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The Efficiency Theory of Wages[1]

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THE point of departure of this article, as well as the sense in which the word *efficiency* is used, will be made plain by reviewing a few points in Professor Marshall's treatment of the theory of wages.

After noting that the "phrase, the 'efficiency of workers,' has some ambiguity," Professor Marshall defines efficiency-wages as "earnings measured, not as time-earnings are with reference to the time spent in earning them, and not as piecework earnings are with reference to the amount of output resulting from the work by which they are earned, but with reference to the exertion of ability and *efficiency* required of the worker" (*Principles*, 4th ed., pp. 630–631). So far as I am aware, this description of efficiency wages is as near as Professor Marshall approaches to a formal definition of efficiency. He has recorded his objection to "bold and rigid definitions" in economics as not compatible with the nature of the subject matter, and as being the origin of mis-

[1] Several technical points in this paper are more fully treated in an article on "The Differential Law of Wages" which will appear in an early number of the *Journal of the Statistical Society*.

understandings and a "frequent source of controversies that have diverted energy from constructive work, and have hindered the progress of the science." In case of doubt as to the meaning of terms, he has told us to "look to the context for a special interpretation clause" (Bk. II., p. 121).

The fifth chapter of the fourth book of the *Principles of Economics* opens with this sentence:—"We have next to consider the conditions on which depend health and strength, physical, mental and moral. They are the basis of industrial efficiency, on which the production of material wealth depends." The most general conclusion as to the relation of efficiency to wages is reached in Book VI., p. 630:—"We may then regard competition, or, to speak more exactly, economic freedom and enterprise, as tending to make time-earnings in occupations of equal difficulty and in neighbouring places not equal, but proportionate to the efficiency of the workers."

The sense in which the term efficiency is used in this article is the sense in which it is used by Professor Marshall in these quotations, and the problem which it is proposed to investigate is the following:—How can the generalisation that wages are proportionate to efficiency be inductively tested? Is the truth of the generalisation limited to the hypothetical state of perfect competition, or is it approximately reached in the industrial world?

Incidentally, it may be observed that we have not thus far come nearer to a solution of this problem than to offer an artless paralogism having its origin in the ambiguity of the term efficiency. At one time the term is used to signify the physical, mental and moral qualities of the labourer, his "general sagacity and energy," and at another time it is taken to signify the measure of the labourer's product. The theory is then propounded that efficiency—implicitly in the first sense—determines the amount of reward, and this is thought to be proved by slipping into the second sense in which actual product is taken as the measure of efficiency. The net result of laborious reasoning is that the labourer gets what he gets.

Preliminary to any attempt to give a statistical test of the doctrine that wages are distributed according to efficiency, we must know two things: (1) the actual distribution of wages in some large group of labourers; (2) the law of the distribution of efficiency among the selected group of workers. These two desiderata we shall consider in order.

The volume on *Employees and Wages*, published by the

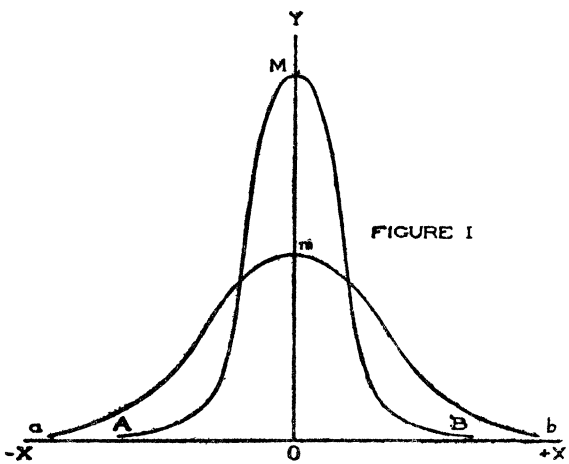
Census Bureau of the United States, in 1903, offers a large range of wage statistics relating to American manufactures, tabulated according to wage groups. For example, it is recorded that in the woollen mills in 1900 wages ranged from less than $2.00 per week to $72.50 per week, and the numbers receiving wages within these limits is given for every fifty-cent group. Thus, out of a representative total of 4,608 in all occupations in that industry, 301 received between $9.00 and $9.50 per week. Similar tables are given for thirty-two industries. In order to ascertain the distribution of wages in the whole of manufactures, thirty of the thirty-two schedules were combined into a general schedule. In this way the distribution of weekly rates of wages in one-dollar groups was ascertained for a total of 160,055 labourers. The labourers in this case included males sixteen years and over, in all sections, all occupations. The graphical description of this general schedule, between the limits $3.15 per week and $35.95 per week, is given by the zigzag line in the accompanying chart. By referring to the chart we see that the greatest number of labourers, about 15 per cent. of the total number, received between $9.00 and $10.00 per week, and that there was an aggregation of wage-earners at definite weekly rates, *e.g.*, at $7–8, $9–10, $12–13, $15–16, $18–19, $21–22, $24–25, $27–28, $30–31. These weekly rates are equivalent to customary daily rates of $1·25, $1·50, $2·00, $2·50, $3·00, $3·50, $4·00, $4·50, $5·00. The general wage schedule upon which this wage curve is based fulfils the requirements of our first desideratum.

We have next to inquire as to the distribution of efficiency. Professor Marshall's discussion of the "basis of industrial efficiency" suggests the clue to a preliminary hypothesis. According to the quotation from the *Principles of Economics* already given, the "basis of industrial efficiency" is in health and strength, physical, mental and moral. Moreover, "we may say that what makes one occupation higher than another, what makes the workers of one town or country more efficient than those of another, is chiefly a superiority in general sagacity and energy which is not specialised to any one trade" (*Principles*, p. 286). Recent investigations by Prof. Karl Pearson[1] and his co-workers have established that the distribution not only of the chief physical characters in man but of his mental and moral

[1] Karl Pearson, F.R.S., and Alice Lee, D.Sc., On the Laws of Inheritance in Man, *Biometrika*, Vol. II., p. 395. Karl Pearson, F.R.S., On the Relationship of Intelligence to Size and Shape of Head, and to other Physical and Mental Characters, *Biometrika*, Vol. V., p. 106.

qualities as well is described with sufficient accuracy by the normal or Gaussian curve. The particular hypothesis made in this paper is that the distribution of "general sagacity and energy" among labourers, which is itself a balance of efficiency determined by physical, mental and moral qualities, follows the Gaussian law.

The normal or Gaussian curve is represented graphically in Fig. 1 by either of the two curves *A M B*, *a m b*. If from a homogeneous group of men a large number of measurements of any physical character be made, for example of stature, it will be found that the measurements may be arranged in such a way that the relative frequencies of the deviations from the average measurement will, when plotted, produce a curve approximating this type. A deviation in excess of the average stature is measured to the right of point *O*, on the line *ox*, and the cor-



responding frequency of the deviation is then plotted perpendicularly at the limit of the deviation. Similarly, deviations below the average are measured to the left of the point *O*. While the two sides of the curve are generally symmetrically disposed about the maximum ordinate, the concentration of the measurements about this ordinate varies with the standard deviation.[1] The standard deviation of the curve *a m b* is twice that of the curve *A M B*.

[1] The standard deviation of a series of measurements is the square root of the mean square of the deviations of the measurements from the average measurement. If a series of n measurements be represented by $m_1, m_2 \ldots . m_n$ the average of which is A, the standard deviation is $\sigma = \sqrt{\frac{(m_1 - A)^2 + (m_2 - A)^2 + + + (m_n - A)^2}{n}}$

Assuming, then, that "general sagacity and energy" is distributed according to the Gaussian law, we require to know, before we proceed to a statistical treatment, how much on the average the members of a sample population differ from each other. If one thousand individuals are selected at random from a population distributed, in respect to any particular character, according to the Gaussian law, how much will the nth individual differ, on the average, from the $(n+1)$st individual, when the members of the sample population are ranked in the order in which they possess the particular character?

This problem in a generalised form was proposed, in 1902, by Mr. Francis Galton,[1] and was solved by Professor Karl Pearson,[2] who named the problem *Francis Galton's Individual Difference Problem in Statistics.* Professor Pearson reduced his solution to a form by means of which it is possible to compute the difference between any two individuals in a sample population of any size. From his formulas I computed a sample population of one hundred members which will be referred to, in the remainder of this article, as the standard population. By means of this standard population it is possible to make such statements as the following:—Assuming that efficiency is distributed according to the Gaussian law, the difference between the most efficient labourer and his most efficient neighbour, in a standard population of 100, is 0·360964 times the standard deviation of the general population from which the one hundred individuals are taken as a sample. The difference between the 50th and the 51st individual is 0·025066 times the standard deviation. In like manner the difference between any particular individual and the least efficient member of the group can be given. This standard population supplies the second desideratum of our problem.

The ambiguity in the word efficiency, the confusion of qualities in the worker with the measure of his product, has already been adverted to. The fact accounting for this confusion, namely, that with a given degree of efficiency, of "sagacity and energy," the product of the labourer is a function of his environment, is important for our problem. With the distribution of efficiency among a group of labourers according to a given law, the apportionment of their income will vary according

[1] Francis Galton, F.R.S., The Most Suitable Proportion between the Values of First and Second Prizes. *Biometrika,* Vol. I., pp. 385–390.

[2] Karl Pearson, F.R.S., Note on Francis Galton's Problem. *Biometrika,* Vol. I. pp. 390–399.

to the economic environment in which the sub-groups of labourers are placed. It does not follow that because "general sagacity and energy" among a large group of labourers is distributed according to the Gaussian law that, therefore, their income will be distributed according to the same law. Unskilled, unorganised labour works under different conditions from those of skilled, organised labour. The strategic advantage in bargaining enjoyed by the latter group enables them more effectually to exploit their environment, and, consequently, although their income would be apportioned within their group according to the differential efficiency of the members of the group, and, although the income of the unskilled labourers would be similarly apportioned within their group, the fact that the two groups worked at different levels would cause the form of the distribution of individual incomes to vary from the symmetrical Gaussian curve in the direction of a positively skew distribution. It is necessary to make allowance for this factor in our problem.

An approximate measure of the importance of this factor has been attempted in the following way :—For theoretical reasons, which are given in the article on *The Differential Law of Wages*, to which reference has already been made, the range of wages in the general wage schedule of manufactures in the United States was taken as between the limits $3.15 and $35.95 per week. The minimum wage received in the general group was accordingly $3.15, and the average wage of the whole group was $11.43 per week. Of the total wages received by the general group, 35·29 per cent. was divided between the less efficient 50 per cent. of the labourers, the remaining 64·71 per cent. of the aggregate wages was divided among the more efficient labourers. The standard population, composed of one hundred members, was, therefore, divided into a first and second sub-group, the former composed of the fifty least efficient members, receiving 35·29 per cent. of aggregate wages, and the latter, composed of the fifty more efficient members, receiving 64·71 per cent. of the total income. It was assumed that the total wages received by the standard population was one hundred times the average wage in the general wage schedule of the United States, namely, $1143.00.

The series of hypotheses affecting the standard population may now be recapitulated :—

1. Efficiency is distributed among the one hundred members according to the Gaussian law.

2. The sum distributed as aggregate wages equals one hundred times the average wage received by the representative group of American manufacturing labourers, that is to say, $1143.00.

3. The less efficient fifty members receive 35·29 per cent. of the total income, and the more efficient fifty members divide between them 64·71 per cent. of that sum.

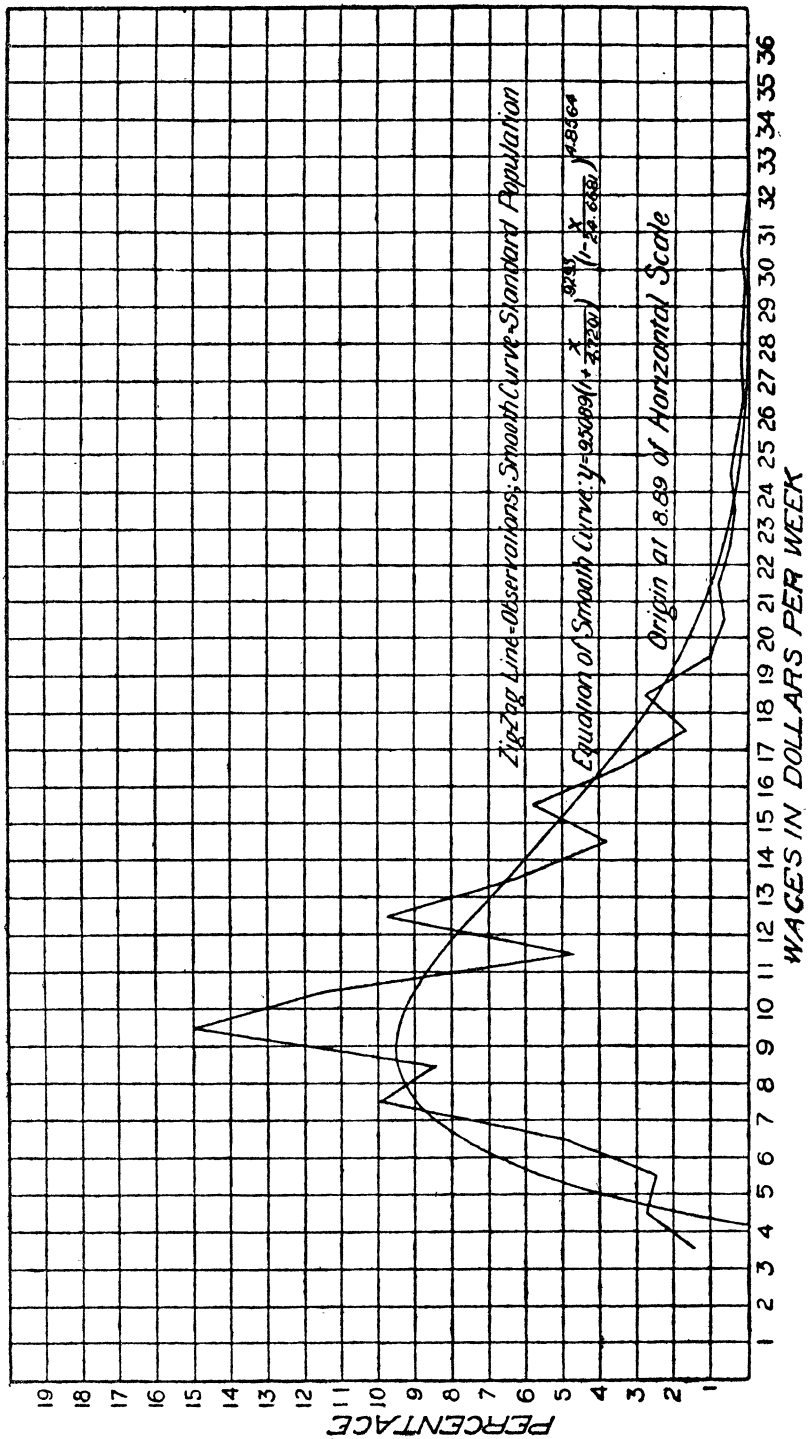
4. The minimum wage of the first sub-group is $3.15, the same as the minimum in the manufacturing schedule.

5. The shares of total wages going to these sub-groups are distributed according to the differential efficiency of the members of the respective sub-groups, that is to say, each member of the standard population receives the minimum wage of his sub-group plus an addition proportionate to his differential efficiency over the least efficient member in his sub-group.

These hypotheses suffice to determine the distribution of wages among the standard population according to the conditions of the efficiency theory of wages. In order to compare this theoretical distribution with the actual distribution in American manufactures, the wages of the standard population were collected into groups of one-dollar range, and the percentages of the total population receiving wages between the several group limits were ascertained. In this way, for example, it was found that 13 per cent. received between nine and ten dollars per week. As the standard population is so very small, the distribution of wages among its members, if plotted in the usual way on the chart, would be represented by a series of scattered points, which would give but a very indefinite idea of the measure in which our theory is borne out by the facts. Instead, therefore, of simply recording these scattered points, the smooth curve that passes as near as possible to these points was computed and is drawn on the chart. The general conformity of the smooth curve to the zigzag line is the measure of the conformity of facts and theory.

Considering the nature of the statistical material, the small size of the standard population, the impossibility of determining accurately the effect of the strategic advantage of organisation, the chances of error in the estimation of the minimum wage, and the imperfect working of competition, it is regarded that the general agreement of theory and fact is remarkably satisfactory.

I may be permitted to suggest two directions in which the theory of the standard population may be used :—


Zig-Zag Line-Observations; Smooth Curve-Standard Population
Equation of Smooth Curve: $y=95.09(1+\frac{x}{27.20})^{.9235}(1-\frac{x}{24.888})^{4.8564}$
Origin at 8.89 of Horizontal Scale
WAGES IN DOLLARS PER WEEK
PERCENTAGE
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

1. Just as the wage group has been broken into sub-groups of unskilled labour and skilled labour, so the whole class in society who receive productive incomes, as contrasted with incomes from capital, may be divided into appropriate sub-groups, and the law of the distribution of their incomes tested according to the efficiency theory. The efficiency theory of profit would lend itself to this form of investigation.

2. The *rationale* of Pareto's curve of distribution of income is suggested. Variations in the value of a have been established for different conditions. A partial, but fundamental cause for the average value of a, as well as for its variations, will probably be found in the economic facts discussed in this paper.

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