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METHODS OF MEASURING THE CONCENTRATION OF WEALTH.

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There may be wide difference of opinion as to the significance of a very unequal distribution of wealth, but there can be no doubt as to the importance of knowing whether the present distribution is becoming more or less unequal. For this purpose we need some method of interpreting such statistics as we have that show the condition of a country at different epochs or of different countries at the same epoch; that is, we wish to be able to say at what point a community is to be placed between the two extremes,—equality, on the one hand, and the ownership of all wealth by one individual on the other. It is the purpose of the present article to discuss some of the methods that have been used, and to suggest an additional one, but not to enter upon a discussion of the reliability of the data used for illustration. Let us consider first the numerical measures, taking up later the graphic representations.

It has been a common practice to construct a table of class divisions of wealth or income with the per cent. of the population falling within each class. An increase in the percentage of the middle class is supposed to show a diffusion of wealth. For example, the following table of income tax returns in England is supposed to show a “tendency of wealth among

the income-tax paying classes to distribute itself in smaller amounts in a larger number of hands":*—

INCOME TAX ASSESSMENTS UNDER SCHEDULE D.

	1877.	1886.	Per Cent. of Increase or Decrease.
Between £150 and £500	285,754	347,021	21.4 (Increase)
" 500 " 1,000	32,085	32,033	nil
" 1,000 " 5,000	19,726	19,250	2.5 (Decrease)
Over 5,000	3,122	3,048	2.3 (Decrease)

It is impossible to tell from such a table whether there has been a concentration or diffusion of wealth because it might be true that the incomes over five thousand pounds, although a smaller proportion of the total number in the second epoch, nevertheless constitute a much larger proportion of the total income.† It should be added, however, that comparisons of fixed classifications are of use in noting the absolute increase or decrease in wealth on the part of the lower classes. It is important to know, for example, that 29 per cent. in 1892 and 39 per cent. in 1901 of the people of Prussia (according to the income tax returns) had more than nine hundred marks a year income.‡

Another method of interpreting a table like the one just quoted has been suggested by Dr. Julius Wolf.§ Instead of paying attention to the change in the percentage of members in a certain class, we should observe, he says, the movement of persons from one class to another. To illustrate his way of attacking the problem, he gives the following table of the number of tax-payers in the Canton Zürich (p. 234):—

* Goschen, *Journal of the Royal Statistical Society*, 1887, vol. 1., p. 600.

† For a further discussion of this point, see Ely, "Evolution of Industrial Society," p. 259.

‡ *Zeitschrift des königlich preussischen statistischen Bureaus*, 1902, p. 246.

§ "Sozialismus und kapitalistische Gesellschaftsordnung" (being Vol. I of "System der Sozialpolitik"), Stuttgart, 1892.

Class. (Dollars.)	Wealth of each Individual in each Class. (Dollars.)	Number of Individuals.
0 to 9	1, 3, 5, 7, 9	5
10 to 24	10, 12, 14, 16, 18	5
25 to 49	25, 28, 31, 34, 37	5
50 to 99	50, 60, 70, 80, 90	5
100 and over	100, 110, 120, 130, 140	5

Now imagine the wealth of each specific individual doubled. The relation between the wealth of individuals has not changed, and hence the degree of concentration must be the same.* The classification will now be as follows:—

Class. (Dollars.)	Wealth of each Individual. (Dollars.)	Number.
0 to 9	2, 6	2
10 to 24	10, 14, 18, 20, 24	5
25 to 49	28, 32, 36	3
50 to 99	50, 56, 62, 68, 74	5
100 and over	{ 100, 120, 140, 160, 180 } { 200, 220, 240, 260, 280 }	10

We find that the movement between classes has been as follows:—

	Number.	Per Cent. of those originally in the Class from which the Move- ment took Place.
I to II	3	60
II to III	3	60
III to IV	5	100
IV to V	5	100

* It has been objected that doubling incomes does not leave individuals in the same relation to each other because (owing to the law of diminishing utility) doubling a rich man's income does not add proportionately as much to his well-being as in the case of a poor man. But this does not affect the argument above, because, according to this view, doubling incomes would tend to diffuse enjoyment, not concentrate it, as Wolf's method would indicate. In the present problem no error will result from confining our attention to nominal incomes.

This indicates a concentration, which we know has not taken place. The root fallacy here is the ignoring of the change in the meaning of a fixed classification with a change in general wealth. Again, if we examine, as he does, the sharing of various classes in the increase in the community's wealth, we find the first class actually lost, the second got 0.1 per cent. of the increase, the third and fourth lost, and the fifth 99.9 secured per cent. Yet there was no change in the concentration.

It is apparent that we need to take account simultaneously of changes in wealth and changes in population. Here also several erroneous methods have been used. Dr. Soetbeer sought to show that there had been no concentration in Prussia between 1876 and 1888 because "the average income of tax-payers in the higher classes had shown no tendency to increase."* But that average income is no safe criterion is shown by the hypothetical case just presented, where the average income of the highest class shows a marked increase, although there was no change in the degree of concentration.

Again, the "triple measure" presented by Mr. George K. Holmes† is not trustworthy. According to this we are to note, first, the average wealth in order to get the plane of distribution, or the relation of the whole population to its total mass of wealth; secondly, the per cent. of people owning wealth shows the "width of distribution"; and, finally, to find the state of the distribution among the possessing class, he takes the median of the number of owners and the median of the amount of wealth, and observes the distance between these medians (p. 146). That distance is the measure of the inequality of the distribution. To the first two parts of this triple measure no objection can be taken, but the last part would lead to error. Its method of application is shown by the following hypothetical case given in the article: A group of 64 persons is divided into 15 classes according to their wealth:—

* *Jahrbücher für Nationalökonomie und Statistik*, 1889, vol. lii. p. 420.

† *Publications of the American Statistical Association*, 1892-93, vol. iii., p. 141.

Class . . .	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8	\$9	\$10	\$11	\$12	\$13	\$14	\$15
Number . .	1	2	3	4	5	6	7	8	7	6	5	4	3	2	1
Wealth of each class }	1	4	9	16	25	36	49	64	63	60	55	48	39	28	15

Here the median of the number is in the centre of the eighth class, or \$8.50, and the median of the amount of wealth is in the ninth class, or at the point represented by \$9.82, and the difference between the two is \$1.32, which is the measure of concentration. The error in this measure lies in the fact that the distance between the medians varies not only with the degree of concentration (for it does this), but also with changes in the total wealth. Let us suppose each individual's wealth in the above case is doubled. The two medians would now be \$17 and \$19.65, and their difference \$2.65, showing an increase in concentration, but by hypothesis the relative position of the members has not changed.

Another method of taking account of changes both in wealth and of population is simply to state in a table the per cent. of total wealth and of total population in each class in each epoch. The following table,* for example, does this with regard to Prussian incomes in the years 1892 and 1901:—

Class.	1892.		1901.	
	Per Cent. of Number.	Per Cent. of Total Income.	Per Cent. of Number.	Per Cent. of Total Income.
Under 900	70.1	41.2	60.5	31.7
900-3,000	26.0	30.0	34.8	35.3
3,000-6,000	2.5	8.6	3.0	9.3
6,000-9,5007	4.2	.8	4.5
9,500-30,5006	7.4	.7	8.1
30,500 and over1	8.6	.2	11.1
	100.0	100.0	100.0	100.0

* Constructed from data in the *Zeitschrift* of the Prussian Statistical Bureau, 1902, the incomes below 900 marks being estimated on the assumption that their average was 700 in both epochs, and that the persons assessed are the same per cent. of the total number of income-receivers as the taxable part of the population (*Einkommensteuerpflichtiger Theil*) is of the total population.

What can such a table tell us? We can make such statements as the following: 70 per cent. of the number in 1892 had 41 per cent. of the income, and 60 per cent. of the number in 1901 had 31 per cent. of the income; but does this indicate concentration or diffusion? If we knew what per cent. of the income was received in 1901 by 70, 26, 2.5, 0.7, 0.6, and 0.1 per cents., respectively, of the number, we could make a comparison. This, it is true, is not a theoretical difficulty, but a practical one due to the insufficiency of data, to be encountered by any method. But suppose we had complete data, could we then interpret the results by this method? The difficulty would be in attempting to comprehend the significance of changes in half a dozen classes, especially when some would indicate diffusion and some concentration. However, we can always arrive at definite though very general results by merely reducing the number of classes to two, the richer and the poorer halves of the community, and noting changes in their proportion of the total income or wealth.* If the poorer half has acquired a larger proportion of the total income, we should probably be warranted in saying that there had been on the whole a movement toward equality. It is apparent, however, that such a measure does not tell the whole story. It covers up some of the changes that may be going on within each half.

The objection that the foregoing method does not tell the whole story is obviated *in part* by Dr. T. S. Adams† in applying to the present problem the measures of dispersion suggested by Mr. A. L. Bowley.‡ For example, assuming the members of a community arranged in order according to their wealth, we find the first and second quartiles, and divide their difference by their sum. This quotient will vary from 0 to 1, and the nearer 1, the greater the concentration. This is the best numerical measure that has as yet been sug-

* Suggested in Ely's "Evolution of Industrial Society," p. 257.

† Adams and Sumner, "Labor Problems," pp. 534 and 538.

‡ "Elements of Statistics," second edition, p. 136.

gested, although it may also hide some of the changes that are going on.

Turning now to the graphic measures, a simple plotting of wealth along one axis and the numbers of the population along another is not satisfactory for the reason that changes in the shape of the curve will not show accurately changes in the relationships of individuals. To escape this objection, one naturally resorts to logarithmic curves. Professor Pareto, in his "Cours d'Économie Politique,"* does this, but in an erroneous way. He represents logarithms of class divisions in wealth along one axis, and the logarithms of the number of persons having more than each class division along the other. The error in this procedure lies in adhering to a fixed classification for two epochs. The number of persons having more than, say, \$10 in each of the two periods of time is, as we have seen, of no significance in the question of degree of concentration when the per capita wealth of the community is growing. It will be found, for example, that plotting such curves for the hypothetical case given on page 212 shows a steeper curve for the second epoch. The method is especially inapplicable to data in which the highest class is given as those having more than a certain amount (as in the table on page 214); for, imagine a community in which the wealth is nearly equally distributed, and then assume that the richest individual becomes a multi-millionaire, with no change in the wealth of the remainder. Professor Pareto's curve would tell us nothing about this change.

If one wishes to use logarithmic curves, the following method would be better: Measure along the horizontal axis cumulated per cents. of the population from poorest to richest, and along the vertical axis logarithms of the *cumulated amounts* of wealth held by the successive per cents. of the population from poorest to richest. In interpreting these curves, it is necessary to pay attention solely to their shape, and to ignore the actual distance from the base line. The

* Vol. ii. p. 304.

steeper the curve, the greater the concentration. It should be noted that according to this method an equal distribution does not give a horizontal line.

However, logarithmic curves are more or less treacherous. Forgetting that they are logarithms, we are apt to think of them as absolute amounts, when plotted. For this reason it is believed that the following graphic method will be found more satisfactory. It takes account of changes in wealth and population, thus putting upon a comparable basis any two communities of the most diverse conditions. Where guessing is necessary, owing to insufficient data, it enables us to do this guessing in the most impersonal way. The method is as follows:—

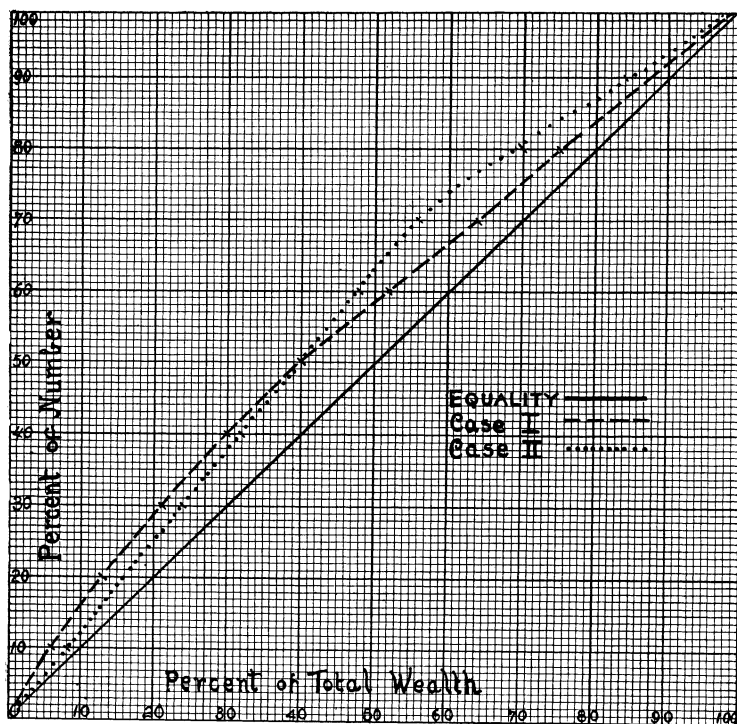
Plot along one axis cumulated per cents. of the population from poorest to richest, and along the other the per cent. of the total wealth held by these per cents. of the population. To illustrate, take a population in which wealth is distributed equally. No matter what the average wealth or size of the population, we should always plot the following sets of figures: * —

The poorest 1 per cent. of the population have 1 per cent. of the wealth.									
"	"	2	"	"	"	"	"	2	"
"	"	3	"	"	"	"	"	3	"
etc.					etc.				

This will give a straight line. With an unequal distribution, the curves will always begin and end in the same points as with an equal distribution, but they will be bent in the middle; and the rule of interpretation will be, as the bow is bent, concentration increases. If we plot in this way the figures for Prussian incomes given on page 214, we get the following results:—

* In practice the method will not be found laborious, because it is not necessary to plot each per cent.

We get curves as follows:—



There has been a tendency toward an equal distribution in the lower half, but a contrary tendency in the upper half. Notice that we can tell from the diagram that in each case the poorest half of the community have 40 per cent., but the poorest two-thirds have a smaller per cent. of the total wealth in the second case.