He does not mention how this survey was taken; but most probably it must have been incorrect.—Mr. *Maitland* gives two accounts of the number of houses within the bills. One carefully taken from the books of all the parishes and precincts belonging to *London*; and another taken from a particular survey in 1737, made by himself with incredible pains. The first account makes the number of houses 85,805. The second account makes it 95,968. And the reason of the difference he observes, is, that many landlords of small places, paying all taxes, they are in the parish books reckoned as so many single houses, tho' each of them contain several houses. See Mr. *Maitland's* History of *London*, 2d Book at the end.—This, perhaps, may be also the reason of the deficiencies which, I suppose, there must be in the survey, mentioned by Dr. *Brakenridge*.—It will be observed presently, that the number of inhabitants in *London* in 1737, was considerably greater than it is now.

(a) Vid. Gentleman's Magazine for 1752, and Dr. *Short's Comparative History of the Increase of Mankind*, p. 38. In page 58 of this last work the author says, that, in order to be fully satisfied about the number of persons to be allowed to a family, he procured the true number of families and individuals in 14 market towns, some of them considerable for trade and populousness; and that in them were 20,371 families, and 97,611 individuals,

or

houses was 7,139, and of individuals 36,169, which gives nearly 5 to a house.—Another

or but little more than $4\frac{3}{4}$ to a family. He adds, that, in order to find the difference in this respect between towns of trade and country parishes, he procured, from divers parts of the kingdom, the exact number of *families* and *individuals*, in 65 country parishes. The number of *families* was 17,208; *individuals* 76,284; or not quite $4\frac{1}{2}$ to a family.—In the place I have just referred to, in the Gentleman's Magazine, there is an account of the number of *houses* and *inhabitants* in *Oxford*, exclusive of the colleges; and in *Wolverhampton*, *Coventry* and *Birmingham*, for 1750. The number of persons to a *house* was, by this account, $4\frac{4}{5}$ in the two former towns, and $5\frac{3}{4}$ in the two latter.—Dr. *Davenant*, from Mr. *King's* Observations, gives $4\frac{1}{3}$, as the number of persons to a *family* for the whole kingdom. See *An Essay on the probable method of making a people gainers by the balance of trade*. From an account with which a friend at *Shrewsbury* has favoured me, it appears, that in that town, in 1750, the number of inhabitants to a *house* was $4\frac{1}{3}$.—Very exact accounts, of which I shall take further notice, prove, that in the parish of *Holy-Cross*, one of the suburbs of *Shrewsbury*, and at *Northampton*, the same proportion is $4\frac{1}{3}$ to a *house* in the former; and $4\frac{3}{4}$ in the latter.—It seems, therefore, that five persons to a house is an allowance large enough for *London*, and too large for *England* in general. From whence it will follow, that Dr. *Brakenridge* has likewise over-rated the number of people in *England*. In a letter to *George Lewis Scott*, Esq; published in 1756, in the *Phil. Transf.* Vol. 49, p. 877; he says, that he had been certainly informed, that the number of houses rated to the window-tax was 690,000. The number of cottages not rated, he adds, was not accurately known; but from the accounts given in, it appeared, that they could not exceed 200,000; and from these data, in consequence of allowing six to a house, he makes the number of people in *England* to be 5,340,000. Perhaps the number of houses in this account is too little.

ther method which Dr. *Brakenridge* took to determine the number of inhabitants in *London* was from the annual number of burials, adding 2000 to the bills for omissions, and supposing a 30th part to die every year. In order to prove this to be a moderate supposition he observes that, according to Dr. *Halley's* Observations, a 34th part die every year at *Breslaw*. But this observation was made too inadvertently. The number of annual burials there, according to Dr. *Halley's* account, was 1174, and the number of inhabitants, as deduced by him from his Table, was 34,000; and therefore a 29th part died every year. Besides; any one may find, that in reality the Table is constructed on the supposition, that the whole number born, or 1238, die every year; from whence it will follow that a 28th part died every year. (a) Dr. *Brakenridge*, therefore, had

Suppose it a million; and let five be allowed to a house; and the number of people in *England* will be five millions: Which, since five to a house is too large an allowance, ought to be considered as, probably, *more* than the true number. — The number of people in *Scotland* and *Ireland*, Dr. *Brakenridge* estimates at three millions. See *Phil. Trans.* Vol. 50, p. 473.

(a) Care should be taken, in considering Dr. *Halley's* Table, not to take the first number in it, or 1000, for so many just born. 1238, he tells us, was the annual medium of births, and 1000 is the number he supposes all living at one year and under. It was inattention to this that led Dr. *Brakenridge* to his mistake.

had he attended to this, would have stated a 24th part as the proportion that dies in *London* every year, and this would have taken off 150,000 from the number he has given. But even this must be less than the just proportion. For let three-fourths of all who either die in *London* or migrate from it, be such as have been born in *London*; and let the rest be persons who have removed to *London* from the country, or from foreign nations. The *expectation* of the former, it has been shewn, cannot exceed 20 years; and 30 years have been allowed to the latter. One with another, then, they will have an *expectation* of $22\frac{1}{2}$ years. That is; one of $22\frac{1}{2}$ will die every year. (a) And, consequently,

It will be shewn in the 4th Essay, that the number of the living, under 20, is given too high in this Table; and from hence it will follow, that more than a 28th part of the inhabitants die at *Breslaw* annually.

(a) The whole number of inhabitants in *Rome* in 1743, was 147,476, and the annual medium of burials for three years, from 1741 to 1743, was 6338. A 23d part, therefore, died every year. See a Treatise in *German*, on the different degrees of human mortality in different situations, by *Susmilch*, first counsellor of his *Prussian Majesty's* Consistory, and member of the Royal Academy of sciences at *Berlin*, p. 15.

In 1761, the whole number of inhabitants in the same town, was 157,452. The annual medium of births, for three years, from 1759 to 1761, was 5167; and of burials 7153. One in 22, therefore, died annually. See *Dr. Short's Comparative History of the Increase and Decrease of Mankind in England and several Countries abroad*, p. 59, 60.

quently, supposing the annual recruit from the country to be 7000, the number of
births

60.—In 1752, the accurate and diligent Mr. *Struyk*, took particular pains to determine the number of inhabitants in *Amsterdam*; and the result of his enquiry was, that very probably it did not amount to 200,000. The annual medium of burials for six years, from 1747 to 1752, was 8247. One in 24, therefore, died annually. See *Susmilch* *ibid.*—At *Amsterdam*, there is a great number of Jews, and their burials are not included in the bills. There must, I suppose, be other deficiencies, and an allowance for these would, I doubt not, increase the proportion of inhabitants, who die annually, to one in 21 or 22.—At *Dublin*, in the year 1695, the number of inhabitants was found, by an exact survey, to be 40,508, (see *Philos. Transactions*, N^o 261). I find no account of the annual burials just at that time; but from 1661 to 1681, the medium had been 1613; and from 1715 to 1728 it was 2123. There can, therefore, be no material error in supposing that, in 1695, it was 1800; and this makes 1 in 22 to die annually.—In 1745 the number of families in the same city appeared, by an exact account laid before the Lord Mayor, to be 9,214. It is highly probable this number of families did not consist of so many as 45,000 individuals. Suppose them, however, 50,000; and, as at this time the medium of annual burials appears to have been 2,360, 1 in 21 died annually: see *Dr. Short's Comparative History*, p. 15, and *New Observations*, p. 228.—These facts prove that I have been very moderate in making only 1 in 22½, including emigrants, to die in *London* annually.—In 1631 the number of people in the *city and liberties* of *London* was taken, by order of the Privy Council, and found to be 130,178.—This account was taken five years after a plague that had swept off near a quarter of the inhabitants; and when, therefore, the town being full of recruits in the vigour of life, the medium of annual burials must have been lower than usual, and the births higher. Could, therefore, the medium

births 3 times 7000 or 21,000, and the *burials* and *migrations* 28,000 (which are all high suppositions), the number of inhabitants will be, $22\frac{1}{2}$ multiplied by 28,000, or 630,000.

I will just mention here one other instance of exaggeration on the present subject.

Medium of annual burials at that time, within the walls, and in the 16 parishes without the walls, be settled, exclusive of those who died in such parts of the 16 parishes without the walls, as are not in the *liberties*, the proportion dying annually obtained from hence might be depended on, as less than the common and just proportion. But this medium cannot be discovered with any accuracy. *Graunt* estimates that two-thirds of these 16 parishes are within the *liberties*; and, if this is right, the medium of annual burials in the *city and liberties* in 1631, was 5,500, and 1 in $23\frac{3}{4}$ died annually; or, making a small allowance for deficiencies in the bills, 1 in 22. — Mr. *Maitland*, in his *History of London*, Vol. II. p. 744, by a laborious, but too unsatisfactory, investigation, reduces this proportion to 1 in $24\frac{1}{2}$; and on the suppositions, that this is the true proportion dying annually, *at all times*, in *London*, and that the deficiencies in the burials (including the burials in *Marybone* and *Pancras* parishes) amount to 3,038 annually; he determines, that the number of inhabitants within the bills was 725,903, in the year 1737.

The number of burials not brought to account in the bills is, probably, now much greater than either Dr. *Brakenridge* or Mr. *Maitland* suppose it. I have reckoned it so high as 6000, in order to include emigrants, and also to be more sure of not falling below the truth.

It will appear in the last Essay, with an evidence little short of demonstration, that, at least, 1 in $20\frac{3}{4}$ die annually in *London*, and that, consequently, the number of inhabitants, if the omissions in the burials are 6000, cannot exceed 601,750.

Mr.

Mr. *Corbyn Morris*, in his useful *Observations on the past growth and present state of the city of London*, published in 1751, supposes that no more than a 60th part of the inhabitants of *London*, who are above 20, die every year, and from hence he concludes that the number of inhabitants was near a million. In this supposition there was an error of at least one half. According to Dr. *Halley's Table*, it has been shewn, that a 34th part of all at 20 and upwards, die every year at *Breslaw*. In *London*, a 29th part, according to Mr. *Simpson's Table*, and also according to all other *Tables of London Observations*. And in *Scotland* it has been found for many years, that, of 974 ministers and professors whose ages are 27 and upwards, a 33d part have died every year. Had, therefore, Mr. *Morris* stated a 30th part of all above 20 as dying annually in *London*, he would have gone beyond the truth, and his conclusion would have been 400,000 less than it is.

Dr. *Brakenridge* observed, that the number of inhabitants, at the time he calculated, was 127,000 less than it had been. The bills have lately advanced a little, but still they are much below what they were from 1717 to 1743. The medium of the annual *births*, for 20 years, from 1716 to 1736, was 18,000, and of *burials* 26,529; and, by calculating from hence on all the same suppositions with those which made 651,580 to be

be the present number of inhabitants in *London*, it will be found that the number then was 735,840, or 84,260 greater than the number at present. *London*, therefore, for the last 30 years, has been decreasing; and though now it is increasing again, yet there is reason to think that the additions lately made to the number of buildings round it, are owing, chiefly to the increase of luxury, and the inhabitants requiring more room to live upon (a).

It should be remembered, that the number of inhabitants in *London* is now so much less as I have made it, than it was 40 years

(a) The medium of annual burials in the 97 parishes within the walls was,

From 1655 to 1664,	—	3264
From 1680 to 1690,	—	3139
From 1730 to 1740,	—	2316
From 1758 to 1768,	—	1620

This account proves, that though, since 1655, *London* has doubled its inhabitants, yet, *within the walls*, they have decreased; and so rapidly for the last 30 years as to be now reduced to one half.—The like may be observed of the 17 parishes immediately without the walls. Since 1730, these parishes have been decreasing so fast, that the *annual burials* in them have sunk from 8,672 to 5,432, and are now lower than they were before the year 1660. In *Westminster*, on the contrary, and the 23 out-parishes in *Middlesex* and *Surrey*, the *annual burials* have since 1660, advanced from about 4000 to 16,000.—These facts prove, that the inhabitants of *London* are now much less crowded together than they were. It appears, in particular, that *within the walls* the inhabitants take as much room to live upon as double their number did formerly.—The very same conclusions may be drawn from an examination of the *christenings*.

ago,

ago, on the supposition, that the proportion of the omissions in the *births* to those in the *burials*, was the same then that it is now. But it appears that this is not the fact.—From 1728, (the year when the ages of the dead were first given in the *bills*) to 1742, near five-sixths of those who were born died under 10, according to the bills. From 1742 to 1752 three quarters: And ever since 1752, this proportion has stood nearly as it is now, or at somewhat more than two-thirds. The omissions in the *births*, therefore, compared with those in the *burials*, were greater formerly; and this must render the difference between the number of inhabitants now and formerly less considerable than it may seem to be from the face of the bills. One reason, why the proportion of the amounts of the *births* and *burials* in the bills, comes now nearer than it did, to the true proportion, may, perhaps, be, that the number of Dissenters is lessened. The Foundling Hospital also may have contributed a little to this event, by lessening the number given in the bills as having died under 10, without taking off any from the *births*; for all that die in this hospital are buried at *Pancrass* church, which is not within the *bills*. See the preface to a collection of the yearly bills of mortality from 1657 to 1758 inclusive, p. 15.

I will

I will add, that it is probable that *London* is now become less fatal to children than it was; and that this is a further circumstance which must reduce the difference I have mentioned; and which is likewise necessary to be joined to the greater deficiencies in the births, in order to account for the very small proportion of children who survived 10 years of age, during the two first of the periods I have specified.—Since 1752, *London* has been thrown more open. The custom of keeping country-houses, and of sending children to be nursed in the country, has prevailed more. But, particularly, the destructive use spirituous liquors among the poor has been checked.

I have shewn that in *London*, even in its present state, and according to the most moderate computation, half the number born die under *three* years of age. But it appears from *Graunt's* (a) accurate account of the births, weddings, and burials in three country parishes for 90 years; and also, from *Dr. Short's* collection of observations in his *Comparative History*, and his treatise entitled, *New Observations on Town and Country Bills*

(a) See *Natural and Political Observations on the Bills of Mortality*, by Capt. *John Graunt*, F. R. S.—See also *Mr. Derham's Physico-Theology*, p. 174, where it appears, that in the parish of *Aynbo* in *Northamptonshire*, tho' the births had been, for 118 years, to the marriages as 6 to 1; yet the burials had been to the marriages only as $3\frac{3}{4}$ to 1.

of

of *Mortality*; that in country villages and parishes, the major part live to mature age, and even to marry.

In the parish of *Holy-Cross (a)*, in *Salop*, it appears from a curious register, which has been kept by the Rev. Mr. *Gorsuch*, the vicar, that of 655 who have died there at all ages for the last 20 years, 321, or near one half, have lived to 30 years of age: And, by forming a Table of Observations from this

(a) This parish contains in it a village which is a part of the suburbs of *Shrewsbury*. It consists of 1400 acres of arable and pasture land; besides 300 acres taken up by houses and gardens. It is six miles in circumference; half of which lies along the banks of the river *Severn*.—I mention these particulars to shew, that it may be reckoned a *country* parish; tho', perhaps, not perfectly so, on account of its nearness to *Shrewsbury*.—The christenings in it exceed the burials a little; and the number of inhabitants (mostly labouring people) has, for the last 20 years, kept nearly to 1050, without any considerable increase.—The register of this parish, from 1750 to 1760, has been published in the 52d volume of the *Philosophical Transactions*, Part I. Art. 25. And a continuation of it from 1760 to 1770, has been lately communicated and read to the Royal Society. It is to be wished, that more such accounts, specifying, as this does, the males and females dying at all ages, were kept in different situations in the country. This is the only one that I have ever heard of. It is kept with particular care and accuracy by Mr. *Gorsuch*; and furnishes very useful *data* for determining the difference in value between town and country lives.—It deserves to be mentioned particularly, that no *foreigners* or *strangers*, who happen to die in this parish, or who may be brought into it to be buried, are entered into the register: Nor are any of the fixed inhabitants omitted, tho' carried out to be buried.

register, in the manner which will be described in the last Essay, I find that a child just born in this parish has an expectation of 33 years; and that in general, under the age of 50, the *expectations* of lives here exceed those in *London*, in the proportion of about 4 to 3.—So great is the difference, especially to children, between living in great towns and in the country.—But nothing can place this observation in a more striking light, than the account given by Dr. *Thomas Heberden*, and published in the *Philosophical Transactions* (vol. LVII. p. 461), of the *increase and mortality of the inhabitants of the island of Madeira*. In this island, it seems, the weddings have been to the births, for 8 years, from 1759 to 1766, as 10 to 46.8; and to the burials, as 10 to 27.5. Double these proportions, therefore, or the proportion of 20 to 46.8, and of 20 to 27.5, are the proportions of the number marrying annually, to the number born and the number dying. Let 1 marriage in 10 be a 2d or 3d marriage on the side of either the man or the woman; and 10 marriages will imply 19 individuals who have grown up to maturity, and lived to marry once or oftener; and the proportion of the number marrying annually the first time, to the number dying annually, will be 19 to 27.5, or near 3 to 4. It may seem to follow from hence, that in this island near three-fourths of those who die have

O

been

been married; and, consequently, that not many more than a *quarter* of the inhabitants die in childhood and celibacy; and this would be a just conclusion were there no increase, or had the births and burials been equal. But it must be remembered, that the general effect of an increase while it is going on in a country, is to render the proportion of persons marrying annually, to the annual deaths, *greater*, and to the annual births *less*, than the true proportion marrying, out of any given number born. This proportion generally lies between the other two proportions, but always nearest to the first (*a*); and, in the present case,

(*a*) In a country where there is no increase or decrease of the inhabitants, and where also life, in its first periods, is so stable, and marriage so much encouraged, as that half of all who are born live to be married, the *annual* births and burials must be equal, and also *quadruple* the number of weddings, after allowing for 2d and 3d marriages. Suppose in these circumstances (every thing else remaining the same) the *probabilities of life*, during its first stages, to be improved. In this case, more than *half* the born will live to be married, and an increase will take place. The births will exceed the burials, and both fall below *quadruple* the weddings; or, which is the same, below *double* the number annually married.—Suppose next (the *probabilities of life* and the *encouragement to marriage* remaining the same) the *prolifickness* only of the marriages to be improved. In this case it is plain, that an increase also will take place; but the *annual* births and burials, instead of being less, will now both rise above *quadruple* the weddings; and therefore the proportion of the born to that part of the born who marry (being by supposition two to one) will be less than the proportion
of

case, it is sufficiently evident that it cannot be much less than two-thirds.

In

of either the *annual* births or the *annual* burials, to the number marrying *annually*.—Suppose again (the *encouragement to marriage* remaining the same) that the *probabilities of life* and the *prolifickness of marriages* are both improved. In this case, a more rapid increase will take place, or a greater excess of the births above the burials; but at the same time they will keep nearer to *quadruple* the weddings, than if the latter cause only had operated, and produced the same increase.—I should be too minute and tedious, were I to explain these observations at large. It follows from them, that, in every country or situation where, for a course of years, the *burials* have been either *equal to* or *less* than the *births*, and both under *quadruple* the marriages; and also that, wherever the burials are *less* than quadruple the annual marriages, and at the same time the births *greater*, there the major part of all that are born live to marry. In the instance which I have considered above, and which occasions this note, the annual births are so much *greater* than *quadruple* the marriages, and at the same time the annual burials so much *less*, that the proportion that live to marry of those who are born, can scarcely be much less than I have said; or two-thirds.

I have shewn how the allowance is to be made for 2d and 3d marriages; but it is not so considerable as to be of any particular consequence; and, besides, it is, in part, compensated by the natural children which are included in the births, and which raise the proportion of the births to the weddings higher than it ought to be, and therefore bring it nearer to the true proportion of the number born *annually*, to those who marry annually, after deducting those who marry a 2d or 3d time.

In drawing conclusions from the proportion of *annual* births and burials, in different situations, some writers on the increase of mankind, have not given due attention to the difference in these proportions, arising from the

In *London*, then, *half* die under three years of age; and in *Madeira* about *two-thirds* of all who are born live to be married. Agreeable to this, it appears also from the account I have referred to, that the *expectation* of a child just born in *Madeira* is about 39 years; or double the expectation of a child just born in *London*. For the number of inhabitants was found, by a survey made in the beginning of the year 1767, to be 64,614. The annual medium of *burials* had been, for eight years, 1293; of *births* 2201. The number of inhabitants, divided by the annual medium of *burials*, gives 49.89; or the *expectation* nearly of a child just born, supposing the *births* had been 1293, and constantly equal to the *burials*, the number of inhabitants remaining the same. And the same number, divided by the annual medium of *births*, gives 29.35; or the *expectation* of a child just born, supposing the *burials* 2201, the number of births

different circumstances of increase or decrease among a people. One instance of this I have now mentioned; and one further instance of it is necessary to be mentioned. The proportion of *annual* births to weddings has been considered as giving the true number of children derived from each marriage, taking all marriages one with another. But this is true only when, for many years, the births and burials have kept nearly equal. Where there is an excess of the births occasioning an increase, the proportion of *annual* births to weddings must be less than the proportion of children derived from each marriage; and the contrary must take place where there is a decrease.

and

and of inhabitants remaining the same. And the true *expectation* of life must be somewhere near the mean between 49.89 and 29.35.

Again: A 50th part of the inhabitants of *Madeira*, it appears, die annually. In *London*, I have shewn, that above twice this proportion dies annually. In smaller towns a smaller proportion dies; and the births also come nearer to the burials. In general; there seems reason to think that in towns (allowing for particular advantages of situation, trade, police, cleanliness, and openness, which some towns may have,) the excess of the burials above the births, and the annual deaths are more or less as the towns are greater or smaller. In *London* itself, about 160 years ago, when it was scarcely a fourth of its present bulk, the births were much nearer to the burials, than they are now. But in country parishes and villages the births almost always exceed the burials; and I believe it seldom happens that so many as a 30th, or more than a 40th part of the inhabitants, die annually (a). In the four provinces

(a) In the year 1733, a survey was taken of the inhabitants of the parish of *Stoke-Damerel* in *Devonshire*, and the number of men, women, and children was found to be 3361. — The *christenings* for the year were 122—the *weddings* 28—*burials* 62.—No more, therefore, than the 54th part of the inhabitants died in the year.—In part of this year an epidemical fever prevailed in the parish. See *Martyn's Abridgment of the Philos. Transactions*, vol.

provinces of *New-England* there is a very rapid increase of the inhabitants; but, notwithstanding this, at *Boston*, the capital, the inhabitants would decrease were there no supply from the country: for, if the account I have seen is just, from 1731 to 1762, the burials all along exceeded the births (a). So remarkably do towns, in consequence of their unfavourableness to health, and the

IX. p. 325.—According to *Graunt's* account of a parish in *Hampshire*, not reckoned, he says, remarkably healthful, a 50th part of the inhabitants had died annually for 90 years. *Natural and political Observations, &c.* Chap. xii.

In 1098 country parishes, mentioned by *Susmilch*, the annual average of deaths, for six years, ending in 1749, was 5255. The number of inhabitants was 225,357. One, therefore, in 43 died annually.—In 106 other parishes, mentioned by him, this proportion was 1 in 50.—In the Dukedom of *Wurtemberg*, the inhabitants, he says, are numbered every year; and from the average of five years, ending in 1754, it appeared that, taking the towns and country together, 1 in 32 died annually.—In another province, which he mentions, consisting of 635,998 inhabitants, 1 in 33 died annually. From these facts he concludes, that, taking a whole country in *gross*, including all cities and villages, mankind enjoy among them about 32 or 33 years each of existence. And this, very probably, may not be far from the truth in the present state of most of the kingdoms of *Europe*. And it will follow, that a child born in a country parish or village, has, at least, an expectation of 36 or 37 years; supposing the proportion of *country* to *town* inhabitants to be as $3\frac{1}{2}$ to 1; which, I think, this ingenious writer's observations prove to be nearly the case in *Pomerania*, *Brandenburgh*, and some other kingdoms.

(a) See a particular account of the births and burials in this town from 1731 to 1752 in the *Gentleman's Magazine* for 1753, p. 413.

luxury which generally prevails in them, check the increase of countries.

Healthfulness and Prolifickness are, probably, causes of increase seldom separated. In conformity to this observation, it appears from comparing the births and weddings, in countries and towns where registers of them have been kept, that in the former, marriages, one with another, seldom produce less than four children each; generally between four and five, and sometimes above five. But in towns seldom above four; generally between three and four; and sometimes under three (a).

I have sometimes heard the great number of old people in *London* mentioned, to prove its favourableness to health and long life. But no observation can be more erroneous. There ought, in reality, to be more old people in *London*, in proportion to the number of inhabitants, than in any smaller towns; because at least one quarter of its inhabitants are persons who come in-

(a) Any one may see what evidence there is for this, by consulting Dr. *Short's* two books already quoted, and the *Abridgment of the Philosophical Transactions*, vol. VII. part iv. p. 46, and *Graunt's* account already quoted, of the births, weddings and burials in three country parishes for 90 years; compared with similar accounts in towns. In considering these accounts, it should not be forgotten that allowances must be made for the different circumstances of increase or decrease in a place, agreeably to the observation at the end of the note in page 194.

to it, from the country, in the most robust part of life, and with a much greater probability of attaining to old age, than if they had come into it in the weakness of infancy. But, notwithstanding this advantage, there are much fewer persons who attain to great ages in *London*, than in most other places where observations have been made.—At *Vienna*; of 22,704 who died in the four years 1717, 1718, 1724, 1725 (a), 109 reached 90 years; that is, 48 in 10,000. But in *London*, for 30 years before 1769 only 35 of the same number have reached this age.—At *Breslaw* it appears, by Dr. *Halley's* Table, that 41 of 1238 born, or a 30th part live to be 80 years of age.—In the parish of *All-saints*, in *Northampton*, an account has been kept ever since 1733 of the ages at which the inhabitants die; and I find that a 22d part die there turned of 80. At *Norwich* a like account has been kept; and it appears, that for the last 30 years, a 27th part of the inhabitants have died, turned of the same age.—According to Mr. *Kerffjeboom's* Table of Observations, published at the end of the last edition of Mr. *De Moivre's* Treatise on the Doctrine of Chances, a 14th part of all that are born live to 80.—And in the parish of *Holy-Cross*, already mentioned p. 183 and p. 192,

(a) Vid. Abridgment of the Philosophical Transactions, vol. VII. part iv. p. 46.—It appears also that more than three-fifths of all who died in these years at *Vienna* were boys and girls, by whom, I suppose, are meant persons under 16.

the *eleventh* part of the inhabitants live to 80 (a).—But in *London*, for 30 years, ending at the year 1768, only 25 of every 1000, who have died, or a 40th part, have lived to this age; which may be easily discovered, by dividing the sum of all who have died during these years at all ages, by the sum of all who have died above 80.

Among the peculiar evils to which great towns are subject, I might further mention the PLAGUE. Before the year 1666, this dreadful calamity laid *London* almost waste once in every 15 or 20 years; and there is no reason to think, that it was not generally bred within itself. A most happy alteration has taken place; which, perhaps, in part is owing to the greater advantages of cleanliness and openness, which *London* has enjoyed since it was rebuilt; and which lately have been very wisely improved.

The facts I have now taken notice of are so important that, I think, they deserve more attention than has been hitherto bestowed

(a) This, however, will appear itself inconsiderable, when compared with the following account: “In 1761 the burials in the district of *Christiana*, in *Norway*, amounted to 6,929, and the christenings to 11,024. Among those who died, 394, or 1 in 18, had lived to the age of 90; 63 to the age of 100, and seven to the age of 101. — In the diocese of *Bergen*, the persons who died amounted only to 2,580, of whom 18 lived to the age of 100; one woman to the age of 104, and another woman to the age of 108.”

See the *Annual Register* for 1761, p. 191.

upon

upon them. Every one knows that the strength of a state consists in the number of people. The encouragement of population, therefore, ought to be one of the first objects of policy in every state; and some of the worst enemies of population are the luxury, the licentiousness, and debility produced and propagated by great towns.

I have observed that *London* is now (*a*) increasing. But it appears that, in truth, this is an event more to be dreaded than desired. The more *London* increases, the more the rest of the kingdom must be deserted; the fewer hands must be left for agriculture; and, consequently, the less must be the plenty and the higher the price of all the means of subsistence.—*Moderate* towns, being seats of refinement, emulation, and arts, may be public advantages. But *great* towns, long before they grow to half the bulk of *London*, become checks on population of too hurtful a nature, nurseries of debauchery and voluptuousness; and, in many respects, greater evils than can be compensated by any advantages (*b*).

Dr.

(*a*) This increase is greater than the bills shew, on account of the omission in them of the two parishes which have been most increased by new buildings; I mean *Marybone* and *Pancrats* parishes. The former of these parishes is, I suppose, now one of the largest in *London*.

(*b*) The mean annual *births*, *weddings*, and *burials* in the following towns, for some of the last years, have been nearly,

At

Dr. Heberden observes that, in *Madeira*, the inhabitants double their own number in 8 $\frac{1}{2}$ years. But this (as you, Sir, well know) is a very slow increase, compared with that which takes place among our colonies in AMERICA. In the back settlements, where the inhabitants apply themselves entirely to agriculture, and luxury is not known, they double their own number in 15 years; and all thro' the northern colonies, in 25 years (a). This is an instance of increase so rapid, as to have scarcely any parallel. The births in these countries must exceed the burials much more than in *Madeira*; and a greater proportion of the born must reach maturity.—In 1738, the number of inhabitants in *New Jersey* was taken by order of the government, and found to be 47,369. Seven years afterwards, the number of inhabitants was again taken; and found to be increased, by procreation only, above 14,000; and very near one *half* of the inhabitants were found

	Births.	Weddings.	Burials.
At Paris, —	19,100	— 4,400	— 19,400
Vienna, —	5,600	—	— 6,800
Amsterdam, —	4,600	— 2,400	— 8,000
Copenhagen, --	2,700	— 886	— 3,300 .
Berlin, for 5 } years, end- } ing at 1751 }	3,634	— 936	— 4,092

(a) See a discourse on *Christian Union*, by Dr. Styles, *Boston*, 1761, p. 103. 109, &c.— See also *The Interest of Great Britain considered with regard to her Colonies, together with Observations concerning the increase of mankind, peopling of countries, &c.* p. 35. 2d edit. *London*, 1761.

to be under (a) 16 years of age. In 22 years, therefore, they must have doubled their own number, and the births must have exceeded the burials 2000 annually. As the increase here is much quicker than in *Madeira*, we may be sure that a smaller proportion of the inhabitants must die annually. Let us, however, suppose it the same, or a 50th part. This will make the annual burials to have been, during these seven years, 1000; and the annual births 3000; or an 18th part of the inhabitants.—Similar observations may be made on the much quicker increase in *Rhode Island*, as related in the preface to the *Collection of the London Bills of Mortality*; and also in the valuable pamphlet, last quoted, on *the Interest of Great Britain with regard to her Colonies*, p. 36.—What a prodigious difference must there be, between the vigour and the happiness of human life in such situations, and in such a place as *London*?—The original number of persons who, in 1643, had settled in *New-England*, was 21,200. Ever since, it is reckoned, that more have left them than have gone to them (b). In the year 1760, they were increased to half a million. They have, therefore, all along

(a) According to Dr. *Halley's* Table, the number of the living under 16, is but a *third* of all the living at all ages.

(b) See Dr. *Styles's* pamphlet just quoted, p. 110, &c.

doubled

doubled their own number in 25 years. And if they continue to increase at the same rate, they will, 70 years hence, in *New-England* alone, be four millions; and in all *North America*, above twice the number of inhabitants in *Great-Britain* (a).—But I am wandering

(a) The rate of increase, supposing the procreative powers the same, depends on two causes: The “encouragement to marriage;” and the “expectation of a child just born.” When one of these is given, the increase will be always in proportion to the other. That is; As much greater or less as the ratio is of the numbers who reach maturity, and of those who marry, to the number born, so much quicker or slower will be the increase. — Let us suppose the operation of these causes such, as to produce an annual excess of the *births* above the *burials*, equal to a 36th part of the whole number of inhabitants. It may seem to follow from hence, that the inhabitants would double their own number in 36 years; and thus some have calculated. But the truth is, that they would double their own number in much less time. Every addition to the number of inhabitants from the births, produces a proportionably greater number of births, and a greater excess of these above the burials; and if we suppose the excess to increase annually at the same rate with the inhabitants, or so as to preserve the ratio of it to the number of inhabitants always the same, and call this ratio $\frac{1}{r}$, the period of doubling will be, the quotient produced by dividing the logarithm of 2, by the difference between the logarithms of $r + 1$ and r ; as might be easily demonstrated. In the present case, r being 36, and $r + 1$ being 37, the period of doubling comes out 25 years. If r is taken equal to 22, the period of doubling will be 15 years.—But it is certain that this ratio may, in many situations, be greater than $\frac{1}{22}$; and, instead of remaining the same, or becoming less, it may increase, the consequence of which will be, that the period of doubling will be shorter than this rule gives it.—According to Dr. Halley’s Table, the number of persons between 20 and

dering from my purpose in this letter. The point I had chiefly in view was, the present state

42 years of age is a third part of the whole number living at all ages. The prolific part, therefore, of a country may very well be a 4th of the whole number of inhabitants; and supposing four of these, or every other marriage between persons all under 42, to produce *one* birth every year, the annual number of births will be a 16th part of the whole number of people. And, therefore, supposing the burials to be a 48th part, the annual excess of the births above the burials will be a 24th part, and the period of doubling 17 years. — The number of inhabitants in *New-England* was, as I have said from Dr. *Styles's* pamphlet, half a million in 1760. If they have gone on increasing at the same rate ever since, they must be now 640,000; and it seems to appear that in fact they are now more than this number. For, since writing the above observations, I have seen a particular account, grounded chiefly on surveys lately taken with a view to taxation and for other purposes, of the number of males, between 16 and 60, in the four provinces. According to this account, the number of such males is 218,000. The whole number of people, therefore, between 16 and 60, must be nearly 436,000. In order to be more sure of avoiding excess, I will call them only 400,000. In Dr. *Halley's* Table the proportion of all the living under 16 and above 60, to the rest of the living, is 13.33 to 20; and this will make the number of people now living in the four provinces of *New-England* to be 666,000. But on account of the rapid increase, this proportion must be considerably greater in *New-England*, than that given by Dr. *Halley's* Table. In *New-Jersey*, I have said the number of people under 16, was found to be almost equal to the number above 16. Suppose, however, that in *New-England*, where the increase is slower, the proportion I have mentioned is only 16 to 20; and then the whole number of people will be 720,000.

I cannot conclude this note without adding a remark to remove an objection which may occur to some in reading
Dr.

state of *London* as to healthfulness, number of inhabitants, and its influence on population. The observations I have made may, perhaps, help to shew, how the most is to be made of the lights afforded by the *London* bills; and serve as a specimen of the proper method of calculating from them. It is indeed extremely to be wished, that they were less imperfect than they are, and extended further. More parishes round *London* might be taken into them; and, by an easy improvement in the parish registers now kept, they might be extended through all the parishes and towns in the kingdom. The advantages arising from hence would be very considerable. It would give the precise law according to which human life wastes in its different stages; and thus supply the necessary *data* for computing accurately the values of all *life-annuities* and *reversions*. It would, likewise, shew the different degrees of health-

Dr. *Heberden's* account of *Madeira*, to which I have referred. In that account 5945 is given as the number of children under seven in the island, at the beginning of the year 1767. The medium of annual births, for eight years, had been 2201; of burials 1293. In six years, therefore, 13,206 must have been born; and if, at the end of six years, no more than 5945 of these were alive, 1210 must have died every year. That is; almost all the burials in the island, for six years, must have been burials of children under seven years of age. This is plainly incredible; and, therefore, it seems certain, that the number of children under seven years of age must, through some mistake, be given, in that account, 3700 or 4000 too little.

fulness

fulness of different situations, mark the progress of population from year to year, keep always in view the number of people in the kingdom, and, in many other respects, furnish instruction of the greatest importance to the state. Mr. *De Moivre*, at the end of his book on the doctrine of chances, has recommended a general regulation of this kind ; and observed, particularly, that at least it is to be wished, that an account was taken, at proper intervals, of all the living in the kingdom, with their ages and occupations ; which would, in some degree, answer most of the purposes I have mentioned. — But, dear Sir, I am sensible it is high time to finish these remarks. I have been carried in them far beyond the limits I at first intended. I always think with pleasure and gratitude of your friendship. The world owes to you many important discoveries ; and your name must live as long as there is any knowledge of philosophy among mankind. That your happiness in this, and every other respect, may continually increase, is the sincere wish of,

SIR,

Your much obliged,

and very humble servant,

Newington-Green,
April 3, 1769.

RICHARD PRICE.

P O S T-

P O S T S C R I P T.

AT *Edinburgh*, bills of mortality, of the same kind with those in *London*, have been kept for many years. I have, since the foregoing letter was written, examined these bills, and formed a Table of Observations from them, as I found them for a period of 20 years, beginning in 1739, and ending in 1758.—As this is a town of moderate bulk, and seems to have a particular advantage of situation; I expected to find the probabilities of life in it, nearly the same with those at *Breslaw*, *Northampton* and *Norwich*; but I have been surprized to observe, that this is not the case. During the period I have mentioned, only one in 42 of all who died at *Edinburgh*, reached 80 years of age; which is a smaller proportion than attains to the same age in *London*. See p. 201.—In general; it appears, that the probabilities of life in this town are much the same, thro' all the stages of life, with those in *London*, the chief difference being, that after 30, they are rather lower at *Edinburgh*.—It is not difficult to account for this. It affords, I think, a striking proof of the pernicious effects arising from uncleanness, and crowding together on one spot too many inhabitants. At *Edinburgh*, Mr. *Maitland* says, “ the build-
“ ings, elsewhere called *houses*, are denomi-
P “ nated

“ nated *lands*; and the *apartments*, in other
 “ places named *stories*, here called *houses*, are
 “ so many freeholds inhabited by different
 “ families; whereby the houses are so ex-
 “ cessively crouded with people, that the
 “ inhabitants of this city may be justly pre-
 “ sumed to be more numerous than those of
 “ some towns of *triple* its dimensions.” See
Maitland's History of Edinburgh, p. 140.

In the year 1748, the whole number of *apartments* or *families* in the city and liberties of *Edinburgh*, was 9064. This Mr. *Maitland* mentions as the result of particular examination, and undoubtedly right. *Ib.* p. 217, 218.—In 1743, an accurate account was taken, by the desire of this writer, of the number of *families* and *inhabitants* in the parish of *St. Cuthbert*. *Ib.* p. 171. The number of *families* was 2370, and of *inhabitants* at all ages, 9731. The proportion, therefore, of *inhabitants* to *families*, was $4\frac{1}{15}$ to 1; and, supposing this the true proportion for the whole town, the number of inhabitants will be $4\frac{1}{15}$ multiplied by 9064, or 37,162.—The yearly medium of deaths in the town and liberties for eight years, from 1741 to 1748, was 1783. *Ib.* p. 220 and 222. And, consequently, *one* in $20\frac{4}{5}$ died annually.

Mr. *Maitland*, tho' possessed of the *data* from which these conclusions necessarily follow, has made the number of inhabitants 50,120, in consequence of a disposition to

exaggerate in these matters, and of assuming, without any reason, a 28th part of the inhabitants as dying annually.

In page 220, he expresses much surprize at finding, that the number of males in this town was less than the number of females, in the proportion of 3 to 4. But this is by no means peculiar to *Edinburgh*.

All I have been saying must be understood of the state of *Edinburgh*, before the year 1758. The bills, for the last 12 years, have been so irregular, and so different from the same bills for the preceding years, and from all other bills, that I cannot give them any credit. Either some particular incorrectness has crept into the method of keeping them; or there has been some change in the state of the town which renders them of no use. Probably the former is the truth.

From the note in p. 203, it appears, that the christenings and burials at PARIS, come very near to equality. This once led me to suspect, that there must be some particular singularity in the state of *Paris*, which rendered it much less prejudicial to health and population than great towns commonly are. But better information has lately obliged me to entertain very different sentiments.—The difference between the births and burials at *Paris*, is much greater than the bills shew.

“ Children here are baptized the instant
 “ they are born ; and, in a day or two af-
 “ terwards, it is the custom to send them to
 “ the adjacent villages to be nursed. A
 “ great number therefore, of the infants born
 “ at *Paris*, die in the country, and these
 “ appear only in the register of christen-
 “ ings.” See a book entitled the *Police*
of France, page 127. And *Buffon’s* Natural
 History, Tom. II. at the end. — “ All the
 “ children also received into the *Foundling-*
Hospital, are immediately sent to be nursed
 “ ed in the country, at a distance from *Paris*,
 “ where they remain 5 or 6 years; at the end
 “ of which time they are brought again to
 “ *Paris*, the boys to be placed in the suburbs
 “ of *St. Antoine*, and the girls at *Salpetriere*,
 “ to be further maintained till they arrive at
 “ the age of twelve years.” *Police of France*,
 p. 81. — The following passage in the same
 writer, containing a further account of this
 Hospital, is important; and, therefore, tho’
 long, I cannot help transcribing it — “ Let
 “ us suppose, that out of 4000 children an-
 “ nually carried into the country, two thirds
 “ may die, during the five years they are de-
 “ stined to remain at nurse; so that only
 “ 1333 would constantly be the annual
 “ number sent back to *Paris*; who, being
 “ kept at the two Hospitals *St. Antoine* and
 “ *Salpetriere* just mentioned, ’till they are 12,
 “ and succeeded by a like number each year,
 “ the

“ the total number composed of all brought
“ in the successive years, would make the
“ constant resting stock to amount to 9331.
“ But of these we will suppose a 5th part
“ to die every year. Yet even then the
“ constant resting stock of children ought to
“ be 7465. How greatly then must we be
“ surprized to find, by the authentic account
“ taken from their own books, only 640
“ boys in the college of *St. Antoine*, and not
“ more than 600 girls at the *Salpetriere* ;
“ so that the resting stock of returned found-
“ lings appears to be no more than 1240,
“ which being deducted from 7465, will make
“ the difference in the deficiencies 6225.
“ What then becomes of these? — Are they
“ reclaimed by their parents? — Or do they
“ perish for want of care? — In answer to
“ which questions it was explained to me ;
“ that as many of the lower class of people
“ were induced to marry, in order to be ex-
“ cused from serving in the militia ; so when
“ these have children, which they are una-
“ ble to maintain, they usually send them to
“ this hospital ; which, therefore, must be
“ looked upon, as not only a charity for the
“ care of exposed and deserted children whose
“ parents are unknown, but also as a public
“ *nursery* for the sustenance of the children
“ of poor people, who, tho’ registered at the
“ office, are often reclaimed from their coun-
“ try nurses by their parents. This accounts,

“ in some measure, for the small stock of
 “ children brought back to the hospital at
 “ *Paris*.—The further difference is suspected
 “ to be owing to the insufficient nourishment
 “ they receive ; as this particular charity, as
 “ well as the General Hospital, adopts that
 “ preposterous method of taking in an un-
 “ limited number, while there is only a li-
 “ mited income for their subsistence.” *Ib.*
 page 83.

These facts prove, that, at the same time that the register of *christenings* at *Paris* must be full, the register of *burials* must be very deficient. Let the deficiencies be reckoned at 4600 ; and, consequently, the annual burials at 24,000. The annual average of weddings, given in p. 203, is 4300 ; and, therefore, the number of persons who marry annually must be 8600. The difference between the christenings and burials is 5000 ; which, therefore, is the number of annual recruits from the country. These, in general, must be persons in mature life. Suppose 3600 of them to marry after settling at *Paris*. Then, 8600 lessened by 3600 or 5000, will be the number of persons born at *Paris* who marry annually ; and 14,100, or near three-fourths of all who are born at *Paris*, will be the number dying annually in childhood and celibacy.

The suppositions on which I have made this computation seem moderate ; but if any
 one

one thinks otherwise, he may make the same calculation on any other suppositions.

The births at *Paris* are above four times the weddings; and it may seem, therefore, that here, as well as in the most healthy country situations, every wedding produces above four children. I have observed nothing like this in any other great town. Many children born in the country are, I suppose (*a*), brought to the Foundling-Hospital, and there christened. This Hospital may likewise occasion a more than common number of illegitimate births. And, besides, some who leave the country to settle at *Paris*, may come thither already married. These are circumstances that will swell the register of births, without having any effect on the weddings. I do not, however, know that any of them take place at *Paris*; and, perhaps, it must be granted, that it is distinguished in this respect from most other towns. Nor can I wonder at this, if it be indeed true, not only, that all married men in *France* are excused serving in the militia from whence draughts are made for the army, but also, that a *fifth* of all the children born at *Paris* are sent to the *Foundling-Hospital* (*b*). These
are

(*a*) “ If the parents of a child brought to this Hospital
“ are known, the register of its baptism must be pro-
“ duced. If the parents are unknown, the child must
“ be baptised after being received.” *Police of France*,
pag. 82.

(*b*) See *Police of France, &c.* p. 83. — This writer
adds, that a *third* of all that die at *Paris* die in Hospitals.

are encouragements to marriage that no other city enjoys. It has been seen that the *Foundling-Hospital*, tho' attended with this effect, is, probably, in the highest degree pernicious; but it is to be wished, that some policy of the same kind with that *first* mentioned, was pursued in this kingdom.—At the end of the 2d vol. of Monsieur *De Buffon's Natural History*, there are Tables formed from the Observations of M. *Du Pre de S. Mour*, of the *French Academy*, containing an account of the ages at which 13,189 persons died in three parishes at *Paris*; and also, of the ages at which 10,805 persons died in 12 country parishes and villages near *Paris*.—According to these Tables, many *more* die in the beginning of life, and much *fewer* in the latter part of life, in the country than in *Paris*. But the circumstances of *Paris*, and the country round it, are such, that no argument can be drawn

“ In the *Hotel Dieu* (a great Hospital situated in the
 “ middle of the city) we may, he says, behold a horrid
 “ scene of misery; for, the beds being too few for the
 “ numbers admitted, it is common to see 4, or 6, or
 “ even 8 in a bed together, lying 4 at one end, and 4
 “ at the other, ill of various distempers in several de-
 “ grees; some bad, others worse; some dying, others
 “ dead.—Above a *fifth* of all admitted to this Hospital
 “ die; the annual numbers admitted being 21,823. The
 “ medium of deaths for three years from 1751 to 1753,
 “ 4650.—The medium of deaths for the same years in
 “ all the Hospitals was 6181.” *Ib.* p. 85.—In our two
 great city Hospitals, *St. Thomas's* and *St. Bartholomew's*,
 about 600 die annually; or one in 13 of all admitted as
 in patients.

from hence in favour of *Paris*. Many of the children dying in the country, are children sent thither from *Paris* to be nursed; and, on the other hand, *many*, perhaps *most*, of those who die in old age at *Paris*, are persons who have removed thither from the country, some to *Hospitals*, and some to places and settlements. It is evident, therefore, that these Tables give a representation of the probabilities of life at *Paris*, which, when compared with those in the adjacent country (*a*), is just the reverse of the truth. Were the children born at *Paris*, who die in the country, to be transferred to the town register; and, on the contrary, the adults born in the country, who die at *Paris*, to be transferred to the country register, there is no reason to doubt, but that the probabilities of life at *Paris*, would be found as low, in comparison with those in the country, as the probabilities of life in *London* are; or, perhaps, much lower. — This observation is applicable, in some degree, to most other great towns; and, in general, on account of the migrations from the country to towns, navies and armies, we may be satisfied, that we err on the side of *defect*,

(*a*) It is for this reason that these Tables, when combined, exhibit justly the *mean* probabilities of life for town and country taken together; and that the Table of the *decrements* of life deduced from them by M. *Buffon* and M. *Du Pre*, agrees nearly with Dr. *Halley's* Table.

whenever

whenever we judge of the probabilities of life in the *country*, from the numbers dying in the several stages of life ; and, on the side of *excess*, whenever, in the same way, we judge of the probabilities of life in *towns*. And this, it is obvious, has a tendency to confirm all that has been said in the preceding Essay, concerning the pernicious effects of great towns on human life.

There are several *ordonances* and *arrets* of council which fix the boundaries of *Paris*, and prohibit all new buildings beyond those boundaries.—The reasons of this regulation, as set forth in one of these *arrets*, are remarkable ; and it will not be improper to recite them.—“ By the excessive aggrandizing of the city, it is said, the air would be rendered unwholesome, and the cleaning the streets more difficult.”—“ Augmenting the number of inhabitants would augment the price of provisions, labour and manufactures.”—“ That ground would be covered with buildings, which ought to be cultivated in raising the necessary subsistence for the inhabitants ; and thereby hazard a scarcity.”—“ The people in the neighbouring towns and villages would be tempted to come and fix their residence in the capital, and desert the country.”—“ And lastly ; the difficulty of governing so great a number of people would occasion a disorder in the *Police*, and give an opportunity

“tunity to rogues to commit robberies and
“murders (a).”

No one can think overgrown cities greater evils than I do. But, yet, I can by no means approve of this policy. The effect of it must be, crowding together too many people within the prescribed boundaries, and rendering a town more the seat of uncleanness, infection and disease.—The number of houses in *Paris* is reckoned to be 28,000 (b), or 30,000; but the number of inhabitants, supposing a 20th part to die annually, cannot be much less than 480,000, or 16 times the number of houses.

It is happy for LONDON, that there have been no laws to restrain its increase. In consequence of being allowed to extend itself on all sides into the country, the inhabitants now take near twice the room to live upon that they did; and it is become less the means of shortening human life. See p. 189, 191, and 201.

In page 203, I have given the annual *medium* of births, weddings and burials at BERLIN, from 1747 to 1751.—In 1747, an account was taken with the utmost care, by the order of the King of PRUSSIA, of the number of inhabitants in this town; and, it was found to be 107,224.—In order to be more certain, a *second* account was taken the

(a) Vid. *Police of France*, p. 130.

(b) *Ibid.*—There are other accounts which makes this number above 50,000.

same year; and the number found the same within 200.—In 1749, the inhabitants were increased to 110,933. Their number, therefore, compared with the annual burials, was as 27 to 1.—This is a higher proportion than could be expected in a town so considerable; and also so much crowded, as to have, at an average, 16 inhabitants in every house. But there is a plain reason to be given for this fact.—BERLIN, for many years, had been increasing very fast, by a conflux of people from the surrounding country and provinces. About the year 1700, the medium of annual burials was no more than 1000. In 50 years, therefore, it quadrupled itself.—In a city increasing with such rapidity, the *ratio* of inhabitants to the annual deaths, must be greatly below the just standard.—Were there now, such accessions to LONDON of deserters from the country, in the beginning of mature life, as would cause the number of inhabitants to increase at the rate of 10,000 every year, it would in 60 years be doubled; and the proportion of inhabitants to deaths would rise gradually, till it came to be about one third greater. BERLIN, we have seen, has, in fact, increased at *more than double* this rate; and, therefore, the proportion of inhabitants dying annually in it is in reality very low.

The ingenious *Susmilch*, to whose curious book, already quoted, I owe my information concerning BERLIN, makes the proportion
of

of people who die annually in *great* towns, to be from $\frac{1}{24}$ to $\frac{1}{28}$; in *moderate* towns, from $\frac{1}{28}$ to $\frac{1}{31}$; and, in the country from $\frac{1}{40}$ to $\frac{1}{50}$.—The observations and facts in this Essay, joined to those which will be found in the 4th Essay, prove, I think, that these proportions may be more truly stated as follows.—*Great* towns, from $\frac{1}{20}$ to $\frac{1}{23}$ or $\frac{1}{24}$. *Moderate* towns, from $\frac{1}{23}$ to $\frac{1}{28}$. The *country*, from $\frac{1}{30}$ or $\frac{1}{35}$, to $\frac{1}{50}$ or $\frac{1}{60}$.—This, however, must be understood with exceptions. There may be *moderate* towns so ill situated, or whose inhabitants may be so crowded together, as to render the proportion of deaths in them greater than in the largest towns: And, of this, EDINBURGH, if it is not now, was 20 years ago, an example.—There may be also *great* towns in which, from a sudden increase, this proportion may be less than in small towns: And of this, I have just given an example in BERLIN.

E S S A Y II.

On Mr. DE MOIVRE'S Rules for calculating the Values of Joint Lives.

THE calculation of the values of *single* and *joint* lives, from given Tables of Observation, being tedious and troublesome; Mr. *De Moivre* has had recourse to two *Hypotheses*, which give easy rules for this purpose; and which, he thought, corresponded with sufficient exactness to Observations.— The first of these *Hypotheses* is, that the probabilities of life decrease, as we advance from childhood to old age, in an *arithmetical progression*; or in such a manner, that the *difference* is always the same, between the number of persons living at the beginning of any one year, and the number living at the beginning of the next following year.— The other *Hypothesis* is, that the probabilities of life decrease in a *geometrical progression*; or in such a manner, that the *proportion* is always the same, between the number of persons living at the beginning of any one year, and the number living at the beginning of the next following year.— All the Tables of Observation shew, that the real law, according to which human life wastes, comes
 much

much nearer to the former *Hypothesis*, than the latter.—In Tables III, IV, and V, in the *Appendix*, it is so near the former *Hypothesis*, that the difference is scarcely worth regarding. According to this *Hypothesis*, therefore, (accommodated to the *Breslaw* Table, in the manner mentioned in the note, page 2.) Mr. *De Moivre* calculated the values of *single* lives; and the rules founded upon it for this purpose are so easy, that an operation which would otherwise take up much time, may be performed almost immediately.

By proceeding on the same principles, the values of *joint* lives might have been calculated; but the rules for this purpose derived from these principles, are far from being equally easy in practice. Here, therefore, Mr. *De Moivre* quitted his *first* Hypothesis; and finding, that the *second* Hypothesis afforded, in the case of *joint* lives, rules that were as easy, as the rules given by the other Hypothesis were in the case of *single* lives, he chose to adopt this *Hypothesis*; believing at the same time, that the values of *joint* lives, obtained by rules derived from it, would not deviate much from the truth. But in this he was greatly mistaken. The values of *two joint* lives obtained by these rules are so wrong, that in finding the present value, in a *single payment*, of one life after another, they generally give results which are near $\frac{1}{4}$ of the true value too great; and about *two-fifths*

ffths too great, when the value is sought in *annual payments* during the joint lives.— These are errors so considerable, that I think it is of particular importance that the public should be informed of them, in order to prevent the inconveniencies and perplexities they may occasion.

Mr. *Simpson* (in the Appendix to his Treatise on the *Doctrine of Annuities and Reversions*) has observed, that Mr. *De Moivre's* rules for finding the values of joint lives are wrong. But I don't know, that it has been ever attended to, that they are *so* wrong as I have found them. Mr. *Simpson's* remarks point out chiefly the errors in these rules, when the values of *three* or more joint lives are calculated by them; but, 'till I was forced to a particular examination of this subject by some difficulties into which I found myself brought by following Mr. *De Moivre* too implicitly, I did not at all suspect, that any such errors as I have mentioned, could arise from these rules, when the values of only *two* joint lives are calculated by them. Mr. *De Moivre*, in consequence of other remarks contained in Mr. *Simpson's Appendix*, altered in the 4th edition of his Treatise some of his rules. It is surprizing he did not see reason at the same time to alter these.

That there may be no doubt about the truth of these observations, I will just mention a few examples of the difference between
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the values of a given reversionary annuity, according to the rules to which I have objected, and the *true* values, according to the exact method of deducing them from Mr. *De Moivre's first Hypothesis*.

Let the proposed annuity be 30*l.*, to be enjoyed for what shall happen to remain of the life of a person now 40 years of age, after the life of another person of the same age. The value of the joint lives (interest being at 4 *per cent.*) is, by problem 2d of Mr. *De Moivre's Treatise on Life-annuities*, 8.964; which subtracted from 13.196, (the value by Table VI, of a single life at 40) gives 4.23; which remainder, multiplied by 30, gives 126.9, or the value of the reversion in a single present payment. And 126.9, divided by the foregoing value of the joint lives, is 14. 16; or, the value of the reversion in annual payments during the joint lives.—But the *true* values are 1101.1 in a *single* payment, by Quest. I. chap. I.; and 110.3, in *annual* payments, by Quest. IV.—The former values, therefore, are near a *quarter* of the true value too great in the *single* payment; and near *two-fifths* too great in the *annual* payments.

The *true* value of the same annuity for a life at 66, after another life of the same age, is, (reckoning interest as before, at 4 *per cent.*) 681. in a *single* payment; and 13.5, in *annual* payments.—But these values, according

Q to

to the Problem just quoted, are 9*l.*, and 21*l.* one of which is near a *third*, and the other above *half* the true value too great.

In *unequal* lives these errors may be no less considerable.—Thus; if the value of the proposed annuity be required for a life at 70, after a life at 30 years of age; it will, by the same Problem, be *l.* 26.5, in a *single* payment; and *l.* 5.1, in *annual* payments during the joint lives. But the *true* values are 17*l.* and *l.* 3.05.

Where 3 or more lives are concerned the errors will be still greater.

The true values of the joint lives, mentioned in these Examples, have been calculated by a rule in pag. 16, of Mr. *Simpson's* Treatise on the Doctrine of *Annuities* and *Reversions*, and explained in note (M) *Appendix*.—To save, however, a great deal of trouble hereafter, I have thought proper to calculate Table VII, which gives the exact values, according to Mr. *De Moivre's* first Hypothesis, of two joint lives, for every five years of human life from, 10 to 70.

This *Hypothesis*, I have observed, does not differ much from the Tables of Observation in the *Appendix*, for *Breslaw*, *Northampton* and *Norwich*. Between the ages of 30 and 40, it gives the values of *single* lives almost the same with the *Breslaw* Table. Under 30, it gives them somewhat *less*; and above 40, somewhat *greater*. But it ought to be remembered,

membered, that wherever it does this, it gives, at the same ages, the values of the *joint* lives also too little or too great; and that, consequently, the results from it, in calculating the values of *Reversions*, and of the *longest* of given lives, come so much nearer to exactness.

The rules to which I have objected are the only ones given by Mr. *De Moivre*, in all the editions of his *Treatise on Life-Annuities*. But it seems, this great mathematician became at last sensible, that they were too incorrect; and, therefore, at the end of the last edition of his *Treatise on the Doctrine of Chances*, pag. 320, (a work which gets into comparatively few hands) he has given other rules which come nearer the truth. But even these rules produce errors so great in many cases, (particularly when combined with the errors of the Hypothesis) that it will be best never to use them.

E S S A Y III.

Of the Method of calculating the Values of Reversions depending on Survivorships.

ALL Questions relating to the values of lives and reversions, are at present of particular importance in this kingdom. Much business is continually transacted in this way; and any considerable errors in the methods of solving such questions, must in time produce very bad consequences.—The design of the following observations is to point out a particular error, into which there is danger of falling, in finding the values of such reversions as depend on survivorships. In doing this, I shall, in order to be as plain as possible, take the following case. “A, aged
 “ 40, expects to come to the possession of
 “ an estate, should he survive B, aged like-
 “ wise 40. In these circumstances he offers,
 “ in order to raise a present sum, to give fe-
 “ curity for 40*l. per annum*, out of the estate
 “ at his death, provided he should get into
 “ posses-

“ possession; that is, provided he should survive B. What is the sum that ought now to be advanced to him, in consideration of such security, reckoning compound interest at 4 *per cent*?”

Mr. *De Moivre*'s directions in his *Treatise on Annuities, Problems* 17th and 20th, lead us to seek the required sum in this case, by the following process.

Find first, the present sum A should receive, for the reversion of 40*l. per annum* for ever after his death; supposing it *not* dependent on his surviving B. The present value of such a reversion is “the (*a*) value of the life subtracted from the *perpetuity*, and the *remainder* multiplied by the annual rent.”—The value of the life is, by Table VI, 13.196. This subtracted from 25, the *perpetuity*, leaves 11.80; which, multiplied by 40, gives 472; the value of the supposed estate, after the life of A. But, as Mr. *De Moivre* observes, the lender having a chance to lose his money, a compensation ought to be made to him for the risk he runs, which is founded on the possibility, that a man of 40 years of age may not survive another person of the same age. This chance is an *equal* chance; and, therefore, half the preceding sum, or 236*l.* is the money which should be advanced now on the expectation mentioned.

(*a*) By *Scholium*, p. 34, and Problem 26th, p. 293, of Mr. *Simpson's* *Select Exercises*.

This solution carries a plausible appearance; and most persons will, probably, be ready to pronounce it right; nor will this be at all wonderful, as so great a master of these subjects as Mr. *De Moivre* appears to have been misled by it. — Nothing more is necessary to prove it to be fallacious, than proceeding in the same way to solve the following similar Question.

“ A, aged 40, offers to give security for
 “ 40*l. per annum*, to be entered upon at his
 “ death, provided it should happen *before* the
 “ death of B, aged likewise 40. What sum
 “ should now be advanced to him for such a
 “ reversion, interest being reckoned at 4 *per*
 “ *cent* ?”

In solving this Problem, agreeably to the method just described, we are to find the value of 40*l. per annum*, to be entered upon *certainly* at the death of A; and then to multiply this value by the chance that A shall *not* survive B, or by $\frac{1}{2}$; and in this way the answer comes out the same with that already given.

Now it may be easily seen, that this must be wrong. The value of a reversion, to be received when a person of a given age dies, cannot be the same, whether the condition of obtaining it is, that he shall die *before*, or that he shall die *after* another person. That is, whether it is provided, that a purchaser, if he succeeds, shall get into possession *sooner* or
later.

later. The reversion in the latter case must, without doubt, be of less value than in the former.

The first Question here proposed, resolves itself into the following general Question.

“What is the present value of a given reversionary estate, to be entered upon after the failure of two lives, provided one *in particular* of them should be the *longest life?*”

Now, the present value of an estate to be enjoyed for ever, after the failure of the *longest* of two lives, is “the value of the *longest* of the two lives, subtracted from the *perpetuity*; and the remainder multiplied by “the annual rent of the estate.”—The value of the *longest* of two lives is, (as is well-known) the value of the two *joint* lives, subtracted from the *sum* of the (*a*) values of the two *single* lives. In the present case, therefore, it is 9.82, (the value of two joint lives at the age of 40 by Table VII,) subtracted from twice 13.196; (the value of a *single* life at the same age by Table VI,) that is, 16.57 year’s purchase. And this subtracted from 25, (the perpetuity) gives 8.43; which, multiplied by 40, gives £337.2, the value of the given estate were it *certainly* to be enjoyed, after the ex-

(a) See Mr. *De Moivre* on Annuities, Problem IV; or Mr. *Simpson’s Doctrine of Annuities and Reversions*, Prob. II.

tion of the longest of two lives both 40; that is, whether *one* or *other* of them failed last. But that A's life in particular should fail last, rather than B's, is an even chance. The true value of the reversion, therefore, is half the last value, or *l.* 168.6.

In like manner. The second Question is the same with the Question, "What is the present value of 40*l.* *per ann.* for ever, to be entered upon after the extinction of two *joint* lives both 40; that is, whenever *either* of them shall fail; provided the first that fails should happen to be A's life in particular?" — And the answer is found by subtracting the present value of the *two joint* lives from the *perpetuity*, and multiplying the remainder by $\frac{1}{2}$, or by the chance that A in particular shall die first: And this will give the required value, *l.* 303.4 (a).

In short. It appears in *both* these cases, that, according to the first method of solution, we are to subtract from the *perpetuity* the value of *one* of the single lives; when, in the *former* case, the value of the *longest* of the two lives; and, in the *latter* case, the value of their *joint continuance*, ought, in reality, to be subtracted. I need not say what prodigious errors may often arise from hence; and how unfit such a method of solution is for practice.

(a) I have, tho' scarcely necessary, given a demonstration of these Solutions in the Appendix, note (N).

Mr. *Simpson*, in p. 322, of his *Select Exercises*, speaks on this subject in the following manner. — “ I have been very particular
 “ on these kinds of Problems; and the more
 “ so, as there has been no method before pub-
 “ lished, that I know of, by which they can
 “ be rightly determined. ’Tis true, the man-
 “ ner of proceeding, by first finding the pro-
 “ bability of survivorship, (which method is
 “ used in my former work, and which a cele-
 “ brated author has largely insisted on in three
 “ successive editions) may be applied to good
 “ advantage, when the given ages are nearly
 “ equal; but then it is certain, that this is
 “ not a genuine way of going to work, and
 “ that the conclusions hence derived are at
 “ best but near approximations.”

This excellent mathematician has here expressed himself much too favourably of the method of solution on which I have remarked. — In both the cases I have specified the ages are equal; and yet, in one of them the error is a good deal above a *third* of the true value, and in the other, a *fifth*: And, it is obvious, that in cases where three equal lives are taken, the errors will be much greater. — Mr. *Simpson’s* Observations in this passage are true only, when applied to a *different* method used by himself, in the 28th and following Problems of his *Treatise on the Doctrine of Annuities and Reversions*. This method is exact when the lives are equal; but,
 it

it gives results which are too far from the truth, when there is any considerable inequality between the lives.

It is with reluctance I have made some of these remarks. Mr. *De Moivre* has made very important improvements in this branch of science; and the highest respect is due to his name and authority. This, however, only renders these remarks more necessary.

In the first Chapter (Questions 10th, 11th, 12th, 14th, &c.) I have given a minute account of the method of finding, in all cases, the values of the reversion which have been the subject of this Essay.

Observations on the proper Method of constructing Tables for determining the Rate of human Mortality, the Number of Inhabitants, and the Values of Lives in any Town or District, from Bills of Mortality in which are given, the Numbers dying annually at all Ages.

IN every place that just supports itself in the number of its inhabitants, without any recruits from other places; or where, for a course of years, there has been no increase or decrease, the number of persons dying every year at any particular age, and above it, must be equal to the number of the living at that age.—The number, for example, dying every year, at all ages, from the *beginning* to the utmost *extremity* of life, must, in such a situation, be just equal to the whole number *born* every year. And for the same reason, the number dying every year at *one* year of age and upwards; at *two* years of age and upwards; at *three* and upwards, and so on; must be equal to the numbers that attain to those ages every year; or, which is the same,
to

to the numbers of the living at those ages. It is obvious, that unless this happens, the number of inhabitants cannot remain the same. If the former number is greater than the latter, the inhabitants must *decrease*; if less, they must *increase*. — From this observation it follows, that in a town or country, where there is no increase or decrease, bills of mortality which give the ages at which all die, will shew the exact number of inhabitants; and also the exact law, according to which human life wastes in that town or country.

In order to find the number of inhabitants; the mean numbers dying annually, at every particular age and upwards, must be taken as given by the bills, and placed under one another in the order of the second column of the 12th Table in the Appendix. These numbers will, it has appeared, be the numbers of the living at 1, 2, 3, &c. years of age; and, consequently, the sum, diminished by half the number born annually (*a*), will be the whole

(*a*) This subtraction is necessary for the following reason.—In a Table formed in the manner here directed, it is supposed, that the numbers in the second column are all living together at the beginning of every year. Thus; the number in the *second* column opposite to 0 in the *first* column, the Table supposes to be all just born together on the first day of the year. The number, likewise, opposite to 1, it supposes to attain to one year of

age

whole number of inhabitants.—In such a series of numbers, the excess of each number above that which immediately follows it, will be the number dying every year, out of the particular number alive at the beginning of the year; and these excesses set down regularly as in the third column of the Table to which I have referred, will shew the different rates at which human life wastes thro' all its different periods, and the different probabilities of life at all particular ages.

It must be remembered, that what has been now said goes on the supposition, that the place whose bills of mortality are given, supports itself, by procreation only, in the number of its inhabitants. In towns this very seldom happens, on account of the luxury and debauchery which generally prevail in them. They are, therefore, commonly kept up by a constant accession of strangers or *settlers*,

age just at the same time that the former number is born. And the like is true of every number in the second column.—During the course of the year, as many will die at all ages as were born at the beginning of the year; and, consequently, there will be an excess of the number alive at the beginning of the year, above the number alive at the end of the year, equal to the whole number of the annual births; and the true number constantly alive together, is the arithmetical mean between these two numbers; or, agreeably to the rule I have given, the *sum* of the numbers in the second column of the Table, lessened by *half* the number of annual births. See Essay I, page 174.

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who remove to them from country parishes and villages. In these circumstances, in order to find the true number of inhabitants, and probabilities of life, from bills of mortality containing an account of the ages at which all die; it is necessary, that the proportion of the annual births to the annual settlers should be known; and also the period of life at which the latter remove.— Both these particulars may be discovered in the following method.

If for a course of years there has been no sensible increase or decrease in a place, the number of annual settlers will be equal to the excess of the annual burials above the annual births. If there is an *increase*, it will be *greater* than this excess. If there is a *decrease*, it will be *less*.

The period of life at which these settlers remove, will appear in the bills by an increase in the number of deaths at that period and beyond it. Thus; in the *London* bills, the number of deaths, between 20 and 30, is generally above double; and between 30 and 40, near triple the number of deaths between 10 and 20: And the true account of this is, that from the age of 18 or 20, to 35 or 40, there is an afflux of people every year to *London* from the country, which occasions a great increase in the number of inhabitants at these ages; and, consequently, raises the deaths for all ages *above* 20, considerably

siderably above their due proportion, when compared with the number of deaths *before* 20. — This is observable in all the bills of mortality for towns with which I am acquainted, not excepting even the *Breslaw* bills. Dr. *Halley* takes notice, that these bills gave the number of deaths, between 10 and 20, too small. This he considered as an irregularity in them, owing to chance; and, therefore, in forming his Table of Observations, he took the liberty so far to correct it, as to render the proportion of those who die to the living in this division of life, nearly the same with the proportion which, he says, he had been *informed* (a) die annually of the young lads in *Christ-Church Hospital*. But the truth is, that this irregularity in the bills was derived from the cause I have just assigned.—During the five years for which the *Breslaw* bills are given by Dr. *Halley*, the births did, indeed, a little exceed the burials; but, it appears, that this was the effect of some peculiar causes that happened to operate just at that time; for, during a complete century from 1633 to 1734, the annual medium of *births* was 1089 (b), and of bu-

(a) See *Lowthorp's* Abridgment of the Philosophical Transactions, vol. III. p. 670.—Dr. *Halley's* information in this instance was not right, as will appear presently; and, therefore, he has by no means sufficiently corrected the irregularity I have mentioned.

(b) See Dr. *Short's* Comparative History, p. 63.

rials 1256 (a). This town, therefore, must have been all along kept up by a number of yearly recruits from other places, equal to about a *seventh* part of the yearly births.

What has been now observed, concerning the period of life at which people remove from the country to settle in towns, would appear sufficiently probable, were there no such evidence for it as I have mentioned; for it might be well reckoned, that these people in general, must be single persons in the beginning of mature life, who, not having yet obtained settlements in the places where they were born, migrate to towns in quest of employments.

Having premised these Observations, I shall next endeavour to explain distinctly, the effect which these accessions to towns must have, on Tables of Observation formed from their bills of mortality. This is a subject proper to be insisted on, because mistakes have been committed about it; and because also, the discussion of it is necessary to shew, how near to truth the values of lives come as deduced from such Tables.

The following general rule may be given on this subject.

(a) It appears from the account in the *Philosophical Transactions*, (Abridgment, vol. VII, No. 380, p. 46, &c.) that from 1717 to 1725, the annual medium of births at *Breslaw* was 1252, of burials 1507; and also, that much the greatest part of the births died under 10 years of age.

If a place has, for a course of years, been maintained in a state nearly stationary, as to number of inhabitants, by recruits coming in every year, to prevent the decrease that would arise from the excess of the burials above the births; a Table formed on the principle, “that the number dying annually, after every particular age, is equal to the number living at that age,” will give the number of inhabitants, and the probabilities of life, too *great*, for all ages preceding that at which the recruits cease; and after this, it will give them *right*.—If the accessions are so great as to cause an *increase* in the place, such a Table will give the number of inhabitants, and the probabilities of life, too *little*, after the age at which the accessions cease (*a*); and too great, if there is a decrease. *Before* that age it will in *both* cases give them too great; but most considerably so in the former case, or when there is an increase.

(*a*) Agreeably to these Observations; if a place increases, not in consequence of accessions from other places, but of a constant excess of the births above the deaths; a Table, constructed on the principle I have mentioned, will give the probabilities of life too low through the *whole extent* of life; because, in such circumstances, the number of *deaths* in the *first* stages of life must be too great, in comparison of the number of deaths in the *latter* stages; and more or less so, as the *increase* is more or less rapid.—The contrary, in all respects, takes place where there is a decrease, arising from the excess of the *deaths* above the *births*.

For example. Let us suppose, that 244 of those born in a town, attain annually to 20 years of age; and that 250 more, all likewise 20 years of age, come into it annually from other places; in consequence of which, it has, for a course of years, been just maintained in the number of its inhabitants, without any sensible increase or decrease. In these circumstances, the number of the living in the town of the age of 20, will be always 244 *natives* and 250 *settlers*, or 494 in all; and, since these are supposed all to die in the town, and no more recruits are supposed to come in; 494 will be likewise the number dying annually at 20 and upwards.—In the same manner; it will appear on these suppositions, that the number of the living, at every age, subsequent to 20, will be equal to the number dying annually at that age and above it; and, consequently, that the number of inhabitants and the decrements of life, for every such age, will be given exactly by the Table I have supposed. But for all ages *before* 20, they will be given much too great. For let 280 of all born in the town, reach 10. In this case, 280 will be the true number of the living in the town at the age of 10; and the recruits not coming in 'till 20, the number given by the bills, as dying between 10 and 20, will be the true number dying annually of the living in this division of life. Let this number be 36; and it will

follow, that the Table ought to make the numbers of the living at the ages between 10 and 20, a series of decreasing means between 280 and (280 diminished by 36, or) 244. But in forming the Table on the principle I have mentioned, 250 (the number above 20 dying annually in the town who were not born in it) will be added to each number in this series; and, therefore, the Table will give the numbers of the living, and the probabilities of life in this division of life, almost twice as great as they really are.—This observation, it is manifest, may be applied to *all* the ages under 20.

It is necessary to add, that such a Table will give the number of inhabitants, and the probabilities of life, equally wrong before 20, whether the recruits all come in at 20, agreeably to the supposition just made, or only *begin* then to come in. In this last case, the Table will give the number of inhabitants, and probabilities of life, too great throughout the whole extent of life, if the recruits come in at all ages above 20. But if they cease at any particular age, it will give them right only from that age; and before, it will err all along on the side of excess; but less considerably between 20 and that age, than before 20. — For example. If, of the 250 I have supposed to come in at 20, only 150 then come in, and the rest at 30; the numbers of the living will be given 100 too high,

at every age between 20 and 30; but, as just shewn, they will be given 250 too high at every age before 20.—In general, therefore, the number of the living *at* any particular age, must be given by the supposed Table, as many too great as there are annual settlers *after* that age; and, if these settlers come in at all ages indiscriminately, during any certain interval of life; the number of inhabitants and the probabilities of life will be continually growing less and less wrong, the nearer any age is to the end of that interval.—These observations prove, that Tables of Observation formed in the common way, from bills of mortality for places, where there is an excess of the burials above the births, must be erroneous, for a great part of the duration of life, in proportion to the degree of that excess. They shew likewise, at what parts of life the errors in such Tables are most considerable, and how they may be in a great measure corrected.

All this I shall beg leave to exemplify and illustrate a little further, in the particular case of *London*.

The number of deaths, between the ages of 10 and 20, is always so small in the *London* bills, that it seems certain few recruits come to *London* under 20; or at least, not so many as before this age are sent out for education to schools and universities. After 20, great numbers come in, 'till 30, and some, perhaps,

perhaps, 'till 40 or 50. — But, at every age after 50, it is probable, that more retire from *London* than come to it.—The *London* Tables of Observation, therefore, being formed on the principle I have mentioned, cannot give the probabilities of life right 'till 40. Between 30 and 40 they must be a little too high; but more so between 20 and 30; and most of all so before 20.—It follows also, that these Tables must give the number of inhabitants in *London* much too great.

Table XII, in the Appendix, is a Table formed in the manner I have explained, from the *London* bills for 10 years, from 1759 to 1768; and adapted to a 1000 born as a *radix*. The sum of the numbers in the second column, diminished by half the number born, is 25,757. According to this Table then, for every 1000 deaths in *London*, there are $25\frac{3}{4}$ as many inhabitants; or, in other words, the expectation of a child just born is $25\frac{3}{4}$; and the inhabitants are to the annual burials, as $25\frac{3}{4}$ to 1. —But it has appeared, that the numbers in the second column being given on the supposition, that all who die in *London* were born there, must be too great; and we have from hence a DEMONSTRATION, that the probabilities of life are given in the common Tables of *London* Observations, too high, for, at least, the first 30 years of life; and also, that the number of inhabitants in *London* must be less, than $25\frac{3}{4}$, multiplied by the annual burials. —The common Tables, therefore, of *London*

Observations, undoubtedly want to be corrected (*a*); and the way of doing this, and in general, the right method of forming genuine Tables of Observation for towns, may be learnt from the following rule.

“ From the sum of all that die annually, “ *after* any given age, subtract the number “ of annual settlers *after* that age; and the “ remainder will be the number of the liv- “ ing *at* the given age.”

This rule can want no explication or proof, after what has been already said.

If, therefore, the number of annual settlers in a town at every age could be ascertained; a perfect Table of Observations might be formed for that town, from bills of mortality, containing an account of the ages at which all die in it. But no more can be learnt in this instance, from any bills, than the whole number of annual *settlers*, and the general division of life in which they enter. This, however, may be sufficient to enable us to form Tables that shall be tolerably exact.— For instance. Suppose the *annual deaths* in a town which has not increased or decreased,

(*a*) The ingenious and accurate Mr. *Simpson* saw that it was necessary to correct the *London Tables*, and he has done it with great judgment; but, I think, too imperfectly, and without going upon any fixt principles, or shewing particularly, how Tables of Observation ought to be formed, and how far in different circumstances, and at different ages, they are to be depended on.

to have been for many years, in the proportion of 4 to 3, to the *annual births*. It will hence follow, that $\frac{1}{4}$ of the persons who die in such a town are *settlers*, or *emigrants* from other places; and not *natives*: And the sudden increase in the deaths after 20, will also shew, agreeably to what was before observed, that they enter after this age. In forming, therefore, a Table for such a town, a *quarter* of all that die at all ages throughout the whole extent of life, must be deducted from the sum of all that die after every given age before 20; and the remainder will be the true number living at that given age. And if, at 20, and every age above it, this deduction is omitted, or the number of the living at every such age is taken the same with the sum of all that die after it, the result will be (supposing *most* of the settlers to come in before 30, and *all* before 40) a Table exact 'till 20; too high between 20 and 30; but nearly right for some years before 40; and after 40 exact again.—Such a Table, it is evident, will be the same with the Table last described at all ages above 20; and different from it only under 20. — It is evident also that, on account of its giving the probabilities of life too great for some years after 20, the number of inhabitants deduced from it may be depended on as somewhat greater than the truth; and more or less so, as the annual recruits enter in general later or sooner after 20.

Let us now consider, what the result of these remarks will be, when applied particularly to the *London* bills.

It must be here first observed, that, at least, one *quarter* of all that die in *London* are *settlers* from the country, and not *natives*.—The medium of annual burials for 10 years, from 1759 to 1768, was 22,956; of births 15,710. The excess is 7246; or near a *third* of the burials.—The same excess, during 10 years, before 1750, was 10,500; or, near *half* the burials. *London* was then *decreasing*. For the last 12 or 15 years it has been *increasing*. This excess, therefore, agreeably to the foregoing observations, was then *greater* than the number of annual settlers; and it is now *less*. I have chosen, however, to suppose the number of annual settlers to be now, no more than a *quarter* of the annual burials, in order to allow for more omissions in the births than the burials; and also, in order to be more sure of obtaining results that shall not exceed the truth.

Of every thousand then who die in *London*, only 750 are *natives*, and 250 are *settlers*, who come to it after 18 or 20 years of age: And, consequently, in order to obtain from the bills a more correct Table than the 12th in the *Appendix*, 250 must be subtracted from every one of the numbers in the second column 'till 20; and the numbers in the third column must be kept the same, the bills always giving these right.—After 20,

the Table is to be continued unaltered ; and the result will be, a Table which will give the numbers of the living at all ages in *London* much nearer the truth, but still somewhat too high.—Such is the 13th Table in the Appendix.—The sum of all the numbers in the second column of this Table, diminished by 500, is 20,750. For every 1000 deaths, therefore, in *London*, there are, according to this Table, 20,750 living persons in it ; or for every single death, $20\frac{3}{4}$ inhabitants. It was before shewn, that the number of inhabitants in *London* could not be so great as 25 times $\frac{3}{4}$ the deaths. It now appears, (since the numbers in the second column of this Table are too high) that the number of inhabitants in *London* cannot be so great as even 20 times $\frac{3}{4}$ the deaths. And this is a conclusion which, I believe, every one who will bestow due attention on what has been said, will find himself forced to receive. It will not be amiss, however, to confirm it by the following fact, the knowledge of which I owe to the particular enquiry, and kind information of Mr. *Harris*, the ingenious master of the Royal Mathematical School in Christ-Church Hospital.—The average of lads in this school has, for 30 years past, been 831. They are admitted at all ages between seven and eleven ; and few stay beyond 16. They are, therefore, in general lads between the ages of eight and 16. They have better accommodations than it
can

can be supposed children commonly have; and about 300 of them have the particular advantage of being educated in the country. In such circumstances it may be well reckoned, that the proportion of children dying annually, must be less than the general proportion of children dying annually at the same ages in *London*. — The fact is, that, for the last 30 years, $11\frac{4}{5}$ have died annually; or *one* in $70\frac{2}{5}$.

According to Table XIII, *one* in 73 dies between 10 and 20, and *one* in 70 between eight and 16. That Table, therefore, probably gives the decrements of life in *London*, at these ages, too little, and the numbers of the living too great: And, if this is true of these ages, it must be true of all other ages *under* 20; and it follows demonstrably, in conformity to what was before shewn, that more people settle in *London* after 20, than the $\frac{1}{4}$ I have supposed; and that from 20 to at least 30 or 35, the numbers of the living are given too great, in proportion to the decrements of life.

In this Table the numbers in the second column are doubled at 20, agreeably to what really happens in *London*; and the sum of the numbers in this column diminished by half the whole number of deaths, gives the *expectation* of life, not of a child just born, as in other Tables, but of all the inhabitants of *London* at the time they enter it, whether that be at birth, or at 20 years of age. The
expecta-

expectations, therefore, and the *values* of *London* lives under 20, cannot be calculated from this Table. But it may be very easily fitted for this purpose, by finding the number of births which, according to the given decrements of life, will leave 494 alive at 20; and then adapting the intermediate numbers in such a manner to this radix, as to preserve all along the number of the living, in the same proportion to the numbers of the dead. This is done in the 14th Table in the Appendix; and this Table may, I fancy, be recommended as better adapted to the present state of *London* than any other Table. The values of lives, however, deduced from it, are in general nearly the same with those deduced by Mr. *Simpson*, from the *London* bills as they stood 40 years ago. The main difference is, that after 52, and in old age, this Table gives them somewhat lower than Mr. *Simpson's* Table.

It has sufficiently appeared, what judgment we are to form of the values of lives thus deduced. During the greatest part of the interval of life, in which the annual recruits that keep up *London* come to it, these values err on the side of *excess*; and after that interval, they err, perhaps, a little on the side of *defect* (*a*), on account of retirements from *London* in the last stages of life.

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(*a*) I have not taken into account the effect of migrations from towns, on Tables formed in the manner I have explained;

The number of inhabitants in *London* may also be learnt from what has been offered,
more

explained; because, towns in general being kept up by recruits from the country, the migrations *from* them are of little consequence, compared with the migrations *to* them.—Thus; in *LONDON*, it appears, from the much greater number of deaths between 40 and 50, than in any other equal interval of life after 10, that more people come to it than leave it, at every age between 20 and 50. After 50, it is probable, that the contrary happens. But, it should be considered, that *emigrants* from *LONDON* after 50, are chiefly persons who, having got fortunes in business, chuse to leave off, and to spend the latter part of their lives in country retirements. But how few are these compared with the multitudes who, tho' possessed of good fortunes, never retire; and with the bulk of the inhabitants in lower stations, who never can be able, without the greatest inconveniencies, to quit the settlements by which they are supported? It is, however, likely, that retirements from *LONDON* are now more numerous than they ever were; and that they have *some* effect on the bills of mortality, and on *Tables* formed from them; by causing these *Tables* to give the number of the living too little, in comparison with the *decrements* of life, at every age, from that at which the migrations *to* and *from London* become equal, to the age at which the latter cease.—To explain this; let us suppose, that none settle in *LONDON* after 50; but that, between 35 and 50, as many come to it as retire from it at all ages after 35; and that these retirements cease at 70. In this case, the *Tables* will give the proportion of the living to the decrements of life too high 'till 35. At 35, this proportion will be given right. After 35, it will begin to be given too low; and this error will increase 'till 50; from which age it will decrease gradually 'till it vanishes at 70: And after 70, the *Tables* will be exactly right again.—This is the exact state of the effect of retirements from *London*, on the *London Table* of Observations. But this effect appears, indeed, to be inconsiderable; for,
after

more nearly than by any method which has been hitherto taken. It cannot, it has been shewn, exceed 20 times $\frac{2}{3}$ the number of annual deaths. Could, therefore, the annual deaths be ascertained, we should know the number of inhabitants within pretty narrow limits. But the omissions in the bills are such, that it is not possible to ascertain, with exactness, the annual deaths. Dr. *Brakenridge* supposed these omissions to amount to 2000 annually. The result of a very minute enquiry by Mr. *Maitland* is, that in the year 1729, they amounted to 3038. But they are probably now much more considerable, than either of these writers have reckoned them (*a*). Let them be 6000; and the number of inhabitants will be 601,750 at most.

All the preceding Observations are, it is plain, applicable to bills of mortality for towns in general; and point out the way of deducing from them genuine Tables of Ob-

after 50, the values of lives by the *London* Table, are continually approaching nearer and nearer to the same values by other Tables; which could not happen were retirements attended with any *great* effect.—It is proper to add, that in summing up, as above-explained, the numbers of the living, in order to find the number of inhabitants in *London*, the circumstance that these numbers may be too small for some years after 40 or 50, in consequence of retirements, is, undoubtedly, much more than balanced by their being given too high between 20 and 40.

(*a*) Vid. Preface to a Collection of the Bills of Mortality from 1657 to 1758, p. 4, &c.

servations,

servations, which shall give the true probabilities and values of lives, and the true number of inhabitants, in the town whose bills are given.—I shall beg leave to confirm and illustrate this, in the particular case of the TOWN of NORTHAMPTON.

In this town, containing four parishes, namely, *All-Saints*, *St. Sepulchre's*, *St. Giles*, and *St. Peter's*, an account has been kept ever since the year 1741, of the number of males and females that have been christened and buried (Dissenters included) in the whole town. And in the parish of *All-Saints*, containing the greatest part of the town, an account has been kept ever since 1735, of the ages at which all have died there.

In 1746, an account was taken of the number of *houses*, and of *inhabitants* in the town. The number of *houses* was found to be 1083; and the number of *inhabitants* 5136.—In the parishes of *All-Saints* and *St. Giles*, the number of *male* and *female heads* of families, *servants*, *lodgers*, and *children*, were particularly distinguished.—The *heads of families* were, 707 males; and 846 females.—*Children*, males 624; females 759.—*Servants*, males 203; females 280.—*Lodgers*, males 137; females 287.—In *St. Peter's*, males 99; females 129.—In *St. Sepulchre's*, *adults* 638; *children* 427. In this parish the sexes were not distinguished.

The

The *Christenings* and *Burials* in the *whole town* for 28 years, from 1741 to 1770, have been as follows.

Christened { Males 2361 } 4649 — Annual medium 155
 { Fem. 2288 }

Buried { Males 2869 } 5747 — Annual medium 191
 { Fem. 2878 }

In the parish of *All-Saints*, from 1735 to 1770, or 36 years,

Christened { Males 1632 } 3242 — Annual medium 90
 { Fem. 1610 }

Buried { Males 1856 } 3690 — Annual medium 102½
 { Fem. 1834 }

Of these died,

Under 2 years of age — 1206

Between 2 and 5 — 276

Between 5 and 10 — 155

Between 10 and 20 — 155

Between 20 and 30 — 297

Between 30 and 40 — 257

Between 40 and 50 — 297

Between 50 and 60 — 300

Between 60 and 70 — 293

Between 70 and 80 — 285

Between 80 and 90 — 155

Between 90 and 100 — 14

Total 3690

A Ta-

A Table formed from these *data* in the manner of Table XII; or, on the supposition, that all who *die* in *Northampton* were *born* there, would give the expectation of a child just born 28.83 years; or, the proportion of the inhabitants to the annual deaths, as 28.83 to 1. It has been shewn, that this proportion, in a place where the burials exceed the births, must be *greater* than the *true* proportion of the number of inhabitants to the annual deaths: And this appears to be the real case. For the bills shew, that, from 1741 to 1750, or for 10 years, about the time when the number of inhabitants was 5136, the annual medium of burials was 197.5; which, multiplied by 28.83, gives 5693; or a 9th part more than the *true* number.

A Table formed in the manner of Table XIII, would give the proportion of inhabitants to the annual deaths, as 26.41 to 1; and this makes the inhabitants 5216; or very near the true number.

The IVth Table, in the *Appendix*, is formed in the same manner with Table XIV, for *London*: And this is the genuine Table of Observations for *Northampton*, from which may be calculated the true probabilities and values of lives, at all ages, in that town.

At NORWICH, bills of mortality, of the same kind with those in *London* and *Northampton*, have been kept for many years. I
have

have been favoured with a copy of these bills for 30 years, from 1740 to 1769. The annual medium of *christenings*, during this period, has been 1057 (a), of *burials* 1206. And from hence, together with the account of the numbers dying in the several decads of life, after 10, I have formed Table V, which shews the true probabilities of life in this town.

The following particulars seem to deserve notice here.

First. Had these Tables been formed from the NORTHAMPTON and NORWICH bills, for no longer time than any 10 years taken together, of the periods I have mentioned; they would have given the probabilities and values of lives nearly the same. These Tables, therefore, are founded on a sufficient number of Observations; and it appears, that there is an invariable law which

(a) In this register all that die before baptism, and also all that are born and die among *Quakers*, *Jews*, &c. are omitted. There are also some other omissions; and the true annual medium of births and burials must be greater than they are given in the bills. But this will have no effect on a Table of Observations, supposing the *proportions* of the births to the burials, and of the numbers dying in the different stages of life, given right.—It is proper I should mention further here, that these bills give only the whole number of children dying under 10, without specifying the numbers dying under two years of age, between 2 and 5, and between 5 and 10, as in other bills. I have, therefore, in forming the Table for NORWICH, supposed the proportions of these numbers the same that they are at NORTHAMPTON.

governs the waste of human life in these towns.—The same remark might be made concerning LONDON (a). See p. 251.

Secondly. An account was taken at SHREWSBURY, in 1750, of the *whole* number of inhabitants; distinguishing, particularly, the number at the age of 21 and upwards.—The former number was 8141; and the latter, 5187.—According to a Table formed for NORTHAMPTON, in the same manner with Table XIII, for LONDON, the whole number of the living is to the number of the living at 21 and upwards, as 26,411 to 16,586; that is, as 8141 to 5113.—According to a like Table for NORWICH, these numbers are to one another, as 24,500 to

(a) Some have entertained a very wrong notion of the imperfections in the LONDON bills. They do, indeed, give the *whole* number of births and deaths much too little; but the conclusions with respect to the probabilities of life in LONDON, and the proportion of inhabitants dying annually, depend only (agreeably to the observation in the last note) on the *proportions* of the numbers dying in the several divisions of life; and these are given *right* in the LONDON bills.—For first. There seems nothing in this case, that can be likely to cause the deficiencies in the bills to fall in one division of life more than in another: But what decides this point is, that these proportions, as given by the bills for *any* ten, or even *any* five years, come out nearly the same with one another; and always very different from the proportions given by other bills.—There are no other variations, than such as must arise from the fluctuations of LONDON, as to increase and decrease; and also from some improvements in its state, which have lately taken place. See Essay I. p. 190, 191, 201.

15,680; that is, as 8141 to 5210. — These Tables, therefore, give the proportion of the *whole* number of inhabitants, to the number of the living at 21 and upwards, almost exactly the same with the true proportion, as it is at SHREWSBURY (a): And this affords a kind of demonstration of the rectitude of the principles on which these Tables have been formed.

In the parish of HOLY-CROSS near SHREWSBURY, an account was taken, in 1760 and 1770, of the *whole* number of inhabitants; distinguishing, *both* times, the number at the age of 70 and upwards; and the *last* time, the number at 10 and upwards: And, I find, that a Table formed from the *Register* of this parish, mentioned p. 192, gives, likewise, *these* numbers as nearly the same as could possibly be expected.

But further.—The number of inhabitants, not reckoning children, in the parishes of *St.*

(a) The annual medium of births at SHREWSBURY, for 7 years, from 1762 to 1768, was 301; of burials, 329. It appears, therefore, that one in $24\frac{3}{4}$ of the inhabitants die annually. But it should be remembered, that in 1766, the small-pox and measles increased very much the mortality in this town; and I find also, that, since 1750, a nursery for *foundlings* from LONDON, was established here; and that in 1768 this nursery contained 660 children and servants. It seems, therefore, probable, that the true *medium* of burials about the year 1750, must have been less than 329; and that the proportion of inhabitants dying annually, may not be much greater than it is at NORTHAMPTON; or 1 in 26.41.

Giles and All-Saints, NORTHAMPTON, was, in 1746, 2460; and the *whole* number of inhabitants in these two *parishes* was 3843. See p. 254.—In the account I have received, the particular age at which the limit of childhood was fixed in taking this survey, is not mentioned; but there is sufficient reason to believe, that it was 21: And, taking this for granted, the number of inhabitants, not children, will come out, (by such a Table for NORTHAMPTON, as Table XIII for LONDON) 2414; or, nearly the same with the number *really* found in these parishes.—Had this number been computed, from a Table formed for NORTHAMPTON, in the manner of Table XII, *Appendix*, it would have come out only 2176. This remark is applicable to the Table for *Breslaw*, formed by Dr. *Halley*, compared with the same Table, corrected for all the ages under 20 (*a*), by the rule, p. 246.

(*a*) I have given Dr. *Halley's* Table in the Appendix just as he framed it. A correction of it might be made from the proportion of births to burials, mentioned p. 239. And it would then appear, that a 25th part of the inhabitants at *Breslaw* die annually; and that half the number born die there under six, as well as at *Norwich*. This Table, as we now have it, makes half live to 16; but the account mentioned in the note, page 240, shews this not to be the truth. It likewise makes the number of inhabitants at SHREWSBURY, above the age of 21, to be 4730; and in the parishes of *All Saints* and *St. Giles* in NORTHAMPTON, 2230. It gives, therefore, these numbers wrong; whereas, as observed above, a corrected Table would give them true.

The

The necessity, therefore, of this correction is verified by facts; and it appears, abundantly, that the Tables I have given for NORTHAMPTON and NORWICH may be depended on.

But, thirdly. In comparing these two Tables, it may be observed, that there is a difference between them in favour of NORTHAMPTON, *fewer* dying there in childhood, and *more* in old age. The same would be found to be true, were the NORTHAMPTON Table to be compared with a corrected BRESLAW Table. It appears, therefore, agreeably to what might have been expected, that NORTHAMPTON, being a small town compared with BRESLAW and NORWICH, is less unfavourable to health and longevity. The difference, however, is not considerable. After the age of 20, there is a striking conformity between all the three Tables, which gives them great weight and authority.

Further. It ought to be particularly noted, that these Tables prove, the *decrements* of life in moderate towns, to be nearly *equal* thro' most of its stages. At NORTHAMPTON it appears that, of a given number of persons alive at 20, the same number die every year 'till 78, without any interruption worth notice, except between the ages of 30 and 40.—A like uniform decrease in the probabilities of life appear in the BRESLAW

and NORWICH Tables; but not so remarkably. It was this circumstance in the BRESLAW Table, that led Mr. *De Moivre* to the *Hypothesis*, described in p. 2, and so often mentioned in this work.—The values of lives, I have said, deduced from this *Hypothesis*, agree so nearly with the same values deduced immediately from the Tables, that it is scarcely worth while to distinguish them. But that every one may be able to judge of this for himself, I have calculated (a) the following Table.

Value of a life at the age	By Breslaw Table.	By Norwich Table.	By Northampton Table.	By Mr. De Moivre's Hypothesis.
12	17.617	17.48	17.20	16.69
20	16.49	16.41	15.93	15.89
30	14.77	15.15	14.85	14.68
40	12.90	13.36	13.10	13.19
50	10.87	11.13	11.25	11.34
60	8.58	8.54	9.02	9.01
70	5.59	5.99	6.26	6.06
75	4.21	4.86	4.79	4.29

It may be observed in this Table, that the values, by the *Hypothesis*, come nearer to the true values by the NORTHAMPTON and NORWICH Tables, than by the BRESLAW Table; and also, that, before the age of 60, they are *all* much higher than the values for

(a) Every calculation of this kind may be made without much labour, by a rule explained in note (O) *Appendix*.

the same ages in LONDON by Table X; the inhabitants of *London*, (as Mr. *De Moivre* observes) being “for causes (a) too well known, “more short-lived than the rest of mankind.” —The *Hypothesis*, therefore, is by no means applicable to LONDON lives. It is proper to add, that neither can it be applied to the valuation of COUNTRY lives.—It appears, from the register of the parish of *Holy-Cross* (b), that the *expectations* of lives there are much greater than the *expectations* by the *Hypothesis*. —The *expectation* there of a life (c)

At 20 is	38	By <i>Hypothesis</i> .	33	In <i>London</i> .	28.9
27	33.9		29.5		25.1
30	32		28		23.6
40	25.7		23		19.6
50	20		18		16
60	14.5		13		12.4
70	10		8		8.8
					From

(a) Doctrine of Chances, p. 347.

(b) See Essay I. p. 191. — I have not given the Table of Observations from whence these conclusions are deduced, because it is possible, some may think 20 years not a period long enough, to afford *data* in this case of sufficient authority. I have in p. 257, mentioned a fact which seems to prove the contrary. It is, however, certain, that the same register continued 10 or 20 years longer, will afford *data* more to be depended on.

(c) The expectation of a child just born in this parish, is 33. At NORTHAMPTON, 25½. At NORWICH, 23¾. In LONDON, 18.—In this parish, 1 in 11 dies at 80, and upwards. In NORTHAMPTON; 1 in 22. In NORWICH; 1 in 27. In LONDON; 1 in 40. See Essay I. p. 200.

From this comparison it appears, that the *Hypothesis*, from 20 to 60, gives nearly the medium

I will add, that the probabilities of life here, appear to be much the same, with the probabilities of life among the ministers and professors in SCOTLAND.—This is a fact of some consequence; and, therefore, I shall beg leave to give a brief account of it.

The mean age at which the ministers and professors enter into benefices and professorships in *Scotland*, is reckoned to be 27. Their number is 974. The establishment among them for providing for their widows, begun on the 25th of *March* 1744; from which time to *November* 22, 1768, 721 have died: That is, 29.23 annually; or 1 in $33\frac{1}{3}$. The *expectation*, therefore, of a life among them, at the age of 27, is $33\frac{1}{3}$; which is nearly the same with the *expectation*, as given above, of a life of the same age in the parish of *Holy-Cross*; and $3\frac{1}{2}$ years more, than the *expectation* of the same age by Tables III, IV and V.—Now, the *expectation* at a given age, being composed of all the probabilities of life from that age to the extremity of life; there arises from hence reason for concluding, that the probabilities of life among the ministers in *Scotland*, cannot differ much in any part of life, from those in this parish.—But there is another fact that confirms this observation.

The annual average of weddings among the ministers and professors in SCOTLAND, for the last 24 years, has been at most 32. The average of married persons among them, for 17 years ending in 1767, had been 667. This number, divided by 32, gives 20.84, the *expectation* of marriage among them; which is $2\frac{2}{3}$ years more than the *expectation* of marriage would be, by *Dr. Halley's* Table, on the supposition, that all 1st, 2d and 3d marriages may be justly considered as commencing, one with another, so early as the age of 30.—The *expectation* of two equal joint lives is to the *expectation* of a single life of the same age, as 2 to 3, by note (L) *Appendix*. It follows, therefore, that among the ministers in *Scotland*, the *expectation* of a single life

medium between the *expectations* of LONDON and COUNTRY lives; and for this reason it is excellently adapted to general use.—After 60, the *expectations* and *values* of lives in LONDON approach nearer and nearer to the *expectations* and *values* of lives in *Northampton*, *Norwich* and *Breslaw*; 'till, at 70, they come to be almost the same. This is a circumstance which, I believe, has not been attended to: And it is the more surprizing, as there is no cause known, which can produce any error in the values of lives after 60, deduced from the LONDON Table, except migrations from *London*; and the effect of these must be to *diminish* these values.

The following observations will, perhaps, account for this.

It has been proved, that at least *half* the inhabitants of LONDON, turned of 20 years of age, are *emigrants* to LONDON from the country. So great a change as that, from the *country* air and modes of life, to the air and modes of life in *London*, must be parti-

cularly
 life at 30 cannot be less than 31.26. Most probably it is more; on account of the later commencement of marriage in the situation of the *Scotch* ministers. — I reckon also, that 27 must be less than the mean age at which they enter their benefices and professorships; meaning by it, not the age on each side of which equal numbers enter; but the age at which, the excess of the interval of time taken to enter on one side, is just such as to compensate the greater numbers who enter on the other side. See the conclusion of note (F) *Appendix*.

cularly hurtful to these persons; and, therefore, (except *infants*) it is in them, probably, that the pernicious influence of *London* on its inhabitants chiefly appears. They come in at every age 'till near 50; and this is the reason why the deaths continually increase in *London* 'till that age; but, after that age, the inhabitants consisting chiefly of persons, who (like men *used* to drink) have been *seasoned* to *London*, or with whom it does not happen particularly to disagree; the number of deaths becomes less, and the values of lives begin to approach nearer to the common standard in other towns.

There is one more fact which I shall here take notice of; and which deserves more attention than has been hitherto bestowed upon it. I mean; “the difference between the probabilities of life among *males* and *females*, in favour of the latter.”

From the account in p. 254, it appears, that at NORTHAMPTON, tho' more *males* are born than *females*, and nearly the same number die; yet the number of living *females* is greater than the number of *males*, in the proportion of 2301 to 1770, or 39 to 30. This cannot be accounted for, without supposing, that *males* are more short-lived than *females*.—One obvious reason of this fact is, that *males* are more subject to untimely deaths by accidents of various kinds; and also, in general, more addicted to the excesses and irregularities

which shorten life. But this is by no means the *only* reason. For it should be observed, that at NORTHAMPTON the number of *female* children was, in 1746, greater than the number of male children, in the proportion of 759 to 624.—The greater mortality of males, therefore, takes place among *children*.—But this, together with the greater mortality in general of males at all ages, will more particularly appear from the following recital of facts.

In the parish of *Holy-Cross, Salop*, the ingenious Vicar, Mr. *Gorsuch*, in 1760, and again in 1770, took the number of male and female inhabitants turned of 70. In 1760, the number of females turned of this age, was 35; of males, 8. In 1770, these numbers were, females, 35; males, 26. And for the last 10 years 11, out of 365 have died between the ages of 85 and 102; and they were all females.

At BERLIN, it appeared, from the accurate account which was taken of the inhabitants in 1747, and which has been mentioned in p. 219, that the number of *female* citizens exceeded the number of *male* citizens, in the proportion of 459 to 391: And yet, out of this smaller number of males, more had died, for 20 years preceding 1751, in the proportion of 19 to 17 (*a*).

(*a*) Vid. *Sufmilch*, p. 8, and p. 32, &c. where a minute account is given of the number of males and females at BERLIN in 1747; and also, of the numbers of each sex that had died from 1722 to 1750.

At

At EDINBURGH, in 1743, the number of *females* was to the number of *males*, as 4 to 3; (See Essay I. p. 211) but the females that died annually, from 1749 to 1758, were to the males, in no higher proportion than $3\frac{1}{3}$ to 3. Before 1749, the bills give the totals of burials, without distinguishing them into the totals of males and females dying every year.

Mr. *Kerfeboom*, in his Essay on the numbers of people in HOLLAND, informs us, that from the Tables of assignable Annuities for lives in HOLLAND, which had been kept there for 125 years, wherein the ages of the persons dying are truly entered; it appears, that females have, in all accidents of age, lived about 3 or 4 years longer than the same number of males. See *Philosophical Transactions* abridged, Vol. IX, p. 326.

In Volume the 7th of the *Philosophical Transactions* abridged, Part IV, p. 46, &c. there is an account of the numbers of *male* and *female* still-born children and chryfoms, and of boys and girls under 10, of married men and married women, and of widows and widowers, who died for a course of years at *Vienna, Breslaw, Dresden, Leipsic, Ratisbon*, and some other towns in GERMANY.

He that will take the pains to examine these accounts will find that, though in these towns the proportion of males and females born is no higher than 19 to 18, yet the pro-

proportion of boys and girls (*a*) that die is 8 to 7; and that, in particular, the *still-born* and *chrysom males*, are to the *still-born* and *chrysom females*, as 3 to 2.

In these accounts it appears also, that of 7270 *married* persons who had died in these towns (*b*), 4336 were *married men*, and but 2934 *married women*; that is, *three married men* died to *two married women*.—The scheme for making provision for the widows and orphans of the ministers in SCOTLAND, has obliged them to keep an account of the number of *weddings* among them, and the number of *widows* left annually; and it appears, from the *reports* of the *trustees* for carrying this scheme into execution, that the annual medium of *weddings* (*c*), is (as observed in the note, p. 264) at most 32. And the annual

(*a*) In the accounts from *Breslaw* it is particularly mentioned, that by *boys* and *girls* are meant children to 10 years of age, of whom, for 8 years from 1717 to 1725, *seven males* died to *six females*, exclusively of the *still-born* and *chrysons*.

(*b*) In *Breslaw* alone, for the eight years mentioned in the last note, 1891 *married men* died, to 1196 *married women*; that is 5 to 3.—In *Dresden* alone, for five years, these numbers were 1080 and 849.

(*c*) The annual medium of *weddings*, among the ministers admitted to benefices, has been, for 24 years from the commencement of the scheme, 27. Besides these I find there have been 4 *weddings* annually among them, *before* admission to benefices. The whole annual medium, therefore, is no more than 31. But I have supposed it 32, in order to go upon more sure grounds.

medium

medium of widows, who have come upon the scheme for 24 years, is 20. Of 32 marriages then contracted annually, 20 become extinct by the deaths of *husbands*; and but 12 by the deaths of *wives*. That is; among the ministers and professors in SCOTLAND, 20 married men die to 12 married women; or 5 to 3. It appears, therefore, that there is the chance of 3 to 2, and in some circumstances even a *greater* chance, that the *woman* shall be the survivor of a marriage, and not the *man*. In order to account for this by the difference of age between men and their wives, this difference ought to be at least 12 years (*a*). That is; supposing the mean age at which women marry to be 23, the mean age at which men marry ought to be 35. But this seems to exceed the bounds of credibility; and, therefore, very probably, the greater mortality of males must operate in this case.

It is further observable in the accounts from *Germany*, to which I have referred, that the number of *widows* dying annually, is four times the number of *widowers* (*b*); and, as
widows

(*a*) The chance of survivorship between two persons aged 21 and 34, is nearly 3 to 2 in favour of the former. There is the same chance of survivorship between 25 and 37; and 28 and 39. This may be learnt from Problem XVI, in Mr. *De Moivre's* Treatise on *Life-Annuities*.

(*b*) In *Dresden* alone, the number of *widows* who died, in four years, was 584. The number of *widowers*, 149. That is; 4 to 1.---It appears from note (*b*) in the last page,

widows are certainly, one with another, several years younger than *widowers*; it may be concluded from hence, that the number of the former in life together could not be less than five times the latter.—This fact is likewise confirmed, by the observations which have been made among the ministers in *Scotland*. At the commencement of the scheme which I have so often had occasion to mention, an account was taken of the number of the *widows* of ministers in the whole country. 364 were counted; and, probably, the true number was greater. See p. 95 and 96. On the contrary; the number of *widowers* among the ministers has, one year with another, been scarcely 90; that is, not so much as a *quarter* of the number of *widows*.—It may be easily seen, and it would not be difficult to demonstrate, that neither the greater number of persons left widows, nor any probable supposition concerning the greater frequency of marriages among widowers, can completely account for this, without admitting the *greater mortality of males*.—This, therefore, appears, on the whole to be a fact

page, that the chance of survivorship in this city in favour of the *wife*, is less than among the ministers in *Scotland*.—Does not this fact afford a reason, additional to that mentioned above, and in p. 92, 93, 94, &c. and notes (A) and (F), for *believing*, or at least *suspecting* that the number of *widows* on the *Scotch* establishment, if marriage does not decline among the ministers, will not at last be found to be so little as 400?

well

well established : And it follows from it, that in order to calculate the values of *Life-Annuities* and *Reversions* with exactness, there ought to be distinct Tables of the probabilities of life for *males* and *females*. But there are no such Tables extant ; nor, indeed, has it been suspected, that there is so much occasion for them as the facts I have mentioned seem to shew. All that is necessary to obtain the proper *data* for forming such Tables is, that the *sexes* as well as the *ages* of the dead should be specified in the bills ; and this is an improvement of bills (a) of mortality which would give little trouble, and which, therefore, I hope, will be some time or other made.

It has been observed, that the author of nature has provided, that more *males* should be born than *females*, on account of the particular waste of *males*, occasioned by wars and other causes. Perhaps it might have been added, that this provision had also in view, that particular weakness or delicacy in the constitution of males, which makes them more subject to mortality ; and which, con-

(a) This improvement would be rendered more complete, by distinguishing the *males* that die, under the denominations of *married men*, *widowers*, and *batchelors* ; and the *females*, under the denominations of *married women*, *widows*, and *virgins*.—The use I have made of some accounts of this kind which have been kept in *Germany*, shews that this would be of considerable service.

sequently,

sequently, renders it necessary, that more of them should be produced, in order to preserve in the world a due proportion between the two sexes.

In the course of this Essay, it has often appeared, that I have been particularly indebted to an information which I have received from NORTHAMPTON.—I should be inexcusable, did I not mention, that I owe this information to Mr. *Lawton*, an ingenious gentleman in that town, who has preserved the bills of mortality there with much care, and been very obliging in communicating them to me.—It is much to be desired, that like accounts were kept in every town and parish. It would be extremely agreeable to learn from them the different rates of human mortality in different places, and the number of people and progress of population in the kingdom. The trouble of keeping them would be trifling; but the instruction derived from them (*a*), would be very important.—I have already proposed one improvement of such accounts. I will add, that they would be still more useful, did they give the ages of the dead after 10, within periods of *five*, instead of *ten* years. — During every period, so short as *five* years, the decrements of life may, in constructing Tables, be safely

(*a*) See Essay I. p. 207, 208.

taken to be *uniform*. But this cannot be equally depended on, in periods so long as ten years.

There is yet another improvement of these accounts, which I will take this opportunity to mention. They should contain not only a list of the distempers of which all die, like that in the *London bills*; but they should specify particularly the numbers dying of these distempers, in the several divisions of life. Accurate registers of mortality kept in this manner, in all parts of the kingdom; and compared with records of the seasons, and of the weather, and with the particular circumstances which discriminate different situations, might contribute, more than can be easily imagined, to the increase of *physical* knowledge. — But to proceed no farther in these Observations; I shall now beg leave to shut up this whole work with the following general reflexion.

I have represented particularly, the great difference between the probabilities of human life in towns and in country parishes; and from the facts I have recited, and the observations I have made, it appears, that the further we go from the artificial and irregular modes of living in great towns, the fewer of mankind die in the *first* stages of life, and the more in its *last* stages. The
lower

lower animals (except such (a) as have been taken under human management) seem in general to enjoy the full period of existence allotted them, and to die chiefly of old age: And were any observations to be made among *savages*, perhaps the same would be found to be true of them.—DEATH is an evil to which the order of providence has subjected every inhabitant of this earth; but to man it has been rendered unspeakably more an evil than it was designed to be. The greatest part of that black catalogue of diseases which ravage human life, is the off-spring of the tenderness, the luxury, and the corruptions introduced by the vices and false refinements of civil society (b). That delicacy which is injured

(a) Calves are the only animals taken under our peculiar care immediately after birth; and, in consequence of then administering to them the same sort of physic that is given to *infants*, and treating them in other respects in the same manner, it is probable, that more of them die soon after being born, than of *all* the other species of animals, which we see in the same circumstances. See the *Comparative View of the State and Faculties of Man with those of the Animal World*, p. 23.—It is, indeed, melancholy to think of the havock made among the human species by the unnatural *customs* as well as the *vices*, which prevail in polished societies. I have no doubt, but that the custom, in particular, of committing infants, as soon as born, to the care of *foster-mothers*, destroys more lives than the sword, famine and pestilence put together.

(b) The ingenious and excellent writer quoted in the last note, observes, that the whole class of diseases which arise from catching cold, are found only among the civilized part of mankind, p. 51.—And, concerning that

jured by every breath of air, and that rottenness of constitution which is the effect of intemperance and debauchery, were never intended by the author of nature; and it is impossible, that they should not lay the foundation of numberless sufferings, and terminate in premature and miserable deaths. — Let us then value more the simplicity and innocence of a life agreeable to nature; and learn to consider nothing as savageness but malevolence, ignorance and wickedness. The order of nature is wise and kind. In a conformity to it consists health and long life; grace, honour, virtue and joy. But nature turned out of its way will always punish. *The wicked shall not live out half their days.* Criminal excesses embitter and cut short our *present existence*; and the highest authority has taught us to expect, that they will not only kill the *body*, but the *soul*; and deprive of an EVERLASTING EXISTENCE.

loss of all our higher powers which often attends the decline of life, and which is so humiliating to human pride; he observes, that it exhibits a scene singular in nature, and that there is the greatest reason to believe, that it proceeds from adventitious causes, and would not take place among us if we led natural lives, p. 62.



A P P E N D I X.

Note (A). See Question III. Page II.

L ET E be any given expectation of life; and $\frac{4E-x}{4E} \times px$ will be the number of persons alive at the end of x years, arising from p persons left annually as widows, (or added annually to a town or society) at the age whose *expectation* is E . The *maximum*, therefore, is always pE —. In Mr. *De Moivre's Hypothesis*, E is always $\frac{1}{2}$ the difference between the given age and 86. See the note page 2, and the latter end of the note in page 37. See likewise the beginning of the First Essay, and note (L) in this Appendix, where the investigation of this rule will be given.

It will not be amiss to give the following example of the application of this rule.

At the time of the commencement of the scheme, among the ministers and professors in SCOTLAND, for making provision for their widows, it was necessary, that a calculation should be made of the number of widows that would be upon the scheme at the end of every year, till they came to a *maximum*, on the supposition that, (agreeably to what particular enquiry had shewn to have happened for many preceding years,) 20 new widows would be left every year. In order to make this calculation, let 4 of the 20 widows be supposed to

be under 32 years of age when left; and let 28 be supposed their mean age. Let the same number be left between 32 and 39, and let 35 be their mean age; between 39 and 47, and 43 their mean age; between 47 and 57, and 52 their mean age; between 57 and the extremity of life, and 63 their mean age. The number in life together, to which, in 10 years, 4 widows left annually at the age of 28 will grow, is, by the rule, (E being 29)

$$\frac{116-10}{116} \times 40, \text{ or } 36.55$$

—The number alive at the end of 20 years, will be $\frac{116-20}{116} \times 80, \text{ or } 66.2.$

At the end of 30 years, the number alive will be 89; of 40 years, 104.82; of 58 years 116—These numbers, found in the same way, for the 2d class, (E being 25.5), at the end of 10, 20, 30, 40, and 51 years, will be 36.7—64.31—84.7—97.25—102—For the 3d class, (E being 21.5) at the end of 10, 20, 30, 40, and 43 years, 35.34—61.4—78.13—85.6—86—For the 4th class, (E being 17) at the end of 10, 20, 30, and 34 years, 34.11—56.47—67—68—For the 5th class, (E being 11.5) at the end of 10, 20, and 23 years, 31.3—45.2—46—The whole number, therefore, consisting of all the classes, will come to a *maximum* nearly in 58 years; and the totals in life, at the end of 10, 20, 30, 40, 50, and 58 years, will be 173.37—293.58—364.83—401.67—418.

These determinations suppose none to marry. In 10 years, from 1757 to 1767, I have been informed, that but 9 widows married. Let us then suppose, that one widow of the first class marries every year; and let all that marry, be supposed to continue, one with another, 5 years in widowhood before they marry. On these suppositions,

the foregoing totals will, at the end of the same periods of years, be 169.23—282—347.5—380.47—394.

These calculations are made from Mr. *De Moivre's* Hypothesis. Had they been made exactly from Dr. *Halley's* table, or any other of the Tables I have given at the end of this work, except the *London* one, the results would have been very nearly the same.—It appears, that the probabilities of life, among these widows, are greater than those given by these tables. See the Last Essay, pages 263, 264 266, 270, &c. The effect of this must be, to raise the *maximum* without sensibly increasing the numbers in life for the first 20 or 25 years: and its effect *may* be, to raise the *maximum*, and at the same time even to diminish these numbers.

Twenty-five years have now elapsed since the commencement of this scheme; and the number of widows living every year have, in fact, corresponded to the last numbers I have given, as nearly as could possibly be expected in an affair of this nature.

Note (B). Question VI. Page 21.

LET r signify the sum of 1*l.* and its interest, for one year. The value of a life, whose complement is n , being (by Mr. *De Moivre* on *Annuities*, 4th edition, page 14. and p. 100.)

$$\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4}, \text{ \&c.}$$

the present value of the remainder of it after *two* years must be

$$\frac{n-3}{nr^3} + \frac{n-4}{nr^4}, \text{ \&c.}$$

which is equal to $\frac{1}{r^2} \times \frac{n-2}{n} \times$

$$\frac{n-3}{n-2r} + \frac{n-4}{n-2r^2} + \frac{n-5}{n-2r^3}, \text{ \&c.}$$

Now $\frac{1}{r^2}$ is the present value of 1*l.* due at the end of two years. $\frac{n-2}{n}$ is the probability that a life, whose complement is n , shall continue two years, and $\frac{n-3}{n-2r} + \frac{n-4}{n-2r^2} + \frac{n-5}{n-2r^3}, \text{ \&c.}$ is the value of a life two years older than the life whose complement is n . And, therefore, (since any number of years less than n may be substituted for two years) the first rule given in this question is right.

The same process, applied to joint lives, will demonstrate what is said in the *Scholium*.

Note

Note (C). See Question VII. Page 22.

LET the complements of any two assigned lives be n and m . The present value of the first possible payment of an annuity to be enjoyed by the life whose complement is n , provided *both* lives continue 7 years, and the life, whose complement is n , survives the other *after* that term, is the probability, that the life of the expectant shall continue 8 years, and the other life 7 years and then fail in the 8th year, multiplied by $\frac{1}{r^8}$, or by 1*l.* discounted for 8 years.—The probability that the life of the *expectant* shall continue 8 years is $\frac{n-8}{n}$. The probability that the *other* life shall continue 7 years is $\frac{m-7}{m}$. The probability that it shall continue 7 years, and fail in the 8th year, is $\frac{m-7}{m} \times 1 - \frac{m-8}{m-7} = \frac{1}{m}$. The probability, therefore, that the life of the *expectant* shall continue 8 years, and the other life 7 and fail in the 8th year, is $\frac{n-8}{n} \times \frac{1}{m}$; and the present value of the first possible payment of the annuity supposed, is $\frac{n-8}{nr^8} \times \frac{1}{m}$.

See *The Doctrine of Annuities* by Mr. Simpson, p. 6—15, or his *Select Exercises*, p. 315, &c.—
In like manner, the present value of the 2d payment, at the end of the 9th year, may be found
to

to be $\frac{n-9}{nr^9} \times \frac{m-7}{m} \times 1 - \frac{m-9}{m-7}$, or $\frac{n-9}{nr^9} \times \frac{2}{m}$.

and the present value of all the possible payments,

$$\frac{1}{r^7} \times \frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}, \text{ \&c.}$$

But this series is equal to $\frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times$

$$\frac{n-8}{n-7r} \times \frac{1}{m-7} \times \frac{n-9}{n-7r^2} \times \frac{2}{m-7} + \frac{n-10}{n-7r^3} \times$$

$$\frac{3}{m-7}, \text{ \&c. Now } \frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \frac{2}{m-7},$$

\&c. is the value of an annuity for a life seven years older than the expectant, after another life seven years older than the life whose complement

is m . $\frac{n-7}{n} \times \frac{m-7}{m}$ is the probability that both

the assigned lives shall continue 7 years. And $\frac{1}{r^7}$

is the value of 1*l.* due at the end of 7 years. The rule, therefore, given for solving this question, is right.

This demonstration, as well as that in the last note, is, for the sake of more ease and clearness, applied to the hypothesis of an equal decrement of life. It does not, however, depend upon it, but may be applied to any table of observations.

Note (D). Question IX. Page 29.

LET the complement of any two assigned lives be n and m , and the given term be *seven* years, as in note (C). The probability that the former life (supposed to be the life in expectation) shall last 8 years, is, by Mr. *De Moivre's* Hypothesis, $\frac{n-8}{n}$; and the probability that the latter life shall fail in 8 years, is $\frac{8}{m}$; and the first payment of the annuity mentioned in this question, depends on the happening of *both* these events, the probability of which is $\frac{n-8}{n} \times \frac{8}{m}$.

The present value, therefore, of the first possible payment of the annuity is $\frac{n-8}{nr^8} \times \frac{8}{m}$.—In like manner; the present value of the *second* possible payment is $\frac{n-9}{nr^9} \times \frac{9}{m}$; and of all the payments, $\frac{n-8}{nr^8} \times \frac{8}{m} + \frac{n-9}{nr^9} \times \frac{9}{m} + \frac{n-10}{nr^{10}} \times \frac{10}{m}$, &c. But $\frac{n-8}{nr^8} \times \frac{8}{m} = \frac{n-8}{nr^8} \times \frac{1}{m} + \frac{n-8}{nr^8} \times \frac{7}{m}$; and $\frac{n-9}{nr^9} \times \frac{9}{m} = \frac{n-9}{nr^9} \times \frac{2}{m} + \frac{n-9}{nr^9} \times \frac{7}{m}$. The foregoing series, therefore, is equal to the two series's $\frac{1}{r^7} \times$

$$\frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}, \text{ \&c. and}$$

$$\frac{1}{r^7}$$

$$\frac{1}{r^7} \times \frac{n-8}{nr} \times \frac{7}{m} + \frac{n-9}{nr^2} \times \frac{7}{m} + \frac{n-10}{nr^3} \times \frac{7}{m}, \&c. \text{ or}$$

$$\text{to } \frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times \frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times$$

$$\frac{2}{m-7} + \frac{n-10}{n-7r^3} \times \frac{3}{m-7}, \&c. + \frac{1}{r^7} \times \frac{7}{m} \times \frac{n-7}{n} \times$$

$$\frac{n-8}{n-7r} + \frac{n-9}{n-7r^2} + \frac{n-10}{n-7r^3}, \&c. \text{ which is the very}$$

rule given for solving this question, as will appear from notes (B) and (C).

Note

Note (E). See the Scholium to Quest. X.

ACCORDING to the calculations, the time in which the first yearly payment of a reversionary *annuity* becomes due, is the end of the year in which the event happens that entitles to it, however little or much of the year may then happen to be unelapsed. And this, likewise, is the time when a reversionary *sum* becomes due. Those who know how the calculations of the values of reversions are instituted, must know this. But an annuity, the first payment of which is to be made at the same time with another payment of a sum in hand, sufficient to buy an equal annuity, is worth one year's purchase more than the sum. For instance. Reckoning interest at 4 per cent. and r being 1*l.* increased by its interest for a year, or 1.04, $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$, &c. = 25*l.* is the present value of an estate of 1*l.* per annum for ever. That is, it is the value of it, supposing the first rent of it is to be paid a year hence.—If the first rent is to be received immediately, or at the same time with another payment of 25*l.* it is worth one year's purchase more, or equivalent to 26*l.*—I have not found, that any of the writers on annuities and reversions, have attended to this observation. It suggests a correction necessary to be applied to the common solutions of several important problems: particularly to the 21st and 22d in Mr. *Simpson's Treatise on Annuities*, and the 26th, 27th, 32d, 33d, and 40th problems in his *Select Exercises*; and to all other problems of the same kind in other writers. There

can be no great occasion for being more explicit. It will not, however, be amiss to add the following demonstration: — $\frac{1}{n}$ is the present probability that a life, whose complement is n , will fail in any one assignable year of its duration. $S \times \frac{1}{nr} + \frac{1}{nr^2}$

$+ \frac{1}{nr^3}$, &c. (n), or the present value of $1l.$ per annum for n years, multiplied by $\frac{S}{n}$, is the present

value of the sum or legacy denoted by S , payable at the failure of the given life. Therefore, (n being 56; the life 30; interest 4 per cent. $r=1.04$; the sum 25*l.*) the value of the expectation, by Mr. *De Moivre's* hypothesis, is 9.919.

Further. The value of $1l.$ to be received at the end of a year, provided the life whose complement is n fails, is the probability of the failure of the life multiplied by $1l.$ discounted for a year, or

$1 - \frac{n-1}{n} \times \frac{1}{r}$. In like manner; the value of $1l.$ to be received at the end of 2 years, if the same

life fails in 2 years, is $1 - \frac{n-2}{n} \times \frac{1}{r^2}$. And, therefore, the value of all the possible payments of an estate or annuity of $1l.$ for ever, to be entered

upon after the given life, is $1 - \frac{n-1}{n} \times \frac{1}{r} + 1 - \frac{n-2}{n} \times \frac{1}{r^2} + 1 - \frac{n-3}{n} \times \frac{1}{r^3}$, &c. (n) $+ \frac{1}{r^{n+1}} +$

$\frac{1}{r^{+2}}$

$$\frac{1}{r^n + 2}, \&c. \text{ or } \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}, \&c. \leftarrow \frac{n-1}{nr} + \frac{n-2}{nr^2} +$$

$\frac{n-3}{nr^3}, \&c.$ that is, the value of the life subtracted

from the perpetuity; or, in this example, 14l. 68s. 4d. (the value of a life at 30) subtracted from 25; that is, 10l. 31s. 6d. But 10.316 is to 9.919, in the same ratio with 104 to 100, or 26 to 25, agreeably to the rule in the *Scholium*.

Note

Note (F.) Question XIII. Page 44.

WHEN I here call 48 the mean age of all married men, and 40 the mean age of married women, I do not intend to suppose, that there are as many married persons who exceed these ages, as there are who fall short of them. It is likely that the latter are most numerous; and it is necessary that this should be the case, to render the supposition I make just—If all marriages commenced at 33 for the man, and 25 for the woman, one half of them would be dissolved by the time the men were 50, and the women 42; for (by the *Hypothesis*, and also nearly by the *Breslaw*, *Norwich*, and *Northampton* tables) there is an equal chance for the joint continuance of two lives, whose ages are 25 and 33, *seventeen* years. Forty-two and fifty then would be properly the mean ages at which widowhood would commence; meaning by these, “the ages on each side of which equal numbers are left widows and widowers.”—But, tho’ in this case half the marriages of every year would be dissolved in 17 years, they would not be *all* dissolved in twice that time. So far would this be from happening, that about a 7th part would continue beyond twice 17 years; nor would it be *certain*, that they would be all dissolved till near the extremity of the possible extent of life. Tho’, therefore, an equal number of marriages would be dissolved, or an equal number of widows and widowers left *before* 50 and 42, and *afterwards*, yet the ages of the latter would, one with another, much more exceed 50 and 42, than the ages of the
the

the former (that is, of the widows and widowers left *before* 50 and 42) would fall short of them. And the number of marriages also in the world, among persons of greater ages than these, would be much fewer than among persons of lesser ages—In other words: the period, at which the marriages that have been contracted are half dissolved, is not the period at which the number of marriages constantly existing is equally divided, but this period falls some years sooner; and the period I have in view, falls in that part of the interval between these two periods, where the greater ages of the marriages on one side, are just enough to compensate (in such a calculation as that I have given) their deficiencies in number, compared with the number of marriages on the other side.

In short. Suppose 35 marriages every year, between persons 33 and 25. In 12 years there would be half as many in the world, as could possibly arise from such a number of yearly weddings. In 17 years, half every set would be extinct. The *expectation* of every marriage would be 19 years, by prob. 21 of Mr. *De Moivre's Treatise on Annuities*, or by the note p. 299: that is, taking them all together, they would exist just as long as an equal number of *single* persons, supposed to be sure of living just 19 years, and no more; or, as long as an equal number of single persons, all 48 years of age, supposed to be subject to the common laws of mortality. One with another, then, they will be all extinct in 19 years: the marriages which continue beyond this term, tho' fewer in number, enjoying among them just as much *more* duration, as those that fall short of it enjoy *less*. *Widows*, then, at a medium, will commence widowhood at 44 (that is, 25 increased by 19)

U

years

years of age, and *widowers* at 52. The values, therefore, of the lives of the *former*, when they commence widowhood, will, one with another, be the same with the value of a life at 44; or, (reckoning interest at 4 per cent.) 12.5 years purchase, in present payment, (the annuity to begin at the end of a year); and their *expectations* of life will be 21 years, or half the difference between 44 and 86. The value of the lives of the *latter* will be 10.92, and their *expectation* 17 years—The whole number of marriages constantly existing, which would result from 35 supposed to commence annually, would be 19×35 , or 665; and 53 years (the difference between 33 and 86) would be the time in which they would increase to this number—The chance of survivorship would be 69 to 53, by prob. 18th, Mr. De Moivre on *Annuities*; that is, in 53 years, 35 relicts of these marriages would be left every year, and the number of *widows* would be to the number of *widowers*, as 69 to 53; or 19.8 *widows* would be left annually, and 15.2 *widowers*. The *maximum* of widows in life together, if none married, would be 21×19.8 , or 416; and they would increase to this number in 114 years (or 61 years after the number of marriages had attained to a *maximum*)—The *maximum* of *widowers* would be 15.2×17 , or 258; and they would increase to this number in 106 years.

An easy method may be hence deduced of solving the question which occasions this note—If the number of the members of the establishment I have supposed, is 665, and the mean ages at which marriage may be deemed to commence are 25 and 33, 19.8 widows will (it has just appeared) be left every year; and the values of their lives, when they commence widowhood, will be, one

with another, $12\frac{1}{2}$ years purchase. An annuity of 20*l.* will, therefore, be worth, to each widow, 250*l.* and 19.8 such annuities must be worth 4950*l.* which, consequently, is the annual income necessary for the support of the establishment, the first payment to be received immediately: or *l.* 7.44 from each of the 665 members; which answers nearly to the determination in page 44.

In the Last Essay, p. 296, it has been shewn, that observations determine the chance of survivorship in favour of the wife in marriage, to be really so great as 3 to 2; and in some circumstances greater. I have also there observed, that in order to account for this, from the difference of age between men and their wives, this difference must be at least 12 years, and the mean ages of all who marry annually, must be supposed to be about 23 and 35. In this case, 19, as before, will nearly be the *expectation* of all marriages. The mean age at which widows and widowers will commence such will be 42 and 54. The number of annual marriages necessary to keep up 665 marriages constantly existing, will be 35. The number of widows left annually, by such a number of marriages, will be 21; and the values of their lives, at the time they commence widowhood, will be 12.85 years purchase by Table VI: and therefore, the whole annual income necessary for the support of the supposed establishment, will be 5397*l.* or an annual payment, beginning immediately, of *l.* 8.11 from each member—The number of widows on such an establishment will, in 63 years, grow, if none marry, to 462; and the number of widowers to 224.—It may be depended on, that all this would happen as far as Dr. *Halley's* table, or the tables for *Norwich* and *Northampton*, exhibit the true state of human mortality.

Among the ministers and professors in SCOTLAND, the number of married men being 667, or nearly that here mentioned, the number of annual weddings has, for many years, been at an average 32, and the number of widows left annually 20; and, therefore, the chance of survivorship in favour of the wife, as 20 to 12, or 5 to 3. See Essay IV. p. 269. This is not more different, from the results I have given, than might have been expected; and the chief reason of the difference is, that the *expectations* of *single* and *joint* lives among the ministers and their wives in SCOTLAND, are greater than those given by Dr. *Halley's*, and the other tables of observation——These tables give the expectations of lives as they are among the bulk of mankind in moderate towns. The expectations of lives among the better sort of men, living mostly in country villages and parishes, are much greater. The fact is, that among the ministers in *Scotland*, the expectation of a *single* life at the age of 27, is three years and an half greater; and, consequently, of *joint* lives, above two years greater, than the same expectations by Dr. *Halley's* Table. Ibid, page 264.

I cannot help just mentioning another remark here.——It may be observed, that supposing no second marriages, and, at the same time, that the odds for the woman's surviving in marriage is 3 to 2, the number of *widows* in the world would be *double* the number of *widowers*. But it has been found, in fact, that the number of widows is five times the number of widowers. How this is to be accounted for, I have shewn in the Essay just referred to, page 270, 271.

Note (G). Question XIV. Page 43.

LET r be $1l.$ increased by its interest for one year; t the given time or number of years for which the assurance is to be made; $a, b, c, \&c.$ the *probabilities* taken out of a table of observations, that the person whose age is given shall live 1, 2, 3, &c. years; and P the probability that he shall live t years. Then $\frac{1-a}{r} + \frac{1-b}{r^2} + \frac{1-c}{r^3}, \&c.$

$$(t-1) + \frac{1-P}{r^t} + \frac{1-P}{r^{t+1}} + \frac{1-P}{r^{t+2}} \&c. = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}, \&c. (t) - \frac{a}{r} + \frac{b}{r^2} + \frac{c}{r^3}, \&c. (t-1) + \frac{P}{r^t} + \frac{1-P}{r^t} \times \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}, \&c.$$

will be the exact value of an annuity to be entered upon at the failure of the given life, provided it happens in t years. And the rule is nothing but this value expressed in words. In a similar manner may be demonstrated the other rule for finding the values of assurances, for a given time, on two joint lives, or the longest of two lives.

Note (H). Question XV. page 56.

LET r signify as before; S the given sum to be assured; t the given time; N and n the number of the living in the table of observations, at the age of A and B respectively; $A, B, C, \&c.$ and $a, b, c, \&c.$ the number of the living in the table, at the end of $1, 2, 3, \&c.$ years from the ages of A and B ; $D, D, D, D, \&c.$ and $d, d, d, d, \&c.$ the decrements of life in the table, at the end of $1, 2, 3, \&c.$ years from the same ages. Then, by reasoning in the same manner with Mr. *Simpson*, in p. 316, &c. *Select Exercises*, it will appear that $S \times$

$$\frac{A \times d}{Nnr} + \frac{B \times d}{Nnr^2} + \frac{C \times d}{Nnr^3}, \&c. (t) + S \times \frac{Dd}{2Nnr} + \frac{Dd}{2Nnr^2} + \frac{Dd}{2Nnr^3}, \&c. (t) = \frac{S}{n} \times \frac{Ad}{Nr} + \frac{Bd}{Nr^2} + \frac{Cd}{Nr^3}, \&c. (t) + \frac{S}{2N} \times \frac{Dd}{nr} + \frac{Dd}{nr^2}, \&c. (t).$$

This is the exact answer to Question XV. and the rule is as near an approximation to it as there is reason to desire.

In the same manner, retaining all the same symbols, it may be found, that the answer to Question XVI. is

$$S \times \frac{Dd}{2Nnr} + \frac{Dd}{Nnr^2} + \frac{D+D \times d}{Nnr^3} + \frac{D+D+D \times d}{Nnr^4} (t), \&c. + S \times \frac{Dd}{2Nnr^2} + \frac{Dd}{2Nnr^3} + \frac{Dd}{2Nnr^4}, \&c. (t-1)$$

$$(i-1) = \frac{S}{nr} \times \frac{Dd}{Nr} + \frac{\overline{D+D \times d}}{Nr^2} + \frac{\overline{D+D+D \times d}}{Nr^3} \dots$$

$$\&c. (t-1) + \frac{S}{2N} \times \frac{Dd}{nr} + \frac{Dd}{nr^2} + \frac{Dd}{nr^3}, \&c. (t).$$

$$\text{But } \frac{D}{Nr} + \frac{D+D}{Nr^2} + \frac{D+D+D}{Nr^3}, \&c. (t-1) \text{ is the}$$

same with the excess of the value of an annuity *certain* for a number of years less by one year than the given term, above the value of an annuity on the life of A, for the same number of years; from whence the reason of the rule for solving this question, may be easily discovered.

Note (I). Page 119, &c.

LET t be any given term of years; p the value of $1l.$ due at the end of the given term; A the value of an annuity certain for the same term; n the *complement* of a given life; G the value for the given term, of two joint lives, both equal to the given life; (to be found by Quest. VI.) P the perpetuity; r , $1l.$ increased by its interest for one year.

Then $A - G \times n + t \times p \times P - A \times P \times r$ will be the present value of $1l.$ $2l.$ $3l.$ &c. (t) payable at the end of 1, 2, 3, &c. (t) years; but subject to failure when the given life fails.

If such a course of payment is to begin immediately, and to be made at the beginning of every year, till $t + 1$ payments are made in t years; add to the preceding value, the value increased by unity of an annuity on the given life for t years, found by Question VI. and the *sum* will be the value sought. And this value, divided by the present value of what may happen to remain of the given life after t years, found by Question VI. will give the *standing annuity* to which such a series of increasing annual payments, beginning immediately, will entitle, for the remainder of the given life after t years.

With the assistance of this theorem, all that is said in page 119, &c. may be investigated. It would be too tedious to enter into a more minute account.

Note

Note (K). Page 149.

LET d signify the *difference* between the *complements* of the youngest and the oldest life in the body of Annuitants, here described, at the time they enter; let S signify the sum of these *complements*; n any given number of years not greater than $\frac{S}{2} - \frac{d}{2}$; and x the ratio of the whole

number of Annuitants to $\frac{S \times d}{2}$. Then

$x \times d$ will be the number that will die the 1st year;

$x \times d + \frac{2d}{S}$ the number that will die the 2d year;

$x \times d + \frac{4d}{S} + \frac{4d}{S^2}$, 3d year;

$x \times d + \frac{6d}{S} + \frac{8d}{S^2} + \frac{8d}{S^3}$, 4th year;

$x \times d + \frac{8d}{S} + \frac{12d}{S^2} + \frac{16d}{S^3} + \frac{16d}{S^4}$, 5th year;

and $x \times nd + \frac{d}{S} + \frac{d}{S^2} + \frac{d}{S^3} + \frac{d}{S^4} + \frac{d}{S^5} + \frac{d}{S^6} + \frac{d}{S^7} + \frac{d}{S^8} + \frac{d}{S^9} + \frac{d}{S^{10}} + \frac{d}{S^{11}} + \frac{d}{S^{12}} + \frac{d}{S^{13}} + \frac{d}{S^{14}} + \frac{d}{S^{15}} + \frac{d}{S^{16}} + \frac{d}{S^{17}} + \frac{d}{S^{18}} + \frac{d}{S^{19}} + \frac{d}{S^{20}} + \frac{d}{S^{21}} + \frac{d}{S^{22}} + \frac{d}{S^{23}} + \frac{d}{S^{24}} + \frac{d}{S^{25}} + \frac{d}{S^{26}} + \frac{d}{S^{27}} + \frac{d}{S^{28}} + \frac{d}{S^{29}} + \frac{d}{S^{30}} + \frac{d}{S^{31}} + \frac{d}{S^{32}} + \frac{d}{S^{33}} + \frac{d}{S^{34}} + \frac{d}{S^{35}} + \frac{d}{S^{36}} + \frac{d}{S^{37}} + \frac{d}{S^{38}} + \frac{d}{S^{39}} + \frac{d}{S^{40}} + \frac{d}{S^{41}} + \frac{d}{S^{42}} + \frac{d}{S^{43}} + \frac{d}{S^{44}} + \frac{d}{S^{45}} + \frac{d}{S^{46}} + \frac{d}{S^{47}} + \frac{d}{S^{48}} + \frac{d}{S^{49}} + \frac{d}{S^{50}} + \frac{d}{S^{51}} + \frac{d}{S^{52}} + \frac{d}{S^{53}} + \frac{d}{S^{54}} + \frac{d}{S^{55}} + \frac{d}{S^{56}} + \frac{d}{S^{57}} + \frac{d}{S^{58}} + \frac{d}{S^{59}} + \frac{d}{S^{60}} + \frac{d}{S^{61}} + \frac{d}{S^{62}} + \frac{d}{S^{63}} + \frac{d}{S^{64}} + \frac{d}{S^{65}} + \frac{d}{S^{66}} + \frac{d}{S^{67}} + \frac{d}{S^{68}} + \frac{d}{S^{69}} + \frac{d}{S^{70}} + \frac{d}{S^{71}} + \frac{d}{S^{72}} + \frac{d}{S^{73}} + \frac{d}{S^{74}} + \frac{d}{S^{75}} + \frac{d}{S^{76}} + \frac{d}{S^{77}} + \frac{d}{S^{78}} + \frac{d}{S^{79}} + \frac{d}{S^{80}} + \frac{d}{S^{81}} + \frac{d}{S^{82}} + \frac{d}{S^{83}} + \frac{d}{S^{84}} + \frac{d}{S^{85}} + \frac{d}{S^{86}} + \frac{d}{S^{87}} + \frac{d}{S^{88}} + \frac{d}{S^{89}} + \frac{d}{S^{90}} + \frac{d}{S^{91}} + \frac{d}{S^{92}} + \frac{d}{S^{93}} + \frac{d}{S^{94}} + \frac{d}{S^{95}} + \frac{d}{S^{96}} + \frac{d}{S^{97}} + \frac{d}{S^{98}} + \frac{d}{S^{99}} + \frac{d}{S^{100}}$

$+ \frac{4d}{S^3} + \frac{8d}{S^4} + \frac{12d}{S^5} + \frac{16d}{S^6} + \frac{20d}{S^7} + \frac{24d}{S^8} + \frac{28d}{S^9} + \frac{32d}{S^{10}} + \frac{36d}{S^{11}} + \frac{40d}{S^{12}} + \frac{44d}{S^{13}} + \frac{48d}{S^{14}} + \frac{52d}{S^{15}} + \frac{56d}{S^{16}} + \frac{60d}{S^{17}} + \frac{64d}{S^{18}} + \frac{68d}{S^{19}} + \frac{72d}{S^{20}} + \frac{76d}{S^{21}} + \frac{80d}{S^{22}} + \frac{84d}{S^{23}} + \frac{88d}{S^{24}} + \frac{92d}{S^{25}} + \frac{96d}{S^{26}} + \frac{100d}{S^{27}} + \frac{104d}{S^{28}} + \frac{108d}{S^{29}} + \frac{112d}{S^{30}} + \frac{116d}{S^{31}} + \frac{120d}{S^{32}} + \frac{124d}{S^{33}} + \frac{128d}{S^{34}} + \frac{132d}{S^{35}} + \frac{136d}{S^{36}} + \frac{140d}{S^{37}} + \frac{144d}{S^{38}} + \frac{148d}{S^{39}} + \frac{152d}{S^{40}} + \frac{156d}{S^{41}} + \frac{160d}{S^{42}} + \frac{164d}{S^{43}} + \frac{168d}{S^{44}} + \frac{172d}{S^{45}} + \frac{176d}{S^{46}} + \frac{180d}{S^{47}} + \frac{184d}{S^{48}} + \frac{188d}{S^{49}} + \frac{192d}{S^{50}} + \frac{196d}{S^{51}} + \frac{200d}{S^{52}} + \frac{204d}{S^{53}} + \frac{208d}{S^{54}} + \frac{212d}{S^{55}} + \frac{216d}{S^{56}} + \frac{220d}{S^{57}} + \frac{224d}{S^{58}} + \frac{228d}{S^{59}} + \frac{232d}{S^{60}} + \frac{236d}{S^{61}} + \frac{240d}{S^{62}} + \frac{244d}{S^{63}} + \frac{248d}{S^{64}} + \frac{252d}{S^{65}} + \frac{256d}{S^{66}} + \frac{260d}{S^{67}} + \frac{264d}{S^{68}} + \frac{268d}{S^{69}} + \frac{272d}{S^{70}} + \frac{276d}{S^{71}} + \frac{280d}{S^{72}} + \frac{284d}{S^{73}} + \frac{288d}{S^{74}} + \frac{292d}{S^{75}} + \frac{296d}{S^{76}} + \frac{300d}{S^{77}} + \frac{304d}{S^{78}} + \frac{308d}{S^{79}} + \frac{312d}{S^{80}} + \frac{316d}{S^{81}} + \frac{320d}{S^{82}} + \frac{324d}{S^{83}} + \frac{328d}{S^{84}} + \frac{332d}{S^{85}} + \frac{336d}{S^{86}} + \frac{340d}{S^{87}} + \frac{344d}{S^{88}} + \frac{348d}{S^{89}} + \frac{352d}{S^{90}} + \frac{356d}{S^{91}} + \frac{360d}{S^{92}} + \frac{364d}{S^{93}} + \frac{368d}{S^{94}} + \frac{372d}{S^{95}} + \frac{376d}{S^{96}} + \frac{380d}{S^{97}} + \frac{384d}{S^{98}} + \frac{388d}{S^{99}} + \frac{392d}{S^{100}}$

will be the whole number dying in n years. When

n is greater than $\frac{S}{2} - \frac{d}{2}$, this series is greater than

the whole number dying in n years; but in all other cases it gives this number exactly, supposing the probabilities of life to decrease uniformly.

In the present instance, the youngest life being 30, and the oldest 60, the two complements are 56 and

26. $S = 82$. $d = 30$. $\frac{Sd}{2} = 1230$. And therefore $x =$

$\frac{33,333}{1230} = 27.1$. Take $n = 30$ years, and the fore-

going series will be $27.1 \times 900 + 318.2 + 7.242 + .164 = 33.214$, which is a little greater than the whole number dying in 30 years, but at the same time less than the whole number of Annuitants.

Note (L). See Effay I. Page 169, 170,
172.

THE *sum* of the probabilities that any given lives will attain to the end of the 1st, 2d, 3d, &c. *years* from the present time to the utmost extremity of life (for instance, $\frac{45}{40} + \frac{44}{40} + \frac{43}{40}$, &c. to $\frac{1}{40} = 22\frac{1}{2}$ for lives of 40, by the *hypothesis*) may be called their *expectation*, or the number of payments due to them, as *yearly annuitants*. The sum of the probabilities that they will attain to the end of the 1st, 2d, 3d, &c. *half years* (or, in the particular case specified, $\frac{91}{92} + \frac{90}{92} + \frac{89}{92} + \frac{88}{92}$, &c. = $\frac{91}{2}$ *half years*, or $22\frac{3}{4}$ *years*) is their expectation as *half yearly annuitants*. And the sums just mentioned of the probabilities of their attaining to the end of the 1st, 2d, 3d, &c. *moments* (equal in the same particular case to 23 *years*) is properly their *expectation of life*, or their *expectation* as annuitants secured by land.

Mr. *De Moivre* has omitted the demonstrations of the rules he has given for finding the *expectations* of lives, and only intimated, in general, that he discovered them by a calculation deduced from the method of fluxions. See his *Treatise on Annuities*, page 66. It will, perhaps, be agreeable to some to see how easily they are deduced in this method, upon the hypothesis of an equal decrement of life.

Let x stand for a moment of time, and n the complement of any assigned life. Then $\frac{n-x}{n}$, $\frac{n-2x}{n}$, $\frac{n-3x}{n}$, &c. will be the *present* probabilities of its
con-

continuing to the end of the 1st, 2d, 3d, &c. moments; and $\frac{n-x}{n}$ the probability of its continuing to the end of x time. $\frac{n-x}{n} \times \dot{x}$ will therefore be the *fluxion* of the sum of the probabilities, or of an *area* representing this sum, whose *ordinates* are $\frac{n-x}{n}$, and *axis* x .—The *fluent* of this expression, or $x - \frac{x^2}{2n}$, is the sum itself for the time x ; and this, when $x=n$, becomes $\frac{1}{2}n$, and gives the *expectation* of the assigned life, or the sum of all the probabilities just mentioned for its whole possible duration.—In like manner: since $\frac{n-x^2}{n^2}$ is the probability that two equal joint lives will continue x time, $\frac{n-x^2}{n^2} \times \dot{x}$ will be the *fluxion* of the sum of the probabilities. The *fluent* is $x - \frac{x^2}{n} + \frac{x^3}{3n^2}$, which, when $n=x$, is $\frac{n}{3}$, or the *expectation* of two equal joint lives.—Again: since $\frac{n-x}{n} \times \frac{2x}{n}$ is the probability that there will be a survivor of two equal joint lives at the end of x time, $\frac{n-x}{n} \times \frac{2x}{n} \times \dot{x}$ will be the *fluxion* of the sum of the probabilities; and the *fluent*, or $\frac{x^2}{n} - \frac{2x^3}{3n^2}$ is (when $x=n$) $\frac{1}{3}n$, or the *expectation* of survivorship between two equal lives; which, therefore, appears to be equal to the

expecta-

expectation of their joint continuance. The expectation of two *unequal* joint lives, found in the same way, is $\frac{m}{2} - \frac{m^2}{6n}$, m being the *complement* of the oldest life, and n the *complement* of the youngest.

The whole expectation of survivorship is $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{3n}$. The expectation of survivorship, on the part

of the oldest is $\frac{m^2}{6n}$; and the expectation, on the

part of the youngest, is $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{6n}$. It is easy to

apply this investigation to any number of joint lives, and to all cases of survivorship.

It may be observed, concerning the first of the fluents here given, that it expresses not only the expectation of a given life for the time x , and therefore its whole expectation when $x=n$, but likewise, the number of persons alive, to which one person added annually to a society, at a given age, will increase in x time.—Thus: Suppose one annuitant, whose age is 28, (and whose *complement* of life, therefore, is 58, or *expectation* of life 29)

to come upon a society every year; the number of annuitants alive, deduced from hence, will, in x years, be $x - \frac{x^2}{4 \times 29}$, or $\frac{4 \times 29 - x^2}{4 + 29} \times x$; and, there-

fore, the number of annuitants alive, deduced in the same time from p annuitants left annually at

the same age, will be $\frac{4 \times 29 - x^2}{4 \times 29} \times px$.—In like

manner, the 2d fluent, or $\frac{x^3}{3n^2} - \frac{x^3}{n} + x$, gives the

number

number of marriages in being together, that will, in x years, grow out of *one* yearly marriage, between persons of *equal* ages, whose complement of life is n . If they are of *unequal* ages, and the complement of the oldest life is m , and of the youngest

n , this number will be $\frac{x^3}{3nm} - \frac{n+m \times x^2}{2nm} + x$. And

if the number of years is required, in which any given number of yearly marriages, between men and women at given ages, will increase so far as to be in any given proportion to the greatest number that can possibly grow out of such marriages, this expression must be made equal to the *expectation* of the joint lives, or of each marriage, multiplied by the fraction expressing the given proportion; and the root of the equation will be the answer. Thus: it may be found, that one marriage every year, between persons 33 and 25 years of age, would in 10 years increase to 8.35; in 15 years, to 11.38; and in 53 years, to 19, or their greatest possible number: and, consequently, that 35 such yearly marriages would, in 10 years, increase to 292; in 15 years, to 398; and in 53 years, to 665.—And if it is enquired in what number of years 35 such yearly marriages would increase to half the number in being together, possible to be derived from them, the value of x , in the cubic

equation $\frac{x^3}{3nm} - \frac{n+m \times x^2}{2nm} + x = \frac{m}{2} - \frac{m}{6n} \times \frac{1}{2}$, must be found; which, in the present instance, is nearly 12.

I have, in some parts of this work, had occasion to make such deductions as these. See note (A), p. 277; and note (F), p. 288; and Questions III. and XIII.

Note

Note (M). Essay II. Page 226.

LET r signify $1l$, increased by its interest for one year.

V the PERPETUITY.

n the difference between the age of the youngest life, and 86; or its *complement*.

m the complement of the oldest life.

P the value (in Table II.) of an annuity certain for m years.

And the exact value of any two given joint lives, according to the hypothesis of an equal decrement

of life, will be $V - \frac{V+1}{n} \times \frac{P}{n-m-2v-1} \times \frac{P}{m}$

$+ 2v$. Example:

Let the ages be 27 and 38; and the rate of interest 4 *per cent*. Then $n = 59$. $m = 48$. $V = 25$.

$P = 21.195$. $n - m - 2v - 1 = -40$. $n - m -$

$\frac{P}{2v-1} \times \frac{P}{m} + 2v = 50 - 17.660 = 32.340$. And

$V - \frac{V+1}{n} \times \frac{P}{n-m-2v-1} \times \frac{P}{m} + 2v = 25 - \frac{26}{35}$

$\times 32.340 = 10.748$, the value of two joint lives whose ages are 27 and 38.

Note

Note (N). Essay III. Page 232.

IT is plain, that the purchaser of A's right, as stated in the first of the questions, to which this note refers, cannot get into possession, till the year when A and B shall be both dead; nor then, unless A happens to die *last*. Now, supposing the common complement of life n ; the probability that A and B shall be *both* dead at the end of the *first* year, and A die *last*, is $1 - \frac{n-1}{n} \times 1 - \frac{n-1}{n}$
 $\times \frac{1}{2} = \frac{1}{2} - \frac{n-1}{2n} - \frac{n-1}{2n} + \frac{(n-1)^2}{2n^2}$. — In like manner, the probability that they shall be *both* dead at the end of the 2d, 3d, &c. years, and A survive, is $\frac{1}{2} - \frac{n-2}{2n} - \frac{n-2}{2n} + \frac{(n-2)^2}{2n^2}$; $\frac{1}{2} - \frac{n-3}{2n} - \frac{n-3}{2n} + \frac{(n-3)^2}{2n^2}$, &c. The *present* value, therefore, of the 1st, 2d, 3d, &c. rents of the reversionary estate is $\frac{1}{2r} - \frac{n-1}{2nr} - \frac{n-1}{2nr} + \frac{(n-1)^2}{2nr^2}$, $\frac{1}{2r^2} - \frac{n-2}{2nr^2} - \frac{n-2}{2nr^2} + \frac{(n-2)^2}{2n^2r^2}$, $\frac{1}{2r^2} - \frac{n-3}{2nr^3} - \frac{n-3}{2nr^3} + \frac{(n-3)^2}{2n^2r^3}$, &c. Supposing r to signify *1l.* increased by its interest for a year; and the estate to be *1l. per annum*. And the *sum* of these terms continued *in infinitum*, is the value *required*. — But $\frac{1}{2r} + \frac{1}{2r^2} + \frac{1}{2r^3}$, &c. is *half*
the

the perpetuity. And $\frac{n-1}{2nr} + \frac{n-1}{2nr} - \frac{\overline{n-1}^2}{2n^2r} +$
 $\frac{n-2}{2nr^2} + \frac{n-2}{2nr^2} - \frac{\overline{n-2}^2}{2n^2r^2} + \frac{n-3}{2nr^3} + \frac{n-3}{2nr^3} - \frac{\overline{n-3}^2}{2n^2r^3}, \&c.$

is *half* the value of the *joint* lives, subtracted from *half* the sum of the values of the two *single* lives; that is, *half* the value of the *longest* of the two lives.

A similar demonstration may be applied to the other question.

Note (O). Essay IV. Page 262.

LET r be 1%. increased by its interest for one year.

Let S represent any given interval of time, or number of years, during which the decrements of life in a table of observations continue equal.

a the number of the living in the table at the beginning of the first year of that interval.

b the number of the living in the table at the beginning of the year immediately following the same interval.

P the value of an annuity certain for S years.

p the value, in Table I. of 1%. due at the end of S years.

Q the value, in Table VI. of an annuity for the life of a person whose age wants S years of 86.

N the value, in strict agreement with the given table of observations, of an annuity on the life of a person whose age is S years greater than the age at which the interval of equal decrements begins. Then

$Q + \frac{b}{a} \times \overline{P - Q}$ will be the value, according to the table of observations, of an annuity for S years, on a life of the same age with that at which the interval of equal decrements begins. And

$Q + \frac{b}{a} \times \overline{P - Q} + pN$ will be the value of an annuity on the whole duration of that life.

When S represents *one* year, Q vanishes, and the last expression becomes $\frac{b}{ar} \times \overline{1 + N}$; which is the

rule for finding, from the value given of any life, the value of a life one year younger.

These Theorems save much labour in calculating the values of life-annuities from tables of observations.

The first of them, with its investigation, may be found in page 341, 3d edition, of *Mr. De Moivre's Treatise on the Doctrine of Chances*. But it is necessary to observe, that the direction *Mr. De Moivre* has given for finding the value of Q is wrong. In consequence of calculating agreeably to this direction, he gives the value of a life at the age of 42, by *Dr. Halley's* table, greater than the value of the same life by his own hypothesis; whereas, it is evident, that the probabilities of living after 42, being all along less in *Dr. Halley's* table, than in the hypothesis, the value of the life must be also less.

The mathematical reader may easily satisfy himself, that the value of Q ought to be taken from Table VI. as I have directed.

An easy and accurate method of finding the values of single lives, agreeably to any given table of observations, is given by *Mr. Dodson* in his *Mathematical Repository*, vol. II. page 161.

There is also in *Mr. Simpson's Select Exercises*, page 273, a very easy rule for approximating to the values of single lives, according to *Dr. Halley's* table. But this rule must not be depended on; for I have found it $\frac{1}{2}$ a year's purchase, and sometimes three-quarters of a year's purchase wrong.

To prevent the danger of mistaking the Theorem I have given, I have thought proper to subjoin the following example.

Let the table of observations be the *Bristol* Table, or Table III. The value of a life at 78, by this

Table, is $\frac{49}{58r} + \frac{41}{58r^2} + \frac{34}{58r^3}$, &c. to the end of life. The number of terms in this series being small, it may be easily found to be 3.514, supposing interest at 4 *per cent.* and $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c. being the values, in Table I. of 1*l.* at the end of 1, 2, 3, &c. years.—From 78 to 74 the decrements of life continue equal; and therefore $S=4$. $a=98$. $b=58$. $P=3.6298$, by Table II; $p=.8548$, by Table I; $Q=1.406$, by Table VI; $N=3.514$. $P-Q+pN=5.227$; and $Q+\frac{b}{a} \times \overline{P-Q+pN} = 4.500$, or the value of a life at 74.

From 74 to 70 there is another interval of equal decrements; and, by a like easy operation, the value of a life at 70 will be found to be 5.595.

T A B L E I.

The present Value of 1*l.* to be received at the end of any number of years, not exceeding 100; discounting at the rates of 3, 3½, 4, 4½, 5 and 6 per cent. compound interest.

	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
1	,970874	,966184	,961538	,956938	,952381	,943396
2	,942596	,933511	,924556	,915730	,907029	,889996
3	,915142	,901943	,888996	,876297	,863838	,839619
4	,888487	,871442	,854804	,838561	,822702	,792094
5	,862609	,841973	,821927	,802451	,783526	,747258
6	,837484	,813501	,790315	,767896	,746215	,704961
7	,813092	,785991	,759918	,734828	,710681	,665057
8	,789409	,759412	,730690	,703185	,676839	,627412
9	,766417	,733731	,702587	,672904	,644609	,591898
10	,744094	,708919	,675564	,643928	,613913	,558395
11	,722421	,684946	,649581	,616199	,584679	,526788
12	,701380	,661783	,624597	,589664	,556837	,499669
13	,680951	,639404	,600574	,564272	,530321	,468839
14	,661118	,617782	,577475	,539973	,505068	,442501
15	,641862	,596891	,555265	,516720	,481017	,417265
16	,623167	,576706	,533908	,494469	,458112	,393646
17	,605016	,557204	,513373	,473176	,436297	,371364
18	,587395	,538561	,493628	,452800	,415521	,350344
19	,570286	,520156	,474642	,433302	,395734	,330513
20	,553676	,502566	,456387	,414643	,376889	,311805
21	,537549	,485571	,438834	,396787	,358942	,294155
22	,521893	,469151	,421955	,379701	,341850	,277505
23	,506692	,453286	,405726	,363350	,325571	,261797
24	,491934	,437957	,390121	,347793	,310058	,246979
25	,477606	,423147	,375117	,332731	,295303	,232999
26	,463695	,408838	,360689	,318402	,281241	,219810
27	,450189	,395012	,346817	,304691	,267848	,207368
28	,437077	,381654	,333477	,291571	,255094	,195630
29	,424346	,368748	,320651	,279015	,242946	,184557
30	,411987	,356278	,308319	,267000	,231377	,174110
31	,399987	,344230	,296460	,255502	,220359	,164255
32	,388337	,332590	,285058	,244500	,209866	,154057

TABLE I. Continued.

	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
33	,377026	,321343	,274094	,233971	,199873	,146186
34	,366045	,310476	,263552	,223896	,190355	,137912
35	,355383	,299977	,253415	,214254	,181290	,130105
36	,345032	,289833	,243669	,205028	,172657	,122741
37	,334983	,280032	,234297	,196199	,164436	,115793
38	,325226	,270562	,225285	,187750	,156605	,109239
39	,315754	,261413	,216621	,179665	,149148	,103056
40	,306557	,252572	,208289	,171929	,142046	,097222
41	,297628	,244031	,200278	,164525	,135282	,091719
42	,288959	,235779	,192575	,157440	,128840	,086527
43	,280543	,227806	,185168	,150663	,122704	,081630
44	,272372	,220102	,178046	,144173	,116864	,077009
45	,264439	,212659	,171198	,137964	,111297	,072650
46	,256737	,205468	,164614	,132023	,105997	,068538
47	,249259	,198520	,158283	,126338	,100949	,064658
48	,241999	,191806	,152195	,120898	,096142	,060998
49	,234950	,185320	,146341	,115692	,091564	,057546
50	,228107	,179053	,140713	,110710	,087204	,054288
51	,221463	,172998	,135301	,105942	,083051	,051215
52	,215013	,167148	,130097	,101380	,079096	,048316
53	,208750	,161496	,125093	,097014	,075330	,045582
54	,202670	,156035	,120282	,092837	,071743	,043001
55	,196767	,150758	,115656	,088839	,068326	,040567
56	,191036	,145660	,111207	,085013	,065073	,038271
57	,185472	,140734	,106930	,081353	,061974	,036105
58	,180070	,135975	,102817	,077849	,059023	,034061
59	,174825	,131377	,098963	,074497	,056212	,032133
60	,169733	,126934	,095060	,071289	,053536	,030310
61	,164789	,122642	,091404	,068219	,050986	,028598
62	,159990	,118495	,087889	,065281	,048558	,026989
63	,155330	,114487	,084508	,062470	,046246	,025453
64	,150806	,110616	,081258	,059780	,044044	,024012
65	,146413	,106875	,078133	,057206	,041946	,022653
66	,142149	,103261	,075128	,054742	,039949	,021370
67	,138009	,099769	,072238	,052385	,038047	,020161
68	,133989	,096395	,069460	,050129	,036235	,019020
69	,130086	,093136	,066788	,047971	,034509	,017943
70	,126297	,019986	,064219	,045905	,032866	,016927

T A B L E I. Continued.

	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 par Ct.	6 per Ct.
71	,122619	,086943	,061749	,043928	,031301	,015969
72	,119047	,084003	,059374	,042037	,029811	,015065
73	,115580	,081162	,057091	,040226	,028391	,014212
74	,112214	,078418	,054895	,038494	,027039	,013408
75	,108945	,075766	,052784	,036836	,025752	,012649
76	,105772	,073204	,050754	,035250	,024525	,011933
77	,102691	,070728	,048801	,033732	,023357	,011258
78	,099700	,068336	,046924	,032280	,022245	,010620
79	,096796	,066026	,045120	,030890	,021186	,010019
80	,093977	,063793	,043384	,029559	,020177	,009452
81	,091240	,061636	,041716	,028287	,019216	,008917
82	,088582	,059551	,040111	,027068	,018301	,008412
83	,086002	,057538	,038569	,025903	,017430	,007936
84	,083497	,055592	,037085	,024787	,016600	,007487
85	,081065	,053712	,035659	,023720	,015809	,007063
86	,078704	,051896	,034287	,022699	,015056	,006663
87	,076412	,050141	,032968	,021721	,014339	,006286
88	,074186	,048445	,031700	,020786	,013657	,005930
89	,072027	,046807	,030481	,019891	,013006	,005595
90	,069928	,045224	,029309	,019034	,012387	,005278
91	,067891	,043695	,028182	,018215	,011797	,004979
92	,065914	,042217	,027098	,017430	,011235	,004697
93	,063994	,040789	,026055	,016680	,010700	,004432
94	,062130	,039410	,025053	,015961	,010191	,004181
95	,060320	,038077	,024090	,015274	,009705	,003944
96	,058563	,036790	,023163	,014616	,009243	,003721
97	,056858	,035546	,022272	,013987	,008803	,003510
98	,055202	,034344	,021416	,013385	,008384	,003312
99	,053594	,033182	,020592	,012808	,007985	,003124
100	,052033	,032060	,019800	,012257	,007604	,002957

TABLE II.

The present Value of an Annuity of One Pound,
for any Number of Years not exceeding 100,
at the several Rates of 3, 3½, 4, 5, and 6l.
per Cent.

Ye.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
1	.9708	.9662	.9615	.9523	.9433
2	1.9133	1.8997	1.8860	1.8594	1.8333
3	2.8286	2.8016	2.7750	2.7232	2.6730
4	3.7170	3.6731	3.6298	3.5459	3.4651
5	4.5797	4.5151	4.4518	4.3294	4.2123
6	5.4971	5.3286	5.2421	5.0756	4.9173
7	6.2302	6.1145	6.0020	5.7863	5.5823
8	7.0196	6.8740	6.7327	6.4632	6.2097
9	7.7861	7.6077	7.4353	7.1078	6.8016
10	8.5302	8.3166	8.1108	7.7212	7.3600
11	9.2526	9.0015	8.7604	8.3064	7.8868
12	9.9540	9.6633	9.3850	8.8632	8.3838
13	10.6349	10.3027	9.9856	9.3935	8.8526
14	11.2960	10.9205	10.5631	9.8986	9.2949
15	11.9379	11.5174	11.1183	10.3796	9.7122
16	12.5611	12.0941	11.6522	10.8377	10.1058
17	13.1661	12.6513	12.1656	11.2740	10.4772
18	13.7535	13.1897	12.6592	11.6895	10.8276
19	14.3238	13.7098	13.1339	12.0853	11.1581
20	14.8774	14.2124	13.5903	12.4622	11.4699
21	15.4150	14.6980	14.0291	12.8211	11.7640
22	15.9389	15.1671	14.4511	13.1630	12.0415
23	16.4436	15.6204	14.8568	13.4885	12.3033
24	16.9355	16.0584	15.2469	13.7986	12.5503
25	17.4131	16.4815	15.6220	14.0939	12.7833

APPENDIX.

TABLE II. Continued.

Yr.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
26	17.8768	16.8904	15.9827	14.3751	13.0031
27	18.3270	17.2854	16.3295	14.6430	13.2105
28	18.7641	17.6670	16.6630	14.8981	13.4061
29	19.1884	18.0358	16.9837	15.1410	13.5907
30	19.6004	18.3920	17.2920	15.3724	13.7648
31	20.0004	18.7363	17.5884	15.5928	13.9290
32	20.3887	19.0689	17.8735	15.8026	14.0840
33	20.7657	19.3902	18.1476	16.0025	14.2302
34	21.1318	19.7007	18.4111	16.1929	14.3681
35	21.4872	20.0007	18.6646	16.3741	14.4982
36	21.8322	20.2905	18.9082	16.5468	14.6209
37	22.1672	20.5705	19.1425	16.7112	14.7367
38	22.4924	20.8411	19.3678	16.8678	14.8460
39	22.8082	21.1025	19.5844	17.0170	14.9490
40	23.1147	21.3551	19.7927	17.1590	15.0462
41	23.4124	21.5991	19.9930	17.2943	15.1380
42	23.7013	21.8349	20.1856	17.4232	15.2245
43	23.9819	22.0627	20.3707	17.5459	15.3061
44	24.2542	22.2828	20.5488	17.6627	15.3831
45	24.5187	22.4955	20.7200	17.7740	15.4558
46	24.7754	22.7009	20.8846	17.8800	15.5243
47	25.0247	22.8994	21.0429	17.9810	15.5890
48	25.2667	23.0912	21.1951	18.0771	15.6500
49	25.5016	23.2766	21.3414	18.1687	15.7075
50	25.7297	23.4556	21.4821	18.2559	15.7618
51	25.9512	23.6286	21.6174	18.3389	15.8130
52	26.1662	23.7958	21.7475	18.4180	15.8613
53	26.3749	23.9573	21.8726	18.4934	15.9069
54	26.5776	24.1133	21.9929	18.5651	15.9499
55	26.7744	24.2641	22.1086	18.6334	15.9905

TABLE II. Continued.

Ye.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
56	26.9654	24.4097	22.2198	18.6985	16.0288
57	27.1509	24.5504	22.3267	18.7605	16.0649
58	27.3310	24.6864	22.4295	18.8195	16.0989
59	27.5058	24.8178	22.5284	18.8757	16.1311
60	27.6755	24.9447	22.6234	18.9292	16.1614
61	27.8403	25.0674	22.7148	18.9802	16.1900
62	28.0003	25.1839	22.8027	19.0288	16.2170
63	28.1556	25.3004	22.8872	19.0750	16.2424
64	28.3064	25.4110	22.9685	19.1191	16.2664
65	28.4528	25.5178	23.0466	19.1610	16.2891
66	28.5950	25.6211	23.1218	19.2010	16.3104
67	28.7330	25.7209	23.1940	19.2390	16.3306
68	28.8670	25.8173	23.2635	19.2753	16.3496
69	28.9971	25.9104	23.3302	19.3098	16.3676
70	29.1234	26.0004	23.3945	19.3426	16.3845
71	29.2460	26.0873	23.4562	19.3739	16.4005
72	29.3650	26.1713	23.5156	19.4037	16.4155
73	29.4806	26.2525	23.5727	19.4321	16.4297
74	29.5928	26.3309	23.6276	19.4592	16.4431
75	29.7018	26.4067	23.6804	19.4849	16.4558
76	29.8076	26.4799	23.7311	19.5094	16.4677
77	29.9102	26.5506	23.7799	19.5328	16.4790
78	30.0099	26.6190	23.8268	19.5550	16.4896
79	30.1067	26.6850	23.8720	19.5762	16.4996
80	30.2007	26.7488	23.9153	19.5964	16.5091
81	30.2920	26.8104	23.9571	19.6156	16.5180
82	30.3805	26.8700	23.9972	19.6339	16.5264
83	30.4665	26.9275	24.0357	19.6514	16.5343
84	30.5500	26.9831	24.0728	19.6680	16.5418
85	30.6311	27.0368	24.1085	19.6838	16.5489

TABLE II. Continued.

Ye.	3 per Ct.	3½ per Ct.	4 per C.	5 per Ct.	6 per Ct.
86	30.7098	27.0887	24.1428	19.6988	16.5556
87	30.7862	27.1388	24.1757	19.7132	16.5618
88	30.8604	27.1873	24.2074	19.7268	16.5678
89	30.9324	27.2341	24.2379	19.7398	16.5734
80	31.0024	27.2793	24.2672	19.7522	16.5786
91	31.0703	27.3230	24.2954	19.7640	16.5836
92	31.1362	27.3652	24.3225	19.7752	16.5883
93	31.2002	27.4060	24.3486	19.7859	16.5928
94	31.2623	27.4454	24.3736	19.7961	16.5969
95	31.3226	27.4835	24.3977	19.8058	16.6009
96	31.3812	27.5203	24.4209	19.8151	16.6046
97	31.4380	27.5558	24.4431	19.8239	16.6081
98	31.4932	27.5902	24.4646	19.8323	16.6114
99	31.5468	27.6234	24.4852	19.8403	16.6145
100	31.5989	27.6554	24.5050	19.8479	16.6175

TABLE III.

Shewing the Probabilities of the Duration of Life, as deduced by Dr. *Halley* from Observations on the Bills of Mortality of BRESLAW.

Ages	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages	Persons living.	Decr. of Life.
1	1000	145	31	523	8	61	232	10
2	855	57	32	515	8	62	222	10
3	798	38	33	507	8	63	212	10
4	760	28	34	499	9	64	202	10
5	732	22	35	490	9	65	192	10
6	710	18	36	481	9	66	182	10
7	692	12	37	472	9	67	172	10
8	680	10	38	463	9	68	162	10
9	670	9	39	454	9	69	152	10
10	661	8	40	445	9	70	142	11
11	653	7	41	436	9	71	131	11
12	646	6	42	427	10	72	120	11
13	640	6	43	417	10	73	109	11
14	634	6	44	407	10	74	98	10
15	628	6	45	397	10	75	88	10
16	622	6	46	387	10	76	78	10
17	616	6	47	377	10	77	68	10
18	610	6	48	367	10	78	58	9
19	604	6	49	357	11	79	49	8
20	598	6	50	346	11	80	41	7
21	592	6	51	335	11	81	34	6
22	586	7	52	324	11	82	28	5
23	579	6	53	313	11	83	23	4
24	573	6	54	302	10	84	19	4
25	567	7	55	292	10	85	15	4
26	560	7	56	282	10	86	11	3
27	553	7	57	272	10	87	8	3
28	546	7	58	262	10	88	5	2
29	539	8	59	252	10	89	3	2
30	531	8	60	242	10	90	1	1

TABLE IV.

Shewing the Probabilities of Life at NORTH-AMPTON. See page 255, 256.

Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.
0	1149	300	31	428	7	62	187	8
1	849	127	32	421	7	63	179	8
2	722	50	33	414	7	64	171	8
3	672	26	34	407	7	65	163	8
4	646	21	35	400	7	66	155	8
5	625	16	36	393	7	67	147	8
6	609	13	37	386	7	68	139	8
7	596	10	38	379	7	69	131	8
8	586	9	39	372	7	70	123	8
9	577	7	40	365	8	71	115	8
10	570	6	41	357	8	72	107	8
11	564	6	42	349	8	73	99	8
12	558	5	43	341	8	74	91	8
13	553	5	44	333	8	75	83	8
14	548	5	45	325	8	76	75	8
15	543	5	46	317	8	77	67	7
16	538	5	47	309	8	78	60	7
17	533	5	48	301	8	79	53	7
18	528	6	49	293	9	80	46	7
19	522	7	50	284	9	81	39	7
20	515	8	51	275	8	82	32	6
21	507	8	52	267	8	83	26	5
22	499	8	53	259	8	84	21	4
23	491	8	54	251	8	85	17	4
24	483	8	55	243	8	86	13	3
25	475	8	56	235	8	87	10	2
26	467	8	57	227	8	88	8	2
27	459	8	58	219	8	89	6	2
28	451	8	59	211	8	90	4	2
29	443	8	60	203	8	91	2	1
30	435	7	61	195	8	92	1	1

TABLE V.

Shewing the Probabilities of Life at NORWICH.
See page 256, 257.

Ages	Persons living.	Decr. of Life.	Ages	Persons living.	Decr. of Life.	Ages	Persons living.	Decr. of Life.
0	1185	320	32	392	6	63	174	9
1	865	160	33	386	6	64	165	9
2	705	60	34	380	6	65	156	9
3	645	32	35	374	6	66	147	9
4	613	23	36	368	6	67	138	9
5	590	20	37	362	6	68	129	9
6	570	16	38	356	6	69	120	9
7	554	13	39	350	7	70	111	9
8	541	11	40	343	6	71	102	8
9	530	9	41	337	6	72	94	8
10	521	7	42	331	6	73	86	8
11	514	6	43	325	7	74	78	8
12	508	6	44	318	7	75	70	8
13	502	5	45	311	7	76	62	7
14	497	5	46	304	7	77	55	7
15	492	5	47	297	7	78	48	6
16	487	5	48	290	7	79	42	5
17	482	5	49	283	7	80	37	5
18	477	5	50	276	7	81	32	4
19	472	5	51	269	7	82	28	4
20	467	6	52	262	7	83	24	4
21	461	6	53	255	8	84	20	3
22	455	6	54	247	8	85	17	3
23	449	6	55	239	8	86	14	3
24	443	6	56	231	8	87	11	2
25	437	6	57	223	8	88	9	2
26	431	7	58	215	8	89	7	2
27	424	7	59	207	8	90	5	2
28	417	7	60	199	8	91	3	2
29	410	6	61	191	8	92	1	1
30	404	6	62	183	9	93	1	1
31	398	6						

TABLE VI. (a).

Shewing the present Values of an Annuity of *1*l. on a single life, according to Mr. *De Moivre's* hypothesis; and, therefore, nearly, according to the probabilities of life at *BRESLAW*, *NORWICH*, and *NORTHAMPTON*. See p. 2, and p. 262.

Age.	3 per Ct.	$3\frac{1}{2}$ per Ct.	4 per Ct.	$4\frac{1}{2}$ per Ct.	5 per Ct.	6 per Ct.
8	19,736	18,160	16,791	15,595	14,544	12,790
9	19,868	18,269	16,882	15,672	14,607	12,839
10	19,868	18,269	16,882	15,672	14,607	12,839
11	19,736	18,160	16,791	15,595	14,544	12,790
12	19,604	18,049	16,698	15,517	14,480	12,741
13	19,469	17,937	16,604	15,437	14,412	12,691
14	19,331	17,823	16,508	15,356	14,342	12,639
15	19,192	17,707	16,410	15,273	14,271	12,586
16	19,050	17,588	16,311	15,189	14,197	12,532
17	18,905	17,467	16,209	15,102	14,123	12,476
18	18,759	17,344	16,105	15,015	14,047	12,419
19	18,610	17,220	15,999	14,923	13,970	12,361
20	18,458	17,093	15,891	14,831	13,891	12,301
21	18,305	16,963	15,781	14,737	13,810	12,239
22	18,148	16,830	15,669	14,641	13,727	12,177
23	17,990	16,696	15,554	14,543	13,642	12,112
24	17,827	16,559	15,437	14,442	13,555	12,045
25	17,664	16,419	15,318	14,340	13,466	11,978
26	17,497	16,277	15,197	14,235	13,375	11,908
27	17,327	16,133	15,073	14,128	13,282	11,837
28	17,154	15,985	14,946	14,018	13,186	11,763
29	16,979	15,835	14,816	13,905	13,088	11,688
30	16,800	15,682	14,684	13,791	12,988	11,610
31	16,620	15,526	14,549	13,673	12,855	11,530
32	16,436	15,367	14,411	13,553	12,780	11,449
33	16,248	15,204	14,270	13,430	12,673	11,365

(a) This Table is the same with Mr. *De Moivre's* Table of the values of single lives, published in his *Treatise on Life Annuities*, and carried as far as the age of 79 to three places of decimals by Mr. *Dodson* in his *Mathematical Repository*, vol. ii. p. 169.

TABLE VI. Continued.

Age	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
34	16,057	15,039	14,126	13,304	12,562	11,278
35	15,864	14,871	13,979	13,175	12,449	11,189
36	15,666	14,699	13,829	13,044	12,333	11,098
37	15,465	14,524	13,676	12,909	12,214	11,003
38	15,260	14,345	13,519	12,771	12,091	10,907
39	15,053	14,163	13,359	12,630	11,966	10,807
40	14,842	13,978	13,196	12,485	11,837	10,704
41	14,626	13,789	13,028	12,337	11,705	10,599
42	14,407	13,596	12,858	12,185	11,570	10,490
43	14,185	13,399	12,683	12,029	11,431	10,378
44	13,958	13,199	12,504	11,870	11,288	10,263
45	13,728	12,993	12,322	11,707	11,142	10,144
46	13,493	12,784	12,135	11,540	10,992	10,021
47	13,254	12,571	11,944	11,368	10,837	9,895
48	13,012	12,354	11,748	11,192	10,679	9,765
49	12,764	12,131	11,548	11,012	10,515	9,630
50	12,511	11,904	11,344	10,827	10,348	9,492
51	12,255	11,673	11,135	10,638	10,176	9,349
52	11,994	11,437	10,921	10,443	9,999	9,201
53	11,729	11,195	10,702	10,243	9,817	9,049
54	11,457	10,950	10,478	10,039	9,630	8,891
55	11,183	10,698	10,248	9,829	9,437	8,729
56	10,902	10,443	10,014	9,614	9,239	8,561
57	10,616	10,181	9,775	9,393	9,036	8,387
58	10,325	9,913	9,527	9,166	8,826	8,208
59	10,029	9,640	9,275	8,933	8,611	8,023
60	9,727	9,361	9,017	8,694	8,389	7,831
61	9,419	9,076	8,753	8,449	8,161	7,633
62	9,107	8,786	8,482	8,197	7,926	7,428
63	8,787	8,488	8,205	7,938	7,684	7,216
64	8,462	8,185	7,921	7,672	7,435	6,997
65	8,132	7,875	7,631	7,399	7,179	6,770
66	7,794	7,558	7,333	7,119	6,915	6,535
67	7,450	7,234	7,027	6,831	6,643	6,292
68	7,099	6,902	6,714	6,534	6,362	6,010
69	6,743	6,565	6,394	6,230	6,073	5,779
70	6,378	6,210	6,065	5,918	5,775	5,508

TABLE VI. Continued.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
71	6,008	5,865	5,728	5,596	5,468	5,228
72	5,631	5,505	5,383	5,265	5,152	4,937
73	5,246	5,136	5,029	4,926	4,826	4,636
74	4,854	4,759	4,666	4,576	4,489	4,324
75	4,453	4,373	4,293	4,217	4,143	4,000
76	4,046	3,978	3,912	3,847	3,784	3,664
77	3,632	3,575	3,520	3,467	3,415	3,315
78	3,207	3,163	3,111	3,076	3,034	2,953
79	2,776	2,741	2,707	2,673	2,641	2,578
80	2,334	2,309	2,284	2,259	2,235	2,188
81	1,886	1,867	1,850	1,832	1,816	1,783
82	1,429	1,411	1,406	1,394	1,384	1,362
83	0,961	0,955	0,950	0,943	0,937	0,925
84	0,484	0,483	0,481	0,479	0,476	0,472
85	0,000	0,000	0,000	0,000	0,000	0,000

Y

TABLE

TABLE VII.

Shewing the Value of an Annuity on the joint continuance of two lives, according to Mr. *De Moivre's Hypothesis*; and, therefore, nearly according to the probabilities of life at BRESLAW, NORWICH, and NORTHAMPTON. See *Essay II.* and p. 2, 3, 226, 262.

Age of the youngst.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
10	10	15.206	13.342	11.855
	15	14.878	13.093	11.661
	20	14.503	12.808	11.430
	25	14.074	12.480	11.182
	30	13.585	12.102	10.884
	35	13.025	11.665	10.537
	40	12.381	11.156	10.128
	45	11.644	10.564	9.646
	50	10.796	9.871	9.074
	55	9.822	9.059	8.391
	60	8.704	8.105	7.572
65	7.417	6.980	6.585	
	70	5.936	5.652	5.391
15	15	14.574	12.860	11.478
	20	14.225	12.593	11.266
	25	13.822	12.281	11.022
	30	13.359	11.921	10.736
	35	12.824	11.501	10.402
	40	12.207	11.013	10.008
	45	11.496	10.440	9.541
	50	10.675	9.767	8.985
	55	9.727	8.975	8.318
	60	8.632	8.041	7.515
	65	7.377	6.934	6.544
	70	5.932	5.623	5.364

TABLE VII. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
20	20	13.904	12.341	11.067
	25	13.531	12.051	10.840
	30	13.098	11.711	10.565
	35	12.594	11.314	10.278
	40	12.008	10.847	9.870
	45	11.325	10.297	9.420
	50	10.536	9.648	8.880
	55	9.617	8.879	8.233
	60	8.549	7.967	7.448
	65	7.308	6.882	6.495
	70	5.868	5.590	5.333
25	25	13.192	11.786	10.621
	30	12.794	11.468	10.367
	35	12.333	11.095	10.067
	40	11.776	10.655	9.708
	45	11.130	10.131	9.278
	50	10.374	9.509	8.761
	55	9.488	8.766	8.134
	60	8.452	7.880	7.371
	65	7.241	6.826	6.440
		70	5.826	5.551
30	30	12.434	11.182	10.133
	35	12.010	10.838	9.854
	40	11.502	10.428	9.514
	45	10.898	9.936	9.112
	50	10.183	9.345	8.620
	55	9.338	8.634	8.018
	60	8.338	7.779	7.280
	65	7.161	6.748	6.373
	70	5.777	5.505	5.254

TABLE VII. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
35	35	11.632	10.530	9.600
	40	11.175	10.157	9.291
	45	10.622	9.702	8.913
	50	9.955	9.149	8.450
	55	9.156	8.476	7.879
	60	8.202	7.658	7.172
	65	7.066	6.662	6.294
	70	5.718	5.450	5.203
40	40	10.777	9.826	9.014
	45	10.283	9.418	8.671
	50	9.677	8.911	8.244
	55	8.936	8.283	7.710
	60	8.038	7.510	7.039
	65	6.951	6.556	6.198
	70	5.646	5.383	5.141
45	45	9.863	9.063	8.370
	50	9.331	8.619	7.987
	55	8.662	8.044	7.500
	60	7.831	7.332	6.875
	65	6.807	6.425	6.080
	70	5.556	5.300	5.063
50	50	8.892	8.235	7.660
	55	8.312	7.738	7.230
	60	7.568	7.091	6.664
	65	6.623	6.258	5.926
	70	5.442	5.193	4.964
55	55	7.849	7.332	6.873
	60	7.220	6.781	6.386
	65	6.379	6.036	5.724
	70	5.291	5.053	4.833

T A B L E VII. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
60	60	6.737	6.351	6.001
	65	6.043	5.730	5.444
	70	5.081	4.858	4.653
65	65	5.547	5.277	5.031
	70	4.773	4.571	4.385
70	70	4.270	4.104	3.952

TABLE VIII.

Shewing the Probability of the Duration of Life in LONDON, deduced by Mr. *Simpson* from observations on the bills of mortality in LONDON for 10 years, from 1728 to 1737.

Ages	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.
0	1000	320	27	321	6	54	135	6
1	680	133	28	315	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	
26	327	6	53	141	6			

TABLE IX.

Shewing the *Expectations* of Life in LONDON, according to the preceding Table. See Mr. *Simpson's Select Exercises*, p. 255.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
1	27.0	28	24.6	55	14.2
2	32.0	29	24.1	56	13.8
3	34.0	30	23.6	57	13.4
4	35.6	31	23.1	58	13.1
5	36.0	32	22.7	59	12.7
6	36.0	33	22.3	60	12.4
7	35.8	34	21.9	61	12.0
8	35.6	35	21.5	62	11.6
9	35.2	36	21.1	63	11.2
10	34.8	37	20.7	64	10.8
11	34.3	38	20.3	65	10.5
12	33.7	39	19.9	66	10.1
13	33.1	40	19.6	67	9.8
14	32.5	41	19.2	68	9.4
15	31.9	42	18.8	69	9.1
16	31.3	43	18.5	70	8.8
17	30.7	44	18.1	71	8.4
18	30.1	45	17.8	72	8.1
19	29.5	46	17.4	73	7.8
20	28.9	47	17.0	74	7.5
21	28.3	48	16.7	75	7.2
22	27.7	49	16.3	76	6.8
23	27.2	50	16.0	77	6.4
24	26.6	51	15.6	78	6.0
25	26.1	52	15.2	79	5.5
26	25.6	53	14.9	80	5.0
27	25.1	54	14.5		

TABLE X.

Shewing the Value of an Annuity on *One* Life, according to the probabilities of life in LONDON. See Mr. *Simpson's Select Exercises*, p. 260.

Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.	Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.	Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.
6	18.8	16.2	14.1	31	14.8	12.9	11.4	56	10.1	9.1	8.4
7	18.9	16.3	14.2	32	14.6	12.7	11.3	57	9.9	8.9	8.2
8	19.0	16.4	14.3	33	14.4	12.6	11.2	58	9.6	8.7	8.1
9	19.0	16.4	14.3	34	14.2	12.4	11.0	59	9.4	8.6	8.0
10	19.0	16.4	14.3	35	14.1	12.3	10.9	60	9.2	8.4	7.9
11	19.0	16.4	14.3	36	13.9	12.1	10.8	61	8.9	8.2	7.7
12	18.9	16.3	14.2	37	13.7	11.9	10.6	62	8.7	8.1	7.6
13	18.7	16.2	14.1	38	13.5	11.8	10.5	63	8.5	7.9	7.4
14	18.5	16.0	14.0	39	13.3	11.6	10.4	64	8.3	7.7	7.3
15	18.3	15.8	13.9	40	13.2	11.5	10.3	65	8.0	7.5	7.1
16	18.1	15.6	13.7	41	13.0	11.4	10.2	66	7.8	7.3	6.9
17	17.9	15.4	13.5	42	12.8	11.2	10.1	67	7.6	7.1	6.7
18	17.6	15.2	13.4	43	12.6	11.1	10.0	68	7.4	6.9	6.6
19	17.4	15.0	13.2	44	12.5	11.0	9.9	69	7.1	6.7	6.4
20	17.2	14.8	13.0	45	12.3	10.8	9.8	70	6.9	6.5	6.2
21	17.0	14.7	12.9	46	12.1	10.7	9.7	71	6.7	6.3	6.0
22	16.8	14.5	12.7	47	11.9	10.5	9.5	72	6.5	6.1	5.8
23	16.5	14.3	12.6	48	11.8	10.4	9.4	73	6.2	5.9	5.6
24	16.3	14.1	12.4	49	11.6	10.2	9.3	74	5.9	5.6	5.4
25	16.1	14.0	12.3	50	11.4	10.1	9.2	75	5.6	5.4	5.2
26	15.9	13.8	12.1	51	11.2	9.9	9.0				
27	15.6	13.6	12.0	52	11.0	9.8	8.9				
28	15.4	13.4	11.8	53	10.7	9.6	8.8				
29	15.2	13.2	11.7	54	10.5	9.4	8.6				
30	15.0	13.1	11.6	55	10.3	9.3	8.5				

T A B L E X I.

Shewing the Value of an Annuity on the joint continuance of Two Lives, according to the probabilities of life in LONDON. See Mr. *Simpson's Select Exercises*, p. 266.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
10	10	14.7	13.0	11.6	20	20	12.8	11.3	10.1
	15	14.3	12.7	11.3		25	12.2	10.8	9.7
	20	13.8	12.2	10.8		30	11.6	10.3	9.2
	25	13.1	11.6	10.2		35	10.9	9.8	8.8
	30	12.3	10.9	9.7		40	10.2	9.2	8.4
	35	11.5	10.2	9.1		45	9.5	8.6	7.9
	40	10.7	9.6	8.6		50	8.8	8.0	7.4
	45	10.0	9.0	8.1		55	8.1	7.5	6.9
	50	9.3	8.4	7.6		60	7.4	6.9	6.4
	55	8.6	7.8	7.1		65	6.7	6.3	5.9
15	60	7.8	7.2	6.6	70	6.0	5.7	5.4	
	65	6.9	6.5	6.1	75	5.2	5.0	4.8	
	70	6.1	5.8	5.5	25	25	11.8	10.5	9.4
	75	5.3	5.1	4.9		30	11.3	10.1	9.0
	15	13.9	12.3	11.0		35	10.7	9.6	8.6
	20	13.3	11.8	10.5		40	10.0	9.1	8.2
	25	12.6	11.2	10.1		45	9.4	8.5	7.8
	30	11.9	10.6	9.5		50	8.7	7.9	7.3
	35	11.2	10.0	9.0		55	8.0	7.4	6.8
	40	10.4	9.4	8.5		60	7.3	6.8	6.3
45	9.6	8.8	8.0	65		6.6	6.2	5.8	
50	8.9	8.2	7.5	70		5.9	5.6	5.3	
20	55	8.2	7.6	7.0	75	5.1	4.9	4.7	
	60	7.5	7.0	6.5	30	30	10.8	9.6	8.6
	65	6.8	6.4	6.0		35	10.3	9.2	8.3
	70	6.0	5.7	5.4		40	9.7	8.8	8.0
	75	5.2	5.0	4.8					

TABLE XI. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
30	45	9.1	8.3	7.6	45	65	6.3	5.8	5.4
	50	8.5	7.8	7.2		70	5.6	5.3	5.0
	55	7.9	7.3	6.7		75	4.9	4.7	4.5
	60	7.2	6.7	6.2	50	50	7.6	6.8	6.2
	65	6.5	6.1	5.7		55	7.2	6.5	6.0
	70	5.8	5.5	5.2		60	6.7	6.1	5.7
	75	5.1	4.9	4.7		65	6.2	5.7	5.3
35	35	9.9	8.8	8.0	70	5.5	5.2	4.9	
	40	9.4	8.5	7.7	75	4.8	4.6	4.4	
	45	8.9	8.1	7.4	55	55	6.9	6.2	5.7
	50	8.3	7.6	7.0		60	6.5	5.9	5.5
	55	7.7	7.1	6.6		65	6.0	5.6	5.2
	60	7.1	6.5	6.1		70	5.4	5.1	4.8
	65	6.4	6.0	5.6	75	4.7	4.5	4.3	
	70	5.7	5.4	5.1	60	60	6.1	5.6	5.2
75	5.0	4.8	4.6	65		5.7	5.3	4.9	
40	40	9.1	8.1	7.3		70	5.2	4.9	4.6
	45	8.7	7.8	7.1		75	4.6	4.4	4.2
	50	8.2	7.4	6.8	65	65	5.4	5.0	4.7
	55	7.6	6.9	6.4		70	4.9	4.6	4.4
	60	7.0	6.4	6.0		75	4.4	4.2	4.0
	65	6.4	5.9	5.5		70	70	4.6	4.4
	70	5.7	5.4	5.1	75		4.2	4.0	3.9
	75	5.0	4.8	4.6	75		75	3.8	3.7
45	8.3	7.4	6.7						
50	7.9	7.1	6.5						
55	7.4	6.7	6.2						
60	6.8	6.3	5.8						

T A B L E XII.

Shewing the Probabilities of Life in LONDON, on the supposition, that all who die in LONDON were born there. Formed from the Bills, for 10 years, from 1759 to 1768. See page 245.

Ages	Persons living.	Decr. of Life.	Ages	Persons living.	Decr. of Life.	Ages	Persons living.	Decr. of Life.
0	1000	240	31	404	9	62	132	7
1	760	99	32	395	9	63	125	7
2	661	42	33	386	9	64	118	7
3	619	29	34	377	9	65	111	7
4	590	21	35	368	9	66	104	7
5	569	11	36	359	9	67	97	7
6	558	10	37	350	9	68	90	7
7	548	7	38	341	9	69	83	7
8	541	6	39	332	10	70	76	6
9	535	5	40	322	10	71	70	6
10	530	4	41	312	10	72	64	6
11	526	4	42	302	10	73	58	5
12	522	4	43	292	10	74	53	5
13	518	3	44	282	10	75	48	5
14	515	3	45	272	10	76	43	5
15	512	3	46	262	10	77	38	5
16	509	3	47	252	10	78	33	4
17	506	3	48	242	9	79	29	4
18	503	4	49	233	9	80	25	3
19	499	5	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

TABLE XIII.

Shewing the *true* Probabilities of Life in LONDON
till the Age of 19. See page 249.

Age.	Persons living.	Decrements of life.
0	750	240
1	510	99
2	411	42
3	369	29
4	340	21
5	319	11
6	308	10
7	298	7
8	291	6
9	285	5
10	280	4
11	276	4
12	272	4
13	268	3
14	265	3
15	262	3
16	259	3
17	256	3
18	253	4
19	249	
20	494	
21	487	
&c.	&c.	

The numbers in the second column to be continued as in the last Table.

T A B L E X I V.

Shewing the *true* Probabilities of Life in LONDON for all Ages. Formed from the bills for 10 years, from 1759 to 1768. See page 251.

Ages.	Perfons living.	Decr. of Life.	Ages.	Perfons living.	Decr. of Life.	Ages.	Perfons living.	Decr. of Life.
0	1518	486	31	404	9	62	132	7
1	1032	200	32	395	9	63	125	7
2	832	85	33	386	9	64	118	7
3	747	59	34	377	9	65	111	7
4	688	42	35	368	9	66	104	7
5	646	23	36	359	9	67	97	7
6	623	20	37	350	9	68	90	7
7	603	14	38	341	9	69	83	7
8	589	12	39	332	10	70	76	6
9	577	10	40	322	10	71	70	6
10	567	9	41	312	10	72	64	6
11	558	9	42	302	10	73	58	5
12	549	8	43	292	10	74	53	5
13	541	7	44	282	10	75	48	5
14	534	6	45	272	10	76	43	5
15	528	6	46	262	10	77	38	5
16	522	7	47	252	10	78	33	4
17	515	7	48	242	9	79	29	4
18	508	7	49	233	9	80	25	3
19	501	7	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

OBSERVATIONS

ON

TABLES I. and II.

THESE Tables may be met with in most of the books that treat of compound interest and annuities; but there has been, in this work, so much occasion for referring to them, that it was necessary to save the reader the trouble of turning to other books for them.

The 2d, 3d, 4th, &c. numbers in the *Second* Table, are only the *sums* of the first 2, 3, 4, &c. numbers in the *First* Table. This Table, therefore, is the foundation of the *Second*; and, indeed, of all the common tables of compound interest; and, with the help of it, almost all the questions in compound interest may be easily answered.

The following specimen of this may, I think, be of considerable use.

QUESTION I. “ To what *sum* or *annuity*
 “ will any given *sum* or *annuity*, now to be
 “ laid up for improvement, at a given rate
 “ of compound interest, increase, in a given
 “ number of years ?”

ANSWER. Divide the given *sum* or annuity by the value of *1l.* payable at the end of the given number of years, and the *quotient* will be the answer.

Example. Let the given *sum* be *50l.* and the given time 18 years. The rate of interest *4 per cent.*—The present value, at *4 per cent.* of *1l.* payable at the end of 18 years is, by Table I. *.4936*; and *50l.* divided by this value, gives *101.296*, or *101l. 5s.* the *sum* to which *50l.* will increase in 18 years. In like manner; *2l. per annum*, the first payment of which is to be made a year hence, will increase (interest supposed the same) in 18 years, to an *annuity* of *l.4.05*: for *2l.* the given annuity, divided by *.4936*, gives *l.4.05*, or *4l. 1s.*

QUESTION II. “ To what *sum* will a
“ given *annuity* amount, in consequence of
“ being forborn and improved, at a given
“ rate of compound interest, for a given
“ number of years ?”

ANSWER. From the *increased* annuity, found by the last Question, subtract the *given* annuity; and multiply the *remainder* by the PERPETUITY, and the *product* will be the answer.

Example. *2l. per ann.* improved at *4 per cent.* compound interest, will, by the last Question, increase, in 18 years; to *l.4.05 per ann.* *2l.* subtracted from *4.05*, leaves *2.05*,
which,

which, multiplied by 25, the *perpetuity*, gives *l.* 51.25, or 5*l.* 5*s.*, the *amount* in 18 years. In the same manner it may be found, that 10*l.* *per ann.* (interest being the same) will amount, in 41 years, to 998*l.*

It should be remembered, that the PERPETUITY is 33.33, — 28.57, — 25 — 20, — or 16.666, according as interest is reckoned at 3, — 3½, — 4, — 5 or 6 *per cent*: And that the *annuity* meant in all these Questions is an annuity, the first payment of which is to be made at the end of a year.

QUESTION III. “ In what number of
“ years will a given *sum* or *annuity* increase
“ to another given *sum* or *annuity*, in conse-
“ quence of being improved at a given rate
“ of interest ?”

ANSWER. Divide the *original sum* or *annuity* by the *increased sum* or *annuity*; and look for the *quotient*, or the number nearest to it, in Table I; and the number of years corresponding to it will be the answer.

Example. Let the *sum* be 50*l.* The increased *sum* *l.* 101.29. The rate of interest, 4 *per cent*. The former *sum* divided by the latter gives .4936, which stands opposite in the Table to 18 years, or the time in which 50*l.* will gain the required increase.—In like manner, it may be found, that 18 years is the time in which 2*l.* *per ann.* will increase to 1.4.05 *per ann.*

QUESTION

QUESTION IV. "In what time will any given annuity amount to a given sum, in consequence of being forborn and improved, at a given rate of compound interest?"

ANSWER. Divide the given sum to which the annuity must amount by the PERPETUITY. Add the given annuity to the quotient; and by the quotient so increased, divide the given annuity; and this second quotient, found in Table I. will shew the answer.

Example. A person owes 1000*l.* and resolves to appropriate 10*l.* per annum of his income towards discharging it. In what time will such an appropriation, in consequence of being improved at 4 per cent. amount to a sum equal to the debt?—1000*l.* divided by 25 gives 40*l.* 10*l.* added to 40*l.* makes 50*l.*; and 10*l.* divided by 50*l.* gives .2000, which in the Table stands opposite to 41 years, the required time.

In the same manner it will appear, that the same annuity, if improved at 5 per cent. will amount to 1000*l.* in 37 years.

QUESTION V. "In what time will a given principal be annihilated, by taking out of it, at the end of a year, a given sum, and after that, the same sum annually, together with its growing interests?"

Z.

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ANSWER. In the same time plainly in which an equal annuity would amount to the *given principal*.

A person, therefore, possess'd of 1000*l.* capital, bearing interest at 4 *per cent.* would, by Question IV. reduce it to nothing in 41 years, by taking out of it 10*l.* the first year, and as much more every following year, as would be necessary, together with the interest of the remaining capital, to make his annual income constantly 50*l.*

Remark. The sum to which a *given* annuity will amount in a given time, is the same with the value of an annuity for the given time, equal to the *given* annuity increased by the yearly interest of the amount. That is, 1000*l.* is the value of 50*l. per ann.* for 41 years, at 4 *per cent.*: And the same sum is likewise the value of 60*l. per annum,* for 37 years, at 5 *per cent.* The reason is plain: 1000*l.* it has appeared, would, in consequence of being put out to these different rates of interest, be just sufficient to pay the annuities.

I have been the more explicit in these rules, because they point out a very easy method of deducing and examining all I have said, in different parts of this work, and particularly in Chap. III. concerning the increase of money at interest.—I will just mention one instance.

400,000*l.*

400,000*l.* *per annum*, applied in the manner supposed in Questions IV and V. would annihilate 55 millions, bearing interest at 5 *per cent.* in 42 years.

In 1716, when the *sinking fund* was established, the public debts were near this sum, and bore 5 *per cent.* interest. This fund then, had but 400,000*l.* of it been inviolably applied to the annihilation of the public debts, would, in 1758, have discharged all the debts contracted *before* 1716.—And it may be further found very easily, by the answer to Question IV. that had it been suffered to go on in its operation, and been applied, *since* 1758, to the redemption of only 3 *per cents* at *par*, it would by this time have discharged 104 millions; and seven years hence, 140 millions.—The assertion, therefore, in page 164, is strictly true. But the following proof of that assertion will, perhaps, be more clear and striking.

Suppose an annuity of 400,000*l.*, beginning in 1716, to have been applied till 1730, to the annihilation of debts bearing interest at 5 *per cent*; from 1730 to 1743, to the annihilation of debts bearing interest at 4 *per cent.* and from 1748 to 1771, to the annihilation of debts bearing interest at 3 *per cent.* In the first of these periods the annuity would have increased to 800,000*l.*; in the *second*, to 1,600,000*l.*; in the *last*, to 3,200,000*l.*—

In the present year, therefore, the nation

might have been eased of above *three millions per annum* in taxes. And, at the same time, (supposing all the same measures taken in other respects) it would have enjoyed that very *sinking fund* it now has; and no detriment could have arisen to the public, from any applications of it to current expences.

As I am now again on the national debt, I will beg leave to add the following proof of another observation on this subject, in page 140.

The disbursements, on account of any loans, will be the same, whatever different interests they bear, supposing a provision made for discharging the principal, by applying to that purpose surplusses bearing to one another the same proportions with those interests.

For Example. Let a million be borrowed at *3 per cent.* and let a fund be charged with it which brings in *6s. per cent. per annum* more than the interest; or *33,000l.* instead of *30,000l. per annum.* This surplus, applied in the manner I have explained, will annihilate the *principal* in *81* years, as may be gathered from *Question V.* At the end of this time, the disbursements on account of the loan will be *33,000l.* multiplied by *81*; that is, *2,673,000l.*

Let us again suppose a million borrowed at *6 per cent.* and let a fund be charged with it producing a surplus of *12s. per cent. per annum.*

annum. Such a fund, besides paying the interest, will discharge the principal in 41 years; and the disbursements on account of the loan, will be 66,000*l.* multiplied by 41; that is, 2,706,000*l.* or very nearly the same with the disbursements on account of an equal loan at 3 *per cent*.

It appears, therefore, agreeably to the observation to which I have referred, that were the public, in raising money, to adopt the plan I have proposed, it would be of little consequence what interest was given for money. The practicability of such a plan is self-evident, for it cannot be less easy to apply the *interest* of a sum, to the payment of a debt, than the sum itself: and this plan requires no more.—One particular advantage attending it, already hinted, I will beg leave here to repeat. By keeping the *stocks* steadily at or near *par*, that fluctuation in them would be in a great measure prevented, which now produces so many evils; and which, with the aid of annual lotteries, will, I fear, in time, ruin all honest industry, and turn us into a nation of sharpers and gamblers.

DIRECTIONS for finding the VA-
LUES of Two JOINT LIVES
of given Ages, by TABLE VII.

IF both the ages are given in the Table,
the value wanted will be found imme-
diately by inspection.

If the ages are not given in the Table, it will
be best to proceed in the following manner.

Suppose the rate of interest 4 *per cent.* and
the proposed ages 40 and 66.—It will
appear, from inspecting the Table, that the
value sought would be 6.556, were the age
of the elder life 65; and 5.383, were it 70.
Since, therefore, it is 66, the value must be
the *first* of four arithmetical means between
6.556 and 5.383, or 6.322.—For the
same reason, had the ages of the elder been
68, the value would have been the 3d arith-
metical mean between 6.556 and 5.383, or
5.854.—In like manner, were the proposed
ages 43 and 65, the value would be the 3d
arithmetical mean between 6.556 (the value
of two joint lives whose ages are 40 and 65)
and 6.425, (the value of two joint lives
whose ages are 45 and 65) or 6.478.

Again, let the ages be 43 and 66. That
is, let it be supposed, that neither of the
proposed ages is given in the Table.

The values corresponding to the ages

$$\left. \begin{array}{l} \{40\} \\ \{45\} \end{array} \right\} \text{ and } \left. \begin{array}{l} \{66\} \\ \{66\} \end{array} \right\}, \text{ are } \left. \begin{array}{l} \{6.322\} \\ \{6.200\} \end{array} \right\}.$$

The

The value, therefore, corresponding to the ages 43 and 66, must be the 3d mean between 6.322 and 6.200, or 6.250.—

N. B. The 1st, 2d, 3d, and 4th of four arithmetical means between two numbers are found by subtracting $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ of the *difference* between the two numbers, from the *greatest* of them.

Thus. The difference between 6.556, and 5.383, is 1.173. One fifth of this difference is .234; which, subtracted from 6.556, leaves 6.322; the first of 4 means between 6.556 and 5.383.—In like manner; the difference between 6.322 and 6.200 is .122. *One* fifth of this difference is .024; and, therefore, three-fifths of this difference is .072, which, subtracted from 6.322, leaves 6.250, the *third* arithmetical mean between 6.322 and 6.200.

In order to avoid trouble, if the ages are nearly equal, a year or two may be added to the least, and as much subtracted from the greatest; and the value taken by inspection. But if one of them much exceeds the other, it will in general be sufficient to take the nearest number in the Table for the lesser.

The mean between the values at 3 *per cent.* and 4 *per cent.* may be taken for the value at $3\frac{1}{2}$ *per cent.* without any error of consequence. And the like may be said of the values at $4\frac{1}{4}$ *per cent.*

The values of the *longest* of two lives is found by subtracting the value of the *joint*
 4 lives

lives from the *sum* of the values of the two *single* lives.—Thus, the values of two single lives, whose ages are 25 and 30, are by Table VI. (interest reckoned at 4 *per cent.*) 15.31 and 14.68. The sum of these two values is 29.99; the value of the joint lives is (by Table VII) 11.46; and this value; subtracted from 29.99, gives 18.53, or the value of an annuity on the longest of the two lives.

The value of *two* joint lives being given, the value of *three* joint lives may be found by the following rule, taken from Mr. *Simpson's Select Exercises*, page 279.

Let A be the youngest, and C the oldest of the three proposed lives. Take the value of the two joint lives B and C, and find the age of a *single* life D of the same value. Then find the value of the *joint* lives A and D, which will be the answer.

Example. Let the three given ages be 25, 30, and 40, and let the rate of interest be 4 *per cent.* Then the value of the two oldest joint lives B and C, will (by Tab. VII.) be 10.428, answering, in Tab. VI. to a single life D of 54 years of age. And the value of the joint lives A and D, which is 8.917 year's purchase, will be the value sought.

F I N I S,