Mathematics in Economics: Achievements, Difficulties, Perspectives

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I am deeply excited by that high honour which fell on my lot and I am happy for the opportunity to appear here as a participant of this honorable series of lectures.

In our time mathematics has penetrated into economics so solidly, widely and variously, and the chosen theme is connected with such a variety of facts and problems that it brings us to cite the words of Kozma Prutkov which are very popular in our country: “One can not embrace the unembraceable”. The appropriateness of this wise sentence is not diminished by the fact that the great thinker is only a pen-name.

So, I want to restrict my theme to the topics which are nearer to me, mainly to optimization models and their use in the control of the economy for the purpose of the best use of resources for gaining the best results. I shall touch mainly on the problems and experience of a planned economy, especially of the Soviet economy. Certainly even within these limits I will succeed in considering only a few problems.

1. Specific peculiarities of the problems considered

Before discussing methods and results I think it will be useful to talk about the specific peculiarities of our problems. These are distinctive for the Soviet economy and many of them appeared already in the years just after the October Revolution. Then for the first time in history all main means of production passed into the possession of the people and there arose the need for the centralized and unified control of the economy of the vast country. This need appeared in very complicated and social conditions and met with some specific peculiarities. The following problems are related both to the economic theory and to the practice of planning and control.

1) First of all, the main purpose of economic theory was altered. There appeared a necessity to shift from study and observation of economic processes and from isolated policy measures to systematic control of the economy, to the common and united planning being based on the common aims and covering a long time horizon. This planning must be so detailed as to include specific tasks to individual enterprises for specific periods and to that common consistency of the whole this giant set of decisions was guaranteed.

It is clear that a planning problem of such scale did appear for the first time, so its solution could not be based on the existing experience and economic theory.

2) Economic science must yield not only conclusions on general economic problems concerning the national economy as a whole but also serve as the basis for solutions concerning single enterprises and projects. So it needs the proper information and methodology to provide decisions that are in accordance with general goals and interests of the national economy. Finally, it must contribute not only general qualitative recommendations but also concrete quantitative and sufficiently precise accounting methods which could provide the objective choice of economic decisions.

3) Together with material flows and funds in capitalist economies there are also studied and directly observed such important economic indices as prices, rents, interest rate in their static and dynamic properties. The indices mentioned serve as the background for all economic calculations, for aggregation, for the construction of the synthetic indices. It became clear that a, consistently planned economy cannot do without indices characterizing the analogous aspects. They could not be observed here and were given as normatives. The problem of their calculation was however not restricted only by technical aspects of calculation and statistics. It is important that in the new conditions similar indices received a quite different sense and significance, and some problems as to their nature, role and structure arose. For example, it was unclear and open to discussion whether a land rent should exist in a society where land is in the possession of the people or whether such an index as the interest rate has a right to exist.

4) The previous problems are displayed in one more peculiarity of the planned economy. Obviously the economy of such scale and complexity cannot be quite centralized ‘up to the least nail’ and a valuable part of decisions should be retained for the lower levels of the control system.

The decisions of different control levels and from different places must be linked by material balance relations and should follow the main object of the economy.

The problem is to construct a system of information, accounting, economic indices and stimuli which permit local decision-making organs to valuate the advantage of their decisions from the point of view of the whole economy. In other words to make profitable for them the decisions profitable for the system, give a possibility to check the validity of the work of local organs activity also from the point of view of the whole economy.

5) New problems of control of the economy and new methods put forward the question of the most efficient structural forms of control organization.

Some changes of these forms have taken place both owing to the tendency to perfection of the control system as well as to changes in the economy itself, connected with the increase of its scale, the increasing complexity of connections and with new problems and conditions. The problem of the most efficient structure of a planning system has also a scientific aspect, but its solution is not well advanced.

6) Some complex problems of economic control were generated by the contemporary development of the economy, by the so-called scientific-technical revolution. I mean the problems of prediction and control in conditions of large shifts in the weights of different branches, of the rapid changes in production and technology, national economy. The problems of estimating technical innovations and the general effect of technical progress. The problems of ecology connected with the deep changes of the natural environment under the influence of human activity, the prospects of exhausting the natural resources. The prediction of social changes and their influence on the economy. The changes in presence of contemporary computational technique, means of communication, managerial devices and so on.

Most of these problems arise also in countries with capitalist economy but in socialist economy they have their own difficulties and peculiarities.

There existed neither experience nor sufficient theoretical foundation for the solving of these hard problems.

The economic theory of Karl Marx became the methodological background of the new created Soviet economic science and of the new control system. A number of its important and fundamental statements on general economical situations turned out to be applicable immediately to a socialist economy. However a practical use of Marx' ideas needed serious theoretical research. There was no practical economic experience under the new conditions.

These problems were being solved practically by governmental bodies and economic executives. They were being solved under the first years of the state in difficult conditions of the Civil war, Devastation and postwar reconstruction. Nevertheless the problem of building up an effective economic mechanism was resolved. I have no possibilities to describe it in detail but I just wish to point out that the system of planning organs was created on the initiative of the founder of our state V. Lenin and simultaneously on the same initiative a system of economic accounting (hozraschet) was introduced which gave a certain financial form of balance and control of separate economic activities.

An evidence of significant efficiency of this mechanism lies in the great improvements of the economy, successful solution of the industrialization problem, of the economic problems of state defence before and during the Second World War, the postwar reconstruction and further development.

The system of planning and economic organs was, improved and altered in connection with new problems. The generalization of this experience built a stock in anticipation of economic theory of the planned socialist economy.

At the same time in our country the necessity of further improvements of the control mechanism, some defects in the use of resources, incomplete realization of the potential advantages of the planned economy were pointed out repeatedly. It was obvious that

such improvements should be based on new ideas and new means. This led to the natural idea to introduce and use quantitative mathematical methods.

2. The new methods

The first attempts to use mathematics in the Soviet economic researches were made in the 20-ies. Let me name the well-known demand models of E. Slutsky and A. Konjus, the first growth models of G. Feldman, the 'chesstable' balance analysis done in the Central Statistical Department, which was later developed both mathematically and economically using the data of the US economy by W. Leontiev. The attempt of L. Jushkov to determine rate of investment efficiency received a profound continuation researches of V. Novojilov. The above mentioned researches had common features with the mathematical direction in Western economic science which developed at the same time and was presented in the works of R. Harrod, E. Domar, F. Ramsey, A. Wald, J. von Neumann, J. Hicks et al.

Here I would like to talk mainly about optimization models which appeared in our country in the late 30-ies (and later independently in USA) and which were in a certain sense the most suitable means to treat the problems I have mentioned.

The optimizing approach is here a matter of prime importance. The treatment of the economy as a single system, to be controlled toward a consistent goal, allowed the efficient systematization of enormous information material, its deep analysis for valid decision-making. It is interesting that many inferences remain valid even in cases when this consistent goal could not be formulated, either for the reason that it was not quite clear or for the reason that it was made up of multiple goals, each of which to be taken into account.

For the present the multi-product linear optimizing models seems to be mostly used. I suppose that now it is spread in economic science not less than for instance Lagrange equations of motion in mechanics.

I see no need to describe in detail this well-known model which is based on the description of an economy as a set of main kinds of production (or activities, – the term of professor T. Koopmans), each characterized by use and production of goods and resources. It is well-known that the choice of optimal program i.e. of the set of intensities of these activities under some resource and plan restriction gives us a problem to maximize a linear function of many variables satisfying some linear restrictions.

This reduction has been described too many times so that it can be treated as well-known. It is more important to show those of its properties which determine its wide and various use. I can name the following ones:

a) Universality and flexibility. The model structure permits various forms of its application, it can describe very different real situations for extremely different branches of economy and levels of its control. It is possible to consider a series of models where

necessary conditions and restrictions are introduced step-by-step while the needed descriptive precision is not reached.

In more complicated cases when the linearity hypothesis significantly contradicts the problem specifics and we must take into account non-linear inputs and outputs, indivisible decisions and non-deterministic information. Here the linear model becomes a good 'elementary block' and the take-of point for generalizations.

b) Simplicity. In spite of its universality and good precision the linear model is very elementary in its means which are mainly those of linear algebra, so even people with very modest mathematical training can understand and master it. The last is very important for a creative and non-routine use of the analytical means which are given by the model.

c) Efficient computability. The urgency of solving extremal linear problems implied an elaboration of special, very efficient methods worked out both in USSR (method of successive improvements, method of resolving multipliers) and in USA (well-known simplex-method of G. Dantzig), and a detailed theory of these methods. An algorithmic structure of the methods has allowed later to write corresponding computer codes and nowadays modern variants of the methods on modern computers can rapidly resolve problems with hundreds and thousands of restrictions, with tens and hundreds of thousands of variables.

d) Qualitative analysis, indices. Together with the optimal planning solution the model gives valuable devices of qualitative analysis of concrete tasks and of the whole problem. This possibility is given by a system of indices for activities and limiting factors which is found simultaneously with the optimal solution and is in accordance with it. Professor T. Koopmans named them 'shadow prices', my term was 'resolving multipliers' since they were used as an auxiliary device for optimal solution finding like Lagrange multipliers. However shortly after their economic meaning and importance for analysis were realized, and they have been named in economic treatment objectively-determined valuations (Russian version gives an obreviation 'o.o.o.'). They have the sense of value indices of goods and factor equivalence, intrinsically determined for a given problem, and showing how the goods and factors can be exchanged in fluctuations of extremal state. Thus these valuations give an objective way of calculating accounting prices and other economic indices and a way of analysing of their structure.

e) Concordance of the means with the problems. Though separate firms and even government bodies in states of capitalist economy successfully used these methods their spirit corresponds closer to the problems of socialist economy. Evidence of their efficiency is in their successful application to a number of concrete problems of economic science and operations research. They have such large-scale applications as the long-term planning of some branches of Soviet economy, territorial allocation of agricultural production. Now we are discussing problems of model complexes including the model of long-term planning of the whole national economy. These problems are investigated in special large research institutes – Central economic-mathematical

institute in Moscow (headed by academician N. Fedorenko) and Institute of economic science and industry organization in Novosibirsk (headed by academician A. Aganbegjan).

It is necessary to point out as well the current position of optimal planning and mathematical methods in theoretical investigations of Soviet economic science. The linear model has proved a good means of simplest logical description for problems of planning control and economic analysis. It has contributed to significant advancements in pricing problems. For instance, it has given justification of accounting basic funds in production prices, principles of accounting the use of natural resources. It is given also a quantitative approach to reflecting the time factor in investments. Note that a model describing a simple economic index has sometimes a rather sophistical mathematical form (We can mention here as an example a model for the use of a stock of equipment from which the structure of amortization payments was derived).

A problem that needs to be pointed out especially is that of decentralized decisions. The investigation of a two-level model complex leads us to the conclusion that in principle the decentralization of decisions with observance of the total object of the complex is possible by the means of a correct construction of objects in local models. We must point out here a brilliant mathematical formalism of the idea of decomposition given by G. Dantzig and Ph. Wolfe. The value of their paper of 1960 is far from the limits of the algorithm and its mathematical foundation. It gave rise to a lot of active discussions and various treatments in the whole world and particularly in our country.

Together with input-output analysis and optimization models as a result of the activity of a large community of scientists the economic theory and practice was provided with such analytical tools as statistics and stochastic programming, optimal control, simulation methods, demand analysis, social economic science and so on.

Summing up we say that as result of about 15 years of intensive development and spreading of the mentioned methods we have some significant results.

3. Difficulties

The level of development and especially that of applications may cause however a feeling of dissatisfaction. The solving of many problems was not completed. Many applications are episodical, they don’t became regular and are not united into a system. In the most complicated and perspective problems, such as those of national planning, have up till now effective and generally acceptable forms for the realization not been found. The attitude to these methods like to many other innovations went sometimes from scepticism and resistance through enthusiasm and exaggerated hopes to some disappointment and dissatisfaction.

Certainly we can say that the results are not too bad for such a short period of time that has passed. We can refer to the longer periods of widespreading of many technical innovations or to physics and mechanics where some theoretical models are not realized in spite of two-hundreds-year experience. However we prefer to mention some concrete

problems to clarify the main difficulties and their causes and to outline some ways to overcome them. Difficulties arise both from the specific features of the object under investigation and from defects in the researches and their practical realization.

The economic matter is a difficult object for a formal description in view of its complexity and pecularity. The models emphasize only a few of its aspects and take into account the real economic situation very roughly and approximately, so as a rule it is difficult to estimate the correctness of the descriptions and inferences.

So in spite of the above mentioned universality of the model and its generalizations a routine approach is often non-efficient. A work on each serious model and its practical application demands hard research elaboration with joined efforts of economists, mathematicians and specialists in the concrete field, but even in successful cases the widespreading of the model needs several years, especially for testing and improving of practical instructions.

It is especially important to test the influence of the difference between the model and reality on the obtained result and to correct the result or the model itself. This part of work is not often observed.

The hard thing in a model realization is to receive and often to construct necessary data which in many cases have considerable errors and sometimes are completely absent, since none needed them previously. Difficulties of principle lie in the future prediction data and in the estimation of industry development variants.

The computation of optimal solution has its difficulties as well. In spite of the presence of efficient algorithms and codes practical linear programs are not too simple since they are very large. The difficulties grow significantly when the linear model is modified by any of its generalizations.

It was mentioned that theoretically in the linear model there is a perfect accordance and harmony of the optimal solution and the estimating indices and stimuli based on o.o.o. However real decisions and the work of local bodies are evaluated not by the theoretical indices but by actual prices and methods of assessment which are not so simple to replace. Even if one branch or region adopts its proper indices the disharmonies will appear on boundaries with its neighbours. Moreover various parts of the economic system are described by mathematical models with difficult success and they have not always distinct quantitative characteristics. Thus the industrial production is described better than demand and consuming preferences. At the same time in a wide statement of the plan optimization problem it is natural to tend not only to least possible use of resources but as well to a structure of production which is optimal for consumers. This condition complicates the correct choice of objective function.

Certainly the situation is not hopeless. For instance, one can use an idea of extremal state (i.e. of a state which cannot be improved all-round, ‘efficient decision’ of A. Wald), which is pithy enough. Then one can make a compromise of a few criteria or be less

rigorous and solve the industrial part of the problem by optimization methods and the consuming one by the traditional expert methods. One can try to use econometrics, – too many 'can' mean that the problem is very far from solution.

In planning the idea of decentralization must be connected with routines of linking plans of rather autonomous parts of the whole system. Here one can use a conditional separation of the system by means of fixing values of flows and parameters transmitted from one part to another. One can use an idea of sequential recomputation of the parameters, which was successfully developed by many authors for the scheme of Dantzig-Wolfe and for aggregative linear models.

A solution of newly appearing economic problems, and in particular those connected with the scientific-technical revolution often cannot be based on existing methods but needs new ideas and approaches. Such one is the problem of the protection of nature. The problem of economic valuation of technical innovations efficiency and rates of their spreading cannot be solved only by the long-term estimation of direct outcomes and results without accounting peculiarities of new industrial technology, its total contribution to technical progress.

The accounting methods based on mathematical models, the use of computers for computations and information data processing make up only one part of the control mechanism, another part is the control structure. Thus success of the control depends on to what extent and how there is guaranteed in the system the possibility of personal interest in correct and complete information, in proper realization of decisions achieved. The construction of such interest and of checking systems is not an easy job either.

Moreover, in order to achieve a real spread of the new methods it is necessary that they be studied and mastered by the people who are employed in planning and economic science. It is necessary to reorganize this system, to overcome some psychological barrier, to shift from many-years-used routines to new ones.

For this purpose we have an educational system which serves to acquaint the planning administration up to top level with the new methods. The accounting reorganization usually is combined with the introduction of computerbased information systems. It is clear that such a recognization of methods and consciousness is difficult and time-spending.

4. Perspectives

In spite of mentioned difficulties I am looking optimistically on the prospects of wide spread of mathematical methods, especially those of optimization, in economic science and in all-level economic control. It can give us a significant improvement of planning activity, better use of resources, increment of national income and living standards.

The difficulties of modelling and data creation can be overcome like similar difficulties were overcome in the natural and technical sciences. My hope is based on the more and more intensive steam of research for new methods and algorithms in this field, on the

fact of appearance of new theoretical approaches and problem statements, on a series of concrete studies of general and special problems concerning separate economic branches, on the fact that a whole army of talented young researchers work now in this field. A significant progress is now being made in the development of computer hard- and software and their mastering.

The mathematicians, economists and practical managers have achieved a better mutual understanding.

The favourable conditions for the work in this field were given by well-known important statements on control methods and their improvements which were made in last years by our authorities.

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