PROBLEMS OF ECONOMIC DYNAMICS AND PLANNING

Essays in honour of

MICHAŁ KALECKI

PWN—POLISH SCIENTIFIC PUBLISHERS
WARSZAWA

1964

CONTENTS

BIOGRAPHY OF MICHAŁ KALECKI

Michał Kalecki was born in Łódź on June 22nd, 1899. His father was an owner of a small spinning-mill and later a book keeper. In 1917 he finished a Grammar School in Łódź and in the same year started to study at Warsaw Polytechnic. In 1919 he was called up and as a result of the interruption was able to continue his studies only two years later, this time at the Polytechnic in Gdańsk. In 1923 shortly before he should have finished, he had to interrupt his studies because his father had lost his means of livelihood. Amongst many other casual jobs by which he was able to earn his living was collaboration with a servicing enterprise gathering data on the financial position of establishments applying for credit (so-called credit enquiry); this collaboration was for Kalecki the first experience of its kind in practical economic analysis. His interest in economic problems had begun while he was studying at the Polytechnic. During these years he became acquainted amongst others with the main lines of the economic theory of Karl Marx. Marx' schemes of reproduction made a particularly powerful impression on Kalecki. There is a certain paradox in the fact that Kalecki—later one of the most consistent theorists of inadequate effective demand in capitalism came across Marx' schemes for the first time in the book by Tugan-Baranowski, the originator of the theory of disproportions which utterly ignores the problem of inadequate demand in capitalist economy. The interesting part of Tugan's conception is however the stress on investments as a factor in the realization of capitalist profit even then when they do not serve the increase of consumption in the subsequent period.

Kalecki's first publications on economics were devoted mainly to an analysis of the business cycle and to a description of the structure of big enterprises and their activity in home and world markets. An attempt to found a newspaper entitled "Koniunktura włókiennicza" (The Textile Market) in Łódź ended with the first issue. However, for many years he wrote for "Przegląd Gospodarczy" (The Economic Review) and "Polska Gospodarcza" (Polish Economy).

Kalecki went to Warsaw in 1927 and at the beginning earned his living with temporary jobs. Only at the end of 1929 did he obtain work at the Institute of Research on Business Cycles and Prices, directed by Professor Edward Lipiński. Taking part in the current work of the Institute, Kalecki conducted reasearch into aspects of the business cycle and published his results in the Institute bulletin "Koniunktura gospodarcza" (The Business Cycle). At the same time, his attention was turned to problems of considerably broader scope and degree of generality. Together with one of the most prominent Polish economists and statisticians of the inter-war period, Ludwik Landau, he conducted research within the Institute into the Polish national income. The results of these studies are two books written jointly: *Szacunek dochodu* *społecznego w 1929 r.* (An Estimate of the National Income in 1929), Warsaw 1934 and *Dochód społeczny z r. 1933 i podstawy badań periodycznych nad zmianami dochodu* (The National Income in 1933 and the Foundations of Periodic Research into the Changes in Income), Warsaw 1935. Another smaller contribution written jointly by these authors was *Wahania cen i kosztów a wahania produkcji przemysłowej w Polsce* (The Fluctuations in Prices and Costs and Fluctuations of Industrial Production in Poland), 1935.

In 1933 in the Institute's series of publications, Kalecki published a book which after a certain period of time, gained a lasting place in the history of economic thought. *Próba teorii koniunktury* (An Attempt at a Business Cycle Theory) is his own theoretical summary of his economic interests over ten years. There is no doubt that, apart from current analyses, the path that Kalecki took to reach his own theory of the business cycle led through Marxist schemes of reproduction as well as the above mentioned research on the volume and division of the national income. The final impulse which led to the writing of *Próba teorii koniunktury* was a reading of Tinberger's study, *Ein Schiffbauzyklus*? which suggested to Kalecki the mathematical construction of the general cycle theory.

It is possible to present briefly Kalecki's main idea on the theory of business cycles as follows: with an incomplete use of productive capacity, gross profits are determined by the volume of the consumption and investment expenditure of capitalists—although it may appear that the reverse is true—that gross profits determine expenditure. The common-sense opinion that the more capitalists consume, the less they invest, holds true only for individual capitalists: it does not, however, hold true for the whole class.

The key to an understanding of the business cycle fluctuations is the process of investment. Investment improves the economic situation during the gestation period opening a valve whereby additional purchasing power is received into the national economy. However, the increase in the productive apparatus and thus in production as a result of a new investment, leads to a decline in the rate of profits and further to a breakdown of the boom. Kalecki saw one of the greatest paradoxes in the capitalist system in the fact that an increase in the productive apparatus, and thus an enrichment of society, carries in itself the seeds of crisis during which this enrichment proves to be only potential, for a certain portion of the productive apparatus is unused and may be set into motion only during the next turn of the business cycle[1].

---

[1] The changes which Kalecki introduced into his model in several later publications amount only to a more developed and somewhat amended presentation of the motivation of the investment decisions of capitalist entrepreneurs and to the introduction of stocks-movement as a factor of the business cycle. These changes are not vital to the fundamental framework of the model and, in the opinion of some, even destroy the clarity of the original model. Kalecki himself maintained that the changes introduced by him constituted a certain progress, for they consolidated his theory and brought it nearer to reality.

Kalecki's theory shed some light on the role of additional purchasing power, produced by a budget deficit, an export surplus and other similar means, which enable capitalists to attain profits exceeding their own purchase of goods and services. It also gave a basis for a new conception of the economic role of a bourgeois state, as well as the role of expansion on external markets[2].

Deeply anti-capitalist in its basic assumptions, Kalecki's conception may be regarded as a development of certain of Marx ideas, expounded in the third volume of *The Capital*, and, at the same time, as a solution to the problems with which Róża Luksemburg struggled twenty years earlier.

Kalecki's theory might have played an important political role in counteracting the burgeois tendencies aiming to shift the burden of crisis onto the shoulders of the working class by reducing wages. These tendencies were, of course, strongly attacked by Marxists. The Marxists' arguments were mainly limited to the social aspects of the problem without sufficiently clear, economic proof that this lowering of prices cannot even be considered a "capitalist method of overcoming the crisis", since it is not a method at all. Such a proof could have been found in Kalecki's writings. He explained that as a result of wage reductions the workers' income decline; this however does not increase the total amount of profits (and accumulation), but contributes to the fall in the national income and a deepening of the crisis.

Unfortunately the main line of Kalecki's theory was not understood properly. And though Kalecki was, in the thirties, closely connected with the leftist socialist movement (he published several articles in "Przegląd Socjalistyczny" (The Socialist Review) under the assumed name of Henryk Braun), he was criticized from the political standpoint by some Marxists. The tendency of suppressing anything that had a touch of "Luxemburgism" prevailing then among Marxists might have been the reason for these criticisms.

In 1935 John Maynard Keynes published his *General Theory of Employment, Interest and Money*, in which he broke with the traditional attitude of hiding one's head in the sand, peculiar to academic economics up to that time, taking as the

---

[2] Kalecki dealt with this item most precisely in his *Theory of Economic Dynamics* where we read: The connection between "external" profits and imperialism is obvious. The fight for the division of existing foreign markets and the expansion of colonial empires, which provide new opportunities for the export of capital associated with the export of goods, can be viewed as a drive for export surplus, the classical source of "external" profits. Armaments and wars, usually financed from budget definits, are also a source of this kind of profits. M. Kalecki, *Theory of Economic Dynamics*, London 1956, p. 52.

[3] Kalecki's work entitled *Płace nominalne i płace realne* (Nominal Wages and Real Wages), Warsaw 1939, includes a detailed exposition of his theory of wages. This work, published by the Institute of Social Economy with an introduction by L. Krzywicki, was written at the request of the then Ministry of Labour and Welfare, in connection with the appearance of J. Wątecki's book called *Sztywne płace źródłem bezrobocia* (Fixed Wages as a Source of Unemployment).

starting-point of his deliberations the fact of the existence of depressions, of mass unemployment and of the difficulty of maintaining the equilibrium in a capitalist economy. In inadequate demand he perceived the chief restraint on the growth of capitalist production, and in investment—the main-spring of its growth. The nature of the analysed subject itself—disproportions in the capitalist economy as a whole—caused the appearance in Keynes' reasoning of certain analogies with the ideas of the earlier classical economics (particularly the macro-economic approach) and even with Marxist economics (particularly the denial of Say's law and a grasp of the role of means of production in the dynamics of capitalist production) and with the views of Róża Luksemburg (the significance of inadequate demand). Keynes' approach to certain basic problems in capitalist economy quickly earned him an unusual fame and this in consequence had some influence on the attitude towards Kalecki's ideas. In spite of the basic differences in the attitudes and conclusions of Keynes—a doctor of capitalism with the help of modern methods, and Kalecki—a convinced critic of capitalism, attention began to be paid to the similarities of certain concepts. This is the beginning of Kalecki's paradoxical and perhaps unique international career as a pupil and interpreter of Keynes. Keynes himself treated him as such, in all good faith, for up to the end of his life he had never heard of Kalecki's work published before his *General Theory*[4].

In this way, as a result of particular historical conditions, Kalecki was pressed formally into the Keynesian current, but with his unequivocal socialist attitude, the adjective "Left-wing" was usually added to the name "Keynesist".

Before Keynes' "General Theory" was published Kalecki's theory did not draw much attention either in Poland or in western academic circles. In the year in which *Próba teorii koniunktury* was published, Kalecki presented his theory in a report at the conference of the International Econometric Society in Leyden, (Netherlands). Nothing points here to the fact that the beginning of what was later called "revolution" was observed in it. However, his paper was published in the Society's periodical

[4] An article written after Keynes' death ("Economic Journal", Vol. 57, 1947) by E. A. G. Robinson who stood far from Keynesism, drew attention for the first time in print to the fact that Michał Kalecki came to conclusions, similar to those of Keynes, quite independently. Four years later the American econometrist Klein, author of the book *The Keynesian Revolution* published in 1947, did so more fully. In 1951 on the occasion of the appearance of Harrod's book on Keynes, he wrote: "Recently, after having reexamined Kalecki's theory of business cycle (in an article by Kalecki published in "Econometrica", July 1935) I have decided that he actually created a system that contains everything of importance in the Keynesian system, in addition to other contributions... yet I believe that he has a theory of employment that is the equal of Keynes'. Kalecki's theory attracted attention for reasons largely unrelated to its revolutionary statement on the theory of employment, and he certainly lacked Keynes's reputation or ability to draw world-wide attention; hence his achievement is relatively unnoticed. Some respects in which Kalecki's model is superior are that it is explicitly dynamic; it takes income distribution as well as level into account; and it makes the important distinction between investment orders and investment outlays". ("Journal of Political Economy", October 1951).

in the July 1935 issue, and in addition he was recommended to the Rockefeller Foundation as a candidate for a scholarship.

After he had received the scholarship for a year, Kalecki went to Sweden in order to make scientific contact with Myrdal and other economists and there grew the intention of writing a book of a general nature, but particularly on the extension of the profit theory. When he had already begun to dictate the book to his wife, he received news of the appearance of a book by Keynes, solving certain questions in a similar way. So he gave up work on his own book and went to England where he worked at the London School of Economics.

In London, Kalecki made friends with the German refugee, Erwin Rothbarth. Rothbarth was fascinated by Kalecki's concept of the business cycle; taking the mathematical model of Kalecki's cycle as a basis, he tried to create a uniform theory of the economic dynamics of capitalism, embracing trend and long waves as well as the cycle. The work was never completed for the author died on the anti-fascist front towards the end of the war. But from the two friends conversations were born a number of ideas which were later absorbed and developed in Kalecki's studies on the dynamics of capitalist economy.

Kalecki used his stay, first and foremost, for studies with the aim of consolidating and enlarging his first book on the cycle. At the beginning of 1937 he published a new version of his book in the Review of Economic Studies, comparing it with the assumptions of Keynes' theory. Towards the end of this year he published in "Economica" a contribution entitled *The Principle of Increasing Risk.* The principle presented here, as Kalecki underlined, is an expansion and generalization of certain ideas voiced earlier in Marek Breit's German article[5]. Early in 1938 Kalecki published a theoretical and statistical article on the working class share in the national income (*The Distribution of the National Income*, "Econometrica", April 1938); in this article appears the important difference in attitude between Kalecki and Keynes, especially in the manner of the treatment of the working class and the analysis of the role of capitalist monopolies. The final remark of Kalecki's study reads unequivocally: "The results arrived at in this essay have a more general aspect. A world in which the degree of monopoly determines the distribution of the national income is a world far removed from the pattern of free competition. Monopoly appears to be deeply rooted in the nature of the capitalist system: free competition, as an assumption, may be useful in the first stage of certain investigations, but as a description of the normal state of capitalist economy it is merely a myth".

Kalecki left London to study in other countries. He spent several months in Paris, studying the results of Blum's economic policy, amongst other things for a statistical verification of his theory of real and nominal wages, explained in detail

[5] M. Breit, *Ein Beitrag zur Theorie des Geld- und Kapitalmarktes* ("Zeitschrift für Nationalökonomie"), Band VI, Heft 5.

in later publications, but already contained in a shorter form in the first book on the cycle.

At the beginning of 1938 Kalecki went to Cambridge. He had already visited that dynamic centre of economic thought during his stay in London, establishing scientific contact with Joan Robinson, Piero Sraffa, R. F. Kahn and others. Towards the end of 1937 he met Keynes but this meeting did not serve as the beginning of an intellectual exchange, or closer scientific collaboration. Quite apart from the differences in their social attitudes, their personal situations were diametrically opposite. Keynes was absorbed in the search for historical analogies: he was collecting material for a biography of Newton and marvelled that, at first, only people below forty understood Newton's discoveries. Kalecki had completely different worries: he was looking for a job and Keynes offered to help him.

The fact that, at the end of 1936, his closest friends and collaborators, Marek Breit and Ludwik Landau, were discharged from the Institute of Research on Business Cycles had led Kalecki to the decision to stay in England longer. The reason for their discharge was a report, by them, on the country's economic position. The then Vice-premier and Minister of the Treasury, Eugeniusz Kwiatkowski, demanded the disciplinary discharge of the authors. To manifest his protest Kalecki resigned from the Institute, and published a statement to that effect in the Warsaw press.

Kalecki used the first part of his stay in Cambridge—apart from attending Sraffa's seminars and establishing other scientific contacts—to prepare the book, *Essays in the Theory of Economic Fluctuations*: this appeared in 1939. The book included the three mentioned above essays on the division of the national income, the principle of growing profit and the theory of the cycle with substantial alterations; and also three others not published before: *Investment and Income*, *Money and Real Wages* and *The Long-term Rate of Interest*. In this way, the theory of business cycles was shown on the wide background of the economic problems of the capitalist system. It was just here that Kalecki most explicitly made use of the notion of the Keynesian theory in the broadest sense, which includes also his own theoretical views. At the same time, however, the Marxist origin of his theory was shown clearly in this book as perhaps in no other. In the essay on investment and income, he underlined the basic similarity between his equilibrium equation and the Marxist schemes of expanded reproduction. However, he noted at the same time that Marx did not follow up the problem (at least in the second volume of *The Capital* of what happens when investment outlays do not provide the dynamic equilibrium. In Kalecki's opinion, Róża Luksemburg dealt with this problem in *Akumulacja Kapitału* (The Accumulation of Capital), where she affirmed that if capitalists save, profits may be realized only when the defined volume of investments correspond to them. R. Luksemburg was of the opinion that in pure capitalism, such a correspondence over a long period is impossible; and this causes the necessity of the external expansion of capitalism. Kalecki notes that Róża Luksemburg's theory cannot be accepted as a whole, but that the necessity of covering saving surpluses with investments was grasped probably more clearly by her than by anyone else before (...) the publication of Keynes' *General Theory*[6].

In January, 1940 Kalecki obtained a job at the Oxford Institute of Statistics. This institute, directed by Professor A. L. Bowley, in co-operation with his deputy, the German Social Democrat F. Burchardt, brought together a circle of prominent, often left-wing economists, like J. Steindl, T. Balogh, W. Goldmann and others during the war. Like the whole Institute, Kalecki occupied himself with statistical and economic studies on different aspects of war economy in Great Britain (many papers by Kalecki were published in the Institute Bulletin). Amongst these publications by Kalecki, his works on the subject of rationing which were very radical from the point of view of the social distribution of the consumption income and the degree of the state's regulatory role, deserve particular attention. Kalecki's plan, published in its developed form in 1941, provoked lively interest with opposition expressed by conservative circles. The development of the war situation led to a gradual introduction of rationing in accordance with a scheme not so very different from that of Kalecki.

In 1943 Kalecki published a critical analysis of Beveridge's plan (*Economic Implications of the Beveridge Plan*) and pointed out that the plan went only half-way and, above all, that a high margin of unemployment was assumed in it.

Some other important essays written by Kalecki while at the Institute appeared in two collected volumes: *The Economics of Full Employment* (first published at the end of 1944, the book ran to several editions) and *Essays in War Economics*.

Kalecki also presented the results of his research in monographical lectures given at Oxford University of which the Institute of Statistics was a part.

In connection with his studies on war economy and a policy of full employment, there appeared Kalecki's paper *The Political Aspects of Full Employment* ("Political Quarterly", 4/1943). In this article Kalecki's critical attitude to the possibility of a lasting assurance of full employment in capitalism clearly appeared. Publishing this article in Polish twenty years later, Kalecki added in the foreword the following comment: "Were my predictions of that time correct? Perhaps, but as is usual with historical predictions, not necessarily in every detail. In the article, having analysed the character of the resistance by big business to an improvement of the business cycle by means of state outlays, I forecast in the future some relief of these crises by this method, but not that they would be completely prevented. I stated further that, against a background of state intervention in the course of the business cycle, there would appear a phenomenon which I called "the political business cycle".

[6] Obviously, from intrinsic point of view it would be more fair to compare R. Luxemburg's theory of accumulation with Kalecki's *Próba teorii koniunktury* of 1933. The connection of the latter with R. Luxemburg's theory is significantly more clear and direct than her connection with Keynes' theory, both on account of the basic theoretical assumptions and the social attitude of the authors.

It seems that the present actual state of affairs correspond in the main to this prognosis. On the other hand, support of a policy of full employment in armaments is tied up in my article with integral Fascism. It turned out however that armaments could play an important role in combating mass unemployment even without "Fascism taking over". [*Szkice o funkcjonowaniu współczesnego kapitalizmu*, (Essays on the Functioning of Modern Capitalism), Warsaw 1962, from the Foreword].

Beside his work at the Institute, Kalecki continued his purely theoretical studies. A book entitled *Studies in Economic Dynamics* (1943) contained the results of these studies begun earlier in Cambridge. Amongst other theoretical contributions it is worthwhile to mention: *A Theory of Technical Progress* (1941) and *The Supply Curve of an Industry under Imperfect Competition* (1940).

Towards the end of the war Kalecki formulated suggestions for the French government on the subject of rationing and control over the economy. In March 1945, he left Paris for Montreal where he conducted research into the problems of post-war economic reconstruction and into the problems of full employment in the International Labour Office.

Between July and October, 1946 Kalecki was in Warsaw acting as an economic counsellor to the Central Planning Office and to the Ministry of Finances.

At the end of 1946, with the approval of the Polish authorities, he took up the post of deputy director of a section of the economic department in the United Nations Secretariat in New York. The chief purpose of this department was to work out the World Economic Reports.

While working for the UN, Kalecki spent two months in 1951 in Israel as an economic expert (vide: his *Report on Main Current Economic Problems of Israel*, 1951). A similar invitation from Mexico met with a refusal from the UN Secretariat, undoubtedly on political grounds.

McCarthyist tendencies made themselves felt even in the UN Secretariat. In the spring of 1954 a chapter from the report on the Chinese Peoples' Republic was completely disfigured—despite Kalecki's protests. Somewhat later, a reorganization of the UN Secretariat was used to limit Kalecki's influence on the contents of reports worked out there. Under such circumstances, after obtaining the consent of the Polish authorities, Kalecki tendered his resignation and left the United States at the end of 1954. At the beginning of 1955, he gave lectures at Oxford and Cambridge during his return journey to Poland.

After his return to Poland, Kalecki's activities followed two main lines.

The first one was the work in an advisory capacity to the economic planning bodies. From May, 1955 to March, 1957 he was an economic counsellor with the Office of Ministers' Council and subsequently with the Planning Commission. During 1957–1960 he was the chairman of the Commission for Perspective Planning. In this capacity he worked out the first version of the perspective plan of Poland's economic growth in the years 1960 to 1980. He was one of the vice chairmen of the Economic Council from February, 1957 until it ceased to exist at the beginning of 1963. He plays an active role in the work of the Council for Mutual Economic Assistance (CMEA), particularly as chairman of the Polish delegation to the Economic Commission of CMEA. A wide range of expert advice, his programmes, plans and also several publications were the result of this line of work.

The other line, in many aspects difficult to distinguish from the first one, is the research and teaching activity, even more intensive and diverse in recent years. It is possible to divide Kalecki's recent scientific interests into three groups: the problems of capitalist countries, the theoretical problems of socialist economy and finally the problems of developing countries.

In 1955–1961 Kalecki led a group in the Department of Economic Sciences of the Polish Academy of Science, conducting research into contemporary capitalism. The results of the work of this group were issued in the series *Studia z zakresu koniunktury współczesnego kapitalizmu* (Studies in the Business Cycle of Contemporary Capitalism, four issues were printed between 1957 and 1960). These shed some light on, amongst others, the problems of the business cycle in the U.S.A. and in certain countries in Western Europe during the years 1950–1955, structural transformations in the economy of capitalist countries, the recessions in the period 1956–1958 and the role of capital export from industrialised to developing countries.

In 1956 the Central Qualification Commission bestowed on Kalecki the first academic title of his life—Professor. In the following year he was elected associate member of the Polish Academy of Science.

In 1961 Kalecki gave up his work in the Department of Economic Science in the Polish Academy and went to the Central School of Planning and Statistics, where he gives lectures in the theory of business cycles, the growth of capitalist economy and in the theory of the growth of socialist economy.

Amongst theoretical publications devoted to the problems of capitalist economy, his study *Observations on the Theory of Growth* ("Economic Journal", March, 1962) had vital significance. Already in earlier publications, and particularly in the *Theory of Economic Dynamics* (English editions 1954 and 1956, translations into Italian, Japanese, Spanish, Polish), which was a synthesis of two of Kalecki's earlier books, he emphasised that long-term development is not self-explanatory and inseparable from capitalist economy; but that it must be supported by innovations coming, in some way, from outside and embracing also discoveries of new raw material sources. The above mentioned study is devoted to a theoretical substantiation of his own conception: and in it he gave his opinion on certain aspects of the theory of economic growth as formulated by R. F. Harrod. In opposition to Harrod's attitude that the contradiction of capitalist economy is expressed in fluctuations around the line of trend, Kalecki set the thesis that the contradiction of the capitalist system lies deeper, that "this system cannot get out of the impasse of fluctuations around the stable position, if such 'semiexogenical' factors such as animating investment by innovations, do not support economic growth".

Another group of problems which have been a subject of interest for Kalecki for many years were the problems of the development of those countries of the so-called "third world". He carried on systematic research in this sphere while he was working for UN and summed it up in an essay *The Financing of Economic Development*, published for the first time in 1955.

In the years 1959 and 1960 Kalecki spent several months in India where he worked out a memorandum for the Indian government on the problems of financing the third five-year plan.

In a similar capacity he went to the Cuban Republic where he prepared a document on the foundations of Cuba's economic plan. For several years Kalecki has conducted seminars on the economy of developing countries which are attended by people working for organizations dealing with developing countries and by research workers from various centres and institutes. From 1962 he has been a chairman of the Scientific Council of the Department of Economic Problems in Developing Countries created by The Central School of Planning and Statistics and Warsaw University.

In the spring of 1963 he delivered a paper on long-term planning at a plenary session of an international conference in Geneva, devoted to the problems of the "third world".

The problems of the theory of socialist economy, particularly the theory of growth, have occupied a central position in Kalecki's scientific work during the past ten years. His formula of socialist economy growth, whose basic aspects had already been presented in 1956 in a paper delivered at the Second Congress of Polish Economists, played a key role in theoretical discussions in Poland and in a deepening of the understanding of a number of actual processes. Kalecki's model was later improved and developed repeatedly into the systematic *Zarys teorii wzrostu gospodarki socjalistycznej* (Outline of the Theory of Growth in the Socialist Economy; the Polish edition appeared in 1963). Kalecki's theory of growth of socialist economy supplies convincing evidence of the fertility of the basic methodological assumptions of Marxism, primarily by the demonstration that an analysis of functional interrelations appearing in the model cannot be carried out without a consideration of the basic parameter—production relations. Apparently insignificant differences between the model which Kalecki used for an analysis of the factors of economic growth in socialism and certain models in the West, hide, in fact, the deep contradiction which is immediately visible when the formal interrelations are subjected to an analysis, fully conscious of the specific aspects of the assumptions connected with the system. This theoretician of inadequate effective demand who examined in such a penetrating way the capitalist world not being a subject to the rules of common sense deals again, in his studies on socialism, with a normal world in which the growth of social wealth depends in a natural way on its reproduction ability with aggregate demand adjusted, according to a plan, to the production potential.

Emphasising the superiority of the socialist planned economy with regard to the possibilities of taking full advantage of the social growth potential, Kalecki consistently opposed voluntarism in planning the rate of growth, primarily through the setting of too high a rate of investment. Hence the role that was played in his theory by an analysis of the limiting factors, the so-called "investment barriers", the ignoring of which would have led, in spite of all intentions, to a lowering of the long-term rate of growth below the actual potential. For this reason, Kalecki formulated the "principle of allowing for the level of consumption in the short period", a principle which, if not observed, would lead to negative results not only from the social point of view, but also from the purely economic point of view, by an unfavourable "feed-back effect" on the labour efficiency in the widest sense.

The attention with which Kalecki examined the limiting factors did not in any way lead him to the fatalistic conclusion that passive adjustment to the "barriers" is necessary, but rather to seek real means to remove them by way of raising the effectiveness of managing the economy. "Observation of the principle of the reality of the plan and of allowing for the level of consumption in the short period, leads to moderation in the setting of the rate of growth. Avoidance of waste and care over the effectiveness of investment permit it to be maintained at a relatively high level in spite of this—these words from the article *O podstawowych zasadach planowania wieloletniego* (The Basic Principles of Long-term Planning), "Życie Gospodarcze" No. 24/1963 characterize his general approach to the problem.

Kalecki devoted his energies in the recent period to the theoretical foundations and practical methods of effectiveness calculation in a socialist economy. His studies devoted to the theory of choice between investment variants, and particularly to the methodology of determining the so-called recoupment period amongst others *Uogólnienie wzoru efektywności inwestycji* [(A Generalization of the Model of Investment Effectiveness), written jointly with M. Rakowski] played an important role in the preparation of suitable planning instruction. In this sphere one ought to note the article *Zagadnienie optymalnej struktury spożycia* (The Problem of Optimal Consumption Structure), "Gospodarka Planowa" No. 6/1963, which treats the problem of the choice of the optimum, from the point of view of minimizing social outlays, structure of consumption in a perspective plan. This concept may play an important role in the long-term price theory in socialist economy.

A vital feature of Kalecki's way of seeing the problems of economic calculation in socialist economy is the concrete and consistent application of the principle of a national effectiveness criterion and the subordination to it of partial criteria. This amongst other things, allows many technical fetishes resulting from just such a sectoral point of view to be counteracted.

The problem of shaping the macro-economic proportions over a longer period of time, occupied a central place in Kalecki's approach to the methods of raising efficiency in socialist economy. This finds expression, amongst others, in the position which he took up in 1956–1958 in a period of broad discussion on changes in the economy functioning model. While he appreciated the role of positive changes in the management system (particularly the development of workers' participation in management) and played personally an active part in the preparation of these changes—at the same time Kalecki emphasised the decisive significance of a proper and realistic formation of basic proportions in the process of growth.

written by

Tadeusz Kowalik

Paul A. Baran and Paul M. Sweezy
U.S.A.

NOTES ON THE THEORY OF IMPERIALISM

The Marxian theory of imperialism, as developed chiefly by Hilferding, Rosa Luxemburg, and Lenin and since accepted with but few modifications by most Marxists, has served at least three major purposes. First, it provides a theory of international relations within the capitalist world, encompassing not only relations between advanced and underdeveloped countries but also among the advanced countries themselves. Second, it contributes to the clarification of the development of social and political conditions within the various capitalist countries, both advanced and underdeveloped. And third, it purports to provide an important part of the explanation of strictly economic tendencies and trends within the advanced capitalist countries. In this third connection, two points have been usually stressed. The unequal relations between the developed and underdeveloped countries result in the establishment of terms of trade which greatly favor the former at the expense of the latter. In this way wealth is transferred from the poor countries to the rich, and the disposable surplus of the rich—which can be used to support parasitic classes, a "workers' aristocracy", as well for normal purposes of capital accumulation—is vastly expanded. But imperialism, by putting capital export at the very center of the economic stage[1], is also supposed to provide a crucially important outlet for the surplus of the rich countries. In the terminology of bourgeois economics, capital export expands effective demand and thereby raises income and employment above what they otherwise would have been. It is this last aspect of the traditional theory of imperialism which seems to us to be in particular need of rethinking in the light of conditions existing today, nearly half a century after publication of Lenin's classic work. As we hope to make clear, even within the confines of a brief exploratory essay, the problem is very much more complicated than Marxists have been wont to think, and the breadth and depth of its ramifications can hardly be exaggerated.

1 "Under the old type of capitalism", Lenin wrote, "when free competition prevailed, the export of commodities was the most typical feature. Under modern capitalism, when monopolies prevail, the export of *capital* has become the typical feature". (*Imperialism the Highest Stage of Capitalism,* Chapter 4).

I

At the outset it must be stressed that the familiar national aggregates—Gross National Product, national income, employment, etc.—are almost entirely irrelevant to the explanation of imperialist behavior. In capitalist societies, these are *ex post* calculations which play little if any causal role[2]. Nor does it make any difference whether the "costs" of imperialism (in terms of military outlays, losses in wars, aid to client states, and the like) are greater or less than the "returns", for the simple reason that the costs are borne by the public at large while the returns accrue to that small, but usually dominant, section of the capitalist class which has extensive international interests. If these two points are kept firmly in mind, it will be seen that all liberal and Social Democratic efforts to refute Marxian—or for that matter any other predominantly economic—theories of imperialism on the ground that in some sense or other it "doesn't pay" have no claim to scientific standing[3].

All of which is only another way of saying that the relevant actors on the imperialist stage are classes and their subdivisions down to and including their individual members. And this means in the first instance the dominant classes in the most advanced capitalist countries to which the less developed and underdeveloped countries stand in various relations of subordination. In terms of the total system, these are the classes which have the power of initiative: they are, so to speak, the independent variables. The behavior of other classes—including the subordinate classes in the dominant countries as well as both the dominant and the subordinate classes in the subordinate countries—is primarily reactive. One of the most important tasks of a theory of imperialism is therefore to analyze the composition and interests of the dominant classes in the dominant countries.

At the expense of some oversimplification, we can say that the traditional Marxist view has been that the imperialist ruling classes are made up of industrialists and bankers and that a certain characteristic evolution has taken place in the relations between the two groups. In the first phase—up to the closing decades of the 19th century—the industrialists played the leading role. Their interests in the underdeveloped countries were of two kinds: as sources of cheap food and raw materials

[2] To be sure, depressions and mass unemployment have pushed capitalist governments into armaments' expansion, aggressive foreign policy and even war, but the analysis of these crucially important problems is a task of the general theory of monopoly capitalism which is obviously much broader than the classical "pure" theory of imperialism.

[3] It should perhaps be added that in addition to being based on a fatal methodological error, these alleged refutations of economic theories of imperialism usually rely on arguments which can only be described as nonsensical. In this connection a good recent example is Hans Neisser's *Economic Imperialism Reconsidered*, "Social Research", Spring 1960. Neisser would like to compare what the capitalist world is like today with what it would have been like "if western economic penetration of the rest of the world had stopped at the beginning of the nineteenth century". (p. 73.). That this involves a wholly fanciful and arbitrary invention of a century and a half of world history does not trouble him in the least.

which would have the effect of raising the rate of surplus value and lowering the organic composition of capital, thus doubly boosting the rate of profit, and as markets for manufactured goods which would help to solve the realization problem. Both these ends would best be served by free trade and free competition which could be counted upon to turn the underdeveloped countries into complementary appendages of the advanced countries.

The second phase, beginning around 1880 or so, is characterized by the dominance of finance capital. Concentration and centralization of capital lead to spread of the corporate form, of stock markets, etc. In this context, bankers (investment bankers in the United States) seize the initiative, promote mergers and monopolies over which they establish their dominance, and thus become the leading echelon of the capitalist class. Since the bankers deal in capital rather than in commodities, their primary interest in the underdeveloped countries is in exporting capital to them at highest possible rates of profit. This end, however, is not furthered by free trade and free competition. Finance capitalists in each imperialist country want to establish an exclusive domain out of which they can keep their rivals and within which they can fully protect their investments. Hence the vigorous revival of empire-building—somewhat in abeyance since mercantile days—in the closing decades of the 19th century. There is, of course, no implication that export of capital is in conflict with the aims of the preceding period—raw materials and markets—for, on the contrary, they complement each other nicely. It is only that in the Hilferding–Lenin theory it is the export of capital which dominates imperialist policy.

This theory, taken together with Lenin's very important Law of Uneven Development, worked well in explaining the main lines of development of the world economy and of world politics in the period before the First World War. Since then, however, certain changes in the characteristics of the ruling classes in the dominant countries have taken place which need to be taken into account in the development of the theory.

II

One can no longer today speak of either industrialists or bankers as the leading echelon of the dominant capitalist classes. The big monopolistic corporations, which were formed and in their early years controlled by bankers, proved to be enormously profitable and in due course, through paying off their debts and plowing back their earnings, achieved financial independence and indeed in many cases acquired substantial control over banks and other financial institutions. These giant corporations are the basic units of monopoly capitalism in its present stage; their (big) owners and functionaries constitute the leading echelon of the ruling class. It is through analyzing these corporate giants and their interests that we can best comprehend the functioning of imperialism today.

In size, complexity of structure, and multiplicity of interests the corporate giant of today differs markedly from the industrialist or the banker of an earlier period. This ean be most graphieally illustrated by an aetual ease, and for this purpose we can hardly do better than select Stàndard Oil of New Jersey (hereafter referred to as Standard or Jersey). This eorporation was the earliest of its kind anywhere in the world; it is today the second largest industrial corporation in the world (seeond only to General Motors); and its international ramifieations are at least as complicated and far reaehing as those of any other eorporation. It shows in clearest and most developed form the "ideal type" to which hundreds of other giant corporations, both in the United States and in the other advanced capitalist eountries, are more or less elose approximations.

Here, in brief summary form, are some of the most important data about the size, structure, and operations of Jersey[4].

Size. As of December 31, 1962, Jersey had total assets of $11,488 million. Its aggregate revenues for the year 1962 eame to $10,567 million, and its net income to $841 million (*Form* 10–K).

Geographical distribution of assets and earnings. As of the end of 1958, the percentage distribution of earnings and assets by various regions was as follows (*Notice*):

	Assets	Earnings
U.S. and Canada	67	34
Latin America	20	39
Eastern Hemisphere	13	27
Total	100	100

Rate of return on stockholders' equity. During 1962 the percentage rates of return on stockholders' equity in different regions were as follows (*Annual Report*):

United States	7.4
Other Western Hemisphere	17.6
Eastern Hemisphere	15.0

Number of subsidiaries. As of the end of 1962, Jersey owned 50 pereent or more of the stoek in 275 subsidiaries in 52 countries. The following is a list of the number of such subsidiaries by country of organization (*Form* 10–*K*):

U.S.A.	77	Morocco	2
Canada	37	Switzerland	2
Great Britain	24	Uruguay	2
Panama	17	Venezuela	2
France	12	Algeria	1
Bahamas	8	Dominican Republic	1
Italy	6		

[4] The sources are the company's *1962 Annual Report*, its *Notice of Special Stockholders' Meeting* (October 7, 1959), and its *Form 10-K for the Fiscal Year Ended December 31, 1962*, filed with the Securities and Exchange Commission pursuant to Section 13 of the Securities Act of 1934. These sources are identified as *Annual Report*, *Notice*, and *Form 10–K* respectively.

Sweden	6	Egypt	1
Colombia	5	El Salvador	1
Netherlands	5	Finland	1
Australia	4	Hungary	1
Brazil	4	India	1
Chile	4	Indonesia	1
Germany	4	Kenya	1
Philippines	4	Luxemburg	1
Argentina	3	Madagascar	1
Denmark	3	Mexico	1
Ireland	3	New Zealand	1
Japan	3	Paraguay	1
Neth. Antilles	3	Peru	1
Norway	3	Republic of Congo	1
Austria	2	Singapore	1
Belgium	2	South Africa	1
Bermuda	2	Spain	1
Iraq	2	Surinam	1
Malaya	2	Tunisia	1

Recapitulating by regions, we find that Jersey had 114 subsidiaries in the United States and Canada, 43 in Latin America, 77 in Europe, 14 in Asia, 9 in Africa, and 18 in other regions.

Countries marketed in. According to the *Annual Report*, Jersey sold to "more than 100" countries in 1962.

It would obviously be wrong to expect a corporation like this to behave like a British cotton mill owner interested in getting his raw cotton from abroad at the lowest possible price and in exporting his products to a duty-free India, or like a Rothschild or a Morgan disposing over great amounts of liquid capital and interested in investing it abroad at the highest attainable rate of profit. Standard's interests are much more complicated. Take, for example, the question of exports and imports. Though Standard, through its principal U.S. affiliate, Humble Oil and Refining Company, is one of the biggest producers in the country, the company is definitely not interested in protectionist measures. Quite to the contrary, it is a strong opponent of the present system of controls which limit the importation of fuel oil[5]. "In the interests of consumers, the national economy, and the international relations of our country", states the *1962 Annual Report*, "we hope that these unnecessary controls not only will be relaxed... but will be completely removed". Behind this public spiritedness, of course, lies Standard's interest in having its relatively low-cost Venezuelan subsidiary, Creole Petroleum, sell freely in the lucrative East Coast fuel-oil market.

Or take the question of capital exports. On the face of it, one might be tempted to conclude from the tremendous magnitude and variety of Standard's foreign

---

[5] The existence of these import restrictions is a reflection of the great political power of the oil and gas producing states, especially exercised through the Democratic Party.

operations that over the years the corporation has been a large and consistent exporter of capital. The conclusion, however, would not be justified. From the data presented above, it appears clearly that foreign operations are much more profitable than domestic, and this has been the case since the early days of the corporation. Under these conditions, a small initial export of capital could, and undoubtedly did, expand rapidly through the reinvestment of its own earnings. Not only that. So great have been the profits of foreign operations that in most years even after the needs of expansion have been covered, large sums have been available for remittance to the parent corporation in the United States. The year 1962 may be taken as an example: Standard paid out dividends to its shareholders, the vast majority of whom are resident in the United States, a total of $538 million. In the same year, however, operations in the United States produced a net income of only $309 million. It follows that some 40 percent of dividends plus whatever net investment may have been made in the United States during the year were financed from the profits of foreign operations. Far from being an exporter of capital, the corporation is a large and consistent *importer* of capital into the United States.

The foregoing gives hardly more than a hint of the complexity of Standard's interests. It takes no account of the fact that the oil industry as organized by the giant international corporations is in reality a congeries of businesses: extraction of the raw material from the subsoil, transportation by pipe-line and tanker, processing in some of the most technologically advanced plants in the world, and finally selling a variety of products in markets all over the world. Nor is Standard confined to the oil industry even in this comprehensive sense. It is a large and growing supplier of natural gas to the gas pipe-line companies; it is a major producer of artificial rubber, plastics, resins, and other petrochemical products; and it recently entered the fertilizer business with plans which, according to the *1962 Annual Report*, "will make Jersey an important factor in the world fertilizer industry". Finally, Jersey, like other giant corporations, maintains a large research and development program the purpose of which is not only to lower costs and hence increase profits from existing operations but also to invent new products and open up new lines of business. As an illustration of the latter, we may cite the following from the *1962 Annual Report*: "Food from oil through biological fermentation is an intriguing possibility. Esso Research, in a small pilot plant, has produced a white powder that resembles powdered milk or yeast. It is odorless, has a bland taste, and is high in protein and B vitamins. The first goal is to develop food supplements for animals, but it is hoped that the technique may one day help to improve the diet and health of the world's growing population". Quite a promising market, one must admit.

This is, of course, not the place for a detailed examination of the structure and interests of Standard Oil or any other corporation. But enough has been said, we hope, to carry the conviction that such a huge and complicated institutional "capitalist" can hardly be assumed to have exactly the same attitudes and behavior patterns as the industrial or finance capitalists of classical Marxian theory. But before we explore this subject further, we must ask whether Standard Oil is indeed an ideal type which helps us to distil the essence of capitalist reality, or whether on the contrary it may not be an exceptional case which we should rather ignore than put at the center of the analytical stage.

III

Up to the Second World War, it would have been correct to treat Standard Oil as a sort of exception—a very important one, to be sure, exercising tremendous, and at times even decisive, influence on United States world policy. Nevertheless, in the world-wide scope and ramifications of its operations not only was it far ahead of all others; there were only a handful that could be said to be developing along the same lines. Many U.S. corporations of course had large interests in exports or imports, and quite a few had foreign branches or subsidiaries. In neither respect, however, was the situation much different from what it had been in 1929. Direct investments of U.S. corporations indeed declined slightly between 1929 and 1946[6]. Most of the giant corporations which dominated the U.S. economy were, in the words of "Business Week", "domestically oriented enterprises with international operations" and not, like Standard Oil, "truly world oriented corporations"[7].

A big change took place during the next decade and a half. To quote "Business Week" again: "In industry after industry, U.S. companies found that their overseas earnings were soaring, and that their return on investment abroad was frequently much higher than in the U.S. As earnings abroad began to rise, profit margins from domestic operations started to shrink... This is the combination that forced development of the multinational company"[8]. The foreign direct investments of U.S. corporations increased sharply—from the already cited figure of $7.2 billion in 1946 to $34.7 billion in 1961[9]. While this tremendous jump of course involved actual capital exports by many individual companies, it cannot be overemphasized that for the United States as a whole the amount of income transferred to the United States on direct investment account far exceeded the direct capital outflow. The two series, which can be constructed from official government statistics for the years 1950 and later, are as follows:

[6] The figure was $7.5 billion in 1929 and $7.2 billion in 1946. U.S. Department of Commerce, Office of Business Economics, *U.S. Business Investments in Foreign Countries: A Supplement to the Survey of Current Business*, 1960, p. 1.

[7] *Multinational Companies*, A Special Report, "Business Week", April 20, 1963. It is interesting to note that in the United States, the business press is often far ahead of economists in recognizing, and even attempting to analyze, the latest developments in the capitalist economy.

[8] *Ibid.* The shrinkage of profit margins in the U.S. economy, beginning as early as 1950 and in spite of unprecedentedly rapid technological progress and slowly rising unemployment, is a complete mystery to bourgeois thought, both journalistic and academic. Since it is obviously impossible to pursue this subject within the confines of this essay, we must be content to refer the reader to a forthcoming work, tentatively entitled *Monopoly Capital*, by the present authors.

[9] U.S. Department of Commerce, *Survey of Current Business*, August 1962, p. 22.

Year	Net Direct Investment Capital Outflow ($ Millions)	Direct Investment Income ($ Millions)
1950	621	1.294
1951	528	1.492
1952	850	1.419
1953	722	1.442
1954	664	1.725
1955	779	1.975
1956	1.859	2.120
1957	2.058	2.313
1958	1.094	2.198
1959	1.372	2.206
1960	1.694	2.348
1961	1.467	2.672
Totals.	13.708	23.204

Sources: U.S. Department of Commerce, *Survey of Current Business*, November 1954, pp. 9, 13; August 1955, pp. 18, 20; August 1957, p. 25; August 1959, p. 31; August 1961, pp. 22–23; August 1962, pp. 22–23.

From the figures presented it will be seen that from 1950 through 1961, U.S. corporations were able to expand their direct foreign investments by $27.5 billion while at the same time taking in as income $9.5 billion more than they sent out as capital. Foreign investment, it seems, far from being a means of developing underdeveloped countries, is a most efficient device for transferring wealth from poorer to richer countries while at the same time enabling the richer to expand their control over the economies of the poorer.

But this is not the aspect of the matter which primarily concerns us at the moment. The point is that in the course of expanding their foreign assets and operations in this spectacular way, most of the corporate giants which dominate the U.S. economy have taken the road long since pioneered by Standard Oil. They have become, in "Business Week's" terminology, multinational corporations[10]. It is not enough that a multinational corporation should have a base of operations abroad; its true *differentia specifica* is that "its management makes fundamental decision on marketing, production, and research in terms of the alternatives that

[10] The term seems to have originated with David E. Lilienthal, Director of the Tennessee Valley Authority under Roosevelt and of the Atomic Energy Commision under Truman, and now Chairman of the Development and Resources Corporation which appears to be backed and controlled by the international banking house of Lazard Frères. A paper delivered by Mr. Lilienthal at the Carnegie Institute of Technology in April, 1960, and later published by Development and Resources Corporation, bears the title *The Multinational Corporation.*

are available to it anywhere in the world"[11]. This, of course, is what Standard Oil has been doing since roughly the beginning of the century. The difference is that what was then the exception has today become the rule.

IV

One cannot say of the giant multinational company of today that it is primarily interested, like the industrialist of the 19th century, in the export of commodities; or, like the banker of the early 20th century, in the export of capital. General Motors, for example, produces automobiles for the rapidly expanding European market not in Detroit but in Britain and West Germany; and it probably exports many more from its European subsidiaries to the underdeveloped countries than it does from the United States. In many cases, indeed, the foreign subsidiaries of U.S. companies are large-scale exporters to the U.S. market. In 1957, for example, the aggregate sales (excluding intercorporate petroleum sales) of direct-investment enterprises abroad was $32 billion. Of this amount, more than $3.5 billion (11 percent) was exported to the United States[12]. Considering that aggregate merchandise imports into the United States in 1957 came to $13.2 billion, it is a most striking fact that more than a quarter of this total was supplied by the foreign subsidiaries of U.S. companies. And as for capital export, we have already seen that U.S. multinational companies are on balance massive importers, not exporters, of capital.

What all this means is that one must beware of easy generalizations about the specifically economic interests of the leading actors on the imperialist stage. Their interests are in fact variegated and complex, often contradictory rather than complementary. Subsidiaries of a U.S. company in two foreign countries may both be in a good position to export to a third country. If one gets the business, the interests of the other will be damaged. Which should be favored? Or a certain company produces raw materials through a subsidiary in one country, processes the materials through another subsidiary in a second country, and sells the finished product through yet another subsidiary in the United States. Intercorporate prices can be so fixed as to allocate revenues and profits in any number of ways among the subsidiaries and countries. Which set of prices should actually be selected? These examples illustrate the kind of problem which the top managements of the multinational corporations have to solve every day; and about the only valid generalization one can make is that in every case they will seek a solution which maximizes the (long-run) profits of the enterprise as a whole. And this of course means that whenever necessary to the furtherance of this goal, the interests of particular subsidiaries and countries will be ruthlessly sacrificed. This is admitted with refreshing candor by the authors of the "Business Week" report already cited: "The goal, in the multinational corporation, is the greatest good for the

---

[11] "Business Week's" *Multinational Companies.*

[12] U.S. Department of Commerce, *U.S. Business Investments in Foreign Countries*, p. 3.

whole unit, even if the interests of a single part of the unit must suffer. One large U.S. manufacturer, for example, concedes that it penalizes some of its overseas subsidiaries for the good of the total corporation by forcing them to pay more than necessary for parts they import from the parent and from other subsidiaries. Says one of the company's executives: 'We do this in countries where we either anticipate or already face restrictions on profit repatriation. We want some way to get our money out.' "

A whole treatise could—and should—be written about the way the national interests of the subordinate countries fare under the regime of multinational corporations. Here we will have to be content with one illustration—a case which is less well known that it deserves to be but which we believe to be fully typical. One of the most important natural resources of the Caribbean area is bauxite. Jamaica, Surinam, British Guiana, and the Dominican Republic are all important producers, with operations being organized and controlled by a few U.S. and one Canadian corporate giants. Separate figures on the operations of these subsidiaries are not published. However, the U.S. Department of Commerce does report the profits accruing to U.S. mining companies on their operations in Western Hemisphere dependencies of European countries, at least 90 percent of which must be attributable to bauxite production in Jamaica, Surinam, and British Guiana. Adding a conservatively estimated figure for profits of the Canadian company, profits from operations in these three countries in 1961 were between $70 and $75 million on an investment estimated at between $220 and $270 million[13]. This profit rate of between 26 and 34 percent suggests, in the opinion of Philip Reno, that "this could well be among the most profitable U.S. investment structures in the world." However, this is only part of the story. Commerce Department figures give current costs of U.S. aluminium company operations in the three countries for 1957. Of the total of $81 million, no less than $31 million, or almost 40 percent, are classified under the heading of "Materials and Services." Since it is simply incomprehensible how materials and services could constitute so large a share of the costs of an extractive operation of this kind (more than 50 percent greater than wages and salaries), one can only conclude that this item is artificially padded to cover excessive payments to U.S. shipping, insurance, and other interests. In this manner, profits (and hence taxes) can be kept down and funds can be remitted from the colony to the metropolis.

Nor is even this all. The price of bauxite produced in the United States doubled in the two decades from 1939 to 1959, while the price of bauxite imported from Surinam and British Guiana remained almost the same throughout the whole period. This means that profits which should have been realized by the subsidiary

---

[13] All figures are from an article *Aluminium Profits and Caribbean People*, by Philip Reno, "Monthly Review," October 1963. Mr. Reno spent several months in British Guiana studying the operations of the aluminium companies.

companies and been taxed by the Surinam and British Guiana governments were in fact realized in the United States. At length, however, the parent aluminium companies, with one exception, began to alter this price structure, and here we get a revealing glimpse of the kind of considerations that determine the policy decision of the multinational corporations. In Philip Reno's words: "The prices set on bauxite from all the Caribbean countries except British Guiana did finally begin to rise a few years ago. The explanation lies with the law granting tax concessions to U.S. companies operating in other countries of this Hemisphere through what are called Western Hemisphere Trade Corporations. Instead of a 52 percent corporate income tax, West Hemisphere Trade Corporations pay the U.S. only 25 percent. By raising the price of bauxite, U.S. companies could now reduce their total income taxes. The price of bauxite began to rise for the first time in 20 years, except for British Guiana bauxite mined by Altd, Canada-based and unaffected by Western Hemisphere Trade Corporation maneuvers."

If this is a fair sample of how the underdeveloped countries are treated by the multinational companies, it does not follow that these giant enterprises are any more concerned to promote the national interests of the advanced countries, including even the one in which their headquarters are situated. Quite apart from particular actions—like the Ford Motor Company's remittance abroad of several hundred million dollars to buy out the minority interest in Ford of Britain at a time when the U.S. government was expressing serious concern about the state of the country's balance of payments—a plausible argument could be made that in the last fifteen years U.S. corporations have developed their foreign operations at the expense of, and often in direct competition with, their domestic operations and that these policies have constituted one of the causes of the lagging growth rate of the U.S. economy and hence of the rising trend of unemployment which is now perhaps the nation's number one domestic problem. Whether or not this is really the case—and it would probably be impossible to *prove* either that it is or isn't—it remains true that the decisions and actions of the multinational companies are taken solely with a view to promoting the interests of the companies themselves and that whatever effects, beneficial or injurious, they may have on the various countries in which they operate are strictly incidental.

V

Does this mean that the giant multinational companies have no interests in common on which they can unite? Are there no *general* policies which they expect their governments—and the governments of the dominant imperialist states are indeed theirs—to follow? The answer is that there are common interests and desired general policies, but that for the most part they are not narrowly economic in nature. The multinational companies often have conflicting interests when it comes to tariffs, export subsidies, foreign investment, etc. But they are absolutely united on two things: First, they want the world of nations in which they can operate to be as large as possible. And second, they want its laws and institutions to be favorable to the unfettered development of private capitalist enterprise. Or to put the point in another way, their ideal would be a world of nations in every one of which they could operate uninhibited by local obstacles to their making and freely disposing of maximum attainable profits. This means not only that they are opposed to revolutions which threaten to exclude them altogether from certain areas—as, for example, the Cuban Revolution excluded all U.S. corporations from Cuba—but also that they are adamantly opposed to all forms of state capitalism (using the term in its broadest sense) which might tend to hamper their own operations or to reserve potentially profitable areas of economic activity for the nationals of the countries in question[14]. Their attitude is well expressed in the *1962 Annual Report* of Standard Oil on which we have already drawn for illustrative material: "Both at home and abroad, a greater awareness is needed of the importance of private investment to economic progress. Some countries have shown a trend toward state enterprise both through government participation in new commercial ventures and through nationalization of established private businesses. The interest of these nations will best be served, however, by fostering societies that are based on those principles of free enterprise which have produced the outstanding economic development of many other nations. It is reassuring to see steps taken—such as the Hickenlooper Amendment to the Foreign Assistance Act of 1961—to ensure that economic assistance funds from the United States encourage a climate of progress by emphasizing the importance and protection of private investment in nations receiving aid from the United States." It would be wrong to think that the management of Standard Oil opposes government enterprise in the subordinate countries because of a naive belief that state action is identical with socialism. The explanation is much more rational: government enterprise and state action in these countries generally represent attempts on the part of the native bourgeoisies to appropriate for themselves a larger share of locally produced surplus at the expense of the multinational companies. It is only natural that such attempts should be resolutely opposed by the multinational companies.

The general policy which the multinational companies require of their government can thus be summed up in a simple formula: to make a world safe for Standard Oil. In more ideological terms, this means to protect the "free world" and to extent its boundaries wherever and whenever possible, which of course has been the proclaimed aim of U.S. policy ever since the promulgation of the "Truman Doctrine" in 1947. The negative side of the coin is anti-Communism. The necessary complement is the building up and maintenance of a tremendous global military machine.

---

[14] This does not mean, of course, that they oppose foreign governments' undertaking public works—roads, harbors, public health and education programs, etc., etc.—of a kind that will benefit their own operations. For such beneficent activities they even favor generous "foreign aid" from their own government.

All the major struggles going on in the world today can be traced to this hunger of the multinational corporations for maximum *Lebensraum*. And the connection usually has a direct, immediate, and visible aspect. We cite just two facts relative to Cuba and Vietnam where the essence of present-day imperialist policy can be seen in its clearest form. Under the heading "Standard Oil Co. (New Jersey)", in Standard and Poor's *Standard Corporate Descriptions*, dated July 24, 1961, we learn that "loss of $62,269,000 resulting from expropriation of Cuban properties in 1960 was charged to earned surplus." And from the same company's *1962 Annual Report* we learn that "Jersey continues to look for attractive opportunities both in areas where we now operate and in those where we do not," and that the following are among the measures being taken to implement this policy: "A refinery in which the company will have majority interest is under construction in Malaya, and affiliates have part interests in a refinery under construction in Australia and one that is being planned for Vietnam".

Losses in Cuba, plans for South Vietnam: what more eloquent commentary could there be on the struggles now going on in and around those two little countries on opposite sides of the globe?

Fritz Behrens
D.D.R.

DER DURCHSCHNITTSLOHN BEI OPTIMAL ERWEITERTER REPRODUKTION

Der Durchschnittslohn ist nicht nur ein Stimulant für die Arbeitsproduktivität in ihm kommt auch die Effektivität der sozialistischen Produktionsverhältnisse zum Ausdruck. Seine Bewegungen als Nominallohn, aber auch als Reallohn sind außerdem ein Element des Prozesses der erweiterten Reproduktion der sozialistischen Produktionsverhältnisse und ihrer materielltechnischen Basis. Das ökonomische Gesetz, das die Verteilung der Konsumtionsmittel für die große Masse der Werktätigen im Kapitalismus regelt, ist das Lohngesetz, das auf dem Wert der Arbeitskraft als Ware beruht und bewirkt, daß sich der Anteil der Arbeiterklasse an dem von ihr geschaffenen Wertprodukt in Schranken hält, die der Ververtung des Kapitals angemessen sind. Im Sozialismus tritt an die Stelle des stummen Zwanges der ökonomischen Verhältnisse, der im Kapitalismus auf die Arbeiter einwirkt und sie unter die Botmäßigkeit ihrer Ausbeuter zwingt, die freiwillige Disziplin der Werktätigen. Diese beruht auf einer neuen Arbeitsmoral. Bis aber—wie Marx es ausgedrückt hat—die Arbeit „selbst das erste Lebensbedürfnis" der Menschen geworden ist oder solange die Gesellschaft — um noch einmal mit Marx zu sprechen — noch „in jeder Beziehung, ökonomisch, sittlich, geistig — mit den Muttermalen der alten Gesellschaft" behaftet ist, „aus deren Schoß sie herkommt", so lange wird auch die sozialistische Gesellschaft nicht auf einen ökonomischen Zwang verzichten können, wenn dieser ökonomische Zwang auch grundlegend vom stummen Zwang der ökonomischen Verhältnisse im Kapitalismus verschieden ist. Unter sozialistischen Bedingungen beruht der ökonomische Zwang in der Verteilung nach der Leistung und führt zu der unmittelbaren Interessiertheit der Werktätigen am Ergebnis der materiellen Produktion, die untereinander auch durch die Verknüpfung von Lohnentwicklung und Steigerung der Arbeitsproduktivität erreicht wird.

Die Verteilung nach der Leistung ist eine objektive Notwendigkeit, bis die gesellschaftlichen Produktivkräfte das Entwicklungsniveau erreicht haben werden, das eine Verteilung nach den Bedürfnissen ermöglicht. Bis dahin ist die Verteilung nach der Leistung der notwendige Verteilungsmodus der materiellen Güter, weil sie den Anteil eines jeden Werktätigen am Produkt der gesellschaftlichen Arbeit von der Art und vom Grad seiner Teilnahme an der gesellschaftlichen Arbeit direkt abhängig macht und auf diese Weise persönliche Interessen mit denen der Gesellschaft verbinde. Der Lohn wird durch objektive ökonomische Gesetze bestimmt, die nicht ohne Schädigung der Volkswirtschaft ignoriert werden dürfen. Wird das Gesetz der Verteilung nach der Leistung nicht richtig ausgenutzt oder gar gegen dieses Gesetz verstoßen, so führt das zu einer Verletzung des erforderlichen Verhältnisses zwischen Lohnentwicklung und der Entwicklung der Arbeitsproduktivität und das zeigt sich u.a. über den Lohnanteil der Selbstkosten in der Rentabilitätsentwicklung der Betriebe, aber vor allem in einer Diskrepanz zwischen Warenfonds und Kauffonds. Alle diese — und andere — Symptome bringen Störungen im Prozess der gesellschaftlichen Reproduktion zum Ausdruck.

Soll der Prozess der erweiterten Reproduktion störungsfrei ablaufen und sollen zugleich optimale Ergebnisse der ökonomischen Tätigkeit erzielt werden, so muß ein optimales Entwicklungsverhältnis zwischen Durchschnittslohn und Arbeitsproduktivität geplant und verwirklicht werden.

Soll erweiterte Reproduktion möglich sein, so muß die Abteilung I der gesellschaftlichen Produktion ein Produkt erzeugen, das nicht nur die verbrauchten Produktionsmittel ersetzt, sondern auch für die Erweiterung der Produktion der erforderlichen Produktionsmittel liefert. Die Abteilung II der gesellschaftlichen Produktion muß ein Produkt erzeugen, das wertmäßig und stofflich die individuellen und gesellschaftlichen Bedürfnisse der Werktätigen befriedigt. Daraus folgt, daß das Wertprodukt der Abteilung I größer sein muß als der Wert der verbrauchten Produktionsmittel.

Im Gegensatz zur einfachen Reproduktion, wo Lohn und Reineinkommen der Gesellschaft ganz in die individuelle und gesellschafliche Konsumtion eingehen, so daß das Produkt der Abteilung I der gesellschaftlichen Produktion gleich dem Lohn und dem Reineinkommen der Abteilung II ist, geht bei erweiterter Reproduktion nur ein Teil des Reineinkommens in die individuelle und gesellschaftliche Konsumtion ein, weil ein Teil für die Akkumulation verwendet werden muß. Das Produkt der Abteilung II ist daher nur gleich dem Lohn plus einem Teil des Reineinkommens der Gesellschaft, dem Teil, der in die Konsumtion eingeht.

Es gibt aber bekanntlich zwei Arten der erweiterten Reproduktion:

1. die extensiv erweiterte Reproduktion
2. die intensiv erweiterte Reproduktion.

Gemeinsames Merkmal jeder erweiterten Reproduktion ist, daß ein Teil des gesellschaftlichen Reineinkommens akkumuliert und investiert wird. Extensiv erweiterte Reproduktion erweitert nur das Feld der Produktion. Es werden zusätzliche Arbeitskräfte beschäftigt, ohne daß die Arbeitsproduktivität steigt. Intensiv erweiterter Reproduktion dagegen ist mit wissenschaftlich-technischem Fortschritt verbunden und führt

1. zu einer Steigerung der Arbeitsproduktivität,
2. zu einer Senkung des Gesamtaufwandes an Arbeit bzw. der Selbstkosten.

Maximal erweiterte Reproduktion ist dadurch gekennzeichnet, daß der gesamte Zuwachs an Nettoprodukt akkumuliert wird, d.h.

$$\Delta A = \Delta N \tag{1.1}$$

Daraus folgt

$$A_n = A_0 + \Delta N \tag{1.2}$$

und

$$K_n = K_0. \tag{1.3}$$

Minimal erweiterte Reproduktion hat mit einfacher Reproduktion gemeinsam, daß die Konsumtion maximal. erhöht wird, ohne daß die Akkumulation sich verändert. Sie unterscheidet sich von der einfachen Reproduktion dadurch, daß akkumuliert wird und ist gekennzeichnet durch

$$\Delta K = \Delta N \tag{2.1}$$

Daraus folgt

$$K_n = K_0 + \Delta N \tag{2.2}$$

und

$$A_n = A_0 \tag{2.3}$$

Optimal erweiterte Reproduktion ist eine solche, bei der ein maximaler Zuwachs des Nettoproduktes durch optimale Kombination des Zuwachses von Akkumulation und Konsumtion erzielt wird. Die Akkumulationsrate ist niedriger als bei maximal erweiterter Reproduktion und die Konsumtionsrate ist niedriger als bei minimal erweiterter Reproduktion. Sie ist gekennzeichnet durch

$$\Delta N = \Delta A + \Delta K \tag{3.1}$$

$$A_n = A_0 + \Delta A \tag{3.2}$$

$$K_n = K_0 + \Delta K \tag{3.3}$$

Ein wesentliches Merkmal optimal erweiterter Reproduktion als beste Verbindung von Erhöhung der Akkumulation und wachsender Konsumtion ist eine hohe Effektivität der Akkumulation. Das Wachstum des Nettoprodukts hängt nicht nur von der Rate der Akkumulation, sondern auch von ihrer Effektivität ab. Wenn wir die Rate der Akkumulation mit a, ihre Effektivität mit E bezeichnen, ist

$$\frac{\Delta N}{N} = \frac{A}{N} \cdot \frac{\Delta N}{A} \tag{4}$$

$$= a \cdot E$$

Der Ausdruck (4) ist sehr global und verbirgt als Aggregatausdruck verschiedene Effekte struktureller und anderer Art, die bei näherer Analyse aufgezeigt werden müssen. Doch kommt es uns hier nur darauf an, zu zeigen, daß neben der Rate der Akkumulation ihre Effektivität ein wesentlicher Faktor des Zuwachses des Nettoproduktes ist. Die Effektivität der Akkumulation aber hängt außer vom technisch-organisatorischen Niveau der Wirtschaft, d.h. außer vom Niveau der Arbeitsproduktivität und den Selbstkosten, in entscheidendem Maße von der

Entwicklung der materiellen Konsumtion ab, für die wieder die Entwicklung des Durchschnittslohnes von entscheidender Bedeutung ist.

Diese Überlegungen sind — wie leicht einleuchtet — von großer aktueller Bedeutung.

So richtig es ist, daß die Akkumulationsrate umso höher sein muss, je niedriger die Effektivität der Akkumulation ist, so richtig ist es, daß die Höhe der Akkumulationsrate von der Konsumtionsrate mitbestimmt wird, weil die Entwicklung der Konsumtion ein wichtiges Moment der Effektivität der Akkumulation ist. Je niedriger die Akkumulationsrate ist, umso höher muß nicht nur, sondern *kann* auch die Effektivität der Akkumulation sein.

Hier ist jedoch eine doppelte Einschränkung zu machen.

Erstens gilt der letzte Satz nur, wenn die Konsumtion wächst, und nicht auch dann, wenn die Akkumulationsrate infolge Fehler in der Planung niedrig ist. Wenn ein unverhältnismäßiger Teil des Nettoproduktes für unproduktive Zwecke verwendet wird, kann sich hinter einer hohen Rate der Akkumulation eine niedrige Rate für materielle Investitionen verbergen. Eine übermäßige Erweiterung des Grundfonds außerhalb der materiellen Produktion — Bau von Kulturhäusern, von kostspieligen Verwaltungsgebäuden usw. — geht auf Kosten der erweiterten Reproduktion der materiellen Produktivkräfte. Außerdem kann eine Diskrepanz zwischen Akkumulation und Investition entstehen, wenn die Umlauffonds infolge nicht bedarfsgerechter Produktion anwachsen. In beiden Fällen: Verschwendung von Nettoprodukt und Aufblähung der Umlauffonds ist die effektive Akkumulationsrate niedrig, ohne daß die Konsumtion wächst. Daraus folgt aber, daß auch die Effektivität der Akkumulation niedrig ist, weil keine materiellen Anreize zu ihrer Erhöhung erwachsen.

Dazu kommt *zweitens*, daß eine wachsende und hohe Konsumtion nur dann zu einer hohen Effektivität der Akkumulation führt, wenn sie auch mit einer Höhung des kulturellen Niveaus der Werktätigen verbunden ist, so daß durch vertiefte Einsicht in die ökonomische Notwendigkeit die Einheit von individuellen und gesellschaftlichen Interessen im Handeln der Werktätigen wächst.

Das Niveau der Effektivität der Akkumulation hängt von zwei Grundfaktoren ab:

erstens vom technisch-organisatorischem Niveau der durch die produktive Verwendung des akkumulierten Nettoprodukts neu geschaffenen Anlagen und

zweitens vom technisch-organisatorischem Niveau der bereits vorhandenen produktiven Anlagen vermittelst derer die produktiven Investitionen durchgeführt und die erforderlichen zusätzlichen Umlauffonds produziert werden.

Das technisch-organisatorische Niveau beinhaltet aber eine gesellschaftliche Komponente, die im Sozialismus als materielle Interessiertheit der Werktätigen an der Steigerung der Arbeitsproduktivität und Senkung der Selbstkosten, d.h. an der Entwicklung der materiellen Produktion zum Ausdruck kommt. Die materielle Interessiertheit kann aber ohne eine mit der Akkumulation verbundene Konsumtion nicht zur Entfaltung kommen. Deswegen kommt der Ermittlung einer zwischen der minimal und der maximal erweiterten Reproduktion liegenden optimal erweiterten Reproduktion eine große Bedeutung zu. Hierfür ist die Bestimmung des optimalen Zuwachses des Durchschnittslohnes aber grundlegend, denn die Arbeit der Werktätigen in der materiellen Produktion und besonders der Produktionsarbeiter neben dem ingenieur- und leitungstechnischem Personal ist für die ständige Steigerung der Arbeitsproduktivität entscheidend.

Die Bedeutung des optimalen Zuwachses des Durchschnittslohnes ist also keine rechnerische Angelegenheit, sondern ergibt sich aus wirtschaftspolitischen Entscheidungen, die auf ein optimales Verhältnis des Zuwachses der Konsumtion und der Akkumulation bezwecken. Da unter sozialistischen Bedingungen der Produktion eine Erweiterung der Produktion, somit Akkumulation und Investition der akkumulierten Mittel nicht um des Profites Willen und natürlich auch nicht um der Erweiterung des Produktes Willen stattfindet, sondern zwecks Erweiterung und Verbesserung der Konsumtion, muß die Festlegung der optimalen Akkumulationsrate mit der Festlegung der optimalen Konsumtionsrate identisch sein. Wie sich die einfache Reproduktion verbietet, weil sie den gesellschaftlichen Fortschritt unmöglich macht, so verbietet sich die maximal erweiterte Reproduktion, wenn man von historisch begründeten Ausnahmefällen absieht, weil die Effektivität der Akkumulation von Faktoren abhängig ist, unter denen der materielle Anreiz auf der Grundlage eines wachsenden materiellen und kulturellen Niveaus der Werktätigen eine ganz entscheidende Rolle spielt. Soll eine maximale Effektivität der Akkumulation erreicht werden, so muß die Akkumulation von einer entsprechenden Zunahme der individuellen und gesellschaftlichen Konsumtion begleitet sein, die einen ausreichenden Anreiz zur Ausnutzung aller Reserven zur Steigerung der Arbeitsproduktivität und Senkung der Selbstkosten bietet. Mit anderen Worten: aus der Tatsache, daß der Zuwachs der Nettoproduktes außer von der Rate der Akkumulation von der Effektivität der Akkumulation bestimmt wird, folgt, daß ein maximaler Zuwachs des Nettoprodukts auf die Dauer nur erreicht werden kann, wenn mit einer Akkumulationsrate, die niedriger als die maximale Rate ist, ein solcher Zuwachs der Konsumtion verbunden wird, bei der die Effektivität der Akkumulation maximal wird. Das ist offensichtlich eine optimal erweiterte Reproduktion.

Die optimal erweiterte Reproduktion ist nur dann zu erreichen, wenn die Planung der volkswirtschaftlichen Entwicklung sich auf durch empirische Analyse gewonnene Kennziffern stützt, die die objektiven ökonomischen Prozesse möglichst exakt und umfassend widerspiegeln und wenn die entscheidenden Kennziffern des Nutzeffektes der gesellschaftlichen Arbeit und der Effektivität der Akkumulation in das Programm des Kampfes der werktätigen Massen um die Erfüllung der Pläne aufgenommen werden.

Die optimale erweiterte Reproduktion setzt aber vor allem ein optimales Entwicklungsverhältnis zwischen Durchschnittslohn und Arbeitsproduktivität voraus.

Wenn die Herstellung — Planung und Verwirklichung — eines solchen Verhältnisses auch auf wirtschaftpolitischen Entscheidungen und Maßnahmen beruht, so setzen diese ebenfalls eine genaue Analyse der realen Entwicklungsprozesse voraus. Im folgenden soll nur ein bestimmter Aspekt der Planung dieses Entwicklungsverhältnisses beleuchtet werden. Dabei muß auf die Analyse einer Reihe Voraussetzungen und Nebenumstände hier verzichtet werden. Das ist an anderer Stelle geschehen. Jedoch muß bemerkt werden, daß diese Frage im folgenden nur unter der Annahme behandelt werden soll, daß die Preise nicht mit dem durch Steigerung der Arbeitsproduktivität bewirkten Sinken des Wertes ebenfalls sinken oder — anders ausgedrückt — ,daß die Erhöhung des Reallohnes nur durch Erhöhung des Nominallohnes erzielt wird.

Wir gehen aus von den Formeln

$$lk = \frac{l}{Ap} \tag{5.1}$$

und

$$lk' = \frac{l'}{Ap'} \tag{5.2}$$

Aus (5.2) folgt

$$\frac{1+\frac{\Delta l}{l_0}}{1+\frac{\Delta Ap}{Ap_0}} = 1-\frac{\Delta lk}{lk_0} = lk' \tag{6}$$

$$\frac{l_0+\Delta l}{Ap_0+\Delta Ap} = 1-\frac{\Delta lk}{lk_0} \cdot \frac{l_0}{Ap_0} = lk_n \tag{7}$$

Es ist weiter

$$v = \frac{\frac{\Delta l}{l_0}}{\frac{\Delta Ap}{Ap_0}} \tag{8.1}$$

$$\bar{v} = \frac{\frac{\Delta l}{l_n}}{\frac{\Delta Ap}{Ap_n}} \tag{8.2}$$

Daraus folgt

$$\frac{v}{\bar{v}} = \frac{l_n}{l_0} : \frac{Ap_n}{Ap_0} = lk' \tag{9}$$

Somit ist

$$\frac{l'}{Ap'} = \frac{v}{\bar{v}} = lk' \tag{10}$$

Diese Ausdrücke bringen aber nur sehr allgemein das Entwicklungsverhältnis von Durchschnittslohn und Arbeitsproduktivität zum Ausdruck. Sie lassen zwar erkennen, daß dieses Entwicklungsverhältnis die Lohnkosten beeinflußt, ohne aber die Entwicklung ihrer wesentlichen Bestandteile zu zeigen. Wenn wir den Ausdruck lk bzw. lk' präzisieren, haben wir

$$lk_n = \frac{B_n - Pm_n}{(1+m_n)Q_n} \tag{11}$$

$$= \frac{N_n}{(1+m_n)Q_n}$$

$$lk' = \frac{B_n - Pm_n}{(lk_0 + m_n lk_0)Q_n} \tag{12}$$

Daraus ergibt sich für die Rate des Reineinkommens

$$m_n = \frac{(B_n - Pm_n!)Ap'}{l' \cdot lk_0 \cdot Q_n} - 1 \tag{13}$$

$$= \frac{N_n Ap'}{l' \cdot lk_0 \cdot Q_0} - 1$$

Wir haben außerdem

$$a = l - \frac{\frac{K_n}{L_n}}{G_m} \tag{14}$$

$$k = l - \frac{\frac{A_n}{L_n}}{G_m} \tag{15}$$

Wenn wir statt $G_m = 1+m$ Ausdruck (14) in (14.1) und (15.1) einsetzen, erhalten wir

$$a = l - \frac{\frac{K_n}{L_n}}{\frac{N_n \cdot Ap'}{l' \cdot lk_0 \cdot Q_n}} \tag{14.1}$$

$$\frac{l'}{Ap'} = \frac{a-1}{\frac{K_n Q_n lk_0}{L_n N_n}} \tag{16}$$

$$k = 1 - \frac{\frac{A_n}{L_n}}{\frac{N_n \cdot Ap'}{l' \cdot lk_0 \cdot Q_n}} \tag{15.1}$$

$$\frac{l'}{Ap'} = \frac{k-1}{\frac{A_n Q_n lk_0}{L_n N_n}} \tag{17}$$

Die beiden Ausdrücke (16) und (17) lassen sich für die volkswirtschaftliche Planung verwenden. Um eine Formel für die betriebliche Planung zu erhalten, bezeichnen wir das betriebliche Reineinkommen mit

$$\lambda = B - Sk \tag{18.1}$$

und definieren

$$\lambda' = \frac{\lambda}{L} \tag{18.2}$$

Wir bezeichnen weiter

$$S = Sk - L, \tag{18.3}$$

so daß ist

$$B - S = L + \lambda \tag{18.4}$$

Wir haben dann

$$lk = \frac{B-S}{(1+\lambda')Q} = \frac{L+\lambda}{(1+\lambda')Q} \tag{19}$$

und erhalten weiter

$$lk_n = \frac{B_n - S_n}{(1+\lambda'_n)Q_n} = \frac{L_n + \lambda_n}{(1+\lambda'_n)Q_n} \tag{20}$$

$$\lambda'_n = \frac{(B_n - S_n)Ap'}{l'lk_0Q_n} - 1 = \frac{(L_n + \lambda_n)Ap'}{l'lk_0Q_n} - 1 \tag{21}$$

$$lk' = \frac{(B_n - S_n)}{(lk_0 + \lambda_n lk_0)Q_n} = \frac{L_n + \lambda_n}{(lk_0 + \lambda'_n lk_0)Q_n} \tag{22}$$

Während das gesellschaftliche Reineinkommen ist

$$R = L \cdot \lambda$$

ist das betriebliche Reineinkommen

$$\lambda = L \cdot \lambda'$$

Fritz Behrens

THE AVERAGE WAGE RATE UNDER AN OPTIMUM EXTENDED REPRODUCTION

Summary

The average wage rate is an important factor of efficiency of extended reproduction under socialist production relations. It is based on the cardinal principle of compensation according to performance; this governs the distribution of consumption goods and leads to a direct interest of working people in the results of their work, thus establishing a link between individual and social interests. This principle has to be strictly observed, if the social reproduction is to develop without disturbances. And if it is to yield optimum results, an optimum ratio between the growth in the average wage rate and the productivity of labour has to be planned and realized.

Three kinds of extended reproduction may be distinguished—maximum, minimum and optimum— according to which part of the increment in the net output is going to the accumulation fund. An optimum reproduction is characterized by an optimum combination of increases in accumulation and consumption; this results in a higher efficiency of accumulation.

The lower is the rate of accumulation, the higher has to be its efficiency, and under certain conditions, this may be so, namely if no excessive part of accumulation is devoted to unproductive investment and the rising consumption strengthens the links between individual and social interests.

The efficiency of accumulation is determined by the technical and organizational level of the economy. An important factor which determines that level under socialist production relations, is the material interest of workers in raising their labour productivity and reducing the cost of production. This, however, cannot be obtained unless accumulation is accompanied by a growth in consumption, and for this purpose an optimum increase in the average wage rate has to be fixed.

Thus the importance for the socialist system of an optimum growth in the average wage rate, is not an accounting problem, but lies in the policy decisions aimed at an optimum relation between the growth in consumption and accumulation to be obtained which leads to an optimum extended reproduction. For, given the dependence of growth in the net output on both the rate of accumulation and its efficiency, a maximum growth can be permanently achieved only if the rate of accumulation, being lower than a maximum, is accompanied by a rising consumption which secures a maximum efficiency of that accumulation.

An optimum extended reproduction can be arrived at only if the planning of growth is based on indices which are empirically tested and do reflect the objective economic processes as exactly and comprehensively as possible. But, first of all, it implies an optimum relation between the growth in the average wage rate and the labour productivity to be fixed. Some aspects of fixing that relation are illustrated in a set of mathematical formulae which may be applied to the national economy as a whole and to a single enterprise respectively.

Czesław Bobrowski
Pologne

LES PAYS SOUS-DÉVELOPPÉS AU SEUIL DE LA PLANIFICATION

Dans la mesure où la publication d'un plan pluriannuel peut être considérée en tant qu'un acte significatif, la planification est en voie de devenir monnaie courante dans les pays sous-développés. En effet, la liste des plans pluriannuels publiés par les gouvernements respectifs comporte aujourd'hui plus de 60 postes, dont une dizaine concerne des territoires encore dépendants. A l'appel ne manque qu'une dizaine de pays. Il s'agit là d'un phénomène très récent. En effet, si l'on fait abstraction des territoires dépendants, rares sont les plans publiés à une date antérieure à 1961 et tout à fait exceptionnels ceux faisant suite à un autre plan pluriannuel.

Au cours de la décennie 1950–60 l'adoption par un pays sous-développé d'un plan pluriannuel équivalait à un choix fondamental de l'orientation de toute sa politique économique — on voudrait presqu'écrire: à un choix de la voie de développement. Pour illustrer cette affirmation il suffit je crois, de citer le cas de l'Inde ou celui de la République Arabe Unie. La planification pluriannuelle était précédée, accompagnée ou suivie d'une série de mesures visant à modifier les structures et les institutions de ces pays (il ne s'agit pas là de juger l'éfficacité de ces mesures ni leur bien-fondé mais simplement de constater le lien organique entre le passage à la planification pluriannuelle et l'adoption des mesures de cette sorte). En particulier (aussi bien dans les deux cas cités à titre d'exemple que dans d'autres pays ayant opté pour la planification dans la décennie passée) les plans pluriannuels en règle générale précisaient avec plus ou moins de netteté le dessein d'élargir le secteur d'Etat ou tout au moins de renforcer le dirigisme. Ils ne se limitaient pas à tracer l'avenir en même temps souhaitable et jugé possible mais s'attachaient également à indiquer les moyens à mettre en oeuvre pour atteindre les objectifs des plans.

En est-il de même en ce qui concerne les plans publiés ces temps derniers? Même une analyse très sommaire oblige à répondre par la négative. Et l'étude plus détaillée ne pourrait que renforcer ces conclusions. Arrêtons-nous à certaines caractéristiques faciles à saisir et cependant significatives de la soixantaine de plans figurant sur notre liste, en envisageant notamment l'horizon temporel et l'étendue des plans en question.

En ce qui concerne les plans des territoires dépendants le tableau est d'une simplicité extrême: il s'agit exclusivement de plans sectoriels (secteur public) et d'un horizon temporel généralement modeste puisque jamais supérieur à cinq ans et le plus souvent inférieur à ce délai. On voit très bien les idées, les besoins et l'ambiance ayant présidés à l'établissement de cette sorte de plans. Du point de vue technique il s'agissait de dépasser le cadre trop rigide des budgets annuels et d'assurer une plus grande complémentarité des investissements publics ainsi que des meilleurs conditions de leur exécution. Dans la mesure où ces investissements ont des répercussions sur la cadence de la croissance générale ou sur celle d'un secteur particulier, des objectifs quelque peu plus vastes que ceux mentionnés plus haut pouvaient dans certains cas également entrer en jeu. Du point de vue politique ces plans obéissaient au souci de l'administration coloniale de se tirer honorablement de l'affaire difficile que représentent toujours les dernières années de décolonisation. En même temps la perspective de la décolonisation toute proche imposait tout naturellement des limites à l'extension de ces plans dans le temps et l'espace. Néanmoins pour les futurs pays indépendants, quel que soient les changements d'orientation de la politique économique à venir, ces plans sectoriels et à relativement court terme représentent une étape d'apprentissage qui n'est pas sans valeur appréciable.

C'est un tableau tout différent que nous découvrons en nous tournant vers les plans des pays indépendants. Dans ce groupe les plans limités au secteur public ne représentent qu'un peu plus d'un tiers de l'ensemble. L'horizon temporel est le plus souvent supérieur à cinq ans—dans certains cas il s'approche de dix ans quand il ne les atteint pas. Il est difficile de ne pas trouver ces constatations significatives. Certes, l'allongement de l'horizon temporel n'est pas seulement un objectif qui séduit tout planificateur mais aussi une tendance caractéristique de l'étape actuelle de la planification aussi bien dans les pays socialistes que capitalistes. Et il ne fait aucun doute que la planification sectorielle même étendue à l'ensemble du secteur public représente en quelque sorte un stade inférieur de la planification. Celle-ci ne peut porter tous ses fruits qu'au moment où l'économie dans son ensemble devient soumise—d'une manière ou d'une autre—à l'action du plan, ou tout au moins devient l'objet d'analyse et de réflexion pour le planificateur. Cependant, il est aussi évident—la constatation est presque trop élémentaire—que l'extension du plan à l'ensemble de l'économie ainsi que l'allongement de son horizon temporel comportent des aléas d'envergure.

Ces aléas apparaissent en même temps dans le domaine de l'information (ou si l'on préfère dans celui de la documentation) nécessaire pour l'établissement d'un plan et dans celui des moyens à mettre en œuvre pour réaliser ses objectifs. Dans la mesure où l'élargissement de l'horizon temporel permet de considérer comme variables politiques certaines grandeurs qui à court terme ne sont que des contraintes et données exogènes; dans la mesure où un laps de temps plus long permet de remplacer par l'échellonement des objectifs dans le temps le choix des priorités ou le dosage—combien difficile—des moyens alloués à divers objectifs concurrentiels; dans la mesure enfin, où l'extension du plan à l'ensemble de l'économie permet des choix plus rationnels et des impulsions plus variées et plus riches, il n'y a pas de doute que l'extension des plans dans l'espace et le temps rend beaucoup plus aisé l'établissement de ces plans en eux-même cohérents et pleins d'intérêt. Mais il est infiniment plus délicat de les réaliser.

Ce n'est pas un hasard si les pays socialistes ont suivi une voie caractérisée par l'extension graduelle des plans dans le temps et dans l'espace et s'il ne s'attachent que depuis peu à mettre en oeuvre des plans perspectifs tandis que l'étape des plans quinquennaux fut toujours précédée par une étape d'apprentissage (sous la forme d'établissement des plans sectoriels ou des plans généraux bi- ou tri-annuels). Et dans la mesure où la France peut être à juste titre considérée comme celui des pays capitalistes qui s'est engagée le plus loin dans la voie de la planification, il n'est pas moins significatif de constater que dans ce pays le délai de quatre ans ne sera prolongé à cinq ans qu'à partir du plan prochain (c'est-à-dire le cinquième du nombre).

Faut-il croire que ces pays qui d'emblée ont élaboré des plans généraux pluriannuels, et ceci en dépit des lacunes patentes soit dans le domaine de l'information soit dans celui des moyens d'implantation des plans, ont voulu sauter "l'étape d'apprentissage"? En règle générale une telle conclusion serait fausse. Dans beaucoup, sinon dans la plupart des cas, les plans en question ne sont pas à vrai dire destinés à être réalisés. Dans d'autres ils serviront peut-être de point de départ pour l'élaboration de plans plus limités dans le temps et l'espace, mais en toute circonstance ils ne seront pas directement appliqués. Certains de ces plans ne sont en réalité que des très vagues programmes—s'ils emploient le langage de planification, ils n'en ont pas emprunté la méthode. Ceux qui ont été construits à l'aide de méthodes plus rigoureuses se bornent en règle générale à définir les objectifs ultimes pour l'ensemble de la période planifiée et n'évaluent que d'une manière très globale les moyens nécessaires pour arriver au taux de croissance souhaité. Ils ne s'attachent ni à préciser "le sentier" conduisant aux objectifs ultimes (les étapes du plan) ni à prendre position à l'égard des problèmes institutionnels dont dépendent les chances d'implantation d'un plan donné. En dépit des apparences nous avons donc affaire à quelque chose de très différent de cette notion d'un plan général pluriannuel à laquelle nous ont habitué aussi bien l'économie socialiste que les tentatives de planification les plus sérieuses dont la dernière décennie fut témoin dans certains pays capitalistes développés et sous-développés.

La majeure partie des plans en question doit en effet sa naissance à des conditions politiques au sens étroit de ce terme. Il y en a qui sont le produit de l'indépendance récemment acquise: une tentative de répondre au plus vite à la demande des masses par une promesse d'un avenir radieux ou une tentative de mobiliser ces masses autour d'objectifs ambitieux. Il y en a dont l'élaboration résulte directement du changement d'attitude des Etats-Unis à l'égard de la planification dans les pays sous-développés (on sait qu'au lieu de la combattre les Etats-Unis considèrent aujourd'hui la planification—ou tout au moins ses apparences—comme une condition essentielle pour obtenir l'aide américaine). On en trouvera même qui paraissent être une manoeuvre éléctorale plutôt qu'autre chose ou au contraire une tentative des techniciens d'entraîner dans une certaine voie les gouvernants conservateurs bien décidés à ne pas la suivre. Dans ces conditions ce n'est qu'une fraction infime des plans pluriannuels publiés récemment qui représente une option réelle analogue à ce qui a eu lieu dans des pays comme l'Inde ou la République Arabe Unie.

A certains égards les progrès de la planification dans les pays sous-développés sont donc plus apparents que réels et on peut se demander même si plusieurs pays qui ne sont arrivés qu'au stade de la planification sectorielle ne sont pas en réalité plus avancés dans la voie de la planification que les nombreux pays ayant publié ces temps derniers des plans généraux pluriannuels. Il semble en tout cas légitime d'en tirer deux conclusions: 1) que la règle générale d'après laquelle le développement d'une planification efficace doit passer par une étape de planification sectorielle à horizon temporel assez limité, reste valable sauf cas vraiment exceptionnels; 2) que le problème de la généralisation de la planification de vaste étendue et de grande intensité est loin d'être tranché dans les pays sous-développés.

Ceci nous amène au problème des conditions dans lesquelles va s'effectuer dans les pays sous-développés ce processus inévitable que l'extension et l'approfondissement de la planification représente de l'avis quasi-unanime. Plus précisement il s'agit des conditions *intellectuelles* de ce processus—abstraction faite des facteurs politiques externes et internes qui, bien que décisifs, dépassent le cadre de nos réflexions. A deux égards ces conditions diffèrent d'une manière tranchée de celles que les planificateurs des pays socialistes et ceux des pays occidentaux ont connu en leur temps: 1) il existe aujourd'hui une expérience en la matière—riche, vaste et variée—puisqu'englobant plusieurs pays et embrassant une longue période: 2) ces dernières années ont apportées une moisson abondante de travaux théoriques concernant soit les problèmes du sous-développement, soit même le problème précis de la planification dans les pays sous-développés[1].

L'accès à l'expérience étrangère en matière de planification semble en général assez bien assuré quoique d'une manière assez inégale. En effet, les moyens de transmissions sont nombreux[2], surtout en ce qui concerne les pays capitalistes développés et en premier lieu les anciennes puissances coloniales. Mais les expériences socialistes ainsi que celles des pays sous-développés pratiquant la plani-

---

[1] Les travaux respectifs sont trop nombreux et trop connus pour être cités ici. Mentionnons cependant la série des exposés fait en 1962 au Congrès tenu à Genève sous les auspices des Nations Unies par H. B. Chenery, R. Frisch, M. Kalecki, K. S. Krishnaswamy, P. Massé, G. F. Papanek, I. H. A. Rahman, J. Tinbergen et d'autres.

[2] Formation professionnelle acquise à l'étranger, enseignement des universitaires provenant des pays développés, la participation des experts étrangers dans l'élaboration des plans, les stages dans les organes de planification étrangers et jusqu'à l'aide matérielle qui—même quand elle n'est pas accompagnée de pressions—devient aisément un outil de prosélytisme conscient ou involontaire.

fication depuis un certain temps ne sont plus ignorées. Donc ce n'est pas le problème de *connaissance* qui se pose en premier lieu mais celui de *l'utilité* et de *l'utilisation* des expériences étrangères. Or, à cet égard le cas de la planification socialiste est en quelque sorte l'inverse de celui de la planification occidentale. Les problèmes que les planificateurs socialistes ont eu à affronter sont dans une très vaste mesure analogues à ceux que les planificateurs des pays sous-développés devront résoudre un jour. En effet, parmi les pays socialistes seules la Tchécoslovaquie et l'Allemagne Démocratique étaient industrialisées au départ de l'économie planifiée. Tous les autres pays ont eu à résoudre le problème du sous-développement, agravé d'ailleurs par des dommages extrèmement importants subis du fait de l'intervention étrangère, des mouvements contre-révolutionnaires et des guerres dévastatrices. Cependant, la transformation économique des pays socialistes s'est accomplie dans un cadre institutionnel et politique qui n'est propre qu'à ces pays. Le système de planification socialiste est à tel point intégré dans un régime économique et politique précis que de le copier en dehors de ce cadre apparaît impensable à prime abord. Pour être utilisée dans les pays sous-développés, l'expérience des pays socialistes doit être interprétée et en premier lieu décomposée en deux catégories d'éléments: ceux d'une portée plus générale et ceux qui ne sont valables que dans le cadre socialiste[3]. Or, ce travail considérable n'en est qu'à ses débuts.

Les expériences occidentales de planification sont liées au cadre institutionnel, aux structures et conditions économiques précises d'une manière moins apparente. Le régime socialiste fut le promoteur indéniable de la planification. Or, si le capitalisme moderne à un stade de son évolution a donné naissance à une certaine planification il lui imposait en même temps des limitations. Et ces limitations sont suffisamment sévères pour que de nombreux économistes continuent à contester la compatibilité de la planification et du capitalisme. Pour être subtils et insuffisamment analysés les liens entre la planification et le capitalisme moderne (qui d'ailleurs à l'opposé de ce qui se passe dans le camp socialiste ne sont pas mis en évidence et soulignés par les officiels) sont néanmoins réels et puissants. On pourrait probablement sans grande difficulté retrouver dans ces liens l'explication des différences qui existent entre les méthodes et la fonction de la planification dans des pays capitalistes particuliers[4]. Sans approfondir ce sujet important et passionnant nous nous arrêterons à quelques caractéristiques propres à

[3] Etant donné la prédominance dans les pays sous-développés des économistes de formation occidentale, l'expérience socialiste doit être en quelque sorte aussi "traduite" dans un langage différent et rattachée aux préoccupations directes de ces économistes dont l'attitude est souvent déterminée par leur formation. Les travaux des économistes marxistes de l'Occident (Dobb, Bettelheim et d'autres) ont à ce point de vue une importance certaine.

[4] Les plans des Pays-Bas par ex. sont strictement indicatifs mais les responsables arrivent à y intégrer une politique des revenus ou tout au moins des salaires. En France, par contre, dont les plans comportent une forte dose d'objectifs impératifs l'absence—ou presque—d'une telle politique passe pour la principale faiblesse de la planification en même temps que pour un problème quasi insoluble.

toute planification dans un pays développé et qui réduisent sensiblement la portée de l'expérience de ces pays du point de vue qui nous intéresse.

1) Le cadre institutionnel hérité du passé est considéré par les planificateurs comme une donnée—il ne leur appartient pas d'en proposer la transformation, même si ce cadre représente un obstacle à la croissance. Certes, un plan peut envisager la transformation des structures économiques en tant qu'un "sous-produit" de la croissance mais cette transformation n'est considérée comme un objectif direct du plan qu'exceptionnellement et sur une échelle restreinte. A cet égard les pays sous-développés sont plus proches des pays socialistes que des pays capitalistes développés. Si dans les pays socialistes des changements institutionnels et de structure ont constitués en quelque sorte des préalables à la planifications (préalables de fait ou intellectuels), des modifications dans ce domaine ou la continuation des desseins partiellement réalisés font partie intégrante de la planification socialiste. Nous ne pensons pas que puisse être contestée la thèse suivant laquelle, pour aboutir à des résultats tangibles, la planification dans les pays sous-développés doit considérer le cadre institutionnel comme une *variable politique* et que l'objectif de transformation profonde des structures y est primordial sinon décisif pour la cadence de croissance.

2) Quelque soit le régime et le niveau de développement, l'optimation d'un plan constitue la tâche essentielle d'un planificateur. Mais le terme "optimation" n'a pas le même sens dans les pays développés que dans un pays sous-développé, qu'il soit socialiste ou non. La "fonction de bien-être" aussi discutable qu'elle soit théoriquement en tant que point de départ de la planification, n'est pas en pratique sans une utilité certaine dans des pays déjà développés ayant déjà atteint un niveau de vie élevé et d'épargne réelle. Ceci d'autant plus qu'il s'agit là en règle générale des pays dont le taux de croissance de la population est modéré, sinon faible. Par contre, cette formule est vide de sens pour des pays à population en extension et qui ne peuvent concilier qu'à longue échelle l'objectif d'augmentation du niveau de vie avec celui d'un taux de croissance très élevé. En outre, dans des pays développés la notion d'optimation du plan est en quelque sorte corrigée par celle—imprécise mais combien importante—des "tensions intolérables". Le terme "tension" a d'ailleurs deux significations: d'un côté il désigne la précarité des équilibres partiels que le planificateur se décide à affronter afin de maintenir un taux de croissance élevé et de l'autre les oppositions que le plan peut rencontrer dans tel ou tel autre groupe social. Les pays sous-développés peuvent-ils se payer le luxe de viser délibérement un taux de croissance relativement bas pour éviter les difficultés et tensions pouvant apparaître en cours de route?

3) Ce qui caractérise la position d'un planificateur dans un pays hautement développé devant le problème d'implantation du plan c'est en même temps la pénurie, sinon l'absence, des instruments "forts" et l'extrême abondance d'instruments "faibles"— nuancés et indirects mais somme toute assez efficaces. La pénurie des instruments de la première catégorie relève des limitations que la logique mê-

me d'une économie capitaliste développée impose au planificateur, de son peu de goût pour les instruments impératifs. L'abondance de la deuxième catégorie résulte du niveau élevé du développement économique. A ce niveau l'économie dispose de réserves (capacités productives, stocks, cadres, etc.); l'élasticité des très nombreuses variables est grande ainsi que la mobilité d'un nombre important de facteurs; le réseau des organismes pouvant contribuer à transmettre les impulsions du plan, même quand elles sont nuancées, est développé et diversifié. Si on y ajoute les liens qui existent entre l'appareil gouvernemantal et celui des grandes unités économiques[5], il n'est pas étonnant que même les plans dits indicatifs peuvent avoir une efficacité certaine (en réalité d'ailleurs on retrouve une part "impérative" dans tous les plans). Est-il nécessaire de souligner que les pays sous-développés se trouvent dans une situation nettement opposée—qu'il leur est souvent plus facile de trancher le noeud Gordien que de le défaire?

4) Les informations que le planificateur d'un pays développé traduit en décisions et objectifs du plan sont plus ou moins adéquates à ses besoins. Ceci résulte en partie de ce que l'information économique est abondante mais aussi de ce que ses besoins ne sont pas illimités. Arrêtons-nous à ce dernier aspect qui, en général, attire moins l'attention. Une première limitation de ces besoins ne concerne qu'une partie des plans et notamment ceux qui se bornent à définir les agrégats globaux sans préciser les objectifs détaillés. Dans ce cas-là une bonne statistique moderne suffit entièrement—ce qui évidemment n'est pas le cas d'une planification plus détaillée et qui a besoins de recourir aux enquêtes, aux avis des experts, etc. Une deuxième limitation des besoins d'information—celle-ci pratiquement générale—résulte de ce qu'en plusieures matières les planificateurs des pays développés peuvent recourir (faute de mieux mais sans commettre de grosses erreurs) à cette technique théoriquement très discutable que représente l'extrapolation du trend. En effet, qu'il s'agisse de coefficients techniques, de la structure et de l'élasticité de la demande, du comportement futur des unités économiques et des unités familiales, dans une économie déjà diversifiée et déjà prospère. Il y a de fortes chances que la réalité ne s'écarte pas de manière notable des prévisions extrapolées des séries statistiques longues, précises et maniées par des techniciens hautement qualifiés. Enfin, les planificateurs d'un pays développé sauf cas exceptionnel n'ont pas du tout à se pencher sur le problème des goulots d'étranglement éventuels résultants des capacités organisationnelles insuffisantes. Sans doute les pays sous-développés parviendront un jour à se doter de renseignements statistiques à la mesure des exigences modernes, et ils trouveront des méthodes et techniques permettant d'évaluer sans trop d'erreur ces phénomènes qui échapperont toujours à la statistique (autoconsommation et investissements réalisés entièrement en dehors du marché). Mais pour juger de l'évolution future des

---

[5] Le travail de François Perroux consacré au IVème Plan français signale ce lien d'une manière particulièrement intéressante.

coefficients techniques, de la structure de consommation, etc. les méthodes qui peuvent satisfaire les planificateurs occidentaux ne suffiront pas dans un pays visant à accélérer radicalement sa cadence de croissance ainsi que de transformer la structure de son économie.

Avant d'aborder le dernier problème—celui de la contribution de la science économique—quelques remarques préliminaires paraissent indispensables. Ce n'est pas sur le problème de la valeur intrinsèque des travaux théoriques que nous avons l'intention de nous pencher mais sur celui des rapports entre ces travaux et la planification dans les pays sous-développés. De ce point de vue l'écho qu'une voix est susceptible de provoquer peut être plus important que la voix même. D'autre part, il n'est pas question ici de prendre position à l'égard d'une contribution scientifique particulière, aussi importante soit-elle. Force nous est donc de simplifier—quelque peu abusivement—en parlant de courants ou tendances. Finalement, il paraît inutile de s'étendre sur l'indéniable apport des travaux scientifiques au progrès de la planification dans les pays sous-développés—nous nous attacherons plutôt à en indiquer les limitations et les lacunes.

Les très nombreux et importants travaux consacrés au problème du sous-développement, au mécanisme de croissance et aux obstacles que celle-ci rencontre ont l'indéniable mérite de dissiper les mythes hérités du passé et de permettre aux planificateurs des pays sous-développés d'affronter les tâches qui les attendent sans mauvaise conscience. Il existe cependant un certain revers de la médaille. C'est le propre de la science de chercher à généraliser, de faire abstraction de ce qui paraît secondaire ou exceptionnel. A vrai dire la notion même des pays sous-développés est une généralisation risquée—c'est un ensemble très hétéroclite, beaucoup moins homogène que celui des pays capitalistes hautement développés (c'est à peine une boutade que de dire que les pays sous-développés n'ont qu'un seul trait absolument général : celui de ne pas être développés). Plusieurs obstacles aux processus de développement ont été mis en vedette : le taux insuffisant de l'épargne réelle, la tension inévitable de la balance commerciale, la "barrière alimentaire" résultant d'une plus grande élasticité de la consommation que de la production dans ce domaine, etc. Tout ceci est juste le plus souvent, mais pas toujours. Par ex. dans les pays pétroliers ce n'est pas le niveau de l'épargne ni la pénurie des devises qui constituent le goulot d'étranglement mais la capacité absorbante insuffisante en matière des investissements productifs. Dans certains cas on découvre que le vrai problème n'est pas celui du volume de la consommation alimentaire mais celui de la structure de cette consommation (le Ghana). Dans d'autres—un cas limite peut-être—on est obligé de constater que les facteurs sociologiques et politiques représentent l'unique obstacle à l'accélération de la cadence de croissance (la Guyanne Britannique). Il est normal certes, que les analystes du problème du sous-développement passent outre à ces "anomalies", aussi nombreuses soient-elles. Mais les planificateurs des pays respectifs n'ont pas droit à ce privilège.

En matière de la stratégie du développement deux courants peuvent être décelés. L'un est avant tout soucieux de la rationnalité des décisions et de l'allocation des ressources, en même temps qu'attaché à la notion d'une croissance équilibrée. L'autre, plus attaché au mouvement qu'à tout autre chose, a tendance à considérer toute stratégie du développement comme un pari et attribue une importance primordiale aux "faits accomplis" qui transforment les structures et déclanchent des processus sociologiques et politiques en chaîne. La première tendance n'est pas nécessairement une expression des positions politiques de droite comme la seconde, de celles de gauche. Cependant, on peut craindre que la première — indépendament des intentions subjectives de ses partisans — ne fournisse un excellent prétexte à l'immobilisme et aux tendances conservatives dans les pays sous-développés. C'est ce que croient en tout cas les partisans de la deuxième tendance — y compris l'auteur de ces lignes. Or, c'est le premier courant qui continue à prédominer dans les travaux théoriques publiés sur les problèmes du sous-développement, surtout en langue anglaise.

On mesurera l'importance des travaux consacrés directement aux problèmes de la planification dans les pays sous-développés en se rappellant que dans les pays socialistes la théorie de la planification suivait le travail empirique des planificateurs plutôt qu'elle ne le devançait (il en a été de même par ex. en France). Comme Monsieur Jourdain, le planificateur faisait de la prose avant d'appeller le "maître de philosophie". Cet ordre de choses, historiquement justifié, est aujourd'hui renversé au bénéfice des planificateurs des pays sous-développés, Le problème se pose cependant si les règles de grammaire que l'on propose ne risquent pas de déformer quelque peu le langage.

La planification n'est pas un jeu abstrait — c'est une forme particulière de la politique économique moderne, qu'il soit indicatif ou impératif un plan traite des moyens nécessaires pour atteindre un but — dans des conditions données. Or, ces conditions ne sont pas standarisées — au contraire elles différent d'un pays à l'autre en fonction du niveau de développement, du cadre institutionnel, etc. etc. Il est impossible de tourner ce problème — chaque approche du problème de méthodologie de la planification sous-entend un ensemble de conditions. Et il en est de même en ce qui concerne un niveau d'abstraction très élevé, à cette différence près que le "milieu" dans lequel la planification est censée de se mouvoir est alors peu réaliste, qu'il est une convention simplifiée. Peu importe de classer certains courants de pensée en matière de méthodologie de la planification en tant qu'un reflet des conditions propres aux pays hautement développés ou en tant que des schémas abstraits conçus pour un milieu conventionnel. Sans adaptation nécessaire, parfois radicale, ces méthodes ne pourront pas être appliquées dans une réalité concrète différente du milieu réel ou conventionnel pour lequel elles ont été conçues.

Dans la majorité des travaux théoriques occidentaux la notion de planification est réduite à la seule élaboration du plan, c'est-à-dire à l'allocation des ressources aux objectifs généraux définis (par ex. dans le cadre de la fonction du bien-être) et compte tenu des contraintes. Ceci sous-entend que le problème de l'information d'un côté et celui de l'implantation du plan de l'autre sont par ailleurs résolus. Or, nous avons vu plus haut qu'il n'en est pas ainsi dans les pays sous-développés[6]. Et les contraintes? Certes, il en existe qui sont réellement des données exogènes devant lesquelles le planificateur ne peut que s'incliner, mais combien de goulots d'étranglement peuvent être élargis si le planificateur ose s'attaquer aux institutions existantes, s'il se décide à imposer les privations nécessaires, s'il parvient à provoquer une vague d'enthousiasme ou de dévouement ! L'appareil conceptuel dont se sert un théoricien n'est pas neutre — bien au contraire —, il reflète des attitudes précises à l'égard de la politique économique. Et il ne s'agit pas là d'une querelle de mots mais d'un problème riche en conséquences pratiques. L'attitude — passive ou active — des planificateurs, leur sens plus ou moins grand de responsabilité ainsi que leur influence sur la politique concrète d'un gouvernement donné dépend dans une mesure non négligeable de l'acceptation donnée au terme "planification".

Il ne nous est pas possible dans le cadre de cet article de traiter le fond des problèmes de la méthodologie. Nous nous limiterons donc à signaler une sorte de cas limite en la matière. Il existe des méthodes et techniques qui, à juste titre, peuvent être considérées comme un grand progrès par rapport aux méthodes strictement empiriques — à condition d'être appliquées dans un milieu riche en cadres, en informations, en instruments d'implantation du plan. Nous avons nommé l'emploi des modèles globaux et du tableau des entrées et sorties intersectoriels, la prééminence ou tout au moins la priorité dans le temps de l'approche global (par grands agrégats) par rapport aux projets concrets, l'analyse à l'aide des coefficients globaux d'intensité du capital, etc. Tous ces beaux outils, utils dans une certaine mesure dans les plus développés des pays sous-développés, le sont bien moins dans d'autres. Et ils deviennent même dangeureux, dans la mesure ou l'apparente élégance des méthodes peut voiler un contenu peu consistant. Ce n'est pas sous-estimer les capacités des planificateurs des pays sous-développés que de souligner cette écueil possible — c'est se rendre compte de l'immensité de leur tâche.

---

[6] Dans la planification socialiste les trois phases (documentation, élaboration d'un plan et son implantation) sont considérées comme un tout indissoluble et recouvert par un même vocable, celui de planification.

Cz. Bobrowski

DEVELOPING COUNTRIES ON THE THRESHOLD OF LONG-TERM PLANNING

Summary

The extensive list of developing countries which have published theirlong-term plans, seems to indicate a very important advance in the process of spreading long-term planning. Economic plans applied by dependent territories are, however, short-term, and in most cases limited to the public sector. On the other hand, thc very long-term plans published by the majority of independent countries are generally vague and non-specific. Such plans cannot be considered as a choice of a definite course of development but as an action dictated by short-term political reasons. Long-term planning in developing countries is (except for a few cases) in its early stages and development through the stage of partial and short-term planning seems to be inevitable for these countries.

The main feature of the intellectual conditions under which the above-mentioned process is taking place, is the fact that in contrast to the socialist and advanced capitalist countries developing areas on the threshold of planning have at their disposal:

1) The experience in planning acquired by other countries.

2) Abundant theoretical literature.

The experience of socialist countries is the more precious since it mainly concerns the same problems which developing countries are now faced with i.e. overcoming the lag in development.

This experience, since it was gained under completely different economic and social conditions, if it is to be used, must be divided into elements peculiar to a socialist structure and those which may be more widely applied. This interpretation process is not as yet very advanced. The experience of capitalist countries refers to different institutional conditions and above all to a different stage of development. As a result, such experience is of little value or even useless as far as the following basic problems are concerned:

a) the treatment of institutions as politically variable,

b) the drive towards the optimalization of a plan without excessively reckoning with numerous limitations and strains,

c) overcoming the gaps in information as well as overcoming the poverty of methods for implementing the plan.

With respect to theoretical works dealing with the problems of development, the danger lies in the obvious tendency inevitable in this science, towards generalization, which does not take into consideration a great variety of problems dependent upon conditions characteristic for each country.

At the same time, tendencies prevailing in western theoretical studies are in danger of creating the illusion that formal elegance of methods may be brought into general use in developing countries without detriment to the realism of the plans.

А.Я. БОЯРСКИЙ
СССР

НЕКОТОРЫЕ СООБРАЖЕНИЯ О СХЕМАХ ВОСПРОИЗВОДСТВА

Известно указание В.И. Ленина о том, что для социалистической экономики сохраняет свою силу необходимость определенного соответствия между двумя большими подразделениями общественного производства: I—производство средств производства и II—производство предметов потребления. Известно, что содержание этого соответствия для условий капиталистического хозяйства раскрыто К. Марксом в его схемах простого и расширенного воспроизводства, получивших дальнейшее развитие в работах В.И. Ленина, указавшего их значение в условиях социализма. Для целей практического применения в планировании социалистического народного хозяйства они должны быть развиты и разработаны в деталях. При этом можно с самого начала рассматривать расширенное воспроизводство, имея в виду, что соотношения для простого воспроизводства окажутся охваченными как особый частный случай.

Очевидно, первое, что необходимо для этого сделать, это разобрать схему Маркса с точки зрения того, какие ее элементы и условия сохраняют силу для социалистического хозяйства и какие нет.

Введя подстрочно номера „1" и „2" для всех величин, относящихся к I или II подразделению, и дополнительно к обозначениям Маркса обозначения m_c, m_v, m_m для трех частей, на которые распадается прибавочная стоимость: увеличение постоянного капитала, увеличение переменного капитала и потребление капиталистов, можем соотношение Маркса записать в алгебраической форме следующим образом:

$$v_1+m_{1v}+m_{1m}=c_2+m_{2c}$$

Вновь созданная стоимость I родразделения без обращаемой в нем на увеличение постоянного капитала части прибавочной стоимости должна равняться постоянному капиталу II подразделения, плюс обращаемая в нем на увеличение постоянного капитала часть прибавочной стоимости.

Сразу же видно первое видоизменение схемы применительно к социализму. Здесь нет капиталистов, нет их паразитического потребления. Иначе говоря, отпадает элемент m_{1m} и написанное выше равенство упрощается:

$$v_1+m_{1v}=c_2+m_{2c}$$

Но, если нет $m_m = m_{1m} + m_{2m}$, то имеем

$$m_{1v} = m_1 - m_{1c}$$

Сделав соответствующую подстановку перенесем m_{1c} направо и наше равенство предстанет в виде

$$v_1 + m_1 = c_2 + m_{1c} + m_{2c} = c_2 + m_c$$

Это читается очень просто: вновь созданная в I стоимость равна постоянному капиталу II подразделения, плюс обращаемая во всем народном хозяйстве на увеличение постоянного капитала часть прибавочной стоимости.

Заметим, что вместе с m_m выпало из уравнения m_v: для условий капитализма m_v (как и m_c) черпается из m. Это не вопрос о структуре подразделений, а о том, какой из двух классов потребляет соответствующую стоимость. Для социалистических условий так вопрос не может ставиться.

Теперь обратимся к некоторым особенностям схем Маркса.

Во-первых, в них принимается, что увеличение капитала в I и II черпается только соответственно из m_1 и m_2. Возможность увеличения производства в I за счет m_2 или во II за счет m_1 в них исключается. Это означает, что исключается перелив капиталов, вопросы, связанные с ним, отнесены к III тому *Капитала*. В социалистических условиях этот вопрос, однако, приобретает иной характер. Увеличение средств как I, так и II черпается из общего фонда прибавочного продукта, который во всех своих частях здесь принадлежит одному хозяину—обществу, но важное значение имеет различение его частей по направлению. Однако последнее различение есть различение между соответствующими приращениями: Δc_1 и Δc_2, а не частями самой стоимости по их источнику. Поэтому мы можем отбросить в нашем уравнении промежуточное звено:

$$v_1 + m_1 = c_2 + m_c$$

Введя для всей продукции подразделений P, имеем:

$$P_1 - c_1 = c_2 + m_c$$

И, наконец, учитывая, что $m_c = \Delta_c$

$$P_1 = c_1 + c_2 + \Delta c = c + \Delta c$$

т.е. очень простое соотношение: продукция I должна равняться сумме фонда возмещения перенесенной стоимости c и фонда увеличения постоянного капитала.

Следующей особенностью Марксовых схем является отожествление постоянного капитала с перенесенной стоимостью (придерживаясь его мы и могли писать для приращения постоянного капитала Δc).

Иногда считают, что эта особенность схем не отклоняет их от действительности, если считать время оборота равным одному году или, что сами схемы относятся к циклам, равным одному времени оборота. Однако, легко убедиться в том, что это не так. Пусть время производства составляет везде год и это совпадает со временем оборота переменного капитала и оборотной части постоянного капитала, а время оборота основного капитала — 5 лет. Пусть $c = 1100$, в том числе 1000 основного и 100 оборотного, а $v = 100$, т.е. весь капитал равен 1200. Для простоты оставим в стороне m. Тогда (без учета прибыли) годовая продукция составит:

$$1000 : 5+100+100 = 400 = 300\,c+100\,v.$$

Капитал оборачивается, следовательно, в $1200 : 400 = 3$ года. Но в трехлетней продукции будет $900c+300\,v$, в составе же авансированного капитала мы имеем $1100\,c+100\ v$, т.е. соотношение c и v совсем другое — независимо от того, берем ли мы однолетнюю продукцию или трехлетнюю. Если за 3 года реализуется 300 m, из которых, допустим, 240 обращается на увеличение капитала, то при сохранении структуры авансированного капитала 11 : 1 из них на увеличение постоянного капитала должно быть 220 (в т.ч. 200 на основной) и 20 на увеличение переменного. Новый капитал составит $1320\,c+120\,v$ и за следующие три года продукция без прибыли составит $3\ (1200 : 5+120+120) = 1080\,c+360\,v$.

В составе продукции мы находим дополнительно по сравнению с первым трехлетием 60 v и 180 c. В сумме это действительно равно 240, обращенным на увеличение капитала, но соотношение частей здесь сильно отличается от соотношения 220 : 20. Иначе говоря, если учесть различную оборачиваемость основного и оборотного капиталов, то прибавлением к исходным 900 c (или 300 v — см. продукцию первого трехлетия) дополнительных 220 c (или 20 v) нельзя получить соответственные части продукции второго трехлетия — 1080 c или 360 v (получится 1120 и 320, т.е. на 40 больше c и на 40 меньше v).

Чтобы учесть различие оборотов необходимо ввести некоторый параметр. Это может быть „фондоемкость" — отношение прироста средств производства к приросту продукции. Однако, в отношении переменного капитала вопрос не стоит: в каждом цикле он возвращается целиком. Постоянный же возвращается в зависимости от оборачиваемости (и доли) основного капитала. При введении же такого параметра отпадает необходимость оперировать только циклами, отвечающими одному обороту, можно просто перейти к единице времени — году. В таком случае указанным параметром будет служить попросту стоимость дополнительных средств производства, которые надо вложить, чтобы получить прирост годовой продукции на I, скажем, рубль. Если это f, то введя величину $\Delta c = m_c$, имеем для прироста продукции

$$\Delta P = \frac{\Delta c}{f} = \frac{m_c}{f}.$$

Далее, важной чертой Марксовых схем является их чисто теоретический характер, позволяющий отвлечься от всего, кроме сферы производства. Конечно, не играет роли то, что оставляемую на потребление часть прибавочной стоимости m_m капиталист потребляет не один, а вместе со своими министрами, художниками, генералами, солдатами, полицейскими и ворами. Кроме этого, есть производители услуг, которыми пользуются и рабочие (тех же или других художников, врачей, учителей и т.д.). Введение их в схему потребовало бы перевода соответствующей части v в m, рассмотрение этой части m, как части, которая не является ни потреблением капиталистов, ни накоплением и т.д.

Однако, можно расширительно толковать потребляемую часть прибавочной стоимости m_m, как покрывающую потребление не только капиталистов, но и всей непроизводственной сферы, а ее формальное происхождение из фонда заработной платы в схеме просто игнорировать как налоги и т.п. Для схемы капиталистического воспроизводства это выход. Но он не годится для нас, поскольку мы только что исключили вовсе m_m. Следовательно, необходимо ввести непроизводственную сферу. Но раз мы ее вводим, то надо различать в ней материальное потребление и заработную плату ее работников, различать, с другой стороны, оплачиваемую часть ее услуг и остальное.

Но, в таком случае, надо во II подразделении производство материалов для непроизводственной сферы разбить на производство их для той ее части, услуги которой оплачиваются населением, и остальное. При этом бывает и так, что услуги некоторой из соответствующих отраслей в одной части оплачиваются, в другой — нет (например, наряду с бесплатной медицинской помощью имеются и платные поликлиники). В этом отношении дело обстоит так же, как и с рядом продуктов сферы производства, которые частично используются как средства производства, частично как предметы потребления. В схеме можно учитывать просто отдельно то и другое, разнеся эти части по соответствующим подразделениям. Но, кроме того, и в тех случаях, когда услуги оплачиваются, оплата их как правило не покрывает расходов государства. Однако, и это можно игнорировать при построении схемы, отнеся пропорциональную часть самих услуг к той их части, которая не оплачивается. Если даже допустить, что среди оплачиваемых услуг есть такие, которые перекрывают расходы, то избыток можно считать покрывающим некоторую часть других услуг. В общем итоге оплата во всяком случае не превысит расходов.

Далее условимся согласно принятым схемам баланса народного хозяйства сам объем услуг оценивать суммой материальных затрат и заработной платы, т.е. исходить из того, что никакого прибавочного продукта (как и вообще продукта) в непроизводственной сфере не производится.

Но это еще не все. Социалистическое хозяйство не остается неподвижным или растущим только количественно. Оно развивается в направлении к коммунизму. Это значит, что все большая часть продукции распределяется не по труду, а по потребности и, следовательно, бесплатно. По программе КПСС к 1980 году будут распределяться таким образом жилища, коммунальные услуги, включая городской транспорт, значительная часть продукции общественного питания. Таким образом, для социалистической экономики, развивающейся в направлении к коммунизму, надо ввести в схемы новое деление: производство предметов потребления разделить на производство предметов потребления, оплачиваемых населением, и производство предметов потребления, распределяемых бесплатно.

В результате всего сказанного приходим к следующему делению, так сказать, по подлежащему схемы:

I. Производство средств производства
II. Производство предметов потребления
    a) для населения
        α) платных
        β) бесплатных
    b) для непроизводственной сферы
        α) оплачиваемой части
        β) неоплачиваемой
    Непроизводственная сфера
        α) оплачиваемая часть
        β) неоплачиваемая

В составе стоимости продукции необходимо при этом различать перенесенную стоимость средств производства c, вновь созданную стоимость, распадающуюся на фонд заработной платы v и прибыль m. Из фонда заработной платы при этом некоторая часть обращается на оплату разного рода услуг непроизводственной сферы. Потребление непроизводственной сферы покрывается частично за счет этой оплаты и частично за счет прибыли сферы производства. Остальная часть прибыли обращается на расширение производства и образует $\Delta c = f\Delta P$

В составе стоимости продукции I и II будем иметь: 1) перенесенную стоимость $c = c_1+c_2 = c_1+c_a+c_b = c_1+c_{a\alpha}+c_{a\beta}+c_{b\alpha}+c_{b\beta}$, где подстрочные индексы, как и в других случаях в дальнейшем, означают, что соответствующие величины относятся к указанным выше частям II подразделения. 2) фонд зарплаты

$$v = v_1+v_2 = v_1+v_a+v_b = v_1+v_{a\alpha}+v_{a\beta}+v_{b\alpha}+v_{b\beta}$$

3) прибыль

$$m = m_1+m_2 = m_1+m_a+m_b = m_1+m_{a\alpha}+m_{a\beta}+m_{b\alpha}+m_{b\beta}$$

Мы разложили продукцию всех отраслей производства на три слагаемых, считая, что величина отражает весь труд, затраченный в соответствующих отраслях производства. В действительности продукция II$a\beta$ и IIb может передаваться по себестоимости государству и им передаваться по назначению. Но, тогда будет затруднен анализ, так как дело выглядело бы так, будто прибавочный продукт создается только в некоторой части, а не во всей сфере производства. Значит будем считать, что ведущее все хозяйство государство покупает продукцию II$a\beta$ и IIb как и I, с учетом m, т.е. по полной стоимости. Что касается непроизводственной сферы, то стоимость ее содержания N состоит из материального ее потребления M и фонда заработной платы ее работников Z (куда присоединим также жалование военнослужащих, пенсии, стипендии и т.п.). Таким образом имеем:

Вся продукция сферы производства $P = c+v+m$ и соответственных ее частей (отмечаемых подстрочными указателями) и все расходы непроизводственной сферы $N = M+Z$ и то же с указателями α и β для оплачиваемой ее части и для неоплачиваемой части.

Теперь составим балансовые уравнения. P_1 должно покрыть все производственное потребление и приращение материальных фондов в сфере производства:

$$P_1 = c_1+c_{a\alpha}+c_{a\beta}+c_{b\alpha}+c_{b\beta}+\Delta c.$$

$P_{a\alpha}+N_\alpha$ должно балансироваться с полным фондом заработной платы (включая заработную плату работников непроизводственной сферы):

$$P_{a\alpha}+N_\alpha = v_1+v_{a\alpha}+v_{a\beta}+v_{b\alpha}+v_{b\beta}+Z_\alpha+Z_\beta.$$

$P_{a\beta}$ мы здесь отдельно не балансируем ни с чем — это бесплатно распределяемые блага.

$P_{b\alpha}$ и $P_{b\beta}$ должно отвечать потреблению материалов в соответствующих частях непроизводственной сферы:

$$P_{b\alpha} = M_\alpha$$

$$P_{b\beta} = M_\beta.$$

Развернув левые части по слагаемым и отбросив в этих трех равенствах одинаковые слагаемые справа и слева, напишем их в виде:

$$(*)\quad \begin{cases} v_1+m_1 = c_{a\alpha}+c_{a\beta}+c_{b\alpha}+c_{b\beta}+\Delta c \\ c_{a\alpha}+m_{a\alpha}+M_\alpha = v_1+v_{\alpha\beta}+v_{b\alpha}+v_{b\beta}+Z_\beta \\ c_{b\alpha}+v_{b\alpha}+m_{b\alpha} = M_\alpha \\ c_{b\beta}+v_{b\beta}+m_{b\beta} = M_\beta. \end{cases}$$

Средства государства составляются путем передачи ему прибыли из всех отраслей производства.

При этом вся прибыль должна равняться приращению фондов сферы производства, стоимости содержания неоплачиваемой населением части непроизводственной сферы и стоимости продукции II$_{\alpha\beta}\beta$, которую государство также должно „выкупить" для бесплатного распределения:

$$m_1+m_{a\alpha}+m_{a\beta}+m_{b\alpha}+m_{b\beta} = \Delta c+M_\beta+Z_\beta+c_{\alpha\beta}+v_{\alpha\beta}+m_{a\beta}$$

Отсюда

$$\Delta c = m_1+m_{a\alpha}+m_{b\alpha}+m_{b\beta}-M_\beta-Z_\beta-c_{a\beta}-v_{a\beta}.$$

Подставив это в первое из приведенных выше уравнений, получим после элементарных преобразований

$$M_\beta+Z_\beta = c_2+m_2-v_1-c_{\alpha\beta}-v_{\alpha\beta}-m_{\alpha\beta}.$$

С другой стороны, если подставить M_α из третьего уравнения во второе и выразить из него Z_β, а из четвертого уравнения выразить M_β и найти их сумму, то получим в точности то же. Следовательно, одно из наших равенств является производственным, т.е. баланс между прибылью и ее использованием уже содержится в предыдущих равенствах. В них нет вовсе элементов c_1, $v_{\alpha\alpha}$, $m_{\alpha\beta}$; Z_α; но это не должно нас удивлять. Как известно, c_1 выпадает, так как это — часть продукции I, остающаяся в I. Так же $v_{\alpha\alpha}$ входит в продукцию II, выкупаемую на заработную плату, и выкупается работниками тех же отраслей. Далее, $m_{\alpha\beta}$ есть прибыль, реализуемая отраслями II$\alpha\beta$, чья продукция ,,выкупается" государством для бесплатного распределения. Ресурсы же для этого государство по нашей схеме черпает только в виде прибыли всей сферы производства (налоги и т.п. для простоты предполагаются уже вычтенными их заработной платы и т.д.). Следовательно, прибыль, входящая в состав цены продукции II$\alpha\beta$, увеличивая эту цену, ровно настолько же увеличит ресурсы государства и поэтому из балансовых уравнений выпадает. Наконец, положение с Z_α аналогично $v_{\alpha\alpha}$.

Отсутствие в уравнениях c_1 и $v_{\alpha\alpha}$ не является здесь чем-то новым: вполне аналогичное положение имеет место и в алгебраическом выражении основного соотношения схем Маркса, где все второе подразделение сводится к нашему II$\alpha\alpha$.

Таким образом, вся схема алгебраически отражается системой четырех уравнений (*). Схема же Маркса отражается одним уравнением. Три дополнительных уравнения появляются вследствие:

1) выделения из продукции II подразделения материальных затрат непроизводственной сферы, 2) выделения из последней части, услуги которой оплачиваются населением и 3) выделения из продукции II подразделения части, распределяемой бесплатно. Если первое и второе являются лишь дополнительной конкретизацией схемы, то третье и отражает особенность воспроизводства и период перехода к коммунизму.

Поэтому для более яркого выявления тех особенностей схемы, которые связаны именно с главным содержанием эпохи и ее сравнения со схемой Маркса представляет интерес освободить ее от учета непроизводственной сферы. Для этого достаточно в наших уравнениях (*), поскольку они уже имеются в нашем распоряжении, положить $N_\alpha = N_\beta = 0$, следовательно $M_\alpha = M_\beta = Z_\alpha = Z_\beta = 0$ и также $P_{b\alpha} = P_{b\beta} = 0$ как и все их составные части. Вследствие этого теряют смысл третье и четвертое из уравнений (*). Отбросив в первых двух нулевые члены, получим

$$\begin{cases} v_1+m_1 = c_{\alpha\alpha}+c_{\alpha\beta}+\Delta c = c_2+\Delta c \\ c_{\alpha\alpha}+m_{\alpha\alpha} = v_1+v_{\alpha\beta}. \end{cases}$$

Первое из этих уравнений в точности отвечает алгебраической форме схемы расширенного воспроизводства К. Маркса (в которой можно вместо $m_{1c}+m_{2c}$ писать Δc).

Второе уравнение дополняет ее балансом между фондом заработной платы и платной частью продукции II подразделения.

Первое уравнение в схеме Маркса, как известно, эквивалентно также балансу между фондом заработной платы и всей продукцией II подразделения без той ее части, которая потребляется капиталистом. Для этого надо учесть, что $\Delta c = m - m_v - m_m = m_1 + m_2 - m_v - m_m$. Сделав такую подстановку и прибавив к обеим частям v_2 превратим первое уравнение в

$$v_1+v_2+m_v = c_2+v_2+m_2-m_m = P_2-m_m,$$

что и выражает упомянутый баланс.

В наших же уравнениях второе уравнение, т.е. баланс между фондом заработной платы и платной частью продукции II подразделения, может быть преобразовано в баланс между ее бесплатной частью и „прибылью", предназчаемой на ее выкуп государством.

Для этого выразим из первого уравнения

$$c_{a\alpha} = v_1+m_1-c_{a\beta}-\Delta c$$

и подставив его во второе прибавим к обеим его частям $m_{a\beta}$. После элементарных преобразований (и объединения $m_{a\alpha}+m_{a\beta} = m_2$) представим его в виде

$$m_1+m_2-\Delta c = c_{a\beta}+v_{a\beta}+m_{a\beta}$$

или коротко:

$$m-\Delta c = P_{a\beta},$$

что и выражает упомянутый баланс. В нем хорошо выражен тот простой факт, что хотя бесплатная часть продукции II подразделения распределяется не по труду, но труд и только труд является ее источником.

A. J. Boyarski

CONSIDERATIONS ON THE SUBJECT OF REPRODUCTION SCHEMATA

Summary

THE BASIC relations between the first and the second departments sectors are also valid in socialist economy, but they should be developed and worked over in detail for practical use. Let us start from the schemes of expanded reproduction. The basic task is to find an answer to the question: "Which elements of Marx's schemes are valid in socialism, and which are not?"

The basic equation of the scheme

$$v_1+m_1 = c_c$$

in the socialist economy changes into

$$v_1+m_1 = c_2+m_2$$

In Marx's schemes constant capital is considered as transferred value. This is right providing either that the velocity of capital is one year or that the schemes cover the cycle of one turnover of capital. However in reality this is not so. In order to take into consideration the differences in time during the turnover of capital the parameter of capital intensity should be introduced. The parameter is calculated as the ratio of the increase of means of production to the increase of the volume of production.

The part of the surplus value symbolized by m_m may be treated not only as the consumption of capitalists but also as the consumption of the whole unproductive sphere. The introduced sphere of services should also be reconsidered by dividing them into those paid by the population and the other ones. Thus we come to the following scheme of the national economy:

I. Production of the means of production
II. Production of the means of consumption
- a. for the population
  - α. paid
  - β. unpaid
- b. for the unproductive sphere
  - α. the paid part
  - β. the unpaid part
- c. the unproductive sphere
  - α. the paid part
  - β. the unpaid part

Thus from the new equations received from such a scheme, the surplus value equals the increase in the fund of the productive sphere, the value of maintenance of the part of unproductive sphere unpaid by the population and the value of production in the section IIa, which the state should buy for free use.

Four basic equations appear in socialism instead of the one produced by Marx. These are a result of:

1. the separation of the subdivision of material expenditure of the unproductive sphere from the production in department II

2. the separation of the services paid by the state from the above,
3. the separation of the subdivision of goods distributed freely from the production of department II.

The latter is typical for the period of transition into communism.

The result of the modernized schemes of reproduction in socialist economy is that though the part of production in department II is not distributed according to the input of labour, labour and only labour is the source of its value.

Włodzimierz Brus
Kazimierz Łaski
Poland

GROWTH AT THE FULL EMPLOYMENT OF PRODUCTIVE FORCES

The degree of the utilization of productive forces and their development are one of the basic indicators of the economic effectiveness and the dynamics of every system of production. There is a close interdependence between the degree of the utilization of productive forces, on the one hand, and their development, on the other. By bringing this interdependence out to light it is possible to show certain basic aspects of the superiority of the socialist system over the capitalist one.

The full utilization of productive forces can be best defined as the state at which the national income actually produced reaches the level equal to the national income that can possibly be attained. If we denote the national income actually produced by D and the national income that it is possible to achieve, or the maximum income, by D^{max}, we have:

$$D = D^{max}$$

when the productive forces are fully employed.

However, the simplicity of this definition is only apparent (we shall return to this question later on). But even on the basis of this definition we can and should confront the notion of a full employment of productive forces with the notion of an optimal employment.

By optimal employment of productive forces can be understood not only their full employment but also a situation in which the physical structure of national income (i.e. the structure of consumption and the resultant structure of productive accumulation) is best geared to the existing needs of society and to the degree of the intensity of those needs. Social needs in this context are interpreted broadly as comprising both individual and social preferences assuming that they are somehow reconciled. We can assume, for instance, that individual preferences are included in social preferences if the former do not conflict with the latter which are superior.

A special aspect of the physical structure of national income is the share of consumer goods in income, on the one hand, and the share of the productively accumulated goods, particularly investment goods, on the other. The former satisfy the current consumption requirements, the latter—the future consumption requirements through increased income which they make possible. An optimal physical structure of national income and an optimal employment of productive resources from this point of view are not, of course, tantamount to the proportional degree of the satisfaction of current and future consumption needs but they are tantamount to a compromise between these needs, expressed in an optimum rate of growth. Thus, to the same extent, the problem of the optimal employment of productive forces is not identical with the problem of their full utilization.

These introductory remarks, of course, were not meant to exhaust the whole complex problem of the relationship between the full and optimal employment of productive forces. Their object was to emphasize that the problem of full utilization of productive forces analysed in this paper does not exhaust the problem of their optimal utilization. The latter question is undoubtedly broader and includes the problem of full utilization as a basic, although not the only, element. Both problems become identical if it is assumed that the structure of productive capacity (considering also foreign trade) corresponds fully to the structure of needs in a broad sense.

1. Full Employment of Productive Assets

A full employment of productive forces comprises, first, a full utilization of productive assets and, secondly, a full utilization of labour force. Let us first consider both these problems separately and then in their mutual relationship.

On the surface, it should not be difficult to define precisely the state of the full employment of productive assets. There is a defined productive capacity of particular elements of equipment and the level of production equal to this productive capacity means that the production apparatus is fully employed. Denoting actual income by D and the productive capacity of the production apparatus by $D^{M,\max}$ we have

$$D = D^{M,\max}$$

when productive assets are fully employed.

However, when we go deeper into this definition we encounter, as is usual with definitions, considerable complications. We can distinguish without difficulty, for instance, a factory which is not fully employed or which operates, say, 2 days a week, from a fully employed factory operating at the limit of its productive capacity. However, can we consider as a full employment of productive assets a state in which the shift-coefficient is lower than that technically possible and economically desirable? Or a state in which difficulties in cooperation cause that certain factories (or divisions) produce less than they could and should? Further, it is assumed that in a state of full employment of productive capacity proper input-output relations are preserved among different divisions and branches of production (e.g. the demand for steel at the full employment of the establishments using steel and the supply of steel at the full employment of the productive capacity of steel mills). When these relations are not preserved and when productive capacities of interrelated branches are not properly balanced a full utilization of productive capacity is possible only to the extent determined by various bottlenecks (after allowing for foreign trade). Is it possible, then, to consider as full employment of productive capacity a situation in which the insufficient supply of certain raw materials imposes limitations on the employment of productive capacity in different branches of the manufacturing industry? What should be done, for instance, when stocks and reserves (constituting a part of working capital), exceed, particularly in certain lines, all reasonable limits? It is also quite possible that certain reserves of productive capacity are kept idle on purpose, particularly in some branches of production, in order to ensure a required degree of flexibility.

It is easy to see that situations described above can also appear in a socialist economy. Can we, then, talk about a full employment of productive assets in a socialist economy? Yes, we can, but only in a specific sense. All cases of less than full employment of productive capacity result either from bad organization or from above mentioned disproportions, or are intended. But in a socialist economy there does not appear a specific and very important case of less than full employment of productive capacity due to the lack of aggregate effective *demand*. Only in this sense can we speak of a full employment of productive capacity in a socialist economy and contrast it, from this point of view, with a capitalist economy suffering notoriously from less than full employment of capital assets, which cannot be explained by difficulties on the *supply* side. In the above sense we speak of a full employment of productive capacity when an increase in aggregate demand does not result in increased production at a given level of productive capacity; by the same token, we speak of less than full employment when the reverse is true.

This statement is of great importance for the analysis of growth. Especially the formulae describing the factors of growth of national income, although they merely define certain relationships and therefore can formally be applied to any system of production, are not really applicable to a capitalist system.

In the indirect formula[1] of the type $D = M\frac{1}{k}$ fixed capital assets M are treated as a factor which together with the capital intensity coefficient k determines the level of national income. Accordingly productive investment I is treated as a factor which expands *productive capacity* and in consequence, co-determines an increase in national income. In a capitalist economy, at a less than full employment of productive capacity investment is nevertheless, primarily a factor *determining the demand* for capital goods and also, via the demand for consumer good, a factor determining national income.

Let us assume, that we deal with a purely capitalist economy in which the workers spend all their earnings on current consumption and do not save, and the capitalists, being the symbol of their class, purchase only investment goods, and do not consume.

---

[1] K. Łaski, *Czynniki wzrostu dochodu narodowego w gospodarce socjalistycznej* (Factors of Growth in National Income in a Socialist Economy), "Ekonomista", No. 2, 1960.

In this situation, by definition, wages P would equal consumption K, and investment I would be equal to profits (or savings) O. Indeed, if we have

$$P = K$$

and the national income

$$D = P+O$$

and

$$D = K+I$$

then, of course

$$I = O$$

i.e. the investment in a given period equals the profits in this period.

This, of course, follows from the definition and cannot be questioned. However, the problem consists in a proper interpretation. Kalecki proved that this equality should be understood as meaning that investment determines profits, and not the other way around[2]. At the first glance this seems absurd, because in accordance with common sense it should be expected that it is profits that determine the volume of investment. However, this is not so.

If investment is I, and at the same time there is a given rate of surplus value m—the same in both divisions of social production—then, first of all, wages P_1 in Division I are determined. Indeed, production in Division I is

$$P_1+O_1 = I$$

but $\frac{O_1}{P_1} = m'$, hence $O_1 = P_1 \cdot m'$

and therefore $P_1+P_1m' = I$

$$P_1 = \frac{I}{1+m'}.$$

Thus investment determines the demand for consumer goods which comes from Division I and also the supply, equal to this demand, flowing from Division II to Division I since if

$$P_1 = O_2$$

also

$$O_2 = \frac{I}{1+m'}.$$

The production of Division II equals

$$P_2+O_2 = K$$

$$\text{but } \frac{O_2}{P_2} = m', \text{ hence } P_2 = \frac{O_2}{m'}$$

[2] M. Kalecki, *Theory of Economic Dynamics*, London 1956, p. 53.

and therefore

$$\frac{O_2}{m'}+O_2 = K$$

$$K = O_2\left(\frac{1}{m'}+1\right)$$

$$K = O_2\frac{1+m'}{m'}$$

and thus the total production of consumer goods is $\frac{1+m'}{m'}$ times larger than O_2 which is the production of consumer goods for Division *I*. It is now easy to determine *K* in relation to *I*.

$$K = \frac{I}{1+m'} \cdot \frac{1+m'}{m'} = \frac{I}{m'}.$$

Finally, also income is determined by investment:

$$D = I+K$$

$$D = I+\frac{I}{m'}$$

$$D = \frac{1+m'}{m'}I$$

and $D = \frac{1}{i}I$, where $i = \frac{m'}{1+m'} = \frac{I}{D}$ which is the rate of investment[3].

Very deeply rooted in our minds is the notion that national income in a given period is a given quantity, a sort of a loaf of bread of a given weight. This notion is true on the assumption that productive capacity is utilized to the full. When the loaf is given then the less we earmark for investment the more we can consume, and *vice versa*. The division of the loaf of bread is thus independent of its size and should be considered separately.

When we deal with a capitalist economy in which the productive capacity is not fully employed we cannot assume that national income is given. It increases as the degree of the utilization of the productive capacity increases. And the degree of utilization increases as investment increases, at a given rate of investment (which, as we have already seen depends upon the rate of surplus value). Thus, in a capitalist system national income varies with the size of the part earmarked for investment. Then it is not the rate of investment (and the rate of surplus value) that adjusts itself to the volume of investment at a given income, but national income adjusts itself to investment at a given rate of investment.

[3] Obviously under the term "the rate of investment" the relative rate of investment in the national income is meant here, and in this meaning will the above term be used in all the paper.

In a socialist economy, on the other hand, the level of national income is determined by the full (in the sense defined above) employment of productive capacity. From this point of view, then, income is given and the lesser the share of investment the more remains for current consumption, and *vice versa*. A socialist state determines in its plan the volume of investment and also the amount of saved profits needed for financing the investment. But neither investment, nor savings which are equal to it determine national income; it is determined by the existing productive capacity. When the volume of investment changes, the rate of investment changes, but income does not change.

If the planning authority determines investment as equal to I, then, knowing also the volume of income it simultaneously determines the rate of investment $i = \frac{I}{D^{M,\max}}$, and the rate of surplus product $m' = \frac{i}{1-i}$[4]. It could be said, therefore, that in a socialist economy the national income in a given period is a constant function of investment and the rate of investment in this period

$$D^{M,\max} = I \cdot \frac{D^{M,\max}}{I} = I\frac{1}{i} = \text{const.}$$

The whole problem can be presented graphically. On the axis of abscissae investment is measured as an independent variable and along the axis of ordinates

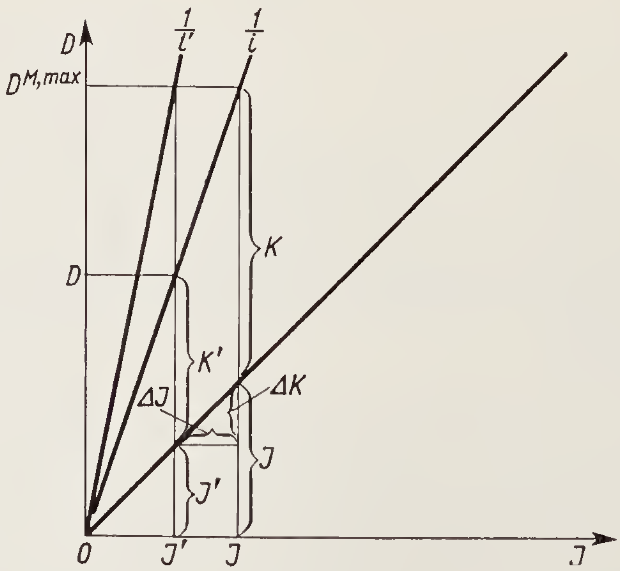


FIG. 1

[4] If $i = \frac{m'}{1+m'}$, then $m' = i\,(1+m')$

$$m' - im = i$$

$$m' = \frac{i}{1-i}$$

national income is measured as a function of investment. $D^{M, \max}$ is the level of national income attainable at the full employment of productive capacity.

To attain $D^{M, \max}$ in a capitalist economy the volume of investment I is required when the rate of investment is i. In no case can the income actually attained exceed $D^{M, \max}$, but it can be less than that. In a capitalist economy this happens, as a rule, if investment is less than I. When investment $I' < I$, the national income $D < D^{M, \max}$ and the decrease in income is larger than the decrease in investment because consumption declines with investment ($K' < K$). Consumption decreases, first of all because, some workers in Division I lose their jobs and earnings and, secondly, because together with them also the workers who previously produced consumer goods for the former lose, in turn, their jobs and earnings.

The Keynesian School—which several scores of years after Marx admitted finally that in a capitalist economy national income may and, in fact, as a rule does remain at a less than full employment level, i.e. below $D^{M, \max}$—points out that the cause is the insufficient propensity to invest i.e. insufficient investment ($I' < I$). From the purely formal point of view this is correct as long as it is recognized that the rate of investment i is given. However, i depends upon the rate of surplus value m'. Therefore it would be more correct to say that at a given propensity to invest (if this rather dubious terminology is to be used) the rate of surplus value, or the rate of exploitation is too high. It would be beyond bourgeois economics, however, to concede this fact, because it would expose the immanent and insuperable contradictions in capitalism. If the rate of exploitation were lowered this would not be without effect on investment which is embarked upon to bring profit. And it is obvious that in theory (ignoring the capitalistic system of production) it is possible to ensure the level of national income equal to $D^{M, \max}$ at any level of investment providing that appropriate rates of investment and of surplus value are found for given investment.

This happens in a socialist economy in which national income reaches the value $D^{M, \max}$ in consequence of the full employment of productive capacity regardless of the volume of investment. When investment declines from I to I' by ΔI then consumption simultaneously increases by the same quantity $\Delta K = \Delta I$. This effect is obtained by adjusting the rate of investment to each given level of investment. Investment I is accompanied by the rate of investment $i = \frac{I}{D^{M, \max}}$, and investment I' is accompanied by the rate of investment $i' = \frac{I'}{D^{M, \max}}$. In line with the change in the rate of investment there will be a corresponding change in the rate of surplus product which for the planning authority in a socialist system is not a final objective but merely a means toward financing planned investment.

So far we have been considering this problem taking the productive capacity as given. However during the process of growth productive capacity changes. For a capitalist economy an analysis of the indirect type and the way in which the role of productive assets is taken into account is useless. This comes out even more clearly when we deal with the process of growth. The indirect factors determining the rate of growth of income are expressed by the Kalecki formula[5]

$$r = \frac{1}{k} i + u - a$$

where

r denotes relative increase in gross national income

k „ the incremental coefficient of capital intensity $\left(\frac{I}{\Delta D}\right)$

i „ the rate of gross productive investment

u „ the coefficient of improvements

a „ the coefficient of decline in national income in consequence of wear and tear of the productive equipment.

In this formula investment is treated as a factor expanding the production apparatus and not as a factor determining the demand. However, when the productive capacity is not fully utilized then investment put into operation (and with this we are concerned in the above formula) does not play any part in determining an increase in national income. For it may happen that investment put into operation, instead of increasing income, will only increase the amount of unused productive capacity. Indeed, there is no reason to believe that in capitalism investment put into operation, that is an increase in productive capacity, determines an increase in national income in a different manner than the productive apparatus in general determines national income in general.

At the bottom of the above formula lies the assumption that the productive capacity is fully employed and therefore it determines the volume of income through the average coefficient of capital intensity. On this assumption—justified in a socialist economy—also investment put into operation determines an increase in income taking into account the incremental coefficient of capital intensity. This formula can then be applied only to such an economy in which investment, via new productive capacity, determines, together with the old productive capacity, the volume of national income.

It follows that between the rate of investment i and the rate of increase in income r there does not exist, by any means, a similar relationship in a capitalist economy as there is in a socialist economy. This matter is directly related to the role of the coefficient u. In a socialist economy where productive capacity is fully employed this coefficient expresses the effect of constant improvements in the utilization of the existing capacity resulting from improved skills of the workers, improvements in organization, a decrease in work stoppage and deficiencies, better cooperation, a greater number of shifts worked, and so on. This is a slow but systematic process and it can be observed in every socialist enterprise. It can be assumed, therefore, that the coefficient u

[5] Cf.: *Z zagadnień teorii dynamiki gospodarki socjalistycznej* (Problems in the Theory of Dynamics of a Socialist Economy) in the collective study: *Zagadnienia ekonomii politycznej socjalizmu* (Problems in the Political Economy of Socialism), Warsaw 1960, 3rd Edition.

is in a socialist economy a constant quantity and, what is of particular importance, that it is always positive.

In a capitalist economy the situation is quite different. It is true that in capitalistic enterprises improvements also take place all the time, sometimes even on a larger scale than in similar socialist enterprises. The point is, however, that since the productive capacity is not fully employed the coefficient u fluctuates considerably and depending on the phase of the business cycle it assumes either positive or negative values. Under these circumstances i, k and a which are given, are accompanied in capitalism by completely different rates of growth in income—r, depending upon the size and sign of the coefficient u. At each breakdown of the business cycle the coefficient u becomes negative, and quite often this factor more than offsets the effect of the factors $\frac{1}{k}i-a$, giving in effeet a decline instead of an increase in national income.

It should be noted, however, that also the role of the coefficients a and k is not identical in different systems of production. The shrinking of productive capacity expressed by the coefficient a is much stronger in a capitalist economy in which the process of absolescence of equipment is not socially controlled and is often subject to the destroying influence of competition. The differences in the role of the coefficient k are of an even greater importance. In a socialist economy it is a variable independent of i (in a certain sense, used in this content) but in capitalism this is not so. For different values of k and at a given distribution of an increase in income between wages and profits, the profit from a given amount of productive investment will also be different (assuming a given degree of their utilization). It is obvious that through the rate of profit this will affect the rate of investment i.

It follows that the above formula cannot be directly applied to a capitalist economy, but that it expresses correctly the process of growth in a planned economy in which, because of the full employment of productive capacity, the coefficient u is relatively constant and positive and in which, therefore, the productive effect of investment $\frac{1}{k} \cdot i$ actually determines (together with the coeffieients a and u) the rate of growth of income. In the capitalist system, on the other hand, it would be more correct to say that the coefficient of improvements u depends upon the rate of economic growth rather than the other way around.

This point should be stressed because sometimes rash statements are made to the effect that the above formula—which, incidentally, displays a considerable formal similarity to the models of the Harrod-Domar type is applicable to every system of production. It is not our intention to enter into the discusion of the theory of growth of a capitalist economy. But every reasonably correct model of this type must take into account the demand side as well as the supply side. With this approach investment cannot be treated exclusively as a factor forming new productive capacity and thus new supply, but should be treated as a factor creating demand. Investment undertaken in a given period of time is a basic factor determining total demand and thus the income produced. In turn, an increase in investment becomes a basic factor determining in a given period of time an increase in aggregate demand (not supply) and in consequence—an increase in income. Only at a certain level of an increase in investment will the effects on the demand side be sufficient for creating income corresponding to the full employment of the productive capacity. It is easiest to emphazise this point by assuming that for a certain period of time productive investment is stabilized on a fairly high level. In a socialist economy this stabilization will be accompanied by an increase (even though at a relatively declining rate) in national income as long as the stabilized investment expands productive capacity. In capitalism, on the other hand, investment stabilized on the same level will be accompanied by the stabilization of total demand and supply, and an increase in productive capacity will simply raise the amount of unused capacity.

This problem can be fairly clearly presented graphically. Let us measure, as before, the volume of investment along the axis of abscissae and the volume of income corresponding to the given level of investments along the axis of ordinates. The relationship between investment in a given period of time and income in the same period is a specific feature of a capitalist economy and is expressed by a simplified linear relationship of the type $D_t = \frac{1}{i} \cdot I_t$. When investment is I'_t then income must be D'_t. Let us measure along the negative part of the axis of abscissae the amount of fixed productive capital assets M and along the axis of ordinates the volume of income that can be achieved when these assets are fully employed. Moreover, let us assume that there is the linear relationship $D_t^{M,\max} = \frac{1}{k} \cdot M_t$, where k denotes both the average and the incremental coefficient of capital intensity. Thus the income that can be attained in year t is $D_t^{M,\max}$ and is greater than D'_t because we have assumed that, as is usual in a capitalist economy, there exists some unused capacity. The segment $D'_t D_t^{M,\max}$ denotes then the amount of unused productive capacity measured by the difference between the actual income and the attainable income.

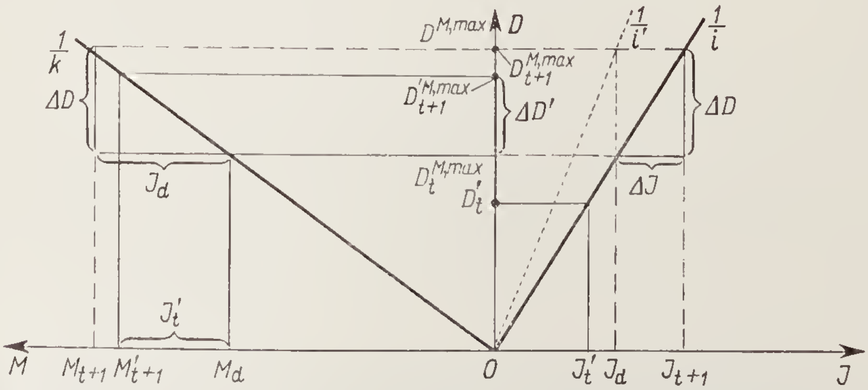


Fig. 2

Let us now assume than in the year $t+1$ investment does not change and is I'_t. Consequently income does not change either, and is D'_t. The productive capacity, however, is in this year $M'_{t+1} = M_t + I_t$ (for simplicity we disregard improvements and the wear and tear of productive equipment), because the increase in productive equipment in the year $t+1$ is determined by investment in the year t.

To the assets M'_{t+1} there corresponding the income $D^{M,\max}_{t+1} = D^{M,\max}_t + \Delta D'$, and $\Delta D'$ denotes an increase in income corresponding to the increase in productive capacity by I'_t. However, owing to the stabilization of investment, there was no increase in income in the year $t+1$, but an increase in unused capacity of the order $\Delta D'$.

It is also easy to see that there is a certain level of investment and a certain rate of their growth which ensure a stable and full employment of productive capacity. The investment in the year t would have to be I_t and in the year $t+1$ they would have to reach I_{t+1} that is they would have to increase by ΔI. Only in this case income in the year $t+1$ would increase by $\Delta D = \frac{1}{i} \cdot \Delta I$ and simultaneously the increase in income in year $t+1$ due to the investment in year t would be $\Delta D = \frac{1}{k} \cdot I_t$. We would have then

$$\frac{1}{i} \cdot \Delta I = \frac{1}{k} \cdot I_t,$$

hence

$$\frac{\Delta I}{I_t} = \frac{1}{k} \cdot i.$$

The right side of this equation is a simplified version of the Kalecki formula expressing the growth of productive capacity without the factors a and u. The left side—characteristic of a capitalist economy—is the increase in investment required for an increase in demand sufficient for ensuring a stable and full utilization of productive capacity. In capitalism, however, even a stable growth in investment is problematic, to say nothing of a stable growth at the rate $\frac{1}{k} \cdot i$.[6]

In a socialist economy, on the other hand, income in a given period, as we have shown, does not depend upon investment in this period. Income depends upon the productive capacity available and, as a rule, fully employed in a given period. If, investment is stabilized then it does not follow, by any means, that income will be stabilized (for simplicity we still ignore the coefficients a and u); only the rate of investment i will change with an increase in income.

For national income in year t equal to $D^{M,\max}_t$ the rate of investment is a function of the level of investment I_t; hence we have $i = \frac{I_t}{D^{M,\max}_t}$. The increase in income in year $t+1$ is ΔD, regardless of the level of investment in year $t+1$ which is I_{t+1}.

[6] Cf. E. D. Domar, *Essays in the Theory of Economic Growth*, New York 1957, p. 97.

This is so because it depends—in accordance with the formula $\Delta D = I_t \cdot \frac{1}{k}$—upon investment in the *preceding period.* If investment in year $t+1$ are stabilized at the level I_t then, the rate of investment will simply change from i to i'. It is possible, of course, that investment will increase by ΔI to I_{t+1} in year $t+1$. In this case the rate i will not change. However, if for some reason it is deemed desirable that investment be raised by more or less than ΔI, or even be lowered, then, anyway, this will not affect the magnitude $D_{t+1}^{M,\max}$ which—we repeat—is determined by the existing productive capacity. Changes occur not in income, but in i and are expressed by different slopes of the straight line with the slope coefficient equal to $\frac{1}{i}$ which is the reciprocal of the rate of investment.

Does it follow that demand does not play any part in the process of growth in a socialist economy? By no means, but its role differs from that played by demand in a capitalist economy. Demand is a primary magnitude only in societies which have lost control over the functioning and the development of their productive resources. In a socialist economy it is not income and the degree of the utilization of productive capacity that are adjusted to total demand but it is the other way around—total demand must fit into the framework determined by the actual level of income which, in turn, depends upon the existing productive capacity[7].

It is obvious that there is always a conflict between the existing needs and the level of income which limits the possibilities of satisfying these needs. These possibilities always lag behind growing social needs and this is a basic stimulus to growth. Each determination of the size of demand is only a current solution of the conflict between social needs and the possibilities of meeting them. The conflict itself will always exist because social needs will never stop growing. The thesis of the saturation of needs is based on the assumption of their stability while in fact they are a product of historical development and, to some extent, a function of the development of production itself. The conflict between the production potential and demand is then a manifestation of a basic conflict characteristic of a given type of production relations and not a result of a high level of the satisfaction of social needs, as some apologists of capitalism allege.

2. *Full Employment of the Labour Force*

In discussing the notion of full employment of productive capacity we have disregarded labour force. We have tacitly assumed that labour force is not a factor limiting the degree of the utilization of capacity. Turning now to the notion of full employment of labour force we assume, for the time being, that the productive apparatus is not a factor limiting the degree of the utilization of capacity.

[7] Limitations on the side of demand, however, may appear in some branches of production in consequence of wrong decisions concerning past investment.

As full employment of labour force can be understood a situation in which all persons able to work can find employment. In spite of its apparent simplicity this statement is not deprived of a double meaning. The notion of "being able to work" depends partly on biological factors, particularly age, and partly on socio-economic factors. Some factors extend the period of occupational activity (particularly the extension of the length of human life), others shorten it (the extension of the period needed for general education and trade training, the system of retirement pensions). Both groups of factors are strongly affected by the development of material production. An essential part is also played by customs, religion and other factors limiting sometimes the possibilities of employment for certain social groups. This is often true as far as women are concerned. Under given social and economic conditions, however, the number of persons able to work is given, with a certain limitation which shall be discussed later.

Under given conditions the length of the working week or the working year is also given. In modern societies the maximum length of the working week is regulated by law and this length—varying in historical development but given in a given period of time—is a point of reference for defining full employment. If, for instance, the length of the working week is 48 hours but a certain portion of the labour force is—against its will—employed only for 32 hours a week then we have a state of less than full employment of the labour force.

In consequence we can determine not only the number of persons able to work but also the potential length of working time during a year. Thus, to a full employment of the labour force there corresponds—at a given level of the productivity of labour and a given distribution of manpower between the productive and non-productive divisions—a certain potential level of national income $D^{Z,\max}$. If the actual income $D = D^{Z,\max}$ then we have a state of full employment.

A developed socialist economy is characterized by the absence of unemployment and a full employment of its labour force, as its essential features. This requires an adaptation of the territorial and the occupational structures of labour power to the trends in the demand for labour. This adaptation is an intended objective of economic planning and, sooner or later, is achieved. As long as there exist structural imbalances we can talk of a full employment of the labour force in principle if, firstly, the number of vacancies is at least equal to the number of jobs wanted and, secondly, if the number of persons seeking work is relatively insignificant in comparison with the labour force.

In our further considerations we shall disregard the territorial and structural distribution of the labour force and consequently the degree of its employment will depend simply on the demand for labour forthcoming from the productive (and non-productive) division. Since we have already assumed that the production apparatus, that is the objective conditions, do not constitute a limiting factor, then in a socialist economy in which the demand for labour is determined by planning no other state than that of full employment is conceivable under the assumed conditions.

The situation is different in a capitalist economy. Here the workers have a right to work and to earn income providing that they simultaneously create profits. The demand for labour is then a derivative of profits and as a rule is below the level of full employment. Both for purely economic reasons and for political reasons capitalism cannot exist without "a reserve industrial army" which appears as a more or less permanent phenomenon in the course of business cycle. The existence of business cycles by itself assumes less than full employment of labour force. Indeed, without unemployment and or part-time employment sudden increases and declines in production and employment would be impossible.

In a socialist economy, because of full employment, an increase in employment is limited by an increase in the labour force. In other words, if the state of full employment is to be maintained then employment must increase at the same rate as the labour force or the population, if it is assumed that the rates of growth of the labour force and of the population are identical.

The balancing of the rates of growth of employment and of the labour force is a necessary condition for maintaining full employment, but it is not a sufficient condition if in the starting period there was a state of less than full employment of the labour force. Let us denote the labour force in the period zero by N_0 and the employment in the period zero by Z . If the labour force and employment are a function of time and grow at the annual rate γ and α respectively then in the year t

$$N_t = N_0(1+\gamma)^t$$

and $$Z_t = Z_0(1+\alpha)^t.$$

Thus we have $N_t = Z_t$ when $N_0 = Z_0$ (i.e. when there was a state of full employment in the starting period) and when $\alpha = \gamma$ (i.e. when the labour force and employment grow at the same rate). If, however $N_0 > Z_0$ then also $N_t > Z_t$ even though $\alpha = \gamma$, and then a state of less than full employment persists in spite of the same rate of growth in the labour force and employment. Unemployment will grow in this case also at the rate $\alpha = \gamma$.

Of course, if even the condition $\alpha = \gamma$ is not satisfied, and $\alpha < \gamma$, then regardless of what the situation was in the starting period unemployment will arise or increase in consequence of the rate of growth in the labour force being faster than the rate of growth in employment.

At full employment the rate of growth in employment cannot, in principle, surpass the rate of growth of the labour force. However at full employment there may exist certain groups of people who under certain circumstances would be willing to start working although they are not actively seeking work. This applies, first of all, to married women and to those branches of economy in which paid work has not been distinctly separated from family labour. It is particularly difficult to determine the degree of the utilization of labour power on privately owned peasant farms. It can be assumed that persons who live on farms and whose productivity of labour is nil (that is those whose labour does not increase production) are actually unemployed. Full employment means then that there does not exist this kind of unused labour force, also known as agrarian overpopulation. However, even if there is no overt agrarian overpopulation there still may exist in agriculture certain surpluses of manpower whose productivity of labour is relatively low in consequence of its not being fully employed during the year. It might be said that this is underemployed labour force. It can partly be transferred (at a certain loss to agricultural production) to non-agricultural occupations, and partly can combine work in agriculture with some other employment. In such cases it is possible to activate a certain part of the population by appropriate economic measures and to increase employment more than it would rise by itself owing to a natural increase in the labour force, (the building of nurseries and kindergartens, the provision of transportation to the place of work, seasonal employment, etc. are the examples of such measures). The activating of these groups raises the coefficient of occupational activeness, that is the ratio of those employed to the total population. Thus, even at full employment, if and as long as this ratio increases, the rate of growth in employment may grow faster than the rate of population increase. However, after the reserves hidden in these marginal groups have been exhausted and a new ratio of employment to population has been achieved the rate of growth of employment must again be adjusted to the rate of growth of the labour force and population.

Under certain circumstances it may happen, however, that the coefficient of occupationel activeness will decline. For instance, when married women who bring up their children go to work because the earnings of the head of the family are insufficient then an increase in these earnings may result in a drop in the employment of married women. An extension of the period of edueation and the lowering of the retirement age would have a similar effect. In all such cases the rate of increase in employment is lower that the rate of growth in the labour force and the population as long as the coefficient of occupational activeness declines. After a new ratio of those employed to the whole population is reached the rate of growth in employment increases and catches up again with the rate of growth in the labour force.

3. Growth at the Full Employment of Productive Forces

We shall now attempt to relate our considerations on the full employment of the production apparatus on the one hand, and of the labour force, on the other. As long as we discuss these two problems independently, a full employment of the productive apparatus may be accompanied by less than full employment of the labour force, and vice versa. To some extent this is a problem of a technical and balance-sheet nature. The production apparatus in order to be fully utilized requires a certain amount of manpower. If both these quantities are adjusted to one another then a full employment of productive resources is possible from the technical and balance-sheet point of view. Whether this possibility is turned into reality will depend to a considerable extent, on the nature of the production relations. In a socialist economy the adaptation of the productive apparatus to the resourccs of manpower is not only a necessary, but also a sufficient condition for a full employment of productive forces.

If the production apparatus is not adjusted to the available labour force and does not provide jobs for all those able to work then even in a socialist system full employment is not immediately attainable. However, this kind of situation is in a socialist system a relict of the past and no matter how serious problems it may produce it is not a characteristic feature of a socialist economy. Appropriate steps will be taken immediately to remedy the situation and sooner or later imbalances will be eliminated.

Generally, if

$$D^{M,\,\max} = D^{Z,\,\max}$$

$$\text{then} \quad D \leqslant D^{\max}$$

i.e. if a full employment of productive resources is possible from the technical and balance-sheet points of view then income can reach the maximum value or be below it. The first alternative occurs in a socialist system and the second—in a capitalist system.

$$\begin{aligned} &\text{If} && D^{M,\max} > D^{Z,\max} \\ &\text{then} && D^{\max} \leqslant D^{Z,\max} \\ &\text{and if} && D^{M,\max} < D^{Z,\max} \\ &\text{then} && D^{\max} \leqslant D^{M,\max} \end{aligned}$$

i.e. if the production apparatus and the labour force are not adjusted, then the maximum value of income corresponds to the full employment of the factor that is at a minimum. Income may, or may not, reach the maximum, as has already been stressed, depending upon the nature of production relations.

Let us assume that in the starting period there exists the state of full employment of productive forces of the type $D = D^{M,\max} = D^{Z,\max}$. If the economy grows at that time then certain conditions must be satisfied to ensure full employment of productive forces also in the future. Let us first assume growth without technical progress with the resultant constant productivity of labour; also the incremental (meaning the productivity of labour in production establishments put into operation every year) and the averagc productivity of labour are equal to one another. Under these circumstances the rate of growth of income, determined by the direct factors, i.e. the living labour and its productivity, is equal to the rate of growth in employment

$$r = \alpha$$

and growth ensures the full utilization of productive resources when

$$\alpha = \gamma$$

i.e. when the rate of growth in employment equals the rate of growth in the labour force or the population with a given coefficient of occupational activeness.

To growth thus defined, at given coefficients k, a, and u, there corresponds a defined rate of investment i. In accordance with the indirect formula we have

$$r = \frac{1}{k} i - a + u$$

and in accordance with the direct formula

$$r = \alpha$$

hence $\frac{1}{k} i - a + U = \alpha$

and hence $i = k(\alpha + a - U)$

We can find the rate of investment graphically by measuring off the rate of growth of the labour force γ along the axis of ordinates. The line parallel to the axis of abscissae intersects the straight line $\frac{1}{k}$ at the point A whose abscissa gives the rate of investment ensuring full employment of productive forces.

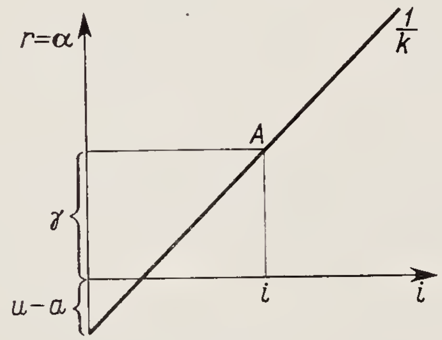


FIG. 3

It should be emphasized that in a capitalist economy growth at a full employment of the productive capacity—even if it were considered possible—does not necessarily ensure a simultaneous full employment of the labour force. Indeed, the postulate by Domar that investment grows in accordance with the formula

$$\frac{\Delta I'}{I} = \frac{1}{k} i$$

means only that demand should grow at the same rate as productive capacity. If this happens and if at the starting point the productive capacity is fully employed, then it will continue to be so. This will be accompanied by some rate of growth in income and if the productivity of labour is constant then it equals the rate of growth of employment

$$r = \alpha$$

If, however, $\alpha < \gamma$ then unemployment results even if there was no unemployment at the starting point.

Full employment of the productive capacity in a capitalist economy is possible, then, at various rates of growth in income, providing that the Domar formula is satisfied. These various rates of growth in income are accompanied, at a constant productivity of labour, by various rates of growth in employment. If to start with we have full employment then an increase in employment cannot, of course be greater than it is made possible by an increase in the labour force, but it can be smaller. Under these circumstances the productive capacity can be fully utilized with less than full employment of the labour force.

In a capitalist economy there are considerable and almost insurmountable difficulties in satisfying the conditions of the Domar formula. This is so because the formula has to be satisfied not at any level, but at the level strictly determined by the balance of the labour force and by the rate of its growth, if full employment is to be maintained (and if—as we have assumed—the productivity of labour is constant).

In a socialist economy both the determination of the rate of growth at which the productive forces are fully employed and its realization by an appropriate rate of investment, strictly defined with our assumptions, is quite feasible, because it is ensured by economic planning. On the other hand, in a capitalist economy, even if it is assumed that this rate of growth and the rate of investment necessary for sustaining it are known, there do not exist the institutional conditions that would transform this knowledge into economic decisions. There is no wonder then, that the rate of growth corresponding to a full employment of productive forces is an unattainable goal in a capitalist economy.

Is it true, though, that for given γ, i.e. for a given rate of growth in the labour force, the rate of growth with a full employment of productive resources is strictly determined? It is easy to see that this conclusion has been arrived at on the simplifying assumption that the productivity of labour is constant. It should suffice to take the less abstract assumption that with technical progress and with constant k the rate of growth of the productivity of labour β is given (thus the previously described situation when $\beta = 0$ is a special case).

Assuming as before that at the starting period productive resources are fully employed the rate of growth of income determined by the direct factors is

$$r = \alpha+\beta$$

where the product $\alpha \cdot \beta$ has been left out as an insignificant magnitude. Hence the rate of growth in employment is

$$\alpha = r-\beta$$

and the condition of growth ensuring a full employment of productive resources is the equation

$$\alpha = \gamma$$

Since, at the same time, according to the indirect formula

$$r = i \cdot \frac{1}{k} - a + u$$

then $i \cdot \frac{1}{k} - a + u = a + \beta$

hence $i = k(a + \beta + a - u)$

and i is again strictly determined.

The rate of investment required under these conditions can be found by a graphical method:

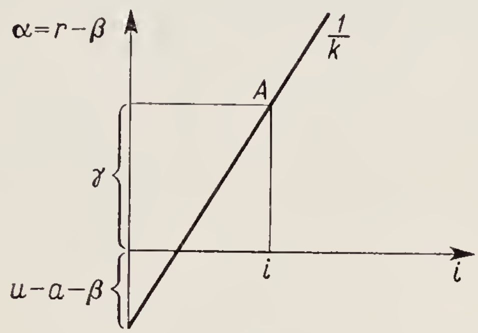


FIG. 4

Measuring off the rate of growth in the labour force γ along the axis of ordinates we find point A whose abscissa gives the rate of investment for growth with given coefficients u, a, β and k.

Thus also after allowing for technical progress and increased productivity there exists a certain rate of growth that makes possible a full employment of productive forces and there also exists a rate of investment needed of attaining the required rate of growth. However, it is less strictly determined than in the case when it is assumed that there is no technical progress. For a given coefficient k, there may be different values of β and thus there may exist different rates of growth satisfying the condition of full employment of productive resources. Therefore, the postulate of the rate of growth thus defined does not mean that there is here no problem of the optimization of the rate of growth of income, all the more so that also the coefficient k is subject to economic choice in connection with the choice of the technique of production. When there is no technical progress the constancy of k means also that $\beta = 0$ and therefore, since the rate of growth of the labour force γ is given, the rate of growth at full employment of productive forces is also given. In this case the rate of growth is not subject to economic choice, but the way of achieving this rate ensuring a full employment of productive resources is subject to choice.

We have already indicated that there are insurmountable obstacles in maintaining this rate of growth in a capitalist economy, even in the simplest case when technical progress is disregarded and when it is assumed that the productive resources are fully employed at the starting period. Very often, however, the problem consists not in maintaining but in achieving the rate of growth ensuring full employment of productive resources. Furthermore, with technical progress and increased productivity of labour the rate of such growth is not uniquely determined, particularly when the coefficient k varies. In such cases the problem is much more complicated. The objective is not to maintain given rates of growth and of investment but to change them so that the productive forces can be fully employed. The problem consists also in choosing an optimum rate of growth under given conditions. To solve these problems the economic system must be flexible in determining the fundamental economic ratios, must know the starting point and the objectives as well as the intermediate stages of the chosen road. From this point of view the superiority of the planned socialist economy over the spontaneous capitalist economy can be most easily seen.

ALAIN COTTA
FRANCE

THE GROWTH OF THE SECTOR OF PRODUCTION

1. *The aggregate analysis of growth is gradually exhausting its potentialities*

THE RECENT theoretical developments which give up summary relationships and sequences (rate of investment, accelerator, average or marginal capital coefficients, ...) to the benefit of a Cobb–Douglas function are proving less and less convincing. We may even ask ourselves whether the latest measurements of growth in the American economy do not definitely call into question the adequacy of this type of analysis.

As we all know, the aggregate production function is represented as follows[1]:

$$V_{(t)} = A \cdot T^{\alpha}_{(t)} \cdot C^{\beta}_{(t)} e^{vt}$$

The introduction of the variable V, is necessary in order to take into account the increases in the national product independant of those of the factors of production T and C. Furthermore, most observations suggest that this variable, which was initially assumed to contain residual and negligible phenomena only, is in actual fact of paramount importance. Solow goes so far as to maintain that the rate of technical progress alone accounts for approximately 80% of the long period growth of the national product of the U.S.A. A similar conclusion is reached by the authors of recent long-term projections for continental Europe, where the value of the coefficient V is situated between 1 and 2%, for the last decade.

---

[1] where Y = National Income or National Product
T = Labour
C = Capital
α and β = Elasticity of the National Product with respect to Labour and Capital
V = Rate of increase in productivity
T = Time
A = Constant of adjustment.

The use of the Cobb–Douglas function has become almost systematic over the past few years, in particular for the elaboration of long-term projections. Much could be said about these techniques which are more attractive than convincing. See in particular: B. F. Massel, *Capital Formation and Technological Change in United States Manufacturing*, "The Review of Economics and Statistics", May 1960; R. Solow, *Technical Change and the Aggregate Production Function*, "Review of Economics and Statistics", No. 39, 1957; *Rapport d'un groupe d'experts*, C. Gruson, Jean Bernard, R. Regul ... in *Méthodes de prévision du développement économique à long terme*, in Informations Statistiques, "Revue de l'Office Statistique des Communautés Européennes", No. 6, 1960.

The values found for this rate of productivity V almost lead to a rejection of the function—and of the analytical approach underlying it—since the quantitative relationships it aims at explaining, turn out to be of secondary importance only. But such values also entail a condemnation of the use of a mere V coefficient. It is hard position indeed, to go on regarding the "Rate of Technical Progress" as a catch-all variablc, whose importance is admitted to be equalled only by our ignorance of it. It is, of course, always unpleasant to have to realize that all our statistical measurements lead to a flat denial of the theory which made them possible. Still, we had better facc the fact, rather than close our eyes and begin once again to harp on the ever-so attractive themes of Innovation and Productivity. By keeping our eyes open we should at least be able to see the dual course which the theory of growth and its associated product—i.e., of fluctuations—ought to take nowadays.

The first should be that of analytical precision, and a measurement of phenomena relating to innovation. It is high time we stopped opposing the "qualitative" argument of technical progress to statistical measurements of past periods of growth. The only way in which we can do this is to begin by making detailed case studies of the changes in the production functions of firms and industrial branches, which, contrary to aggregate measurements, do retain their full meaning. Only a more precise knowledge of long-term changes in technical coefficients can enable us to rid ourselves of Schumpeter's incantations. Not only shall we have to measure these changes, but also obtain such a knowledge of their history as will make them understandable. At a later stage, we shall undoubtedly have to re-examine the essentials of our theory of investment. One cannot fail to notice how little influence the advance of our understanding of the process of growth has had on the analysis of investment. The models of growth have multiplied. Every ideology (Marxian, classical and neo-classical, or Keynesian) now has its own. All are based, more or less directly, on an investment function which remains singularly summary. We still seem to remain faithful to thc traditional image of a private investment decision, which mathematically formalized or not, is based on the marginal calculus. The analysis of public investment, whatever the types of dcvelopment or the nations which it is supposed to represent (developped or under-developped, planned or unplanned economies) is still in its infancy and contents itself to-day with ascribing to the public authorities ultra-marginalist motivations. Finally, no effort has been made to take into account the major discovery for thc second half of this century: the fact that the kind of investment on which growth always hinges is not the production of machinery, but either thc creation of an "infra-structure", or the increase in the sum of human knowledge (total and per capita).

The second course which the theory of growth should take springs from the stalemate in which our all purely global explanations find themselves to-day. Yet, history teaches us that thc development of national aggregates is always diversified. This is the case for national output, total income, the flows of internal and external transactions. This diversity, once it is admitted and recognized, leaves us with a choice between two types of analysis. We can either first study aggregate growth and then try to explain it by splitting it up into its most characteristic components; or we can admit that the development of national aggregates is only the result of a statistical reconstruction, the integration into a whole of those partial flows which are always our first personal experience of reality. In the latter case we ought to reverse the logical sequence of our approach—i.e., we ought to concentrate our efforts on explaining the growth of these partial flows which, once summated—a process which of course has nothing to do with that of arithmetical addition—give us aggregate growth. The essential problem then becomes that of choosing the lines of dissociation characteristic of aggregate flows. It is not insoluble, especially since we already have at our disposal sectoral dissociations. The theoretical reconstruction of aggregate growth can then be carried out from the development of the sectors of production, provided we know its general condition and mechanisms[2]. We shall attempt to do so by adopting the classical distinction[3] between the sectors producing consumer goods and those turning out producer goods.

2. *The Growth of the Sector of Consumer Goods*

The growth of the sector of production has in the western analytical developments of the last twenty years never aroused so much interest as aggregate growth. Our knowledge here, is at best purely historical. However, monographs dealing with the long-term development of the production of certain sectors are beginning to mount up. It should now be possible to give an analytical explanation of the regular patterns which they contain.

2.1. Two types of sectoral development are most frequently met: semi-logarithmetic growth and logistic growth. The former holds for a great number of sectors producing food stuffs. The latter for all the other sectors, particularly those producing durable consumer goods[4]. These two types of adjustment take into account the changes in the rate of growth of a sector through time, and also—notably in the case of the logistic adjustment—the existence of a "ceiling value"

---

[2] This was attempted in a two-sector model of growth presented in Warsaw in November, 1961, cf. *Un Modèle de Croissance Intersectoriel* in "Economie Appliquée", No. 1–2, 1962, pp. 199–227. But, not to mention the fact that in this earlier model the contents of the two sectors are not specified, the justifications given for the pattern of growth assigned to each of them are, at the very least, inadequate.

[3] We have here deliberately left out all the problems relating to the statistical definition of the "sector", and the classification of certain sectors into "producer goods" or "consumer goods".

[4] See S. Kuznets, *Secular Movements in Production and Prices*, New York; L. Dupriez, *Des mouvements économiques généraux*, Louvain 1951; W. G. Hoffmann, *The Growth of Industrial Economics*, Manchester 1958, First Edition 1931; W. W. Rostow, *The Stages of Economic Growth*, Cambridge 1960.

of production. Their use, which has become more and more common, invites the following remarks[5].

(a) In all cases the functions chosen to represent the growth of a sector ($P_{(t)} = a+b \log \cdot t$, $P_{(t)} = \frac{m}{1+a^{b-at}}$, $P_{(t)} = K \cdot A^{(B^t)}$, or Gompertz curves) are only statistical adjustment functions. Real values often deviate from the curve of the mathematical function, whatever the quality of the fit.

(b) It must not be forgotten that all historical developments can always be adjusted to fit several of these functions. The logistic and the Gompertz functions are here often in competition, both being similar means of representating an S-shaped curve. The same possibility holds for the semi-logarithmetic and logistic functions, since the upper part of both curves can, according to the choice of coefficients, present an almost identical shape. In no case then, can an economic content be given without further analysis to a mathematical presentation.

(c) Even if we admit here that the logistic adjustment is often the better one, it appears that no "ceiling", *stricto sensu*, can be assigned to production. Even if the rate of growth of sectoral production follows the pattern specific to logistic development, the existence of a permanent maximum value is rarely noticed. The production of the sector continues to grow slowly in the region of this value, ($m = P_{(t)}$) or else it falls sharply.

(d) Even when it would seem to be ascertainable, the existence of a "ceiling value" is only temporary. If we fit data for fairly distant periods, we are confronted by sudden "leaps" upwards of the ceilings (contained in the changes of the constant m) which alter the whole appearance and significance of the logistic curve. The production of the sector appears then to have obeyed two or several logistic functions. It must therefore be admitted that the shape of the curve is always subject to historical contingency.

(e) Whatever may be the qualities of the adjustments carried out and whatever may be the frequency and the importance of the discontinuities observed, there are at least two historical features common to all the sectors producing consumer goods, irrespective of the economic system in which they take place.

First of all, the changes in the rate of growth of the sector through time. Without forcing history it can be stated that every sector lives through three distinctly different periods. The first is that of its birth: *the period of implantation*, characterized by the small value of its total output, both in absolute and relative terms, and by an increasing rate of growth. The second is *the period of maturity*, in which the absolute and the relative level of production becomes significant while the rate of growth, by this time high, remains fairly constant. In the third period, that of *senescence*, the sector grows at a very slow pace or decreases, while the relative and absolute levels of its production remain high but are decreasing.

[5] On the subject of the logistic trend and its interpretation, see the appendix where secondary mathematical developments have been confined.

Next, we notice immediately that in each of these three stages the rates of growth of the sectors are linked with the rate of growth of the whole economy. The connection is, of course, positive. Therefore, the three periods of each sector are much more clearly defined—and can be better compared—if we measure the rate of growth of a sector at each point of time, in relation to that of the remaining sectors.

It is precisely the regularity of these two patterns which require an explanation. The fact is, that although monographs are mounting up as was mentioned earlier, no theoretical analysis of the logistic development of sectoral production has been really attempted. The only question to which an answer is sometimes sought is not so much that of the plurality of the stages, as that of the reasons for the decrease in, or the slowing down of the rate of growth. A set of two forces is then brought in, named "*expansionary*" and "braking" factors, among which are found pêle-mêle somewhat ill-defined variables: the willingness to innovate, the saturation of demand, public intervention and still others. Even if, admittedly, there is nothing here but a lack of accuracy, a large number of questions still remain unanswered: the differences between the logistic patterns of the various sectors (i.e., the different values of the coefficients m, a, b,) the meaning of the "leaps" of the ceilings, the economic content of the decreasing stage—when it exists. A systematic and general interpretation of the development of sectoral production, remains therefore necessary.

2.2. The birth of a sector is that of a commodity. The appearance and the development of the sector is directly related to the newness of the product and to the way in which the new commodity is accepted socially and is spread throughout the whole economy. The relation is not only one of simultaneity but one of cause-and-effect. *It is the speed with which the commodity is spread in the community which determines the rate of growth of the sector.* An analysis of the speed of diffusion must then precede that of the rate of growth and its changes in the long period.

The notion of propagation or of diffusion, and therefore the measurement of its velocity, is connected with the number of individuals or households who acquire the commodity. For each commodity a "rate of possession" can be defined and measured, namely the ratio of the number of units of the commodity already purchased, to the number of individuals or households at a certain date t. Let the "rate of possession" be called T, so that:

$T_{(t)} = \frac{\Pi_t}{N_{(t)}}$ where $\Pi_{(t)}$: the total number of units of the commodities already in the hands of the consumers at t or "contingent"

$N_{(t)}$: the number of individuals or households at t.

The value of this ratio indicates the extent of the diffusion of the commodity in the economy at time t. The speed of propagation is therefore given by the changes in the ratio through time, i.e. by:

$$T'_{(t)} = \frac{dT_{(t)}}{dt}$$

Whether or not we admit at this stage that the number of individuals and of households varies (i.e., that there is a certain rate of change of the population), the determination of the function $T_{(t)} = f(t)$ enables us to know both the speed of propagation and the changes in the "contingent", in other words, the level of production of the commodity and its changes. The statistical measurement of this function is the same, every time we can rely on long-term series. But it only states in a different form the changes in the level and rate of production. An analytical explanation of the function appears more interesting as soon as we admit a causal relationship between the rate of possession of the commodity and the rate of growth of the sector.

Every function of an economic variable changing with time—however nice it may look—is indeterminate. The change in the rate of possession through time can only be explained by that of another variable, which also varies with time, but in a way which is better understood. One variable springs to mind here: Income. Indeed, as soon as the rate of possession of a commodity at any given date is measured, (and still more clearly, whenever several measurements relating to different points of time are carried out for several commodities), we are immediately tempted to find an explanation of it in terms of differences in income. The collective rate of possession is a function of the average level of income. The higher this is, the higher the rate or rates of possession. If we become aware of this dependence, we are also rapidly led to taking into account the distribution of income. The rate of possession $T_{(t)}$ then can be considered as a function of the level of income and its distribution. We have:

$$T_{(t)} = f(R_{(t)}) + u_{(t)}$$

where $T_{(t)}$ is the rate of possession, $R_{(t)}$ the income of the unit of consumption and $U_{(t)}$, the "uncertainty" variable containing all other determining factors outside income. The representation of this function does indeed vary with the different consumer goods, but, for most of these, follows the general pattern of change of an S-shaped, if not to say logistic curve.

This representation must be both justified and specified[6]. The justification is fairly simple. If we take an instance of durable consumer goods—the motor car, we can easily see the reasons why the increase in the rate of possession in relation to income, leaving aside all external elements contained in $U_{(t)}$, is not linear. The

---

[6] It is assumed in this diagram that there is a maximum rate of possession equal to unity. This is not realistic. Furthermore, the maximum will be different according as one considers individuals or households.

lowest income groups are unable to acquire the commodity, whereas the very high income groups do so without difficulty. It is those members of the community with average incomes who will be most consciously attracted towards the commodity and whose efforts to hold it will be the greater as their total income is lower. The *S*-shape of the curve thus describes the effort of saving of the middle classes who wish to look different from what they really are.

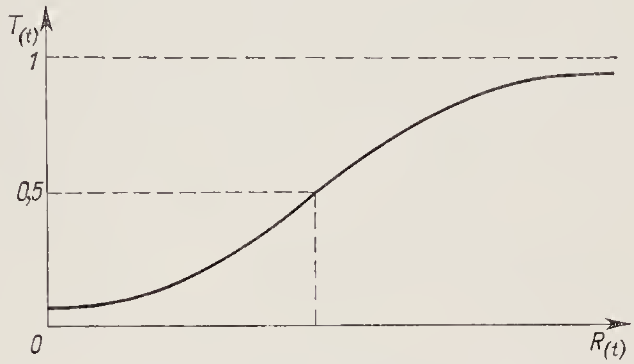


FIG. 1. *Rate of possession of a commodity and distribution of revenue*

Each of the relationships is specific to the commodity whose rate of possession is being analyzed. The diversity of the curves and of the value of the average rate of possession is a result of the influence of numerous factors, and notably of the objective degree of necessity. The higher this is, the higher the rate of possession of the commodity. Other factors nevertheless play an important role, for instance the degree of indivisibility of the commodity.

The speed of diffusion of the commodity through the economy $T'_{(t)}$ is then explained, partly by changes in the level of income and its distribution, partly by possible modifications of the variable $U_{(t)}$.

The increase in the income of households induces that of the rate of possession of the various consumer goods. If

$$R_{(t)} = g(t),\ T_{(t)} = fR_{(t)} = f(g(t)) = \varphi_{(t)}\,.$$

An increase in the numbers of each income group will shift upwards the curve representing the rate of possession. This shift, i.e. the rise in the average rate of possession, bears a direct relation to the rate of growth of the total income both of the community and of each particular group. We cannot here forget movements in prices. It appears very likely that the relative price of a commodity will decrease with time, under capitalism as well as in planned economies, although in different ways. The "real income" effect therefore always contains a "relative price" effect which explains its importance.

A detailed examination of the changes in the rate of possession shows, however, that the role played by changes in income is not entirely determining and that other influences are at play, contained here in the uncertainty variable $U_{(t)}$. This fact becomes clear when we compare the rates of possession of a certain commodity at different points of time, or still better, when we make several comparisons between different communities. In the latter case, we see that the same level of real income can give rise to different rates of possession of any one commodity. The uncertainty variable $U_{(t)}$ indeed, contains several factors which influence the rate of possession independently of income. The most important are the inducements emanating from the sector itself and/or from the public sector, and aiming at increasing the rate of possession at a given level of income.

Among the inducements emanating from the producers one must, of course, mention publicity and the introduction of credit facilities, whose combined effect is to increase the rate of possession by imposing or facilitating the anticipated purchase. Their object can even be to raise the maximum possible rate of possession (which so appeared before the advertising campaign). Everybody knows the recent advertising slogans in the U.S.A.: "no less than two cars per household" and "a television set for everybody". Credit is then the complement of such a policy. It enables desires to materialize more easily.

It often happens that, in addition to these inducements of a purely private nature, we find in Western economies public inducements. There are even cases where the role of the public sector is such that it then offers differences of degree only (and no longer on nature) with the action of the state in planned economies. Entire sectors (the motor car or the electrical household equipment industries...) are considered vital for a steady and rapid growth of the Western economies. This is why the public authorities intervene to forestall local recessions by raising, whenever necessary, the rate of possession of certain commodities. The means of this kind of policy are well known: selective credit policies, fiscal measures, etc.

These two types of inducements, private and public, combine in a very unequal manner, according to the country and the period. It is therefore preferable to introduce them in the form of an uncertainty variable, provided we are aware of its precise content and are thus able to analyse its variations with time[7]. The speed of propagation is, neverthcless, closely related to the growth of average income, to the changes in its distribution and to the modifications of the uncertain variable. The geometric representation of the changes in the rate of possession includes these two influences. The first of these shifts the curve upwards, the second to the left.

---

[7] It appears that in France, the value of the two variables (income and uncertainty factor) is exactly identical ($\frac{1}{2}$, $\frac{1}{2}$). On changes in the rate of possession of motor-cars, see E. Lisle and H. Faure, *Un modèle prospectif du marché de l'automobile*, "Revue du Credoc", No. 4, 1959. This variable $U_{(t)}$ could, incidentally, be more precisely expressed. The public and private inducements which it contains depend (in number and intensity) on the relative importance of the sector. It might not be impossible to replace the variable $U_{(t)}$ by a function $U_{(s)}$, with s expressing the relative importance of the sector.

2.3. The determination of the value of the rate of possession and of its changes through that of the average level of income and of its pattern of distribution appears at first sight to be a satisfactory hypothesis, all the more so since the factors at play beside income are taken into account with the introduction of an uncertainty variable whose content is specified. We may, however, ask ourselves whether this causal relationship is the best that can be found. Some doubt will arise if we realize that the two variables (the rate of possession and income) belong to two distinct fields of analysis: the former to that of stocks, the latter to flows. The concept of the rate of possession involves indeed that of a "contingent" and, for all available commodities as a whole, that of the capital (assets) of the households, since the commodities produced by the sectors here considered, are consumer goods.

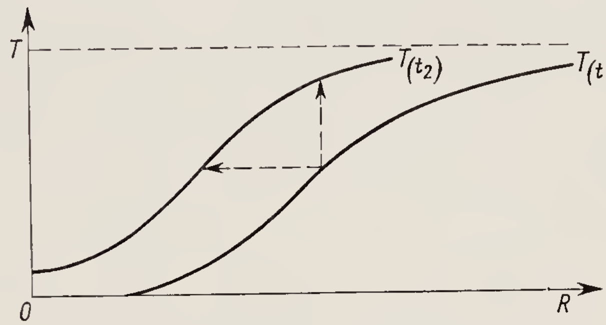


FIG. 2 *The development of the rate of possession*

The relationship which we have just presented is therefore affected by a defect common to all theories of growth: the monopoly held by explanations in terms of flows. It becomes more and more obvious that all the representations and analyses of growth make use only of the notion of flows, income notably. The growth of capital is certainly acknowledged via investment flows and capital coefficients. But it is not given primary importance; it is often even posited without subsequent analysis, as a condition and/or a consequence of an increase in flows.

The monopoly held by flows in economic theory would be justified if it were really possible to leave out capital altogether, that is to say, if all goods produced in each period of the development process were non-durable consumer goods. In this case, total consumption would indeed only be a periodic flow, and investment only a sum of future flows; the structural pattern of consumption would then depend exclusively on that of total income. It is precisely in so far as this kind of situation is witnessed in the economies still at the very early stages of their development, that the contemporary theories of growth provide a good account of 19th century developments.

They are, however, totally inadequate for the 20th century in which growth is essentially characterized by the appearance and mass-diffusion of durable consumer goods. Apart from foodstuffs, whose relative importance is continuously declining, all consumer goods have a lifetime which far exceeds the accounting period of flows: the year. There would be plenty to say here about the growing analytical lack of precision of national accounting aggregates. The level of consumption in particular, can be calculated only at the price of a series of evidently false assumptions regarding the lifetime of goods. Its calculation and its changes, particularly in a period of slump is then no longer really meaningful. What used to be a reasonable approximation some decades ago—and may still be so for under-developped countries—has for developped economies become a major defect which calls into question the validity of the concept of consumption. Even the distinction drawn more and more frequently nowadays between non-durable, semi-durable and durable goods, docs not take a sufficient account of the fact that the accumulation of durable goods by households is becoming the most important phenomenon in the growth of developped countries. One can do so only by positing that the demand for such goods depends on the capital of the household prior to any dependence on their income. The content of this dependence can be specified, only following a preliminary inquiry into the capital of the households and its changes. Let us attempt it by defining the capital of the household as the sum total of the goods held by the unit of consumption. Two large categories of goods can immediately be distinguished: physical commodities (including all the consumer goods whose lifetime exceeds a year) and all financial assets. For each unit of consumption, this capital is characterized by its total size and its composition—its structure—i.e., the two ratios expressing the relative magnitude of the two types of assets.

It appears possible to maintain that the structure of the capital of the household in fact depends on its total size. Three categories of households can here be distinguished and defined by the structure of their assets.

—When the total capital is very small, it consists of two elements assuming almost equal relative importance: on the one hand, a group of material assets of primary utility (furniture, clothing, ...); on the other, a small amount of financial assets constituting a safety reserve and nearly always held in cash[8].

—When the total capital grows the relative weight of the two elements changes, the greater share of the increase in total capital being devoted to the acquisition of material durable goods. There actually exists a "strategic" value of the total capital, beyond which a process of systematic and continuous acquisition of goods takes place. Two remarks must here be made, the first of which concerns the duration of this type of behaviour: this is essentially dependent on the number

---

[8] The problem of the valuation of material assets is here assumed to be solved. The structure of the capital owned by the household can then be measured by the two ratios:

$$\frac{\text{Value of financial assets}}{\text{Total capital}} \qquad \frac{\text{Value of material assets}}{\text{Total capital}}$$

While it is always difficult to give a definite value to long-term (fixed) assets (e.g. houses), the valuation problem is relatively easier in the durable goods for which there is generally a second-hand market. We can then choose between the familiar four types of valuation: market price—replacement costs—accounting value and subjective value.

and the value of the durable goods available in the economy. Nowadays, these are numerous and fairly costly but their total value can be calculated. The duration of the process of accumulation of material assets is a function of this total value *and* of the sums of money periodically used for this purpose. It therefore varies with the households, their desire and ability for periodic accumulation. The second remark concerns the latter variable; the capital of the household can grow in two main ways (if we neglect inheritances): the setting aside of a certain amount from income and borrowing. It is these last two factors which decide, for every household of the same nation and for different nations, how quickly a certain amount of private capital for expenditure can be built up.

Once this capital has been constituted, a second "threshold" appears as soon as the household possesses virtually all the range of material assets available on the market. Subsequent increases in capital either take the form either of sumptuary material assets (additional places of residence, collections, ...) or much more commonly, of a new type of financial assets. These are securities rather than cash, whether they take the form of stocks and shares (Western countries) or of Government bonds only (Eastern countries). As a rule it must be admitted that as total capital grows, an increasing share of it is made up by financial assets.

This very brief analysis of the development of the capital of households appears, sufficient to shed some more light on the process of determination of the rates of possession and therefore of the changes in the rates of growth of the different sectors. Indeed, the rate of possession of a commodity is on average higher, as the capital of the households increases. It then is a function of the average level of capital and its distribution. This distribution itself must be understood and measured in relation to the strategic values of total capital which have just been defined. What we know and can guess about this distribution of capital and above all, about its changes in every nation, enables us to suppose that the major feature of the growth now taking place in most countries is the fact that a high number of households is moving from the first category to the second. The diffusion of durable commodities is the most characteristic indication of the increasing affluence of the community, that is to say, of its growth. The changes in the rates of possession in relation to capital can be represented as follows:

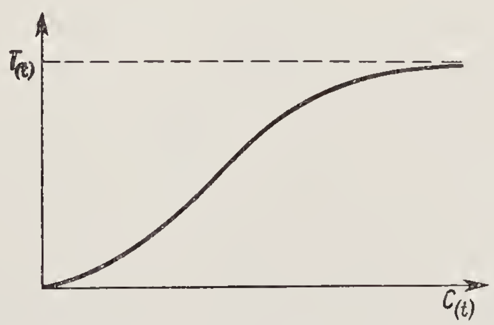


FIG. 3. *Rate of possession and capital of the households*

The rate of possession, which is very low when capital is less than its first strategic value, rises as soon as the threshold has been crossed. The shape of the curve would be totally discontinuous, were it not for the multiplicity of the goods to be acquired in the second stage of the accumulation of assets, the diversity of the order in which they are acquired and the absenteism of a small fraction of the owners of a very large amount of capital.

The increase in the rate of possession $T_{(t)} = f(t)$ is then closely related to that of average capital and still more to its distribution. This causal dependence is much more direct than the dependence on income since this is in reality only one of the possible sources of an increase in capital, the second being the raising of loans. It will enable us to answer the questions arising from the diversity of the logistic patterns of growth of the sectors of production.

2.4. The growth of the output of a sector producing consumer goods reflects the growth of the "contingent", that is to say, for a given population, of the rate of possession. The maximum rate of possession (one unit of the commodity per household or individual, or more) definies then a potential "contingent" which would be achieved by the end of the process of development. At every point of time a real contingent exists which is a measure of the demand which has already been met. The difference between the potential and the actual contingent defines and measures the potential demand, i.e. that fraction which remains to be met. Subsequent changes in the rate of growth can then only reflect changes in the rate at which the potential demand is met. The same holds for the diversity witnessed in the growth of different sectors.

Let us consider the emergence of a sector. When the new commodity appears the potential demand for it is very high but its immediately solvable fraction is limited by the distribution of capital among households. The new commodity is always, if only for a short period, a luxury article regarded and desired as such by the producers and the consumers and is reserved in fact for the owners of a very large amount of capital, i.e. the numbers of our third category. The acquisition of the commodity is therefore possible (only for this) privileged group in the short run. This varies considerably in size with the level of development reached by the economy where the new good is being introduced. The higher the level of development of the nation, the shorter is the stage of implantation of the sector and the higher its rate of growth during this stage. Again, the speed of diffusion will increase with the degree of divisibility of the commodity.

The stage of maturity is that of the diffusion of the commodity throughout the entire community. The new article, at first a luxury, gives rise to a potential market which the various firms wish to exploit. The psychology of "imitators" is well-known nowadays. In this second stage the speed of diffusion, and therefore the rate of growth of the sector will depend essentially on the distribution of capital and its long-period changes as well as on the changes in the relative price of the commodity.

The stage of maturity really begins when the possession of the commodity becomes possible for the households making up the second category, that is to say, with more than a certain amount of capital. Once this has been reached, however, the rate of growth of the sector depends on the distribution of capital and the more equalitarian the distribution, the higher will be the rate of growth. When this is the case the sharply rising part of the logistic curve will be contained between two dates relatively close together. If the distribution is markedly unequal and remains so in the long period, the stage of maturity will be much more spread over time. In the extreme case, where inequality in the distribution of capital is carried to its furthest limit (underdeveloped countries), the growth of the sector is seriously endangered by the absence of any mass-demand. Recourse to import then replaces the domestic creation of a sector. In every case, the rate of growth of the sector depends on that of the average amount of capital, i.e., on that share of income which is not spent on the acquisition of non-durable consumer goods (which is considerably different from the rate of saving given by present calculations), and on the average level of individual indebtedness.

We certainly cannot—as do most analyses conducted in terms of flows—neglect the changes in the relative price of the commodity. This price always falls in the long period on two accounts: first, because of "secondary innovations" which play an appreciably more important role than the simplistic conception of Schumpeterian innovation would like us to admit. The fall in the average cost (excluding profit) of a commodity is a result of enterprise in the general sense of the word, which under every economic system, consists both of management and the introduction of secondary innovations. But the relative price decreases also as a result of the fall in the profit margin. The profit margin inherent in the new commodity is generally high and always above the average. For a number of familiar reasons (the emergence of new producers, price competition, increases in capital costs...) it tends to decline until it reaches a so-called "professional margin" which is nothing else than the equilibrium profit margin of the sector, which closes the market to new competitors while ensuring the financial profitability of the capital already invested in the production process.

This fall in the relative price can follow different patterns. Two policies are possible in a capitalist as well as in a planned economy. Either the relative price can be quickly stepped down, so as to speed up the diffusion of the commodity and raise the rate of growth of the sector, or the action on this price can be more gradual—or even non-existent. The motivations behind these two extreme policies can, of course, be different. In a capitalist economy the firms (whether or not there is an explicit agreement between them) face a choice between the maximation of total profit and that of the rate of profit. In a planned economy the choice is more specific: it concerns the willingness of the authorities to favour or not the "democratization" of the commodity. In both regimes, however, there always is a strong pressure of demand for a faster diffusion of the commodity and for a reduction in the relative price; how and how soon this will take place depends on the degree of resistance to demand offered by the private firms and/or the public authorities.

As soon as the commodity has been acquired by all the individuals or households making up the second category, the third stage, senescence, sets in. The speed of diffusion slows down even if there is an increase in the average level of capital, since the still unfulfilled fraction of demand is diminishing. Furthermore, it becomes increasingly difficult to mobilize potential demand since this should now originate in the members of the third category. Even in a highly developed country there can always be found a certain proportion of individuals and households in an economically precarious condition, a sign of the imperfection of society. In this last stage the rate of growth is therefore closely dependent on the growth of population and on the intensity of replacements.

The development of the production of the sector is largely and permanently determined by the rate of increase in the number of households (or population). During the last stage the influence of this particular factor is more clearly seen. We have assumed so far that the "contingent" of commodities acquired, was a function of the rate of possesion only, the number of households remaining constant. If, however, the latter increases the production of the sector will continue to expand. This fact explains why the most usual pattern of development, during the stage of senescence of the sector, is one of slow growth and rather than complete stagnation—i.e., a situation in which the ceiling value, strictly understood, does not materialize. The production of the sector tends "logistically" towards an exponential curve.

The replacement of durable goods is one of the last phenomena which must absolutely be integrated into our analysis. Its existence has so far been assumed implicitely. Indeed, if the lifetime of consumer goods were infinite, the "contingent" of commodities acquired by the population would grow with an initially increasing and then decreasing rate until it reached a maximum value. Once this had been reached production would cease. The curve of its development, far from conforming to a logistic pattern, would only show an increasing and later decreasing rate of growth. The logistic trend is explained precisely by the demand for replacements. If we call Θ the average lifetime of the commodity, the total output of the sector becomes:

$$P_{(t)} = Pn_{(t)} + Pn_{(t-\Theta)}$$

where $P_{(t)}$ is the total output of the sector
$Pn_{(t)}$ the output sold as first equipment
$Pn_{(t-\Theta)}$ the output sold as "replacement"

The introduction of replacements enables us to explain the increasing period within the stage of maturity of certain sectors: it is the result of the adding-up of two types of demand, first equipment *and* replacements. We can also give a more precise meaning to the ceiling value of the logistic trend: This is equal to the number of replacements, periodically necessary, of the maximum "contingent". Much could be said here about the concept of the average lifetime of a durable commodity. This in fact covers great disparities. Those households with a high amount of capital will replace their goods more frequently than the other categories. Also, the average lifetime can vary as a result of publicity in particular, whose object is to reduce it. Indeed, the shorter the lifetime of the commodity, the higher the rate of growth of the sector will be throughout all its stages.

The introduction of these different variables (population, average lifetime of commodities, relative price...) leads to an explanation of the diversity in the pattern of growth of the various sectors and its discontinuity. The discontinuity of the ceiling value, already mentioned, can nearly always be explained, either by abrupt variations in the number of consumers (opening up or closing of a foreign market) or by sudden changes in the distribution of capital, due itself to modifications in the distribution of incomes, consumption patterns or borrowing facilities. It must be pointed out that this major discontinuity brings into question not the logistic trend itself but one of its common and easily-made interpretations: the notion of the saturation of demand. There is in reality no precise objective limit that could be assigned to the output of a sector turning out consumer goods. The only phenomenon which is sure to occur is the ultimate emergence of a stage of senescence, marked by a very slow rate of growth, stagnation or unequivocal decline. The end of a sector is never mechanistically pre-determined. But there is a basic causal relationship leading from the distribution of capital and its long-term variations to the changes in the rate of growth of output.

3. *The Growth of the Sector of Producer Goods*

The logistic trend is thus only one mean among others of accounting for the long-term pattern of growth of a sector producing consumer goods. Does it apply, even in this limited sense, for the other sectors of production, those which turn out producer goods? If this is the case, it can only indirectly be so because the tendency common to the sectors producing consumer goods will ultimately impose itself on the other sectors. The analysis of this new relationship between the developments of two different types of industry is likely to be more difficult in view of the multiplicity and the diversity of the connections between all the sectors. We can undertake it only by making a simplifying assumption at the outset, which we shall try to remove at a later stage.

3.1. The growth of output in a consumer goods sector implies that of those sectors which supply it with its factors of production. The propagation effect of the development of the consumer goods sector is thus determined by the various coefficients of its production function. We shall assume here that the consumer goods sector (A) acquires from a single producer goods sector (B) the whole of one of the factors which it requires. In view of the fact that the strongest links existing between two different types of sectors originate in the need for equipment goods, we shall here concentrate on this particular connection, which can later be extended to all other factors of production (raw materials in particular).

It is assumed that the equipment goods sector B develops by producing exclusively for the benefit of a consumer goods sector A, whose growth follows a logistic pattern. The growth of output of B is then that of the production of the investment goods necessary for A. Calling k the capital coefficient of sector A, the amount of capital installed in the sector is given by the expression:

$$K_{A(t)} = kP_{A(t)} = k \cdot \frac{m}{1+e^{b-at}} \tag{1}$$

The development of the capital installed in A therefore follows the logistic pattern. This will reflect every acceleration or deceleration of the growth of output, while amplifying them in absolute value since the ratio k connecting capital and output is above unity. The turning point on each curve then corresponds to the same point of time and the successive stages of growth are in the two cases identical. Only the ceiling-value will be different ($m' = mk$).

The output of sector B reflects the investment process carried out by sector A. That is to say:

$$P_{B(t)} = \frac{dK_{A(t)}}{dt} = \frac{a \cdot m' \cdot a^{b-at}}{(1+e^{e-at})^2} \quad (m' = mk) \tag{2}$$

The production of sector B following therefore the type of development outlined below (Diagram 4):

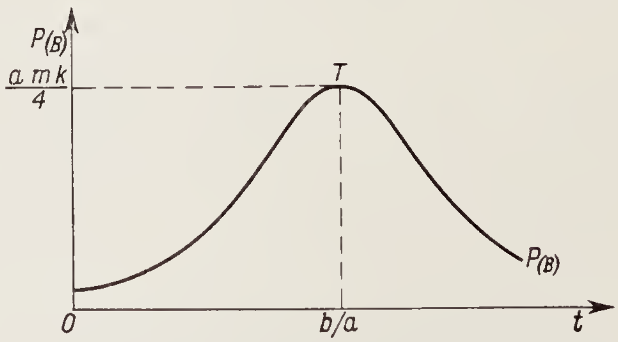


FIG. 4. *The growth of the sector of producer goods*

One can see the essential difference between the types of development of the two related sectors. While the growth of sector A follows its logistic trend, that of sector B is divided into two clearly distinguishable phases. The first one (OT) is a stage of sharp growth, taking place at an increasing rate which corresponds to he rising rate of growth of sector A, i.e., prior to the turning point ($t = b/a$).

The second stage sets in abruptly as soon as this point has been reached, that is, as soon as the output of the sector of consumer goods begins to grow at a decreasing rate. The production of the sector of producer goods B then falls in absolute value. The logistic trend of sector A is therefore propagated into sector B by determining its growth and fall but the type of growth transmitted by A is totally different from the pattern of its own growth.

We must here re-introduce two difficulties, connected with possible variations in the capital coefficients and the demand for replacements. We have, indeed, assumed that the capital coefficients (k) of the sector A remains constant through time. In the long period this assumption cannot be retained. We must on the contrary accept that this coefficient varies, or rather that it falls in the course of time. The fall in the average capital coefficient (induced by that of the marginal coefficient) is in full agreement with the conclusions reached in the monographs devoted to the measurement of their long-term variations. This fall can be regarded as a particular case of the changes in all productivity indices and as one of the reasons for the gradual decrease in average costs.

The peculiarity of this fall lies in the fact that the modification of the capital coefficient of a sector is the result of the combined efforts of the entrepreneurs of this sector and of the producers of the equipment goods which it uses. The growth of sector A benefits from both internal and external economies.

The reduction of the capital coefficient appreciably modifies the development of the capital installed in sector A and therefore also the production of sector B. The growth of this capital ($K_{A(t)}$) becomes less and less close to that of output since the ratio k decreases with time. The absolute value of the production of sector B ($P_{B(t)}$) is less than it was when the ceofficient was constant. The point T is shifted downwards. But nothing is changed in the general pattern of production of the sector B. The decreasing stage only is slightly brought forward in time.

The second difficulty must now be tackled: the demand for replacements of the equipment goods. We shall remove our assumption that the lifetime of the investment goods installed in sector A is infinite and assume it to be equal to λ periods (years). We must accept the fact that the development of the production of sector B will be a function of the requirements of sector A in investment goods, both for increases in capacity and replacements. We have:

$$P_{B(t)} = \frac{dK}{dt} + K_{t-\lambda} \tag{3}$$

whose expression will be different according as we do or do not take into account changes in the capital coefficient. In the most general case the output of the sector B can be seen to be a function of the two variables T and λ. The following conclusions can then be drawn:

a) However large the production of sector A, the development of the output of the sector B depends on the lifetime of the goods which it turns out. Two extreme cases are possible, the first being that in which the lifetime is infinite. Sector B will then regress as quickly as it has grown. Its growth is divided into two violent stages. Generally speaking, any abrupt change in the development of sector A (a leap in the ceiling or a temporary regression) induces changes in the same direction and immediately amplified in sector B. In the other extreme case where the lifetime of the equipment goods is nil, the growth of sector B is similar to that of sector A. It is determined by the ratio of the value of the factor supplied by B to the value of the output of A. In between these two extreme cases the influence of λ can easily be explained. The shorter the life of the equipment, the closer to a logistic trend will be the growth of sector B. Except in the first λ periods, the importance of replacements is always greater than that of additional equipment. When λ is high, on the contrary, the dissociation between the two patterns of growth increases.

b) The lifetime which we must integrate into our analysis is not an objective datum since it includes obsolescence. It varies then with time, during the process of growth of the two sectors. What course will it take? Almost certainly the same as growth. Indeed the stage of emergence of sector $B(OT)$ is that in which competition through innovation prevails. During this stage the elimination of firms gives rise to a new situation in which an attenuation of obsolescence, and therefore an increase in λ, is likely to happen. If this assumption is correct, the growth of the sector B is slowed down and its decline is speeded up.

c) The decline of sector B is limited by the needs of sector A for regular replacement of its capital. This limit value is therefore determined exactly in the same way as the ceiling value of production in sector A, independently even of possible changes in k[9]. The discontinuities in the maximum value of the production of sector A, its slow rise or rapid fall are transmitted to sector via the changes in this limit value.

This brief analysis of the process of growth in the two related sectors in the long period leads to admit that the logistic trend is not always transmitted. The growth of sector B will assume this pattern only if the goods supplied to the sector A have a short lifetime. If we extend the analysis to all the goods which establish a link between the two sectors, we rcach the conclusion that the transmission of the pattern of growth of thc sector of consumer goods A is probable if the sector of producer goods B is a supplier of raw materials and highly improbable if it produces equipment goods. In the latter casc the logistic trend of sector A brings about a pattern of development in sector B markcd by the succesion of two sharply contrasting stages.

3.2 Thc propagation of the growth of sector A into sector B can be more easily analyzed if each sector A represents the total demand for a sector B. In reality we know that this assumption is untenable. Each producer goods sector has to meet

[9] There is a limit value of the production of the sector of producer goods (B) equal to $\frac{K \cdot m}{\lambda}$. The changes in the two variables k and λ can then be reintroduced as a function of time.

the demand of several consumer- goods sectors, each of which follows its own development. A summation of their specific demands would have to take into account so many variables (date of emergence of each consumer goods sector, specific features of its development, leaps in the ceiling...) that no significant result can be a *priori* suggested. We can, however, make the two following remarks.

In the first place, the creation of any consumer goods sector implies the previous existence of a producer goods sector. This unavoidable priority in time is obvious in the case of such factors of production as raw materials. The problem is more complex in the case of these equipment goods which are designed to meet the specific demand and requirements of a particular consumer goods sector. The birth of a sector A always entails the transformation of the production functions of several producer goods sectors up to the point when new goods are created by one of them. The emergence of a new sector A has then at least two consequences: on the one hand, a noticeable increase in the production of the sectors of producer goods, on the other hand variable changes in their production functions, which are unpredictable a priori. These two effects will be more intensely felt if the demand originating in a new sector A is directed towards a small number of producer goods sectors. The analysis of these processes of propagation leads to emphasizing the determining role played by the internal mobility of capital in the producer goods sectors. Growth is an alteration as much as an addition of flows. It consists of a transformation of the already accumulated goods as much as a creation of new ones.

The fact remains, however, that the development of the output of any one producer goods sector depends on that of several consumer goods sectors. How regular this development will be depends on the number of links with the sectors of consumer goods, that is to say, ultimately, on the degree of specificity of the equipment goods supplied. If these are a-specific, that is to say, are required for the production of the greater number of present and future consumer goods (a form of power supply, for instance), the growth of the producer goods sector, while accumulating the fluctuations of each of the consumer sectors, closely resembles that of the whole economy. It is exponential rather than logistic. If on the contrary the producer goods sector supplies such goods as can only be used by a small number of consumer goods industries, our preceding conclusions will apply. Both the short term and the long term fluctuations of the consumer sectors will be propagated and amplified.

4. These few considerations on the growth of the sectors of production offer some interest only if they enable us to explain and organize in a more satisfactory way the growth of a national economy. As a matter of fact, it does appear that both the rate and the pattern of development of a nation and those of sectoral production are closely related.

In the two cases, there is not only a mere interdependency relationship—which would only amount to a tautology—but one of cause and effect. The rate of growth of the national economy turns upon the development of the sectors catering for final demand. Contrary a certain indetermination inherent in current theories or models of growth, we do not think that aggregate growth is permanently stimulated by the creation of *any* new sector. In the long period, saving alone cannot bring about a process of fast and sustained growth. The sectors which are decisive for growth are those which directly satisfy final demand, the consumer goods sectors which are themselves of two types: those which meet the demand of households and those which cater for public demand. Both consist of producers of commodities and suppliers of services. The importance of services should certainly not be neglected. It even tends to grow with the level of development. But a service is always threatened by the appearance of a new durable commodity that would be a total substitute for it. It is really the durable consumer goods making up private and public demand which in any economic system are the stimulants of growth. The rate of growth of a national economy depends on the rate of possesion, both private and public, of the durable goods in existence. This rate defines for each point of time the share of public and private demand which has already been fulfilled, and that which remains to be fulfilled, that is, the potential of growth arising from desires. The fastest periods of aggregate growth are those during which the sectors turning out producer goods are going through the stage of their youth and, above all, that of their maturity—those periods during which the rate of possession rises rapidly through the progressive acquisition of equipment by the households with an medium-sized amount of capital. There are therefore great risks that the rate of growth will diminish when the rates of possession of most durable consumer goods approach their maximum value. If the consumer goods sectors producing for private demand have reached the stage of senescence, where production is mainly for replacements, the aggregate rate of growth can only be low. The possibilities of a stagnation are even close at hand.

In such a situation only a short-term expansion has any chance of "spoiling the picture"—for a limited period only. The expansion can only be temporary and its occurrence due to factors bearing upon the lifetime of goods. Over a long period the rate of growth can be raised in two ways only: either through the appearance of new durable consumer goods for private use, or through a lasting increase in the relative importance of public demand. New durable goods for private use are not very likely to have a lasting effect in an already affluent society, because the rate of possession of the new commodity will rise very quickly—except in the special case of a good characterized by its high value and indivisibility. Only an increase in public demand can then normally have such a lasting effect the creation of goods for collective use. We have here an instance of the ambiguity often inherent in the distinction between "services" and "physical commodities". Most services, and the most important of them, involve the preliminary creation of collective goods (transport, communications, national defence...). Whatever the form it takes, the growing importance of collective goods is inevitable. Even those who are least inclined to accept it will reconcile themselves to it when its necessity becomes obvious to the individuals themselves.

The same relationship can also be established in the analysis of fluctuations. The discontinuous variations in the rates of growth of the consumer sectors are contained in those of the aggregate rate of growth. They can arise from changes in the lifetime of goods, in the capital coefficients, in the ceiling values or, more directly, in the distribution of capital via fluctuations in the borrowing facilities available to households. But in the short term, there is a real that the rate of growth of the consumer goods sectors will be determined without adequate knowledge of the volume of demand. It is mainly in the stage of maturity that the risks of fluctuations are greatest. The private entrepreneurs or, in a planned economy, the public authorities are easily tempted to project into the future the higher rates of growth of the sectors witnessed during this stage, and therefore to embark on excessive investment programmes. The logistic trend, unforeseen or inadequately anticipated, then induces a long period of over-capitalization. The installation of excess capacity appears in the consumer goods sectors—and consequently also in the investment goods sectors—at a time when it becomes obvious that the high rates of growth of some sectors, resulting from a rapid diffusion of durable goods, cannot be maintained. It is in our view scarcely doubtful that here is to be found the most important cause of the recessions which the western economies have experienced over the past twenty years. An examination of the various successive recessions in one country (Great Britain since the war) shows that a contraction of aggregate output is always localized in a sector producing durable consumer goods, whose rate of growth, until then higher than the average rate, decreases abruptly. Furthermore, it can be seen that two successive recessions do not originate in the contraction of the same sector. Sectoral over-capitalization takes time to be re-absorbed—as much time as is required for the rate of possession to move up and therefore justify the actual level of installed capacity. It could not even be absorbed entirely, if it had exceeded the level of capacity required for production during the stage of maturity.

It is therefore an inadequate knowledge of the propagation of the goods into the capital of the individuals and the community which largely explains short-term fluctuations. We can well wonder at this point whether, contrary to what is generally maintained, the recessions of the contemporary period and those of the last century are not similar in nature. We would rather incline to think so and explain the great differences in the course of short-term fluctuations during these two centuries by differences in the goods produced by the consumer goods sectors. The violence of 19th century fluctuations would be explained by the nature of the consumer goods of that period: they were non durable goods and the potential demand for them was therefore constantly very high. Its transistory increases brought about a rise in the demand for the output of the producer goods sectors, where the fluctuations remained localized. To-day's consumer goods, which have become durables, have a demand which can be more easily postponed or anticipated. The amplitude of the fluctuations is therefore reduced—all the more so, as there is no objective limit to public demand. Thus does recession replace depression.

In the last analysis the rate and the pattern of growth depend on the same variable: the changes in the volume and the distribution of public and private capital. Even when the private firms and or the planning authorities appear to have the power of deciding the forms of development by determining the rates of growth of the different sectors, they still remain ultimately dependent on individual choices, which nowadays concern the forms and directions of the accumulation of capital for collective use. Short-term fluctuations are the penalty for the erroneous belief that individuals will consent, or can be made to surrender their power of choice. Because the majority of their choices are decisions on the utilization of assets and not of income-flows, when the level of development rises, a theory of growth must necessarily be a theory of the development of the assets held by the households or the public authorities. The fundamental aspect of socialisation lies in the growing importance of collective goods. The fact that they are under public control, however advisable such control may be on many grounds, is by itself no sufficient justification for their existence. They must also be and remain real substitutes for those consumption goods which can be privately owned. There is a great truth in Adam Smith's assertion that consumption is the sole ultimate object of saving. One should always remember it.

MATHEMATICAL APPENDIX

I. *Let* P be the output of a consumer goods sector following a logistic pattern of growth,

We have (1) $P = \dfrac{m}{1+e^{b-at}}$ $(m,\ b,\ a = \text{constants})$

Let α be the rate of growth of a sector.

We have (2) $\alpha = \dfrac{dP}{dt} = f(t) = \dfrac{ame^{b-at}}{(1+e^{b-at})^2}$

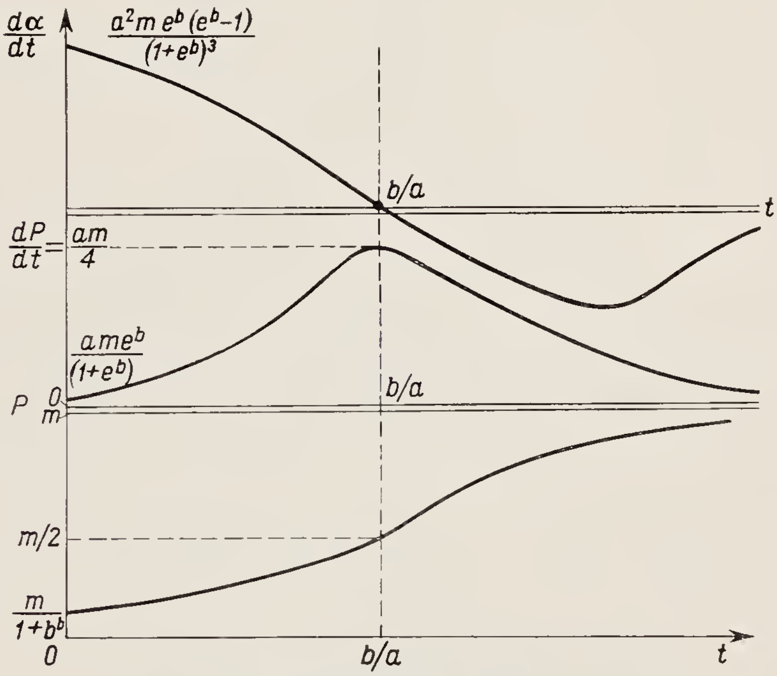
and (2a) $\alpha = \dfrac{dP}{dt} = f(P) = \dfrac{a}{m} \times P \times (m-P) = -\dfrac{a}{m}p^2 + aP$

Let $\dfrac{d\alpha}{dt}$ express the change in the rate of growth of the sector.

We have (3) $\dfrac{d\alpha}{dt} = \dfrac{a^2m \cdot e^{b-at}(e^{b-at}-1)}{(1+e^{b-at})^3}$.

The changes of the three variables through time $\left(P,\ \alpha,\ \dfrac{d\alpha}{dt}\right)$ can be expressed thus:

t	0	b/a	$+\varphi$
P	$\frac{m}{1+e^b}$ turning point →	$m/2$	k
$a = \frac{dp}{dt}$	$\frac{ame^b}{(1+e^b)^2}$ →	$\frac{am}{4}$ →	0
$\frac{da}{dt} = \frac{d^2p}{dt^2}$	$\frac{a^2me^b\,(e^b-1)}{(1+e^b)^3}$	$+0$	-0



Representative curves

K therefore defines the ceiling value of production. Its value is $m/2$ at the instant when the rate of growth of production begins to decline. The ratio b/a is a measure of the length of the increasing period, since the turning point ($p = m/2$) is defined by $t = b/a$. Therefore, the greater is b/a, the slower the process of growth (for a given value of m) and vice-versa.

Let now $\pi_{(t)}$ be the "contingent" of the commodities already produced at instant t. We have:

$$\Pi_{(t)} = \Big|_{t_0}^{t_1} P_{(t)}\,dt$$

$$\Pi_{(t)} = \int_{t_0}^{t_1} \frac{m}{1+e^{b-at}}\, dt$$

Writing $u = e^{b-at}$

$$du = -ae^{b-at}dt \qquad \text{with } dt = -\frac{du}{au}$$

it yields

$$\boxed{\Pi_{(t)} = m(t_1-t_0)+\frac{m}{a}\log\left(\frac{1+e^{b-at_1}}{1+e^{b-at_0}}\right)} \tag{4}$$

If we assume the lifetime of the commodities to be infinite, we have:

$$\boxed{\Pi_{(t)} = mt+\frac{m}{a}\log\left(\frac{1+e^{b-at}}{1+e^{b}}\right)} \tag{4a}$$

If the lifetime is equal to Θ, we can write $t_1 = t$

$t_0 = t-\Theta$

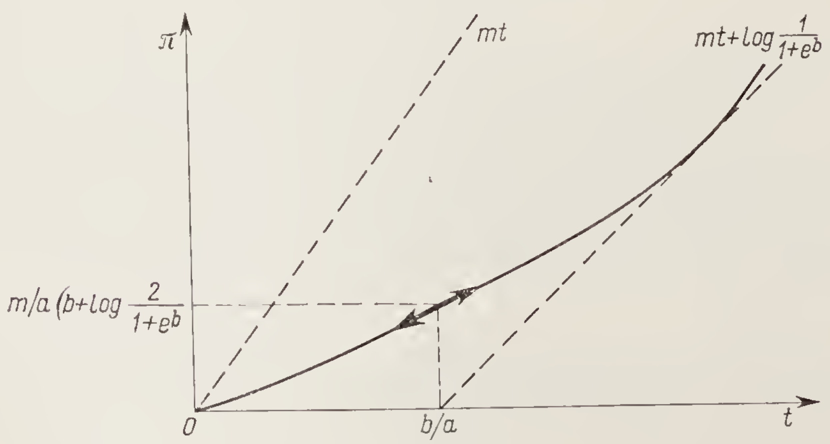
which yields:

$$\boxed{\Pi_{(t)} = m\Theta+\frac{m}{a}\log\left(\frac{1+e^{b-at}}{1+e^{b-a(t-\Theta)}}\right)} \tag{4b}$$

In the case where the lifetime is infinite, the development of the "contingent" of commodities is as follows:

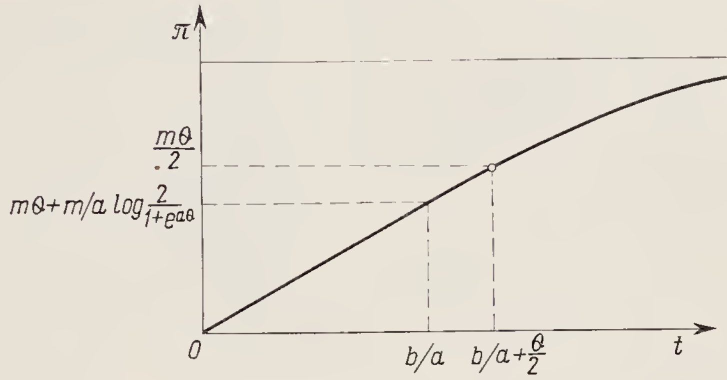
t	0	b/a	φ
$\Pi_{(t)}$	0	$m/a\left(b+\log\cdot\dfrac{2}{1+e^b}\right)$	φ

Representative curve of the "contingent"
Θ = infinite



We can therefore see that if the lifetime of goods is infinite, the development of the "contingent" becomes roughly linear ultimately (kt), i.e., production becomes permanently roughly equal to the ceiling value k. It is the introduction of replacements (Θ) which gives the "contingent" its limit value ($k\Theta$ when $t \to +\infty$). The causal relationship must here be reversed. It is the existence of this limit value of the "contingent" ($k\Theta$) which, with an average lifetime of the goods Θ, yields the ceiling value of production (k).

Representative curve of the contingent
Lifetime = Θ



If the lifetime is nil, the development of the contingent is equal to that of production. We have: $\Pi_{(t)} = p(t)$

Denoting now the rate of possession by $V_t = \dfrac{\Pi(t)}{n(t)}$

If $n(t)$ remains constant, we have:

$V_{(t)} = \dfrac{m}{n}\left[t + \dfrac{1}{a}\log\left(\dfrac{1+e^{b-at}}{1+e^{b}}\right)\right]$	infinite lifetime	(5)
$V_{(t)} = \dfrac{m}{n}\left[\Theta + \dfrac{1}{a}\log\left(\dfrac{1+e^{b-at}}{1+e^{b-a(t-\Theta)}}\right)\right]$	lifetime $= \Theta$	(5a)

The function $V(t) = f(t)$ is therefore of the same type as the function expressing the development of the contingent. The speed of diffusion is therefore given by a logistic function, whose coefficients are directly linked with those of the initial function expressing the development of production.

Given that the rate of possession is related to the capital held by households (average capital and pattern of distribution of this capital when the commodity is introduced, and changes in both the absolute level of capital and its distribution through time), the values of the coefficients k, a, b, of the initial logistic function are explained by the data relative to the capital of households. We only need here to reverse the order of causality.

$$\text{Capital} \to V(t) = f(t) \to \Pi(t) = g(t) \to p(t) = \varphi(t)$$

We can also introduce the changes in the number of households with $n(t) = n_0 e^{it}$ with i = rate of change of the population.

II*. The production of the sector of producer goods can be inferred from that of the consumer goods sector, by assuming one-way relationship between them:

$$P_1(t) = \frac{m}{1+e^{b-at}} = \text{Production of the consumer goods sector.} \quad (1)$$

$$K(t) = k \cdot \frac{m}{1+e^{b-at}} = \text{Installed capacity in the consumer goods sector.} \quad (2)$$

$$\frac{dk(t)}{dt} = \frac{am'e^{b-at}}{(1+e^{b-at})^2} = P_2(t) = \text{Changes in the installed capacity of the consumer goods sector} \quad (3)$$

$$= \text{output of the producer goods sector}$$

$$= P_2(t)$$

With $m' = mk$

k = constant

lifetime of equipment goods = infinite

The geometric representation of the development of production in the producer goods sector is given by one of the previous curves $\left(\frac{dP}{dt}\right)$, with m' being substituted for m.

Let us now assume a change of k, or more specifically, exponential decline throughtime. We then have:

$$k_{(t)} = k_{(0)}\, e^{\varrho t} \qquad \text{with } e^{\varrho} < 1$$

The development of production in the producer goods sector is then given by the following function:

$$P_2(t) = \frac{dk(t)}{dt} = \frac{d(k_0 e^{\varrho t} \cdot m/1+e^{b-at})}{dt} = \frac{e^{\varrho t}[e^{b-at}(a+\varrho)+\varrho]mk_0}{(1+e^{b-at})^2} \quad (4)$$

The shape of the representative curve changes with the values given to ϱ which modifies the influence of the exponential trend on the previous function (3). The value of will be so chosen as to give:

$$e\varrho \neq \text{and} < 1 \quad (0.99 \text{ for instance})$$

We shall finally assume that the lifetime of equipment goods is not infinite but equal to a finite value λ. In this case, we have:

$$P_2(t) = \frac{dk(t)}{dt} + k(t-\lambda)$$

Production for additions to capacity *Production for replacements*

We obtain a function with two variables t and λ and of the following form:

$$\boxed{P_2(t) = \frac{mkoe^{\varrho t}e^{b-at}\,(a+\varrho)+\varrho}{(1+e^{b-at})^2} + \frac{mkoe^{\varrho(t-\lambda)}}{1+e^{b-a\,(t-\lambda)}}} \tag{5}$$

We can write $P_2(t) = \Psi(t)$
$k(t) = Y(t)$

which yields

$$\boxed{\Psi(t) = Y'(t)+Y(t-\lambda)} \tag{5a}$$

We can then obtain $V_{t-\lambda}$ from $Y_{(t)}$ by operating a movement of translation parallel to the t axis and with modulus λ (the curvatures which are obtained are evidently independent of the values of ϱ). The successive transitions from function (3) to functions (4) and (5) show that in practice the value of λ is much more important that that of ϱ (especially if we assign to e^{ϱ} a value close to unity). For $\lambda = 0$, $\Psi(t) =$ logistic trend; for $\lambda = \infty$, $\Psi(t)$ shows the same pattern of development as $Y'_{(t)}$ (function 3). Therefore, the higher is the value of λ, the sharper the contrast between the two stages of the development of the producer goods sector, and *vice versa.*

MAURICE DOBB
GREAT BRITAIN

SOME REFLECTIONS ON THE THEORY OF INVESTMENT PLANNING AND ECONOMIC GROWTH

THE ECONOMIST Wicksteed once said that mathematical modes of statement served to "precipitate the assumptions held in solution in the verbiage of our ordinary disquisitions". What is less commonly recognised is that this virtue may have to be purchased at the price of a quite serious defect: namely that the more formalised is a theory, the more likely is it that corollaries derived from it will be vulnerable because of some implicit assumption concealed behind its formal structure rather than from logical flaws which are more easily detectable.

An example of this would seem to be an assumption implicit in most models of general market equilibrium in recent times (especially in the generalised type of model deriving from Walras), to the effect that the price of any commodity or productive factor which is in surplus supply will fall to zero[1]. A consequential corollary which has done damage in recent decades to clarity of thought about the problems of underdeveloped economies with surplus labour is that there can be no conflict between the objectives of maximising what the classical economists called net 'product', or revenue, and of maximising 'gross product' or 'gross revenue'. Yet it may be remembered that David Ricardo thought otherwise, and that one of the differences between him and Adam Smith was the former's contention that "Adam Smith constantly magnifies the advantages which a country derives from a large gross, rather than a large net income"[2].

As soon as one scrutinises the above assumption as applied to labour, its absurdity immediately becomes evident. One does not need to be an adherent of a subsistence theory of wages (in any rigid sense, at least) to appreciate that wages must have some minimum level if work is to continue at all; since, unlike other categories of income, wages have the special character of an essential input to labour-power (as ores are essential inputs to metal production or textile fibres to cloth-making). Hence labour cannot be realistically treated as simply one among

---

[1] *I.e.*, it will fall to zero if the excess of supply over demand persists despite an initial decline of price to some positive figure. If the latter promotes, e.g., demand-substitution of sufficient magnitude to take up the excess supply, equilibrium will of course be reached at some positive price.

[2] The opening sentence of Chapter XXVI of *Principles of Political Economy and Taxation*: the chapter entitled *On Gross and Net Revenue* (Sraffa edition of *Works and Correspondence of David Ricardo*, Vol. I, p. 347).

a series of n ultimate factors of production. Here classical political economy had more realistic sense in treating labour as unique from the standpoint of cost, and the defectiveness of modern formalism in its treatment of all factors of production and their prices as on a par becomes evident.

At a less formal level, when practical conclusions for policy have been in mind the assumption of which we have spoken has sometimes been translated into the following proposition: that from the social point of view labour should be treated as having a zero social cost so long as there is surplus labour, and that optimal planning implies the assigning of a zero accounting-price to labour. But this proposition (which derives from the very contingent notion, so-called 'opportunity cost') suffers from an analogous defect. In practice it is rarely if ever possible to increase the employment of labour without increasing total consumption. This is partly because an individual who is working a full working week has higher nutritional (and perhaps recreational) needs than one who is idle, and partly for incentive reasons. It is a familiar fact that in unindustrialised, or little-industrialised countries, wages in regular industrial employment are very substantially higher than the average standard of living in the village where labour is underemployed if not actually unemployed. It is also probably the case, under conditions of over-populated peasant agriculture, that the removal of a 'mouth' from the family unit (by migration from village to town) will leave total consumption by the family unaltered: it will merely mean that the remaining members of the family will relax their belts a little and take more from the common bowl now that population-pressure is eased. Such additional consumption consequent on an increase of industrial employment cannot be ignored as a social cost.

Failure to appreciate the distinction between maximising total product (including wage-earners' consumption) and maximising net product or surplus has led to a too hasty, and fallacious, identification of the conditions of so-called static equilibrium and the conditions for growth. To speak more specifically: it has enabled certain corollaries to be drawn from the Theory of Marginal Productivity and to be applied as imperatives for the process of economic development. These corollaries have affected the answers to two questions that are crucial to investment-planning policy: firstly the question of choice of methods of production, or of technique, about which there has been considerable discussion among western economists over the past decade; secondly the question of the distribution of investment between sectors, in particular between production of capital goods and production of consumer goods (the famous Departments I and II of Marx). Analytically these two questions are distinct, though interrelated; but they have been commonly associated as conjoint questions in discussions of economic development and growth.

Traditionally it was assumed by economists in capitalist countries that the answers to both questions followed as direct corollaries from accepted economic theory. As regards choice of technique, this was held to be governed by the principle of comparative costs when factor-prices were determined in accordance with the theory of marginal productivity. According to the existing 'factor endowment' (relative supplies of the factors of production) of a country, the relative marginal productivities of factors would determine factor-prices and hence influence factor-substitution and the choice of technique. At the same time it would determine the comparative costs of different products. Thus in a situation where capital was scarce and labour plentiful the marginal productivity and hence the price of capital would tend to be high, and equivalently the marginal productivity and price of labour would be low. This would encourage a substitution of labour for capital wherever possible by appropriate shifts both in the lines of industrial specialisation and in the methods of production used in any given industry. Lines of production tending naturally to employ a high ratio of labour to capital, (or with a low 'organic composition of capital', in Marx's terminology) would tend to be lower-cost lines than those where the contrary condition prevailed—namely a low ratio of labour to capital (or a high 'organic composition'). In so far as techniques in any given industry were capable of variation, the more labour-using (or 'labour-intensive') technique, which economises on capital, would tend, *ceteris paribus*, to come out as the lower-cost method of production.

On this basis was erected a veritable theory of a hierarchy of stages of development, each stage of development being characterised by a particular state of factor-endowment. At the lowest stage of development, where the economy of a country was characterised by abundance of labour and scarce capital, there seemed to be no possibility of doubt as to the most 'economic' policy to pursue regarding choice of technique and allocation of investment between industries. The principle of comparative cost dictated a concentration on industries that were labour-using and capital-economising and upon methods of production with a similar bias. In the degree that a country, in the course of development, accumulated capital, so that the ratio of capital to labour was appreciably raised, it could graduate towards more capital-intensive techniques and towards investment in industries involving a higher degree of mechanisation (which were usually identified, somewhat loosely, with 'heavy industries'). Here was both a simple and a direct corollary of economic theory as a guide to makers of economic policy. Many no doubt supposed that there could seldom have been a corollary of economic theory that was more certain and so beyond controversy. When a path in conflict with it was taken by Soviet development in the 1930's, economists in Western countries took for granted the uneconomic and probably self-defeating character of this attempt to leap over essential stages of growth. A development-policy of this kind which sacrificed economic rationality on the altar of national aggrandisement or military necessity could only increase the ultimate cost of growth[3].

[3] Mr. Peter Wiles in a recent work, *The Political Economy of Communism* (Oxford 1962), persists in maintaining that any departure from what he calls 'balanced growth' is pointless and uneconomic.

The Achilles heel of this plausible thesis consists in the purely static character of the analysis on which it rests and in its failure to appreciate that the needs of growth can, and do, conflict with the conditions whereby total production, or national income, and also employment are maximised at any given date. There is also the further consideration (on which we shall not dwell here) that the doctrine of comparative costs, if it is to sustain those free trade implications which have been deduced from it since Ricardo's day, must depend on another implicit assumption: namely that changes in the amount of trade undertaken by a country do not exert any appreciable influence on the terms of trade (which is equivalent to assuming that the relevant demand-elasticities are very high). It is a familiar fact that in the case of underdeveloped countries this assumption is least of all justified.

In the discussions of recent years among economists in England and America it has been the view that investment-policy should be judged primarily in terms of its effect on the rate of growth that has formed the main ground of criticism of traditional doctrine (or at least of its corollaries). If the effect of investment policy on growth is adopted as the guiding criterion, substantially different conclusions are reached from those drawn from the comparative-cost-*cum*-marginal-productivity doctrine. In particular, the desirability is indicated of a higher degree of capital-intensity of investment than traditional doctrine prescribes and also the advantage of allocating as large a proportion of investment as possible to the capital goods sector in order to broaden the basis for future investment. This discussion is probably familiar already to most readers, and no more than a summary of the argument and of its main implications will be attempted here.

Analysis of the effects of particular policies on growth will, of course, depend on what is regarded as being the main investment-determinant (or determinants), since the rate of growth is very largely (though not, of course, exclusively) dependent on the rate of investment that an economy can achieve. The older notion that such a determinant is to be sought in some kind of 'savings fund' can certainly not be maintained in conditions of surplus labour; and the notion of an independently given 'savings ratio' as setting a ceiling upon investment is manifestly inapplicable to conditions of a planned economy where the chief components of such a ratio are among the dependent variables of planning policy. But this does not mean that there is no economic 'ceiling' on investment short of a rate of investment that immediately absorbs all unused resources into production (so that the condition of a labour reserve for industrial expansion that we have posited as characteristic of countries at early stages of development disappears). It means merely that we have to look for such limiting factors among the 'real' or basic features of an economy, connected with its conditions of production or its productive structure.

There are two limiting factors which experience has shown to be particularly relevant to underdeveloped economies. Firstly, there is the supply of wage-goods available to meet the consumption-needs of workers employed in the investment sector of the economy (meaning by this a sector that includes both the work of building and construction and the manufacture of constructional materials and equipment used and installed in the new construction-projects). In turn this supply of available wage-goods will depend upon the surplus of production over consumption in the wage-goods industries[4]. Secondly, there is the productive capacity of the industries producing capital goods of all kinds (Marx's Department I)—a productive capacity consisting in the size of the installed capital equipment of this group of industries[5].

As bottlenecks these two factors may well be jointly operative rather than alternatives: they may be always present in the background of every historical situation. Yet it seems likely that in any given situation one of them will be more important that the other; possibly the former of them at early stages of development in underdeveloped countries and the latter at later stages when industrial construction has got well under way and a substantial industrial base has been constructed. At any rate there is no need to argue about their relative priority. This may well vary in different cases as well as changing at different stages of development; and although the practical consequence of emphasising each of them is rather different, there is in this respect no conflict between their respective implications, which can be regarded as constituent elements of any planning policy designed to maximise growth.

At first sight it might seem as though the surplus of wage-goods over the self-consumption of them by their producers bears an analogy with the savings-ratio mentioned above which forms the crux of many theories of growth, in particular those of the Harrod–Domar type. In a sense such an analogy can be found; but it is mainly a formal analogy, since the savings-ratio as customarily conceived is compounded of (and dependent upon) the savings-propensities (or their inverse, the consumption-propensities) of individuals. Viewed concretely in the context we have here indicated, it has an important difference; and attention is at once focussed upon a particular way in which the surplus-ratio may be raised, namely by raising labour-productivity. This is, indeed, the crux of the case for choosing more capital-intensive techniques than the traditional theory allows—a case that has been argued in the past decade by the present writer and by Professor Amartya Kumar Sen[6].

---

[4] These will include agriculture, so that in a predominantly peasant country this surplus will largely depend upon the productivity of peasant agriculture relatively to peasant consumption.

[5] I leave it as an open question whether this should include the production of raw materials ('objects of labour') or be confined to the production of metals and machinery ('instruments of labour'), each of the two main sectors being treated as vertically integrated back to the production of their several raw materials. For many purposes the latter seems to bc the more convenient.

[6] M. Dobb in "Economie Appliquée", 1954, Vol. VII, No. 3; in "Review of Economic Studies", 1955–6, Vol. XXIV, No. 1; and in *An Essay on Economic Growth and Planning*, London 1960. A. K. Sen in "Quarterly Journal of Economics", November 1957, and in *Choice of Techniques*, Oxford 1960. Cf. also W. Galenson and H. Leibenstein in "Quarterly Journal of Economics", August 1955, where, however, it is implied in places that there is advantage in choosing an indefinitely high capital-intensity: this as will be seen above is not so.

It does not follow, because labour-intensive techniques are deleterious to the growth-potential (by keeping labour productivity low), that capital-intensity can with advantage be raised *indefinitely*, since more capital-intensive equipment will tend to be more costly to produce, and at some point this rise in cost will offset (in its effect on the use of a given investment-potential to promote growth) the favourable effect of a rise in productivity of those using this equipment, and hence in the surplus-ratio. There comes an optimum point in the choice of more capital-intensive methods: a point that will tend to come sooner, *ceteris paribus*, the lower is the initial level of real wages, and conversely. In a simplified two-sector model used by the present writer some years ago this point was formally defined by saying that, if p_c and p_i stand for the productivity of labour in the consumer goods (or wage-goods) sector and the investment sector (producing capital goods) respectively, there will tend to be a certain relationship between a rising value of p_c and falling values of p_i ($1/p_i$ being the cost of capital goods). If we write L_c and L_i for the labour force of the two sectors and

$$\frac{s}{w} = \left(\frac{p_c - w}{w}\right)$$

for the ratio of surplus product to wages (= consumption) in the consumption goods sector, the output of capital goods can be seen to depend upon the size of $L_i p_i$, and L_i in turn upon $L_c \cdot s/w$. The condition for maximising $L_i p_i$, and hence the rate of growth of the economy, is that a relation between p_c and p_i should normally be chosen (as one moves along the range of relevant alternatives in the direction of more costly techniques) such that the following condition is fulfilled:

$$\frac{-dp_i}{p_i} = \frac{dp_c}{p_c} \cdot \frac{s+w}{s}.$$

It may be noted that it is only in the unreal case where $w = 0$ that this would be identical with the point where the total output of consumer goods is maximised[7] (and the capital-output ratio minimised) according to the prescriptions of the traditional theory. Total consumption in the immediate future will be smaller, therefore, if investment is governed by this criterion than if less capital-intensive methods had been chosen; so also will employment be smaller. To this extent there is a conflict of objectives. But the conflict is no more than a short-period one. A policy that maximises the rate of increase in investment will *ipso facto* maximise the rate of increase both of total employment and of the output of consumption goods; and in the longer period (which may not be so very long in time) will make the absolute level as well as the increase of employment and consumption greater than if the more cautious and gradual path of development had been taken. For this

[7] That is, identical with the point where the proportionate rise of p_c is *equal* to the proportional fall of p_i.

reason it seems preferable to express the issue in terms of a difference between the short-period and the long-period effects of different investment-policies, rather than as a conflict of objectives as has sometimes been done (e.g. the objective of maximising employment or consumption *versus* the objective of maximising growth). Such a conflict, as we have said, only applies within a certain time-horizon; and beyond it what maximises investment and its rate of increase will also maximise employment and consumption.

It should perhaps be emphasised that what has been said about choice of technique applies on condition that consumption per head (i.e. w in the notation adopted) does not rise proprotionately with the rise of productivity, consequent on choosing a more expensive technique. In a capitalist economy (and *a fortiori*, perhaps, in a peasant economy) there is no guarantee that this will not occur, since the higher productivity will accrue as higher individual incomes (in particular higher profits) which may result in higher consumption-standards and in proportionately higher individual consumption. In countries with a peasant agriculture it is a familiar problem (and itself constituting a barrier to development) that improved agricultural productivity (or alternatively price- or tax-concessions in favour of agriculture) may have little, if any, effect on the marketed surplus of agricultural foodstuffs, but instand exhaust its effect largely in augmenting the self-consumption of peasant producers, or alternatively encouraging them to enjoy more leisure. This is one of the reasons why a high growth-rate policy such as we have described can be expected to be characteristic of planned socialist economies (or at least of economies with a large State sector) and not of free market economies.

Regarding the second of the two limiting factors of which we have spoken, somewhat analogous considerations apply: namely that while a policy of assigning priority to investment in the capital goods sector will cause consumption to grow relatively slowly in the immediate future, by augmenting the investment-potentiality of future years it will eventually enable consumption to increase more rapidly, both absolutely and proportionately, than it could have done if the capital goods sector at earlier dates had grown more slowly. If, of course, the existing level of consumption per head of the labour force has to be regarded as constant (e.g. for efficiency or incentive reasons), then the allocation of investment between the two main sectors is determined for us, within very narrow limits, and there is little or no choice in the matter. Output-capacity in the consumption goods industries must expand in step with total employment; hence the capital goods sector cannot expand faster than the consumption goods sector, unless expansion of the former is accompanied by a shift towards more labour-saving techniques. Expressed in the notation employed above, growth must be so balanced as to observe the equality $L_i = L_c \cdot s/w$: that is, employment in the investment sector can grow no faster than does the surplus production of the wage-goods sector, and (apart from a raising of productivity by rationalised organisation or improved technique) investment must be allocated so as to keep the growth-rates of the two sectors uniform.

But although real wages are subject to a minimum level and even above this level may be causally related to working efficiency, the existing wage-level may have some flexibility at least over limited periods of time. In this case[8] it will be possible to expand the investment sector more rapidly than the rest of the economy; which will have the effect of increasing the relative investment-potential, and hence the rate at which the system can grow, at future dates. It should be noted that, although this will mean (unless technical innovation is sufficiently rapid) that consumption will grow more slowly than employment, this is not inconsistent with a continuing rise in total consumption and even in consumption *per capita* of the population (since the proportion of the whole population employed in industry is rising). Total consumption will, as we have said, increase more slowly in the immediate future than if investment-priority had been given to the consumption goods industries instead of to capital goods industries; but after a certain date in the future total consumption under the high-growth-rate policy will rise above what it would have been under a policy initially more favorable to consumption.

It will have been noted that the simplified model of which we have been speaking is essentially a model in terms of labour and its product, in which capital does not figure separately as a quantity, or as a factor of production: merely capital goods that are products of labour at some previous stage of production and which play the role of aids to labour influencing labour's productivity. The problem of choosing the type of capital good, and the appropriate distribution of labour between the sectors, that promoted maximum growth could have been expressed as a minimum problem in terms of cost-minimising the social cost of maintaining a given rate of growth. In any economy where calculation is in value terms, it will be in this form that the problem will be immediately expressed, at any rate to those taking decisions 'decentrally' at lower levels, such as administrators of particular industries or managers of enterprises. Some interest accordingly attaches to the question as to how our principle applies when expressed in this way? What kind of price-structure is conductive to the taking of the right kind of decision?

At first sight it might seem that, from the nature of our model, the principle must now appear as one of minimising labour input to produce a given quantum of output. But this cannot be so in any *simpliste* interpretation of minimising labour cost; since such a principle can only be applied subject to a certain investment-constraint—that labour is so distributed and methods of production so chosen as to maximise investment (measured in terms of labour-inputs). Otherwise, the principle of minimising expenditure of labour would lead to the use of the most productive known techniques however expensive and capital-intensive, so long as increase in capital-intensity yielded *any* addition, however small, to net productivity (in the notation of our example used above, it would imply choosing the highest possible

[8] Also if technical innovation is sufficiently rapid; or again if the supply of consumer goods and/or capital goods can be augmented by improved terms of trade with other countries or with an agricultural hinterland of the developing economy.

value of p_c when this is interpreted net of the cost of maintenance or replacement of equipment). It follows that cost must be so interpreted as to make some allowance for such an investment constraint (for which purpose, incidentally, capital goods currently produced will need to be priced and aggregated into a total). Such an allowance seems only possible if the use of capital goods is in some way debited with the contribution which it can make to the appearance of a surplus product.

Professor V. V. Novozhilov of Leningrad has suggested a method of pricing that makes an allowance of this kind; and there is some interest, accordingly, in considering how the operation of his method (and the use of 'minimum cost' so interpreted) is related to the principle we have enunciated. To do this was the object (in part) of an article by the present writer in the journal "Kyklos" in 1961 (Vol. XIV, Fasc. 2, pp. 135–150); and the remainder of the present paper will consist of a reproduction of the analysis in the concluding part of that article.

Professor V. V. Novozhilov's proposal is as follows[9]. A ratio which he terms the "marginal effectiveness of investment" is calculated thus. A given quantity of investment funds is allocated according to a uniform ratio at the margin of all uses and in such a way that, when possible investment projects and their variants have been arranged in an order of their effectiveness, all projects yielding an effectiveness-ratio higher than the ratio selected as standard are given priority. When the whole investment fund has been allocated in this way without surplus or deficiency, there will be a given minimum effectiveness ratio at the margin of allocation. This will constitute for the time-being the standard ratio. The ratio in question is defined as that of the reduction of operating cost (or prime cost) resulting from a given increase of investment to the absolute amount of this investment. Thus, where C_1 and C_2 stand for the prime costs respectively in two projects of different technical types, and K_1 and K_2 for the initial capital cost, the effectiveness-ratio will be

$$\frac{C_1-C_2}{K_2-K_1}.$$

Writing the above ratio as r, Professor Novozhilov then proceeds to show that if rK is added to C to represent the social cost of a product (which he calls *narodnokhoziaistvennaia sebestoimost*, or national-economic cost), this will render the cost of a product lowest when produced by the technique, or method of production, that yields an effectiveness-ratio of r. It is to be noted that rK as a magnitude will be independent of the units in which K and C are expressed (i.e. the relative valuation of capital goods and the elements of prime costs); since the larger is K relatively to C, the smaller will be r, and conversely[10].

---

[9] Cf. *Ismerenie Zatrat i ikh Resultatov v Sotsialisticheskom Khoziaistvie* (Comparison of Expenditures and their Results in a Socialist Economy) in *Primenenie Matematiki v Ekonomicheskikh Issledovaniakh* (The Use of Mathematics in Economic Investigations), ed. V. S. Nemchinov, Moscow 1959, pp. 42–213.

[10] V. V. Novozhilov, *loc. cit.*, pp. 112–115.

Thus, suppose that there are three technical variants under consideration such that:

$$K_1 < K_2 < K_3 < K_4 \text{ and } C_1 > C_2 > C_3 > C_4,$$

and

$$\frac{C_1-C_2}{K_2-K_1} > \frac{C_2-C_3}{K_3-K_2} > \frac{C_3-C_4}{K_4-K_3}$$

Let

$$\frac{C_2-C_3}{K_3-K_2} = r;$$

then it will follow that

$$rK_3+C_3 < rK_4+C_4; \text{ also } \leqslant rK_2+C_2 \text{ and } < rK_1+C_1.$$

It follows that if one adopts this principle as the basis of social costing (whether for the purpose of accounting prices only or of fixing actual prices) and alternative methods of production are chosen according to which of them yields the least cost, the result will be the maximum economy of social labour in the qualified sense of which we have spoken (qualified, i.e., by an investment constraint). The inclusion of rK as an element in cost, in addition to C, is a recognition of the latter constraint and is itself a reflection of it in the costing-process.

At first sight this may seem to bear no close relation to the criterion for maximising growth discussed above. Reflection, however, will show, I think, that there is such a connection. Let us first try to express this connection in formal terms, in this way. We have said above that in our model a condition for maximising growth[11] is that

$$\frac{-dp_i}{p_i} = \frac{dp_c}{p_c} \cdot \frac{s+w}{s}$$

$$\left(\text{or alternatively that } \frac{dp_c}{p_c} = \frac{-dp_i}{p_i} \cdot \frac{s}{s+w}\right).$$

It can also be shown that the magnitude $(s+w)/s$ is a measure of the proportional increase in surplus resulting from a proportional rise in p_c: i.e.,

$$\frac{dp_c}{p_c} \cdot \frac{s+w}{s} = \frac{ds}{s}.$$

Now Professor Novozhilov's rK (which we have seen is, as a composite magnitude, independent of the relative valuation of K and C) when expressed as a ratio to C if C consists exclusively of wages (or alternatively as a ratio to that proportion of C which consists of wages) can be shown to be a measure of the relationship in our model between the proportional change of p_c and the proportional change of p_i[12]. This relationship we have just seen is $s/s+w$ when growth is being maxi-

[11] In what we have called elsewhere a "normal" case where the p's at different (vertical) stages of production are approximately uniform.

[12] Since r is equivalent to dp_c/dp_i and rK can be expressed as

$$\frac{dp_c}{dp_i/p_i}.$$

mised. Accordingly, if we write as α the proportion of prime cost that consists of wages, $rK/\alpha C = s/s+w$, since r we have seen is derived by allocating investment so as to have the maximum effect in raising the productivity of labour[13]. For any economic unit (e.g. an industrial enterprise) to which rK is debited as a cost as well as C, that method of producing a given output which minimizes $rK+C$ will be the most profitable (or involve the smallest loss), at whatever level the selling-price may be, *provided* that selling-prices are proportionall to the Novozhilov costprice. But if *only* C is debited to it as an actual cost, that method of production will only be the most profitable if the selling-price is so fixed as to make profit above C when expressed as a ratio to $\alpha C = s/w$: i.e. to make it greater than $rK/\alpha C$ by $s+w/\omega$[14].

In common sense terms the point of this may be expressed in this way. We are comparing the reduced wage-cost of producing a given output with the increased investment-cost of making this reduction; and rK is a measure of this relation. In other words, it measures the economy of labour resulting from more investment against the additional expenditure of labour in the investment sector that is involved thereby. With a given investment-potential for the economy as a whole, the use of more investment in one direction involves reduced investment, and hence a reduced contribution to growth, in some other direction. This reduced contribution to growth in another direction is the addition to surplus that the investment could there have yielded (assuming that surplus is a crucial investment-determinant). If rK is to be an adequate measure of the social cost of using more investment, it must be a measure of the marginal contribution being made in the economy as a whole to the increase in labour-productivity. It follows that for relative prices to be an adequate reflection of social cost, whether they are prices of consumer goods or of capital goods, they must at each stage of production be proportional to C *plus* rK[15].

This when divided by p_c (which in this context would be the equivalent of C if C consisted exclusively of wages) becomes

$$\frac{dp_c/p_c}{dp_i/p_i}.$$

[13] This is subject to a crucial proviso, however: that the output-plan is appropriately fixed. If output is not fixed in a manner consistent with maximising growth, the above equality may not hold, since the allocation of investment is relative to a given pattern of output, and accordingly r may have different values for different output-patterns.

[14] Since

$$s/w = \frac{s/s+w}{w/s+w}.$$

So far as consumer goods arc concerned, prices will only be equilibrium-prices (ignoring direct taxes on wages or saving out of wages) if they are at this level (c.f. the present writer's *Essay*, pp. 91–2, 95–6). It may also be noted that, if selling prices are proportional to $rK+C$ but diverge therefrom, total profit as a ratio to K will not be uniform in all industries.

[15] K will here represent, of course, the value of the capital goods used in the particular production-process in question, not some generalised K avcraged out ovcr production as a whole. The value of r will be derived, however, from a generalised *social* effectiveness-ratio applying to the economy at large.

It has often been supposed that a quantity such as rK can be used to determine the rate of investment itself as well as its optimum allocation. But this is not so. Professor Novozhilov's rK can only be derived on the basis of prior postulation of the amount of total investment (measured, for example, in a given aggregate output of the capital goods sector). Since in the real world planners can never make the volume of investment what they will (but can only influence its rate of change), one need not be unduly worried or surprised that theory should be unable to postulate on *a priori* grounds some optimum rate of investment. If in the real world investment is subject to definite determinants, theory is only being realistic (and is not being arbitrary) in starting from the postulate of a given volume of investment, and then investigating the limits within which, and the means by which, this quantum of investment can be changed over time.

It remains, in conclusion, to make one general observation about the implications of the approach we have outlined for practical problems of economic development. One thing that follows is that what matters from the standpoint of actual policy is not so much what the rate of investment happens to be at some initial date: this will be largely determined by past history, at least so far as the 'ceiling' on it is concerned. What matters most is how that volume of investment is utilised and the difference made by the mode of utilisation to the *rate at which that rate of investment can change.* Investment-allocation must accordingly be thought of, not in terms of equations defining a static equilibrium, but in terms of this rate of change. To take some pre-existing 'savings ratio' and extrapolate it into the future (as is implicitly done in so many 'western' discussions of the limiting factors upon development) tends to give an unduly conservative bias. Any such ratio, based on today's situation or yesterday's, is not the rigidly limiting factor that it is commonly supposed to be, because it can itself be changed by the course of development, if development is planned to that end. Economically backward countries may not be able to 'pull themselves up by their own bootstraps': if, for example, they altogether lack the means of producting machinery themselves, they must inevitably import machinery, at any rate for a time; if they possess a purely subsistance agriculture that yields little or no surplus, they must even import food. But their dependence for development on outside aid is much less, and their ability to develop out of their own resources is much greater, given correct policies, than economists have traditionally allowed. True, such more optimistic perspectives will not emerge from the free operation of market forces, but presuppose planning both as a mechanism of coordination and as a means of imposing a correct order of priorities; and planning if it is to be comprehensive in turn presupposes social ownership of the means of production.

Zofia Dobrska
Poland

A NOTE ON THE RATE OF TECHNICAL PROGRESS IN DEVELOPED AND UNDERDEVELOPED COUNTRIES

The problem of economic underdevelopment has two aspects. The first and basic one is concerned with low living standarts, or simple misery of the prevailing part of humanity, inhabiting those countries, which are underdeveloped. Increasing production capacities as well as production *per capita* is necessary in order to overcome this situation. The second aspect of the problem is concerned with a tremendous span in incomes between backward and developed countries. This span is the source of very serious troubles of economic, social and political nature on a world scale. A high rate of growth of the underdeveloped countries is not sufficient to overcome this span; this rate, moreover, must be higher than the rate of growth of the developed economies.

It is obvious, that the rate of growth in both types of countries is influenced by a number of factors, and that the influence of those factors varies according to circumstances. If we assume full employment of the existing production capacities (that is to say, if we do not take into account possible difficulties in production resulting from the lack of effective demand, and if abstraction is made from the possibility of employing hidden reserves of the economy), then the rate of economic growth will depend on the share of investment in national income and on the effectiveness of investment. The effectiveness of investment will be, in turn, dependent on three factors:

1. the relation between labour and capital available in respective countries that is to say on the rate of substitution of labour by capital, which determines the choice between capital saving and capital consuming techniques,

2. the interindustrial structure of investment, which will influence the capital-output ratio, independent of the rate of substitution between capital and labour in a given economy; this is due to the fact, that alternatives of production techniques in separate industries are limitted,

3. the rate (and character) of technical progress.

The present paper is concerned with the influence of this last factor on the rate of growth in developed and underdeveloped countries, that is to say with possible increasing or decreasing of the span between the standarts of living of the population in both groups of countries.

*

In order to study the influence of technical progress we must start by introducing two, highly unrealistic assumptions, namely, that the rate of substitution between labour and capital, and also the structure of production in both types of countries are univocaly determined, and that they are not subject to change during the period under consideration. Under such assumptions, the capital-output ratio, (determined in the initial period by both factors mentionned above, and thus different for both groups considered), will change during the process of growth only under the influence of technical progress. Thus, we will attempt to answer the question, whether technical progress will influence the capital-ouput ratio in the developed and underdeveloped countries in the same, or in a different way; if we reach the second conclusion we will ask further, whether technical progress will tend to narrow or to widen the span between the rates of growth in both types of countries.

At first sight the answers may seem quite obvious. Nowadays circumstances the achievements of science and technical improvements are equaly accessible to all countries. Although most of those improvements are made in highly developed countries, they can not and are not monopolised by them, except for special cases[1]. Each country can profit from the general achievements and apply them to its own purposes. The opinion is even often heard, that in this respect underdeveloped countries are in a better position than developed oncs, since they can take advantage of the achievements of industrialized nations, without having to bear the costs of laboratories and research institutes. Such a view does not seem quite correct, since in most cases the countries "importing" technical knowledge must either pay for the trade marks, or buy improved capital goods at a price, which containes "excess gaines" of their producers. And even if this is not the case, underdeveloped countries lose advantages connected with competition, offcred by each innovation to its first producer. For all the above reasons it can be said, that although underdeveloped countries are perhaps not in a worst position in this field, they are also not priviledged anyway, what ammounts to assuming, that they have equal access to technical achievements with thc developed countries.

If this is the case, it would scem, that there are no reasons, why technical progress should differ amongst various groups of countrics. In fact, however, the situation is different.

Let us present the problem on a graph (Figure 1), in which thc vertical axis presents the ammounts of capital, and the horizontal axis the ammount of labour ncccessary to producc a givcn amount of a certain commodity. The curve t, called the curve of equal commodity (and often, determined as the production function, which is not correct), presents various technical alternatives, that is to say various

---

[1] Concerning mainly production of military equipment, and having no direct economic meaning.

relations between labour and capital leading to the production of the same ammount of a commodity. The curve t in our example shall present macroeconomic technical alternatives, that is to say, the average technique used in the initial period in the economy as a whole. (Since it is only in the macroeconomic setting, that we have the right to assume, that this curve is an uninterrupted function, i.e., that for each relation between labour and capital a corresponding technique may be found. If we analize production by branches, in which case the ammount of possible technique is always limitted, and the function of equal product interrupted, some of the cases, which are of interrest to us may not occur.) Let us then assume, that the relation between labour and capital on the one hand, and between branch of production with a high and low capital coefficient on the other in an underdeveloped country is such, that technique B is the average technique for the economy as a whole. For a developed country, which has a greater amount of capital versus labour, and a higher proportion of capital consuming industries, the corresponding technique will be technique A, having a high capital coefficient and situated more to the left.

Let us assume moreover, that technical progress which took place in the period of time under consideration, increased to the same degree the effectiveness of both factors of production in all technical alternatives, and thus, that mutual relations of those factors did not change[2]. (This assumption is not indispensable for further reasoning, since the relations considered will appear also under other types of technical progress, but it makes graphical presentation of the problem much easier. On the

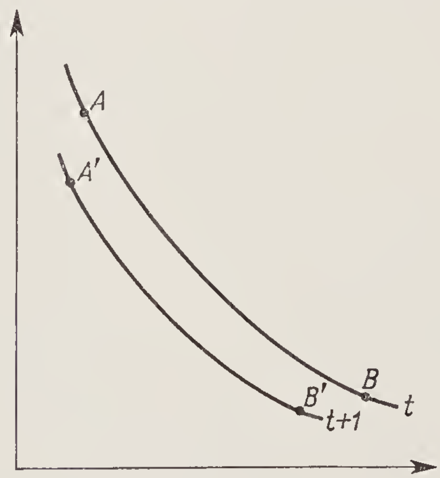


Fig. 1

other hand it is not unrealistic to assume, that both outlays of labour and those of capital per unit of product diminish, owing to technical progress. Empirical research has shown, over and over again, that in the long run the capital coefficient is constant

[2] Harrod used the word "neutral" to determine this type of progress; since in present literature progress under which the capital coefficient does not change is understood by "neutral progress", it would be better to determine it rather as "proportional progress".

in various national economies, in which both in the structure of output by industries. and in the rate of substitution changes occur, which would suggest an increase in the capital output ratio. This would suggest, that technical progress as such was also of a capital-saving character.) The effects of this progress are shown on the curve $t+1$, in which technique A' corresponds to technique A, and technique B' to technique B. As can be seen, if other conditions do not change, and especialy if this is true for the rate of substitution, the countries using both techniques achieve the same advantages from technical progress.

It should be noticed, however, that this is only due to a specific shape of the curve $t+1$, or in other words, due to the assumption, that technical progress under consideration has the same intensity for all "grades of techniques", that is to say, that it is as great for capital consuming techniques as for the capital saving ones. But, as shown by Prof. M. Kalecki (who analysed the problem in his *Outline of the Theory of Growth of a Socialist Economy* from a slightly different aspect), such a shape of the curve of technical progress is not a fixed rule, since technical progress at a certain point may be much higher than at other points.

Figures 2 and 3 illustrate the consequences of such an alternative for the group of countries considered.

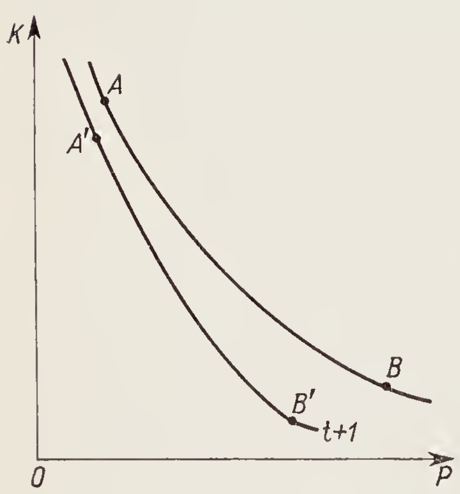


FIG. 2

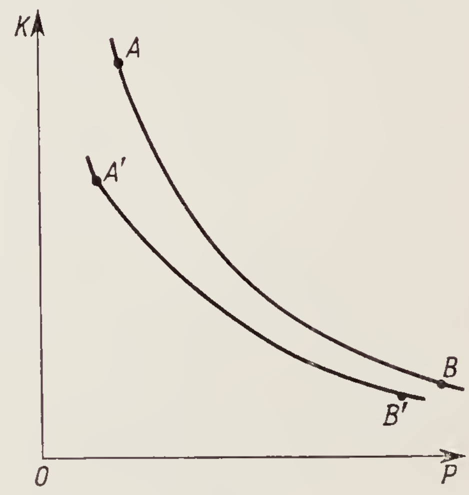


FIG. 3

Figure 2 presents the case, in which technical progress during the period t was extremely fast for capital consuming techniques, whilest the more capital saving was the technique, the lower was its rate. Figure 3 presents the inverse case, in which technical progress is greater for capital saving techniques.

It is obvious, that progress in the field of capital consuming techniques acts in favour of highly developed countries, while progress in the field of capital saving techniques—acts in favour of less developed ones. The fact, that both types of countries have full knowledge as to the ways of introducing technique A' or B' does not change the situation. It is not the question of limitted knowledge, but of limitted possibilities of its application, or rather the question, that it is not expedient to apply this knowledge to all circumstances. In a country, where using the spade for road building is the most "rational" technique, improvements in the construction of buldodgers will not influence the effectiveness of investment, just as in a country, where the scarcity of the labour force makes automatic weaving machines profitable, improving the effectiveness of hand weaving leads to no useful purpose.

Thus the question whether technical progress will influence the existing "span" between developed and underdeveloped countries, and what shall that influence be, can be reduced to the question, whether such a progress is of a uniform character, i.e. whether it is the same in all sectors of technique, or whether it "diverges in some direction".

The analysis of modern technical innovations seems to leave no doubt about the fact, that they are most numerous and most important for those techniques of production, which have a higher rate of capital versus labour. This can be explained not only by the objective fact, that the simpler the tools, the smaller are the possibilities of their improvement. One should notice in the first place the important factor mentionned above, namely that technical innovations are made in the first place in highly developed countries, which actually proceed only along the upper part of the curve of technical alternatives, which has a high rate of substitution of capital for labour. The innovations made in those countries are thus mainly concerned with capital consuming techniques, actually in use there. Capital saving techniques which thus imply relative abundance of the labour force versus capital, and which are not is use in those countries, are therefore not considered by the research institutes even if their improvement could, objectively speaking, lead to the same effects as improvement of capital consuming techniques.[3] And finaly, last but not least, this kind of divergencies of technical progress are influenced by the existing output structure of the separate types of countries. As we know, technical progress is the greatest in new fields of production, such as chemical industry electronics etc. And since highly developed countries have a greater share of those industries in their GNP, progress reached along this line has a greater influence there on the average effectiveness of production, than in underdeveloped ones. There are no reasons to suppose, that any one of the three factors mentionned above will fade away in the near future, so that is should be assumed, that the existing "divergence" of technical progress will be the same in the future.

This seems to lead to the conclusion, that technical progress shall not diminish the gap between developed, and underdeveloped countries, but moreover, *caeteris paribus*, it will widen it.

---

[3] It should be noticed, that in a number of countries, including highly developed ones, research concerning techniques of production best suited to the conditions of backward areas is under way. Its results may hamper this tendency to a certain extent, but they will not stop it altogether.

This pesimistic conclusion, however, is limitted by the method of analysis used thus far, which on the one hand does not take into account the scale of production, and on the other limits the concept of technical progress, following in this last respect a traditional way of thinking. Both those problems should be now considered.

*

It is well known, that rational division of labour within the enterprise, as well as highly efficient and specialised tools can be effectively introduced only if production exceeds a certain minimum level. So called economies of scale which usualy tend to diminish labour and capital input, were one of the basic factors of increasing effectiveness of production in developed economies.

From the strictly formal point of view, techniques of production which are already known, and which are suitable for production on a large scale, form a part of the set of existing techniques, amongst which the investor can make his choice. Thus, their introduction should not be treated as technical progress, but only as a shift along the existing curve of alternative techniques. However, in conditions of underdeveloped countries, theoretical knowledge alone of those technical solutions does not change the fact, that very often they do not enter into the "range of choice" of the investor, for the simple reason, that it is practicaly impossible to apply them. This, in turn may have two reasons: first, a market which is limitted, and second, technical impossibility of obtaining capital needed for this type of production[4].

This situation is somewhat similar to the case presented above, in which actual technical progress was not "effective" from the point of view of underdeveloped countries, since it took place in this section of the curve of technical alternatives, which could not be taken into account by the investors in those countries, owing to the prevailing rate of substitution. In the case, which we are considering now, the technical improvements are not "effective" owing to the limitations of the market, and to "absolute" lack of capital, which makes their application impossible. However, along with the progress of industralization policies, both those "technical bariers", hindering more effective methods of production, will gradualy diminish. The result will be similar to the impact of innovations which makes the introduction of better technical solutions possible.

Let us put it in an other way: owing to economic and social bariers, which hinder the application of known, optimal from the technical point of view, methods of production in underdeveloped countries, the set of technical alternatives at the disposal of those countries is smaller, than in developed ones. Thus, as industralization proceeds, this set will become wider and wider, including those techniques of large scale, which are already in use in developed countries, and independent of new technical solutions, which may appear in the meanwhile. This is why the rate

[4] This last case is typical for small farmer's production and production of craftsmen, which has often a high capital coefficient owing to the use of very unefficient tools. In both cases it is very often impossible to obtain normal credits for production purposes.

of technical progress in underdeveloped countries will be actualy faster, than would seem, basing on the previous reasoning. (When economies of scale are taken into account, the curve of alternative techniques is subject to modification. The lower part of the curve in the case of underdeveloped countries will be higher in relation to the x axis, than in the case of developed ones, and thus the span between this part of the curve and the curve $t+1$ will be greater. This, to a certain extent will counterbalance the divergence of technical progress towards capital consuming techniques which was mentionned above).

*

In the current meaning technical progress is understood as introducing technical innovations which enable to achieve the same output, using an ammount of factors of production smaller than before, or obtaining new production effects by means of the factors given. In this respect progress is always connected with a change of techniques, that is to say with introducing new tools, and that is why—as a matter of fact—it is always connected with investment (independent of, whether new equipment is considered, or old one improved). Owing to this type of interrelation, it should be called "dependent progress", since it can occur owing to investment activity only, and its range is dependent on the size of that activity.

Besides this kind of progress, however, there is also another one, which can be defined as "independent progress" in the sense that it is not so related to the size of accumulation. It is characterized not by a change of the tools, but by achieving better results than previously by applying the same equipment and technical methods. In the first place so called "organizational progress" should be mentionned here, consisting of improvements of managerial methods and of the labour division within the enterprise. Increase of the efficiency of labour connected with the growth of the general level of culture and qualifications of the staff, and enabling to achieve better results on the basis of the same equipment is another kind of independent progress. (Both kinds of independent progress mentionned above could be limitted, in principle, to reducing the labour force employed, in most cases, however, especialy what concerns the "progress of qualifications", they are both labour- and capital-saving.)

Although this kind of progress, and especialy organizationall progress, was often pointed out in literature, it was however very often omitted in model building, since it was either assumed, that it implies small investment outlays enabling to include it into the first category of technical progress, or that its meaning is so small, that it can be omitted[5].

Such assumptions were correct in a number of case. They freed the model analysis from a number of factors of a social, rather than economic character and enabled to show the immediate relations between the rate of growth of national income

[5] The problem of the rise of knowledge and qualifications, as well as the problem of the scale of production, was considered very often not within the analysis of technical progress, but within the framework of "external" and "internal" economies.

and the level of investment and its effectiveness. By the way, when studying a developed economy, which is internaly stabilized, and which developes by way of a steady evolution, one can assume correctly, that independent progress has a constant relation to dependent progress, so that it can be omitted. The situation is the same when we compare two economies which are similar from the structural and institutional point of view; one can take for granted that in both cases the importance of independent progress and the rate of its change are the same.

The matter, however, appears in a different light, when we start investigating problems of development and not only growth, considering a backward country in which the start towards accelerated accumulation is connected with far reaching changes of institutional, social and cultural conditions. In such an economy independent progress will be of vital importance, and its rate may be almost unconnected with the rate of dependent progress.

The rate of organizational progress will be faster than in developed countries, especially if it is considered in a broad sense—not only as improvement of the methods of management within the enterprise, but also as the improvement of relations between the enterprises and within the national economy as a whole. The difference will be very great in the field of raising the efficiency without additional investment, due to the rise in the general level of knowledge and technical abilities of the workers. This last corrolary could be put into doubt, since such a growth takes also place in developed countries. But in the underdeveloped countries there is an additional factor in this respect, independent of the fact, that as a rule, the lower the starting point, the faster is the growth of efficiency resulting from teaching the workers basic technical abilities. Namely, in the developed countries having an old industrial tradition, the greater part of the staff of factories and enterprises consists of experienced workers; this assures their own high efficiency and also makes it easy to brush up professional skills of the new employed. Percent-wise, the ammount of workers having many years of professional experience does not change substantially, since the rotation of labour is smooth. On the contrary, underdeveloped countries, starting on their way of industrialization, either have no skilled workers with long experience, or very few of them. However, as time goes on, the percent of such workers will be steadily increasing, and this alone will be an important factor of the growth of the average productivity of labour.

*

The above remarks were intended to show, that there are certain differencies between the developed and underdeveloped countries and sometimes contradictory tendencies in the field of technical progress. This will last as long, as long there will be substantial institutional and economic differences between those two groups of countries (thus, as long as the division into those two groups will keep its sense). Dependent progress has a higher rate in the "upper range of technique", and that is why it can bring relatively greater advantages to developed countries, while independent progress, on the contrary, is featured rather by a "divergence towards capital-saving techniques", and thus it shall probably have a greater impact on underdeveloped countries. This impact, however, as shown by the previous analysis, will be visible only in those countries, which have broken with social and economic stagnation, engaging on the path of accelerated developement.

It is of course impossible to tell, whether those "divergences" of the various types of techniques will be mutualy balanced, or whether one tendency will be stronger than the other. One essential difference between both kinds of progress should be, however, pointed out, since it has substantial influence on their mutual weight. Dependent progress influences only this part of national income, which results from new investment, while independent progress, on the contrary, leads to the more effective use of both old and new equipment, and that is why it has a much greater range.

This is one of the reasons for supposing, that technical progress is one of the factors acting in the direction of closing the gap between developed and underdeveloped countries.

*

Other factors, which besides technical progress can influence the changes of effectiveness of investment in both groups of countries were omitted in our considerations, namely changes within the interindustrial structure of investment, and changes of the rate of substitution of labour and capital.

The first factor will rather act to the disadvantage of underdeveloped countries, especially in the first period of industrialization, during which the necessity of building up the infrastructure and the basic industries will bring about an increase of the capital-coefficient of investment. The second factor, however, will probably have a stronger influence on the increase of this coefficient in developed, than in underdeveloped countries, since in the last ones, owing to substantial labour reserves, the rate of substitution will probably be subject to smaller change than in the former.

Thus, possible effects of changes in the structure of investment and in the rate of substitution on changes in the coefficient of capital in developed and underdeveloped countries could, to a certain degree, be mutualy cancelled out, and therefore finaly, the effectiveness of investment in both groups of countries will be the more dependent on the rate and character of technical progress.

HENRYK FISZEL
POLAND

THE OVERALL AND PARTIAL OPTIMUM

THIS work shows the means by which coordination is achieved between what are known as the over-all optimum and the partial optimum.

This method is illustrated by an example of the optimum division of raw materials, used for the manufacture of different products, between the various branches. Thus we are concerned with a division of a limited quantity of this raw material—in comparison with the utilized production capacity—between its various applications, with the aim of achieving the optimum, i.e. to maximize the value of the ready product[1]. Let us assume that the following magnitudes are known:

1) a_{ij}^l—the quantity of the i-th raw material utilized for the manufacture of a unit of the j-th product, in the l-th branch, where

$$i = 1$$
$$j = 1, 2 \ldots\ldots m\ (l)$$
$$l = 1, 2 \ldots\ldots s$$

2) p_j^l—prices of a unit of j-th product in the l-th branch. Let us assume, that we are concerned with market commodities, the prices of which depend in a small extent on the changes in production. In other words, these prices are constant through a given not too long period, irrespective of the fluctuations of supply.

3) c_{kj}^l—the quantities of units of the production capacities of the k-th equipment, necessary for the manufacture of a unit of the j-th product in the l-th branch.

4) Denoting the required quantity of the j-th product of the l-th branch by x_j^l, we can express the target function as

$$U = \sum_{l=1}^{s} \sum_{j=1}^{m(l)} p_j^l x_j^l = \text{maximum}$$

The task lies in determining $s.m(l)$ unknowns

$$x_j^l \ /j = 1, 2 \ldots m(l) \qquad l = 1, 2 \ldots s).$$

Apart from the condition $U =$ maximum, the following conditions which follow

[1] We can of course define the criterion of optimality differently—depending on the task we set for ourselves.

from the balance of raw material and the production capacity in each branch, must also be fulfilled:

1) $\sum_{l=1}^{s}\sum_{j=1}^{m(l)} a_{ij} x_j^l \leqslant P_i \qquad i = 1,$

where P_i is the overall quantity of the available raw material,

2) $\sum_{l=1}^{s}\sum_{j=1}^{m(l)} C_{kj}^l x_j^l \leqslant C_k^l,$

where C_k^l = the overall production capacity of the k-th equipment in the l-th branch.

3) $x_j^l \geqslant 0.$

This last condition is of course obvious—production cannot be negative.

In solving this problem, which leads to the usual linear programme, let us allocate the i-th raw material (in this example we assume that we are dealing with one raw material) of the 1 branch, of magnitude

$$V_i^l = \sum_{j=1}^{m(l)} a_{ij} X_j^l$$

where x_j^l are the already known quantities of the j-th product in the l-th branch.

This method, which does not give rise to any doubts from the theoretical point of view, is however, difficult to realize in practice.

Moreover, it assumes that the optimization calculation is made at the level of the Planning Commission; this in itself creates difficulties in the gathering and then in the processing of vast amounts of information.

This approach, moreover, implies a fully centralized economy, in which all decisions, concerning even the most minute details of production, are taken at the highest level. Such a system of administrating the national economy, which is impossible to realize in its most extreme form, is—as has been shown by experience—ineffective from the economical point of view.

The means of reconciling these two contradictory tendencies appearing here—on the one hand, the search for the optimum ought, so to speak, to be universal and on a nationwide scale, while on the other hand, the factors mentioned above favour decentralization, i.e. the calculation should be based on partial optimums—ought therefore to be found.

The question therefore arises of whether partial optimums ensure the general optimum.

This is an essential problem for socialist economies.

The answer tends to be negative since in shifting the calculation to a lower level, the division of means made centrally precedes the optimization calculation, i.e. the partial optimum is calculated with certain limits remaining beyond the optimization calculation.

However, in my opinion, the progress which has been made in the field of econometric methods during the last few years, has made it possible to verify these limits in such a way that the condition of coordination between the partial optimum and the overall optimum has been fulfilled.

Consequently, let us suppose that, in the first approximation the division of raw materials has been completely casual, but that it was made in such a way of course that

$$\sum_{l=1}^{s} P_{il}^{1} = P_i$$

where P_{il}^{1} is the quantity of the i-th raw material (equal to 1) allocated to the l branch; the superscript denotes the successive number of the sector. The particular branches then determine their own optimal programme—according to the principle of maximization of value, i.e.

$$U^l = \sum_{j=1}^{m(l)} p_j^l x_j^l = \text{maximum}$$

—on the basis of the given limits of raw materials which are set for them, and retaining the conditions which result from limitations involving raw materials and production capacities

$$\sum_{j=1}^{m(l)} a_{ij} x_j^l \leqslant P_{il}^{1} \tag{1}$$

$$\sum_{j=1}^{m(l)} c_{kj}^l x_j^l \leqslant c_k^l \tag{2}$$

In order to show that this division is correct we will calculate the price, which let us say is economically justified, of raw materials for each branch. These prices will follow from the solution of the dual programme originating from the original program contained in the above equation and the above set of inequalities. We will exclude limitations connected with the production capacity in so much as the prices of this mass are of no interest to us. Consequently the dual programme will assume the following form:

$$V = \sum_{l=1} P_{il}^{1} u_i^l = \text{maximum}$$

$$\sum_{i=1} a_{ji} u_i^l P_j^l$$

$$u_i^l \geqslant 0,$$

where u_i^l are the corresponding prices of the i-th raw material in each branch. It can be assumed *a priori* that prices expressed in this way will not be uniform for all branches, which suggests that the division of raw materials is not optimal.

Let us assume that when we are considering five branches, and that the relations of prices for these branches calculated from the dual model can be described as follows:

$$u_i^3 > u_i^1 > u_i^4 > u_i^2 > u_i^5$$

(the subscripts denote the number of the branch). This set of prices shows the degree of scarcity of the raw material appearing in the given branch; as the price is greater, so the degree of scarcity will be more acute. The optimal division of raw materials is established when the economically justified prices, in all spheres of application (branches), have been levelled out; it seems that no proof need be provided for this statement.

In this case the central (planning) institute, receiving information about prices calculated on the basis of the limits which it has establihed itself, corrects these limits by decreasing allocations for branches, which have achieved a lower price and increasing allocations for those branches in which these prices are higher. There can be several of these corrections, but there should not be many of them since the gap between prices has already given a sufficient indication of more or less what quantities of raw materials ought to be transferred from one branch to another and from which branches they should be transfered.

Consequently in the example cited above it is obvious that branches 5 and 2 ought to transfer means to branches 3 and 1. It is understood that a new optimal programme should be calculated at the lower level after each new limit, has been set and that the price of raw material is calculated from its dual solution.

Ragnar Frisch
Norway

A GENERALIZED FORM OF THE REFI INTERFLOW TABLE

1. INTRODUCTION

In the theory of the refi model (*re* = real, *fi* = financial) of the Institute of Economics at the University of Oslo, we need an interflow table which is considerably more refined than the usual input–output tables, and even more elaborate than the forms discussed in the Oslo median model (memorandum of 10 October 1956), the interflow table with competitive imports (memorandum of 7 June 1959) and the interflow tables for investment planning (for instance tab. (16.1) in the memorandum of 15 July 1959).

The main problems in the refi-table are to take account of the following complications:

1) The distinction between establishments (plants) and enterprises (firms) in the production. This leads to the concept of *ownership sectors* as distinct from production sectors. The investment and operation decisions rest to a large extent with the ownership sectors.

2) The explicit introduction of *financial objects* (money, credit documents etc.) and the trading in such objects. These operations must be analytically coordinated with the operations in real objects so as to arrive at a coherent common system.

3) The explicit introduction of *financial sectors*, i.e. sectors whose main purpose is to produce and exchange financial objects.

4) A more detailed consideration of the various kinds of *transfers* (taxes, social security contributions and benefits, interests, dividends etc.).

All these desiderata must be weighed against each other so as to arrive at a presentation that is reasonably simple and can be represented in a central table of *two dimensions* (with whatever special appended tables that may be needed).

The main idea of the central table here presented is that it is *general* in the sense that it allows for a variety of *different conventions* on the concrete content of the individual figures. The various special cases are generated by putting some of the elements of the general table equal to zero or equal to some specific concretely determined figures. This general approach is a necessary basis for discussing the best form to be actually used in a specific work on numerical data.

TABLE

			Receiving production sectors (establishments). Current account		Consumer groups. Current account	
			Agriculture, Forestry ...	Accounting sector for foreign trade	Government consumption	Private consumption
			h	ω	G	j
Delivering production sectors (establishments). Current account.	Agriculture Forestry ... Unspecified services	k	X_{kh}	$X_{k\omega}$	C_{kG}	C_{kj}
	Accounting sector for foreign trade Non-competitive imports at domestic market prices.	ω	B_h	B_ω	B_G	B_j
Distributed remuneration to consumer groups.	Government sales of goods and services	o	X_{oh}	$X_{o\omega}$	C_{oG}	C_{oj}
	Wage households Ownership households ...	i	W_{ih}	$W_{i\omega}$	W_{iG}	W_{ij}
Categorized transfers recorded positively for the sector of origin. (Excluding transfers made on rows for consumer groups, ownership sectors and financial sectors).	Direct taxes (positive for the tax paying sector) Indirect taxes (positive for the tax paying sector). Subsidies (negative for the subsidy receiving sector) Social security contributions (positive for the contributing sector) Gifts and similar unilateral transfers (positive for the sector of origin) Interests (positive for the interest paying sector) Dividends (positive for the dividend paying sector) Accounting transfers for balancing the table (positive for the sector of origin)	μ	$T_{\mu h}$	$T_{\mu\omega}$	$T_{\mu G}$	$T_{\mu j}$
Ownership sectors (enterprises as distinct from establishments).	State enterprises Local Government enterprises Big private enterprises in industry and mining ... Single-sector enterprises	e	δ_{eh}	$\delta_{e\omega}$	δ_{eG}	δ_{ej}
Financial sectors (main purpose: trading in financial objects).	State budget Local Government budgets ... Central Bank of issue ...	f	δ_{fh}	$\delta_{f\omega}$	δ_{fG}	δ_{fj}
	GRAND TOTAL		$X_{(h)}$	$X_{(\omega)}$	$X_{(G)}$	$R_{(j)}$

Note. The table is general in the sense that it may be applied under a variety of special certain magnitudes zero by definition, or equal to other preassigned figures.

(2.1).

Domestic gross investment in fixed real capital: In production sectors		Domestic gross investment in fixed real capital: In consumer groups			Rest of the world receiving					
Agriculture ...	Accounting sector for foreign trade	Government administration	Private consumer groups	Net increase in inventories. By sector of origin	Exports at domestic market price	Competitive imports at domestic market price	SUBTOTAL (Actual production in production sectors)	Categorized transfers recorded negatively for the sector of origin. (Same description as in rows μ, but with positive changed to negative and vice versa)	Minus increase in financial objects (see specified list)	GRAND TOTAL
g	ω	G	j	L	A	$-Z$		v	λ	
J_{kg}	$J_{k\omega}$	J_{kG}	J_{kj}	$\dot{L}_k$	A_k	$-Z_k$	X_k	S_{kv}	$-\dot{F}_{k\lambda}$	$X_{(k)}+J_{(k)}$
B^J_g	B^J_ω	B^J_G	B^J_j	B_L	B_A $+X_\omega-$ $-B$	$-B_Z$	X_ω	$S_{\omega v}$	$-\dot{F}_{\omega\lambda}$	$X_{(\omega)}+J_{(\omega)}$
J_{og}	$J_{o\omega}$	J_{oG}	J_{oj}	$\dot{L}_o$	A_o	$-Z_o$	X_o	S_{ov}	$-\dot{F}_{o\lambda}$	$X_{(o)}+J_{(o)}$
W^J_{ig}	$W^J_{i\omega}$	W^J_{iG}	W^J_{ij}	W_{iL}	W_{iA}	$-W_Z$	R_i	S_{iv}	$-\dot{F}_{i\lambda}$	$R_{(i)}$
$T^J_{\mu g}$	$T^J_{\mu\omega}$	$T^J_{\mu G}$	$T^J_{\mu j}$	$T_{\mu L}$	$T_{\mu A}$	$T_{\mu Z}$	T_μ	0	0	T_μ
δ^J_{eg}	$\delta^J_{e\omega}$	δ^J_{eG}	δ^J_{ej}	δ_{eL}	δ_{eA}	δ_{eZ}	...	S_{ev}	$-\dot{F}_{e\lambda}$	$J_{(e)}$
δ^J_{fg}	$\delta^J_{f\omega}$	δ^J_{fG}	δ^J_{fj}	δ_{fL}	δ_{fA}	δ_{fZ}	...	S_{fv}	$-\dot{F}_{f\lambda}$	$J_{(f)}$
$J_{(g)}$	$J_{(\omega)}$	$J_{(G)}$	$J_{(j)}$	$\dot{L}_{()}$	E		...	S_v	$-E$	...

conventions on the concrete content of the figures. The special cases are generated by putting

Work is now being done in a cooperative effort of several Norwegian Government Departments (Ministry of Finance, Ministry of Industry, Ministry of Commerce), the Norwegian Central Bank of issue, the Central Bureau of Statistics and the Institute of Economics at the University of Oslo with a view to building up a better analytical foundation for the national budget—including investment plans—and, in general, for objective discussions on economic policy.

The form of the interflow table which is presented in the sequel, is the outgrowth of many and long discussions that have taken place in the Institute of Economics at the University of Oslo. Valuable contribution to these discussions have in particular been made by the research associates Hans Heli (the role of ownership sectors), Tore Johansen (symbolism), Hans Jacob Kreyberg (general aspects), Jan Serck-Hanssen (competitive imports and the relation between domestic market prices and cif and fob prices) and Tore Thonstad (accounting principles and the typographical shape of the table). Subsequently the results have been reformulated and to a considerable extent modified by me. I am responsible for the table as now presented.

2. GENERAL REMARKS ON THE STRUCTURE OF THE TABLE

The table is built up in rows and columns as indicated in the accompanying tab. (2.1). The rows are grouped together in certain *horizontal parts* and the columns in certain *vertical parts*. The order of succession of these parts are chosen in such a way that the upper left region of the refi table resembles as much as possible the previously used interflow tables that were mentioned in Section 1.

The specification of the individual rows within a certain horizontal part in the refi table and the specification of the individual columns in a certain vertical part is to be decided in each concrete case. But the horizontal parts and the vertical parts are standard. Each part is indicated by a standardized affix, for instance k, ω, i etc. for the horizontal parts and h, ω, j etc. for the vertical parts as exhibited in tab. (2.1). Each of these affixes runs through a certain number of values indicating the individual rows or columns.

It will be found convenient to let the numbering of the individual rows be different from that of the columns. One may, for instance, let the individual row numbers be selected from among 1–499 and the column numbers from among 501–999, and in such a way that if there is a *correspondence* between a certain row and a certain column, the column number should be exactly 500 larger than the row number. If it is not found inconvenient to use four digits, one may instead let the column number be exactly 1000 larger than the corresponding row number. There is no necessity of letting the numbering run continuously. We may for instance number the rows in the first horizontal part 001, 002 etc. and those in the second horizontal part 051, 052 etc. And similarly for the columns.

As a general principle of balancing it may be convenient to require that whenever there is a correspondence between a row and a column, the row sum should be equal to the column sum, and for a row that has no corresponding column, or a column that has no corresponding row, the sum should be zero. This we may call the correspondence principle for balancing. In the general form of the refi table we will, however, not impose these conditions strictly, but allow certain exceptions as explained below. By so doing we may in certain cases let a row sum or a column sum record some essential figure which it seems more logical to have as a marginal total than as a (perhaps negative) balancing item in the interior of the table.

Whatever the exceptions to the correspondence principle are, it will, of course, always be true that the sum of the column sums is equal to the sum of the row sums. This gives a means of numerical checking and will also help to clear up the logic of the exceptions to the correspondence principle. Examples are given below.

The whole interflow table is assumed to apply to *a given period*, say a given year. For this period we assume in essence that the prices of real goods and services are given. They are observed or estimated as the average prices prevailing in the period in question.

Financial objects may be assumed to be traded at the prices actually prevailing. If we do not consider the *chaining* of the refi tables from one period to another in such a way that the value of the stocks of financial objects at any given point of time is logically connected with the interflow tables for each period, we do not need to consider (positive or negative) income elements arising from *changes* in the prices of stocks and bounds and other financial objects. Such elements need therefore not be considered in the table. If it is wanted to do so, it may be done through specifications in appropriate rows and columns in the table. Some special considerations on this aspect of the financial interflow are offered in Section 15 of the memorandum *General theory of the kernel model*, of 7 February 1958.

Depreciation on real capital (leading from the concept of gross investment to that of net investment, and correspondingly from the concept of gross national product to that of net national product) is not considered explicitly in the table, but it could be introduced by the inclusion of an extra row and column. Compare tab. (1.1) in the memorandum of 7 June 1959.

3. THE HORIZONTAL PARTS OF THE TABLE

I. The first horizontal part of the table represents *delivering production sectors*, current account. The classification over the individual rows is here made according to the production sector in which the establishments (plants) are classified. It is not made according to enterprises (firms). The standard affix denoting delivering production sector is k. As an additional sector in this group we consider an accounting sector for foreign trade. Its affix is ω. In the actual numerical work now going on the number of sectors in this part will be between 40 and 50.

II. The next horizontal part represents *consumer groups*. On a given row in this part are recorded the distributed primary factor remunerations that go to the consumer group in question. The fact that these items represent *distributed* remuneration means that we here only consider the amounts that go to households. One or more of these consumer groups pertain, for instance, to ownership groups —and for these only the distributed remuneration is recorded here, while the rest is retained as a surplus in the production sectors (perhaps later to be transferred to an ownership sector as explained below).

On each row the distributed remunerations are broken down according to the sectors where the primary factor input was made (represented by columns, as discussed in Section 4).

On the first row in the horizontal consumer group part we record Government sales of goods and services. In other words we consider the amount of these sales as a remuneration of a primary factor. (This means that these sales will have to be included in the concept of gross national product). This special factor is denoted by the affix 0. We could, of course, have recorded Government sales of goods and services on a production sector's row, but it has been found convenient rather to take them as pertaining to a consumer group.

III. The third horizontal part of the table represents what may be called *categorized transfers*. These are transfers classified according to the special kind of the transfers, for instance direct and indirect taxes, social security contributions, interests, dividends etc. The rows of this horizontal part pertain to transfer items that are recorded as positive numbers for the sector that *makes the payment* (which can be interpreted as the sector that receives a corresponding productive service). The standardized affix for the rows in this part is μ. An example of the specification of these rows is given in tab. (2.1). The last row is a row for accounting transfers that may be needed for the purpose of balancing the table.

In many cases we have a choice of whether we want to let a transfer pass through one of the categories now considered or go directly from the paying to the receiving sector (which may, for instance, be an ownership sector or a financial sector). This is exemplified in Section 6 below.

The interflow table is not constructed so as to exhibit the three dimentional breakdown according to paying sector, receiving sector and the category of the transfer. We can only choose between two out of the three breakdowns. If a three dimensional breakdown is wanted, it must be made in a special table appended to the interflow table.

The horizontal part representing categorized transfers has its counterpart in a vertical part of such transfers as explained under V in Section 4.

IV. The fourth horizontal part represents *ownership sectors*, i.e. enterprises. They are denoted by the standarized affix e (e = enterprise).

A rough breakdown in this horizontal part of the table may, for instance, be:

1. State enterprises.
2. Local Government enterprises.
3. Big private enterprises whose main field of operation is in industry and mining.
4. Non-big enterprises whose main field of operation is in industry and mining.
5. Enterprises whose main field of operation is in shipping and whaling.
6. Other enterprises operating in more than one production sector.
7. Single-sector enterprises, i.e. enterprises each of which only operates in one of the classified production sectors.

V. The fifth and last horizontal part represents the *financial sectors*, i.e. sections whose main purpose is the operation in financial objects. They are denoted by the standardized affix f ($f =$ financial). Some examples of sectors in this part are given in tab. (2.1). A last sector in this part is the Rest of the world. Its row is only used for balancing purposes, when needed.

The ownership sectors and the financial sectors have certain features in common, but concretely they are sufficiently distinct to warrant a separate treatment of them.

4. THE VERTICAL PARTS OF THE TABLE

I. The first vertical part represents receiving production sectors, current account. This part corresponds to the horizontal part I. The standardized affixes for the columns are h and ω.

In a given column in this vertical part are recorded inputs into the sector in question. Some of these inputs may be recorded under the horizontal parts representing delivering sectors or categorized transfers or ownership sectors (when the ownership sector takes over the surplus from the production sector) or financial sectors.

Since there is a choice of recording categorized transfers in a horizontal part or in a vertical part, as explained below under VI, the column sum of a given receiving sector need not be equal to the actual production in the sector. This is why it is denoted $X_{(h)}$ rather than X_h. And similarly for the receiving sector ω.

II. The second vertical part represents the *consumers as receiving units*. The first column (or set of columns) represents Government consumption on current account. It will contain big items. The standard affix is G. If Government consumption on current account is split into headings (for instance in conformity with the state budget or local Government budgets), G will run through a certain number of affixes.

The items W_{iG} are particularly important for $i =$ Wage or salary households.

The rest of the columns in this vertical part, i.e. those with standard affix j, represent private consumer groups. They correspond to the rows i. That is to say, for each value of i there is a corresponding j and *vice versa*.

The balancing of the individual consumer groups may be done in different ways; either vertically or horizontally. Through the accounting transfer row or column any balance from an individual houshold's row may be transferred to its column, or *vice versa*. In whatever way this is done, it is convenient to impose the rule that the sum in the column of a private consumer group should be equal to the sum in its row. This is indicated in tab (2.1) where the same symbol is used for column sum and row sum, i.e. $R_{(j)} = R_{(i)}$ for $i = j$. This sum may be, say, zero or actual consumption in the consumer group in question.

It is also possible to arrange the balancing in such a way that δ_{ej} (for $e =$ single-sector enterprises) records the total saving (real and financial) in the private consumer group j.

Similar remarks apply to Government consumption. Here it will be convenient to impose the condition

$$X_{(o)} = \Sigma_G X_{(G)} \tag{4.1}$$

III. The third vertical part represents the inputs in various *investment directions.*

There are two main vertical sections here: investment in production sectors and investment in consumer groups.

In the former of these sections there is a subsection for investment in the regular production sectors—with standard affix g—and investment in the accounting sector for foreign trade—with standard affix ω. The latter of these subsections is relatively unimportant and nothing much would be lost by putting all elements in this column equal to zero by definition. Occasionally there may, however, occur a concrete item which would find its natural place here. The former of the two subsections is important. Its standard affix g is different from the standard affix h of receiving sectors on current account. There are several reasons for this distinction. One is that the investment directions for production sectors may be either more aggregated or less aggregated than the receiving sectors on current account[1]. In the actual numerical work now being done, there will be between 20 and 25 investment directions in this subsection.

In the section for investment in consumer groups there are also two subsections, one for investment in Government administration—with standard affix G— and one for investment in private household groups—with standard affix j. The former of these subsections may be divided in different special directions, i.e. different values of the affix G. This breakdown may or may not be similar to that used for Government consumption on current account. For the latter of the sub-

[1] In Sections 15 and 16 of the memorandum of 15 July 1959 a great variety of investment directions are considered.

sections one may conveniently use a breakdown similar to that for consumer groups on current account, i.e. j may in both cases run through the same private consumer groups (but the individual column numbers will be different, compare the remarks in Section 2). If such a breakdown is found difficult for lack of data, an aggregation will have to be made, even down to a single column j in the private investment subsection.

In the investment column or columns for private consumer groups we may —if the data are available—record investment in semi-durable or durable consumer goods. An attempt should be made to include at least residential building and private automobiles.

In all the investment columns we allow in principle for the same whole variety of inputs which we considered in the regular production sectors, i.e. labour input and other types of primary factors remuneration (the rows i) and all sorts of taxes and other transfers (the rows μ) as well as surplusses (the rows e and f). In other words each investment direction may in principle be considered itself as a sort of a production sector. In the general form of the table it is very convenient to keep this possibility open. One only has to think of the need for expressing the effect of a diversified system of taxes on special investment directions. If it is not wanted to take advantage of the book-keeping possibilities that are thus kept open, one just has to put some of the items in the investment columns equal to zero.

The concrete meaning of the column sums $J_{(g)}$, $J_{(\omega)}$ etc. will depend on the particular conventions adopted for the recording of the individual items in these columns. For further comments on this point see Section 7.

IV. Net increase in *inventories* is recorded in a separate column (with standard affix L) immediately after the vertical part for investment in fixed real capital. The items in the inventory column are broken down according to the sector of origin (the rows) for the inventory items. The possibility is kept open of considering all sorts of inputs here in the same way we followed for the investments in fixed real capital.

V. The *Rest of the world* as a receiving sector is represented by two columns, one for exports and the other for competitive imports.

As a general rule the items in these columns are recorded in domestic market prices, the correctional term X_ω which is needed in order to correct the values in such a way as to bring the balance in conformity with what the Rest of the world is aetually to be credited or debited for, is entered as one component of the export item on the ω row. This correctional term X_ω may be looked upon as the "total actual product" in the accounting sector for foreign trade. This question is discussed in greater detail in connection with (16.2) in the momorandum of 15 July 1959 (where f was used instead of the now standardized affix ω).

The item B_A represents reexport (at domestic market prices) of non-competitive import items, and B_Z represents competitive imports, if any, into the accounting sector for foreign trade. By the definition of the subtotal X_ω the symbol B, recorded as a component in the same cell as B_A, will be equal to the sum of all the B-items on row ω, i.e.

$$B = \Sigma_h B_h + B_\omega + \Sigma_G B_G + \Sigma_j B_j + \Sigma_g B_g^J + B_\omega^J + \Sigma_G B_G^J + \Sigma_j B_j^J + B_L + B_A - B_Z \quad (4.2)$$

There will always be some practical difficulty in defining concretely what is to be included in the concept of "non-competitive" imports, but the difficulties can be surmounted with a sufficient degree of approximation.

The principle will be to include imports of such *kinds* of goods and services which cannot be conceived of as produced domestically under existing technical conditions and the system of prices for which the interflow table is constructed. It should be noted that it is only a question of whether the *kinds* of goods can be conceived of as produced domestically or not. If a certain kind of good can actually be conceived of as produced at home, it should *not* be classified as non-competitive, even if the domestic *capacity of production* is so limited that it is likely that a smaller or larger quantity of the good will in any case have to be imported. Any such import that may take place because of limited domestic capacity of production should be taken as competitive import. An essential point in the subsequent programming formulation—where the competitive imports represent degrees of freedom—is precisely that the amount of these imports will be determined by taking account of the domestic capacity bounds.

By going through the list of commodities imported and deciding each case according to the above criterion (for Norway say tea, coffee, bananas, certain kinds of machinery etc.), it will be possible to reach sufficiently accurate figures for the non-competitive imports in a given year (actual work in Norway will be done on 700–800 import goods for the year 1955). From these figures one may in the usual way derive non-competitive import *coefficients* (expressed in relation to the actual production in the production sectors or in relation to disposable income in consumer groups etc.).

Categorized transfers for the Rest of the world are recorded either in the horizontal μ part or in the vertical ν part (with $f =$ Rest of the world) and with accounting transfer to the $T_{\mu A}$ or $T_{\mu Z}$ cells (with $\mu =$ accounting transfer row) so that the sum total E of the columns A and $-Z$ becomes equal to the actual net *export surplus* (positive or negative) for which the Rest of the world is to be debited. This amount E is to be compensated through the change in financial objects with the Rest of the world. Compare the remark below under VII on the column sum $-E$ for the aggregation of all the columns of minus increase in financial objects.

VI. The vertical part *categorized transfers* is broken down into individual columns that correspond exactly to those in the horizontal part μ. We thus have a great freedom of choice in deciding whether we want to record such a transfer in a row or in a column. A given item of transfer will either have to be recorded positively in a row or negatively in the corresponding column or *vice versa*.

Since every transfer that is made *from* one sector or group must be made *to* another sector or group, the grand total row sum T_μ for the transfer category μ, must be equal to the grand total column sum for the same category of transfers. That is we have

$$T_\mu = S_\mu \qquad \text{for all } \mu \tag{4.3}$$

But we do not, of course, have any corresponding equality for individual sectors or groups. For instance

$$T_{\mu h} \text{ may be different from } S_{k\nu} \text{ even if } \mu = \nu \text{ and } h = k \tag{4.4}$$

VII. In the last vertical part are recorded changes in the holdings of *financial objects* (financial assets). An actual increase in such a holding is recorded as a negative number. That is, the items record minus the increase. The standard affix for the individual columns here is λ. The following is an example of how the breakdown in the increase in financial assets, i.e. the specification of the individual columns, may be made.

1. Net purchase of existing real caiptal (as distinct from the real investment of the period to which the interflow table pertains). Such a purchase has in this connection to be considered as "financial" even if the underlying object is real.
2. Net increase in cash and deposits in the Central bank of issue.
3. Net increase in the holdings of Government bills.
4. Net increase in the holdings of Government bonds.
5. Net increase in the holdings of non-Government bonds issued by others than the holder.
6. Net increase in the holdings of non-Government bonds issued by the holder.
7. Net increase in the holdings of stocks issued by others than the holder.
8. Net increase in the holdings of stocks issued by the holder.
9. Net increase in special forms of financial capital investment.
10. Net increase in the holdings of banking deposits (the net increase will be positive if the public has actually increased its banking deposits).
11. Net increase in bank loans.
12. Net increase in the holdings of other domestic financial assets.
13. Net increase in the holdings of foreign financial assets.

Since the items recorded in these columns pertain to changes in what any *domestic* sector holds of the various kinds of assets[2], the grand total of all the λ columns, must correspond to the opposite of the change in the financial holdings of the Rest of the World (in its relation to our country). In other words the grand total of all the λ columns in the interflow table must be equal to $-E$, where E is the net export surplus reckoned in the prices at which the rest of the world is to be debited. Compare V of Section 4.

[2] The rest of the world row under f is only a balancing row which may be used if it is wanted to produce zero sums in certains rows and columns.

5. AB SECTOR PRICES VS. PRICES TO BE PAID BY THE RECEIVING SECTORS

In principle the total input into a domestic sector should be reckoned in such a way that this total input includes domestic indirect taxes and similar items. In this sense the total input (to be balanced against the sectors output) is a market price concept, not a factor cost concept.

This, however, still leaves open the question of the particular way in which all the inputs, including indirect taxes and similar items, are distributed over the cells of the column for the receiving sector in question.

Special indirect taxes levied on the production in each separate sector can from the practical statistical viewpoint easily be charged directly to the sector in question (as is done in the Norwegian input–output work). But for the general sales tax such a distribution is not so easy to obtain when this tax is levied on the last stage of the circulation process. In this case it is statistically simpler to charge the sales tax to the special sector "Internal trade" and from this sector to pass it on as part of the inputs that go to the various receiving sectors from the sector "Internal trade". If this is done, we may say that the deliveries from one sector to another are reckoned in "ab sector" prices. (This is done in the Norwegian input–output work).

In point of principle it would be more satisfactory to distribute all indirect taxes over the various delivering sectors. This would be done by including also the general sales tax in the respective h columns for receiving sectors so that these taxes would be passed on as part of the delivery from each sector. This would mean a system of "prices to be paid by receiving sectors" instead of "ab sector prices".

From the programming viewpoint it does not matter very much which one of the two procedures is followed, provided the input to all sectors from the "Internal trade" sector is properly counted (and the input–output coefficients reckoned accordingly). In the "ab sector" system this means a higher input to all sectors from the "Internal trade" sector.

The distinction between domestic market price and *cif* and *fob* prices in relation to the rest of the world is discussed under V, in Section 4.

6. TRANSFERRING SURPLUS FROM PRODUCTION SECTORS

The transferring of surplus from a production sector may be done in a great number of different ways, and the particular way that is chosen will entail different interpretations of the grand total column sum $X_{(h)}$, $X_{(\omega)}$ etc. and the grand total row sums $X_{(k)}+J_{(k)}$, $X_{(\omega)}+J_{(\omega)}$ etc.

Before we discuss these various ways of transferring we will state an equation which in the refi interflow table replaces the rule that the grand total in the column for sector h is equal to the grand total in the row of this sector.

Let

$$Y_h = \Sigma_k X_{kh} + B_h + X_{oh} + \Sigma_i W_{ih} + \Sigma_\mu T_{\mu h} \tag{6.1}$$

be the subtotal in column h up to and including the T-items.

Then by the definition of the column sum—see tab. (2.1)—we have

$$X_{(h)} = Y_h + \Sigma_e \delta_{eh} + \Sigma_f \delta_{fh} \tag{6.2}$$

On the other hand we have by the definition of the grand total on row h

$$X_{(h)} = X_h + \Sigma_\nu S_{h\nu} - \Sigma_\lambda \dot{F}_{h\lambda} - J_{(h)} \tag{6.3}$$

From (6.2) and (6.3) follows

$$X_h - (Y_h - \Sigma_\nu S'_{h\nu}) = \Sigma_\lambda \dot{F}_{h\lambda} + J_{(h)} + \Sigma_e \delta_{eh} + \Sigma_f \delta_{fh} \tag{6.4}$$

The left member of (6.4) is the surplus of sector h in the classical (median model) sense. Hence the right member of (6.4) also expresses this surplus. This right member can now be interpreted in a refi sense. We can break it down into the following three parts

$\Sigma_\lambda \dot{F}_{h\lambda}$ = the financial objects which the production sector h acquires *on its own account*. This is the increase in what may in a large sense be looked upon as the "cash holdings" of the establishments in the sector. (6.5)

$J_{(h)}$ = the gross real investment which the production sector h acquires *on its own account*, including in this real investment both investment in fixed real capital and in inventories. (6.6)

$\Sigma_e \delta_{eh} + \Sigma_f \delta_{fh}$ = the net surplus of the production sector h *over and above* (6.5) and (6.6). This surplus is distributed to the e and f sectors. (6.7)

If we allow the sector to acquire nothing on its own account, the whole surplus will be in the form of (6.7), the sum of which corresponds to the median model surplus concept δ_h.

On the other hand if an accounting rule is adopted whereby (6.7) is put zero, the sector h must invest all its surplus in the form of either (6.5) or (6.6), or in the form of a mixture of these two elements such that their sum is equal to the surplus.

Further, if the sector is not allowed to acquire any financial objects, and (6.7) is by definition put equal to zero, the sector must invest all its surplus in the form of $J_{(h)}$.

Still another alternative is that we let all the surplus in sector h be transferred in the form (6.7) *except* that we put the elements $\delta_{eh} = 0$ for e = single-sector enterprises, meaning by this that we do not record any δ-surplus for the *independent* establishments in the production sector h (i.e. those belonging to single-sector enterprises). In this case the sum of (6.5) and (6.6) would express the total surplus in these independent establishments. And we would also have an expression for the forms in which this surplus is retained by the sector, namely in the financial form (6.5) or in the real form (6.6).

If we allow certain (6.7) items, i.e. surplusses to be transferred to the e and f sectors, these surplusses must, of course, in turn be disposed of in some way. The way in which it is done is expressed by the *horizontal balancing* of the e and f rows. The grand total on each such row, i.e. $J_{(e)}$ and $J_{(f)}$ respectively, can then be looked upon as the gross real investment (including investments in fixed real capital and in inventories) that is required by these sectors *as their property*. And the δ elements on the e and f rows can be looked upon as the surplusses which accrue to the e and f sectors through various activities including the investment directions considered as production activities.

We can throw further light on this way of balancing by considering the equality between the sum of the row totals and the sum of the column totals. Making use of (4.1) and (4.3), we get[3].

$$\Sigma_g J_{(g)} + J_{(\omega)} + \Sigma_G J_{(G)} + \Sigma_j J_{(j)} + \dot{L}_{()} = \Sigma_k J_{(k)} + J_{(\omega)} + J_{(o)} + \Sigma_e J_{(e)} + \Sigma_f J_{(f)} \tag{6.8}$$

This equation expresses the fact that the sum of everything that is invested in all investment directions (the left member of (6.8))—with the interpretation of the elements in the investment direction columns which we have chosen—is equal to the sum of all investments that are acquired as the property of any sector or group (the right member of (6.8)).

In the special case discussed above as an illustration of the horizontal balancing of the e and f rows, the elements $J_{(g)}, J_{(\omega)}$ etc. in the left member of (6.8) are equal to the *actual* gross investments. These we denote with a subscript without parenthesis, i.e.

$$\Sigma_g J_g + J_\omega + \Sigma_G J_G + \Sigma_j J_j + \dot{L} \tag{6.9}$$

This equality between the investment totals symbolized with and without a parenthesis around the subscript need not always hold. It all depends on how we want to perform the horizontal balancing in the e and f rows. We may perhaps decide to distribute the previous grand total $J_{(e)}$ over the various investment direction columns as negative items so as to have the grand total on each e row equal to zero. And similarly on each f row. If this is done, the left member of (6.8), i.e. the sum of the column totals for the investment directions, would only indicate that part of the gross investments which is acquired as the property of the sectors indicated by the affixes in the right member of (6.8) when the terms with $J_{(e)}$ drop out.

If this way of balancing the e and f rows is combined with the procedure of including also the surplus of the independent establishments in the sum (6.7), so that the sum of (6.5) and (6.6) is zero, we may still have $J_{(k)}$, $J_{(\omega)}$ and $J_{(o)}$ different from zero, but the gross investments that are acquired as property by the production sectors could now only be brought about through a decline in the financial hold-

[3] Compare the remark before (4.1). For more precision we could introduce "Households" as an ownership sector, and add $J_{(i)}$ as a term beside $R_{(i)}$.

ings of these sectors. Hence the column totals written in the left member of (6.8) would only represent those investments that are acquired by the production sectors through depletion of the financial holdings of these sectors (plus the unimportant item $J_{(0)}$).

If in the latter case we do not allow the production sectors to make any change in their holdings of financial assets (for instance because we never reckon with any such holdings), all the items in both members of (6.8) would be zero (except $J_{(0)}$).

In all the above cases we may, if we like, include in the investment direction cells of rows e and f a component representing a surplus accruing to the e and f sectors through the investment directions considered as production activities—as we suggested above in the special case first considered when we discussed the horizontal balancing of the e and f rows. But if the gross investments acquired by the e and f sectors as their property is distributed back as negative elements over the e and f rows and in the investment direction columns, the *net* negative elements here would not represent the total investments acquired by the e and f sectors, but only the investments acquired by the e and f sectors through the surplusses in these sectors *exclusive* of the surplusses which these sectors have realized in the investment directions considered as production activities.

Even so, there is much to be said for this way of balancing horizontal the e and f sectors, because we would get a picture of how the surplus of a given e or f sector is *distributed* over gross real investments in the various production sectors (and over the acquisition of financial assets), while we only get an expression for the e and f sectors' *total* gross investment in real capital (and their acquisition of financial assets) if we perform the horizontal balancing in such a way that the gross investment in real capital appears as a grand total.

With these general rules in mind we may consider in more detail some examples of ways of recording transfers of surplusses.

7. A CLOSER EXAMINATION OF TRANSFERS AND THE BALANCING OF THE VARIOUS PARTS OF THE INTERFLOW TABLE

For more precision in the subsequent discussion we will make some additional explanations about the symbols in tab. (2.1). In so doing we will for completeness follow the indication in the footnote 1) to the text immediately before (6.8). That is, we assume that the grand total on row i is denoted $R_{(i)}+J_{(i)}$ instead of simply $R_{(i)}$. We may, if we like, also assume that households is one of the rows in the e part, but this is not necessary. As suggested in Section 3. V, we may also use a row for the Rest of the world, but that is not necessary either. Compare the remark below under (7.15).

We begin by stating explicitly an assumption that was already contained implicitly in the reasoning about (6.1)–(6.4), namely that

$$X_{(k)} = X_{(h)} \text{ when } h \text{ corresponds to } k \tag{7.1}$$

A similar statement must be made for the accounting sector for foreign trade. Following the principle of numbering of rows and columns explained in Section 2, we might for full typographical precision have used a different affix for the row and the column of the accounting sector, instead of the single affix ω. We don't need to do it, however, if we only remember the convention

$$\text{column sum } X_{\binom{\omega}{\text{col}}} = X_{(\omega)} \tag{7.2}$$

equals the item $X_{\binom{\omega}{\text{row}}} = X_{(\omega)}$ that occurs in the grand total of row ω

We also state explicitly the assumption that

$$R_{(i)} = R_{(j)} \text{ when } j \text{ corresponds to } i \tag{7.3}$$

Finally we recall (4.1).

When these assumptions are made, the equality of the sum of row sums and the sum of column sums in tab. (2.1) lead to the equation (6.8) where, however, the item $\Sigma_i J_{(i)}$ now has to be added in the right member so that the equation becomes

$$\Sigma_g J_{(g)} + J_{\binom{\omega}{\text{col}}} + \Sigma_G J_{(G)} + \Sigma_j J_{(j)} + \dot{L}_{()} = \Sigma_k J_{(k)} + J_{\binom{\omega}{\text{row}}} + J_{(o)} + \Sigma_e J_{(e)} + \Sigma_f J_{(f)} + \Sigma_i J_{(i)} \tag{7.4}$$

We will not make any assumption about a correspondence between the J elements in the left and right member of (7.4) similar to the correspondence assumptions (7.1)–(7.3) and (4.1).

To arrive at a precise interpretation of the J items in (7.4) we must consider in general the way in which the various parts of tab. (2.1) are balanced. In this connection there are three different parts of the table to consider.

I. *The correspondence balancing part* consisting of the rows k, ω, o, i and the corresponding current account columns h, ω, G, j.

II. *The vertical balancing part* consisting of the investment columns g, ω, G, j, L and the two Rest of the world columns A and $-Z$, as well as the ν and λ columns.

III. *The horizontal balancing part*, consisting of the rows μ, e and f.

Regarding the correspondence balancing part we note in the first place that by (6.4)—which follows from (7.1) together with (6.1)—we have the interpretation (6.6) of $J_{(h)}$. For complete typographical precision we might have written

$$J_{(k\ \text{cor.}\ h)} \text{ instead of } J_{(h)} \tag{7.5}$$

The symbol to the left in (7.5) means the magnitude $J_{(k)}$ for that value of the row affix k which corresponds to the column affix h. Similarly for $\dot{F}_{h\lambda}$. The simpler notation used in (6.3)–(6.7) is, however, clear enough.

It should be noted that $J_{(h)}$ means the gross real investment which emerges as the *property* of production sector h, regardless of what is the nature of these investments. In principle the ownership distribution of the new investment items may be different from the distribution of these items according to the kind of investment or the production sector where the new capital goods are to be *used.*

In practice certain simplifying conventions in this respect may be made, but such conventions are only specializations, not included in the standard conventions on the symbols of the table.

By a reasoning similar to (6.1)–(6.7) we are by (7.2)–(7.3) and (4.1) led to the interpretations specified in tab. (7.6).

Through (6.5)–(6.7) and tab. (7.6) the balancing of the correspondence part of the table is specified. Even with these standard conventions a great variety of specialized conventions may be made according to what we decide to include in the transfer elements T and S and the surplus elements δ.

Regarding the vertical balancing part we first note that each investment column is considered more or less as a complete production sector that may receive inputs and deliver a total output. The same applies to the inventory column. The inputs in any of these columns may be of all sorts: deliveries from the regular production sectors, for instance J_{kg}, non-competitive imports B_g^J etc. Categorized transfers for the investment directions and the inventory columns can only be recorded on the μ row, but on these rows all categorized transfers can be specified. In principle, surplus items of the categories δ can also be recorded for each investment direction.

TABLE (7.6). *Ways of placing the surplus arising on current account operations in the following sectors.*
Compare also (6.5)–(6.7).

	Accounting sector for foreign trade ω	Sector for Government sales of goods and services, and Government consumption. Compare (4.1)	Consumer group No. *i*. Current account i
The financial objects which the sector acquires *on its own account* (i.e. as its property).	$\Sigma_\lambda \dot{F}_{\omega\lambda}$	$\Sigma_\lambda \dot{F}_{o\lambda}$	$\Sigma_\lambda \dot{F}_{i\lambda}$
The gross real investment goods (produced the same year) which the sector acquires *on its own account* (i.e. as its property), including both investment in fixed real capital and in inventories.	$J_{(\omega)}$ in the sense of $J\binom{\omega}{\text{row}}$	$J(_o)$	$J_{(i)}$
The surplus of the sector over and above the preceeding two items. This surplus is passed on as a transfer to the *e* and *f* sectors.	$\Sigma_e\delta_{e\omega} + \Sigma_f\delta_{f\omega}$	$\Sigma_e\Sigma_G\delta_{eG} + \Sigma_f\Sigma_G\delta_{fG}$	$\Sigma_e\delta_{ej} + \Sigma_f\delta_{fj}$ (*j* corresponding to *i*)

The total output in each investment direction is in tab. (2.1) expressed by the column sums $J_{(g)}$, $J_{(\omega)}\ldots$ We have for generality put a parenthesis around the subscript on these magnitudes, but in all cases where the δ-items are taken to denote actual surplusses to be transferred to ownership or financial sectors, the investment column sums will be identical with the actual gross investments. The same applies to the column sum $\dot{L}_{()}$. Compare thc comments to (6.9). In order to standardize the notation as much as possible, wc will in the following make this convention. I.e. we assume

$$J_{(g)} = J_g, \quad J_{\binom{\omega}{\text{col}}} = J_\omega, \quad J_{(G)} = J_G, \quad J_{(j)} = J_j, \quad \dot{L}_{()} = \dot{L} \tag{7.7}$$

where the symbols without parenthesis around the subscript are the actual real investments in the various directions. In accordance with (7.7) we let the δ-items in the fixed capital investment columns and the inventory column denote actual surplusses to be transferred to ownership or financial sectors.

We assume that the investment directions and the inventory column have no other form of surplus than those of the δ-form. That is, they cannot place their surplus in any of the categories corresponding to (6.5) and (6.6), i.e. to those in the first two horizontal parts of tab. (7.6).

The vertical balancing of the two Rest of the world columns is simple. The sum total of these two columns is equal to the export surplus E (positive, negative or zero) conceived of as that amount for which the Rest of the world is to be debited.

The vertical balancing of the ν columns simply consists in recording the column sums.

Finally, the sum of the column sums in the λ-columns must be equal to the negative of the export surplus, i.e. it must be equal to $-E$. This sum expresses the total (positive, negative or zero) of all financial objects which the domestic sectors have acquired as their property.

Now for the horizontal balancing part.

On each μ row the balancing consists simply in recording the row sums T_μ, the assumption being that the entries in this horizontal part T_μ and those in the vertical part S_ν are made in such a way that (4.3) is fulfilled. For more typographical precision we could write this convention

$$T_\mu = S_\nu \qquad \text{when } \mu \text{ corresponds to } \nu \tag{7.8}$$

For instance, since all direct taxes are recorded on row $\mu =$ direct taxes, or in column $\nu =$ direct taxes[4], the two balancing clements in the column $\nu =$ direct taxes on the row $f =$ statc budget and Local Government budgets together will show the total of all direct taxcs (and direct subsidies, if any)[5].

[4] On the row $\mu =$ direct taxes an actual tax would be recorded as a positive number and that for an actual subsidy would be rccorded as a negative number, while in the column $\nu =$ direct taxcs we will have the opposite rule.

[5] From a formal viewpoint the horizontal μ-part and the vertical ν-part could be looked upon as belonging to the "correspondence balancing part" of the table.

Each of the e rows or f rows is considered only as an accounting sector that can receive payments (i.e. render services) only in the form of surplus transfers δ or in the form of categorized transfers S. If material production services are to be taken account of for any e or f sector, it must be done by a δ-transfer or an S-transfer from a production sector or a consumer group. For any e sector the total surplus that is available and must be placed, is consequently

$$\Sigma_h\delta_{eh}+\delta_{e\omega}+\Sigma_G\delta_{eG}+\Sigma_j\delta_{ej}+\Sigma_g\delta^J_{eg}+\delta^J_{e\omega}+\Sigma_G\delta^J_{eG}+\Sigma_j\delta^J_{ej}+\delta_{eL}+\delta_{eA}+\delta_{eZ}+\Sigma_v S_{ev} \tag{7.9}$$

By the definition of the total on row e in tab. (2.1), the sum (7.9) is equal to

$$\Sigma_\lambda \dot{F}_{e\lambda}+J_{(e)} \tag{7.10}$$

The sum (7.9) expresses the various *sources* of the surplus in sector e and (7.10) represents the two ways in which this surplus can be *placed*. These two ways are completely analogous to (6.5)–(6.6) and the first two horizontal parts of tab. (7.6).

For an e sector there is no concept corresponding to (6.7) or the lower part of tab. (7.6). If we should introduce such a concept for an e sector, it would have to be the *negative* of (7.9), and in this case the surplus of the e sector would by definition be zero. It is more natural to consider (7.9) as a specification of the ways in which the surplus in sector e is created, and (7.10) as a specification of the ways in which it is placed.

An exactly similar reasoning applies to the f sectors.

We can therefore formulate the interpretations given in tab. (7.11).

TABLE (7.11). *Ways of placing the surplus arising on current account operations in the following sectors.*
Compare also (6.5)–(6.7) and tab. (7.6).

	Ownership sector e	Financial sector f
The financial objects which the sector acquires *on its own account* (i.e. as its property). (Since we want to have the column sum of all the λ columns equal to $-E$, we do not record separately the Rest of the world's acquisition of financial objects. This appears as the various λ column sums).	$\Sigma_\lambda \dot{F}_{e\lambda}$	$\Sigma_\lambda \dot{F}_{f\lambda}$
The gross real investment goods (produced the same year) which the sector acquires *on its own account* (i.e. as its property) including both investment in fixed real capital and in inventories. (Since the J items only pertain to *domestic* investments, the Rest of the world has no such investments. Export of fixed capital goods is recorded in the A column and export of previously existing fixed capital goods is recorded in the column λ = existing real capital. Compare Section 4.VII.1).	$J_{(e)}$	$J_{(f)}$

We have now given a complete description of the *standard* meaning of the magnitudes entering into tab. (2.1).

Apart from the meaning of the symbols there is also the question of how the figures are to be entered in the various cells of the table. We will not discuss all possibilities, but only mention how the table will look if the $J_{(e)}$ item is distributed over the various investment columns and similarly for $J_{(f)}$. It is sufficient to indicate that part of the table that is changed. This is shown in tab. (7.12).

TABLE (7.12)

	Domestic gross investment in fixed real capital				Net increase in inventories	Grand total
	g	ω	G	j	L	
e	δ^J_{eg} $-J_{(eg)}$	$\delta^J_{e\omega}$ $-J_{(e\omega)}$	δ^J_{eG} $-J_{(eG)}$	δ^J_{ej} $-J_{(ej)}$	δ_{eL} $-J_{(eL)}$	0
f	δ^J_{fg} $-J_{(fg)}$	$\delta^J_{f\omega}$ $-J_{(f\omega)}$	δ^J_{fG} $-J_{(fG)}$	δ^J_{fj} $-J_{(fj)}$	δ_{fL} $-J_{(fL)}$	0
Grand total	J_g $-\Sigma_e J_{(eg)}$ $-\Sigma_f J_{(fg)}$	J_ω $-\Sigma_e J_{(e\omega)}$ $-\Sigma_f J_{(f\omega)}$	J_G $-\Sigma_e J_{(eG)}$ $-\Sigma_f J_{(fG)}$	J_j $-\Sigma_e J_{(ej)}$ $-\Sigma_f J_{(fj)}$	$\dot{L}$ $-\Sigma_e J_{(eL)}$ $-\Sigma_f J_{(fL)}$	...

In this table $J_{(eg)}$, $J_{(fg)}$ etc. denote how much of the gross investment in the g direction that has been acquired as the property of the e and f sectors respectively. Similarly for $J_{(e\omega)}$, $J_{(f\omega)}$ etc.

The individual row sums e and f in tab. (7.12) will be zero and the sum of all the column sums in the investment directions and in the inventory column will now indicate those parts of the new real investment that was acquired by the production sectors, the accounting sector ω, the sector for Government sales of goods and services and the consumer groups as their own property. In other words we have

$$\begin{aligned}&\Sigma_g[J_g-\Sigma_e J_{(eg)}-\Sigma_f J_{(fg)}]+[J_\omega-\Sigma_e J_{(e\omega)}-\Sigma_f J_{(f\omega)}]+\Sigma_G[J_G-\Sigma_e J_{(eG)}-\Sigma_f J_{(fG)}]+\\&+\Sigma_j[J_j-\Sigma_e J_{(ej)}-\Sigma_f J_{(fj)}]+[\dot{L}-\Sigma_e J_{(eL)}-\Sigma_f J_{(fL)}]=\Sigma_k J_{(k)}+J_{(\omega)}+J_{(o)}+\Sigma_i J_{(i)}\end{aligned} \tag{7.13}$$

This follows from the fact that the sum of column sums must be equal to the sum of the row sums also when tab. (2.1) is reshaped as indicated in tab. (7.12) (and the term $J_{(i)}$ added in the grand total on row i).

The various investments in Government administration will ordinarily not be included either in the left or in the right member of (7.12). In the left member their positive entries are cancelled by the subtraction in the form of $J_{(fG)}$ with f

= State budget and Local Government budgets, (except perhaps for small amounts included in $J_{(\omega)}$ or $J_{(o)}$.

Several of the investment items in (7.13) may ordinarily be put equal to zero by definition, for instance $J_{(\omega)}$. If a small amount is included in $J_{(o)}$, this same amount will ordinarily be equal to the difference expressed by the third bracket in the left member of (7.13).

We will now give some examples of ways of recording categorized transfers and transfers of surplusses.

Take for instance a large gift from abroad accruing to the state budget. It may be recorded in anyone of a number of different ways.

(7.14) One way—and perhaps the simplest—is to enter the amount positively on the μ row gifts in the column A, and at the same time positively on the f row state budget and in the ν column gifts. In this way the relation (4.3) is fulfilled and the result will be that the export surplus—i.e. the total amount by which the Rest of the world is to be debited—is increased by the amount in question. And this amount appears also as an increase of the surplus creating items of the state budget (compare the last term of (7.9) when the equation is applied to the f row state budget). How the state balances this increase, depends on its various activities. It may do it in the $\dot{F}$ or J form (whether the recording is made as in tab. (2.1) or in the reshaped (7.10) form), or it may do it by increasing Government consumption—in which case it would decrease δ_{fG}—or it may do it in other ways. In any case the State budget would be balanced.

(7.15) Another way of recording the foreign gift would be to use the Rest of the world row f, if such a row is included in the table. The gift could then be entered twice on the f row Rest of the world, namely positively in the A column and negatively in the ν column gifts. At the same time the amount would be entered positively on f row State budget in the ν column gifts. This would zero balance the f row Rest of the world, and it would also zero balance the ν column gifts, but E and the State budget surplus would be increased. The question of how the increased State surplus is used, would be the same as under (7.14). This recording via the Rest of the world row is more complicated than that by way of (7.14), so transactions of this sort are no argument in favour of introducing a Rest of the world row.

As another example consider the dividends collected by the State on its holdings of stocks in independent establishments that can be classified under the receiving production sectors h.

(7.16) One possibility is to enter the dividend positively under δ_{fh} (with f = State budget). If so, the effect on the horizontal balancing will appear immediately under $-\dot{F}_{f\lambda}$ or $J_{(f)}$, or be used by the State in some other way.

(7.17) Another possibility is to enter the dividend positively under δ_{eh} (with e = State enterprises). The horizontal balancing will then either appear in $-\dot{F}_{e\lambda}$ or in $J_{(e)}$, or it may be passed on to the State budget by entering the total of the collected dividends negatively under S_{ev} (with $e =$ State enterprises, and $v =$ accounting transfers) and positively under S_{fv} (with $f =$ State budget, and $v =$ accounting transfers). In the latter case the dividends will finish up by effecting the State budget in the same way as under (7.16).

(7.18) A third possibility is to enter the State collected dividends positively under $T_{\mu h}$ (with $\mu =$ dividends) and positively under S_{fv} (with $f =$ State budget and $v =$ dividends). This will not disturb the equality (4.3), and will affect the State surplus in the same way as under (7.16)–(7.17).

(7.19) A fourth possibility is to enter the State collected dividends negatively under S_{kv} (with $k =$ the delivering sector k corresponding to the receiving sector h and $v =$ dividends), and at the same time enter the collected amount positively under S_{fv} (with $f =$ State budget and $v =$ dividends).

Second, consider the dividends collected by the State on its holdings of stocks in combined enterprises.

(7.20) One possibility to record these dividends is to enter them negatively under S_{ev} (with $e =$ the various ownership sectors involved, and $v =$ dividends) and at the same time positively under S_{fv} (with $f =$ State budget and $v =$ dividends). The effect on the State budget would be similar to that under (7.18)–(7.19).

(7.21) Another posibility is to record these dividends negatively under S_{ev} as in (7.20) (with $e =$ the combined enterprises involved and $v =$ dividends) and positively under S_{ev} (with $e =$ State enterprises, and $v =$ dividends). The ensuing surplus on the State enterprises may be handled as under (7.17).

Third, consider in a more general way the recording of any surplus of the establishments in a given receiving production sector h. The surplus is *defined* as the left member of (6.4). Some remarks on the placing of this surplus were made after (6.7), we will now add some further possibilities, also including the sectors mentioned in tab. (7.6).

(7.22) The logically most straightforward way to record the surplus of sector h is to distribute all of it over the δ_{eh} and δ_{fh} cells in the column h (one of the values of e corresponding to single-sector enterprises). In this case the sum of the δ_{eh} and δ_{fh} items becomes equal to the left member of (6.4), and hence

$$J_{(k)} = -\Sigma_\lambda \dot{F}_{k\lambda} \quad k \text{ corresponding to } h \tag{7.22a}$$

The only gross real investment which the delivering production sector k can acquire on its own account, will in this case be what it purchases through depletion of its holdings of financial objects. Compare also the discussion after (6.7).

(7.23) In the case (7.22) it might be natural to record on the e and f rows not only the total surplusses realized in the h columns, but also those realized in the sectors mentioned in tab. (7.6). For the rows ω, 0 and i we would then get equations similar to (7.22a).

As to the surplusses in the investment directions we must remember that the items δ^J_{eg}, $\delta^J_{e\omega}$, δ^J_{eG}, δ^J_{ej} and δ_{eL} cannot be deduced from other individual entries in the table, but will have to be determined by some additional information or conventions and then entered directly on the e and f rows. That is to say, for the investment directions we assume no other form of surplus than the δ-kind. Compare the paragraph after (7.7).

In any case the horizontal balancing of the e and f rows can be made by entering the items $-\dot{F}_{e\lambda}$ and $J_{(e)}$, respectively $-\dot{F}_{f\lambda}$ and $J_{(f)}$, as in tab. (2.1), or by splitting the totals $J_{(e)}$ and $J_{(f)}$ in the way indicated in tab. (7.12).

(7.24) If we follow (7.23) except for the fact that the total surplusses of the *independent* establishments are not recorded as δ-elements, compare (6.4), or, to be more specific, if we put the sum $(\Sigma_e\delta_{eh}+\Sigma_f\delta_{fh})$ for the independent establishments in sector h equal to zero, the independent establishments in sector k (k corresponding to h) would increase the sum of their holdings of financial objects and their real investments by an amount equal to their total surplus.

Now let the symbol $J'_{(k)}$ indicate the real investments retained as the property of the independent establishments in sector k and the symbol $J'_{(k)}$ indicate the real investments that are acquired as the property of the other establishments in sector k so that $J_{(k)} = J'_{(k)}+J''_{(k)}$.

The investments $J''_{(k)}$ can only be brought about through a decline in the financial holdings of the sectors in question, compare (7.22a). That is, we may write

$$J''_{(k)} = -\Sigma_\lambda \dot{F}''_{k\lambda} \tag{7.24a}$$

And similarly for the whole of $J_{(\omega)}$, $J_{(0)}$ and $J_{(i)}$. For all these sectors we will now use the right hand expression corresponding to (7.24a). For independent establishments, on the contrary, we retain the expression $J'_{(k)}$.

If further all $J_{(e)}$ and $J_{(f)}$ items are distributed as in tab. (7.12), the sum of all the column sums in the investment directions g, ω, G, j, L would be equal to

$$\Sigma_k J'_{(k)}-\Sigma_\lambda[\Sigma_k\dot{F}''_{k\lambda}+\dot{F}_{\omega\lambda}+\dot{F}_{o\lambda}+\Sigma_i \dot{F}_{i\lambda}] \tag{7.24b}$$

For instance, if by convention the sectors indicated in the bracket of (7.24b) are not allowed to have any change in their financial holdings, the sum of all the column sums in the investment directions mentioned would be equal to the first term in (7.24b).

The above reasoning will, of course, apply to *any* splitting of the production sectors in two groups, one of which has its total surplus transferred to the e and f rows, and another for which the sum $(\Sigma_e\delta_{eh}+\Sigma_f\delta_{fh})$ is zero.

(7.25) If we want to eliminate the $J_{(k)}$, $J_{(\omega)}$, $J_{(o)}$ and $J_{(i)}$ items altogether, we can, for instance, increase δ_{eh}—and/or δ_{fh}—(with h corresponding to k and with e = the various ownership sectors, including the single-sector enterprise ones and f = the various financial sectors) sufficiently to make up for the amount previously defined as $J_{(k)}$. And similarly for $J_{(\omega)}$, $J_{(o)}$ and $J_{(i)}$ (with households as one of the e-rows). The ensuing items $J_{(e)}$ and $J_{(f)}$ could, if we want to, be distributed over the investment columns as in tab. (7.12). If so, the sum in each of the investment columns g, ω, G, j and L would be zero (if we assume that none of these column sums can be negative).

(7.26) If we proceed as in (7.25) except for the *independent* establishments and if we further use the (7.12) principle, the sum of the column sums in the investment directions would be equal to $\Sigma_k J'_{(k)}$. This is a more general reasoning than the one in the comment immediately after (7.24b).

Jaroslav Habr
Czechoslovakia

A CONTRIBUTION TO THE THEORY OF SLIDING PLANS

INTRODUCTION

There is nothing new in the idea of sliding plans[1]. So far, however, this planning method has not been explored theoretically, nor has it found application in practice. While it was merely "wishful thinking" on the part of many planners for some time past, this proposal has again come up for serious discussion in recent years [1].

The latest achievements of economic science, modern computing machinery, and the considerable experience gained in planning of the traditional type make it now possible to treat the problem of sliding plans in a more realistic way. The old idea is now getting a new impetus thanks to powerful instruments such as linear programming, input-output analysis, economic cybernetics, and electronic computers.

The method of "rigid" plans as represented by Soviet planning technique was based on the proposition that in planning only two alternatives exist: binding plans, represented by directives, and non-binding "plans", amounting to little more than prognoses. Naturally, the latter type was found to be entirely unsuitable for concrete measures in a society building socialism. This proposition, allowing for only two alternatives, has considerably contributed to the difficulties encountered with traditional planning methods. Given the technical impossibility of computing several alternatives, any deviations from the originally assumed conditions had to be corrected "in midstream" by more or less improvised measures.

1. *The Development of Alternative Trajectories*

If we were to establish sliding plans in the conventional rigid way, as is typical for plans covering a fixed number of years (n-year plans), our difficulties would be likely to increase even more. We have arrived, therefore, to the idea of formulating sliding plans as the *development of alternative trajectories.*

This proposal is based on two principles:

1) Plans are drawn up as alternatives within a certain spectrum.

---

[1] Such suggestions were submitted e.g. in the 1947 discussion on planning methods to be adopted under the First Czechoslovak Five-Year Plan.

2) Shifting the plan from the initial to the next planning horizon is again carried out within a certain spectrum.

1) The dominant trajectory in the spectrum is the *planned trajectory*. The whole economic policy is concentrating its attention upon this trajectory in such a way as to secure development along this path. The width of the spectrum is given by the deviations of the *boundary trajectories* from the planned trajectotry. The former represent lines along which the economy might develop in case that economic policy did not succeed in creating exact conditions for the planned development. Here we have in mind the potential effect of exogenous factors, positive or negative, or other disturbing factors. The planned trajectory may be conceived as the result of an *ex ante* optimization, and the boundary trajectories as "optimistic" or "pessimistic" alternatives. Naturally, a spectrum may cover a larger number of alternatives than basic trajectories[2].

The proposed development of alternative trajectories may be applied in practice using different concepts of such trajectories. They may be quantitative (as expressed in quantitatively stated plan targets), or qualitative.

2) The length (period) of the shift and the distance (interval) of the planning horizon will depend on several factors. Among purely economic factors the length of production and investment cycles will be of particular importance. Generally, the distance of the planning horizon exceeds the length of the shift. When the shift period comes to its end (at the end of every year, for instance) the planners will always find a ready spectrum of trajectories so that the new plan may be directly linked up with the stage of growth already achieved.

With the shift of the planning horizon, the basic trajectories will be partitioned again. However, this only applies to those trajectories which have to be taken into account in view of the level actually attained. Evidently, the partition process is not of the partition-tree type, that is, it is not subject to a chain growth. Those trajectories that were not in accordance with the development that has actually occurred will be left out in the subsequent analysis. This reduction in the number of trajectories will take place even if it could be supposed that in the subsequent development actual conditions might coincide with the level that would be reached following some of the dismissed trajectories. The principle will always hold that a certain objective stated *ex ante* may be attained along different trajectories, but a concrete result reached *ex post* may only be realized along a unique trajectory. In other words, the actual path of development eliminates all different simultaneous trajectories from future consideration.

---

[2] Generally, there is no need to tie up the concept of alternative trajectories with optimization, defining one or the other as "better" or "worse". There need not be any evaluation at all. Alternative trajectories may be simply taken as potential development paths derived from the effect of such decisive factors as population growth, technical progress, etc. in fact, they will require a good deal of analytical work.

Our procedure may be demonstrated in the following diagram:
horizon (h) = 2 time units
shift = 1 time unit
number of basic trajectories = 3 (optimal, better, worse; 1, 2, 3)
time axis = T
axis of values of the objective function = E
a) Possibilities of development at origin (t_0)

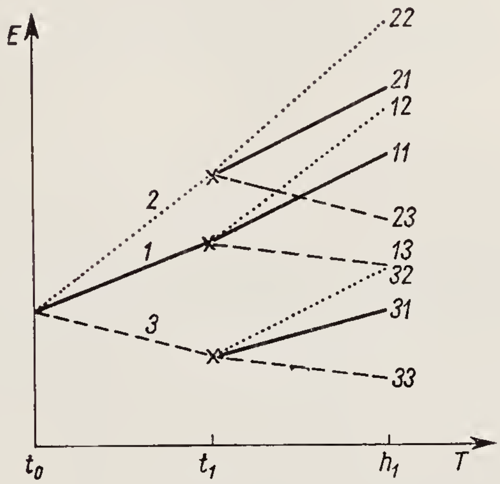


Fig. 1

b) Possibilities of development after the first shift, if actual development proceeded in line with the plan

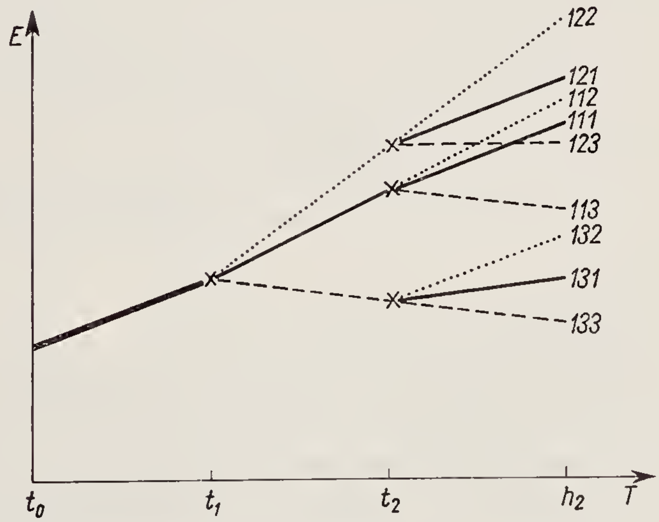


Fig. 2

c) Possibilities of development after the second shift, if in this period development was less favourable than was provided for under the plan

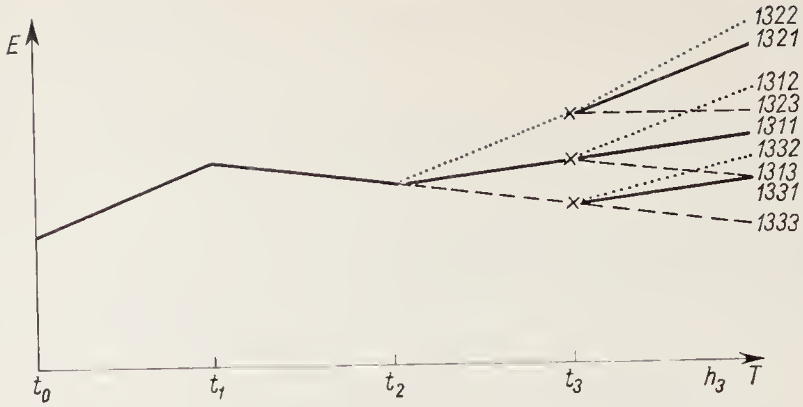


FIG. 3

2. *Model of Growth*

The development of alternative trajectories is capable of being applied over a very wide field: in various types of models, for differing width and density of the trajectory spectrum, for various aggregation of effects (different dimensions), for various echelons of control (firm, sector, or on a national scale). The fact that alternative paths of development with intentional or random changes in the parameters of the model may be simulated by means of efficient computers, considerably facilitates the practical application of this procedure.

In the following we shall demonstrate the idea of a sliding plan with alternative trajectories on a model[3].

This is a macroeconomic, two sector, optimizing model with three basic trajectories. It is based on the input-output analysis combined with linear programming. An element of automatic regulation of the on-off type has been incorporated in the model with a view to achieving a kind of dynamic equilibrium.

Conceptually the model implies a certain strict formalization, but it weakens the rigidity of the procedure in two directions:

1. The results gained by formalized iterations are checked by intraposition of logical analyses with a view to actively influencing the decisive parameters (man→machine→man etc.)[4].
2. The model uses feedback approximations technique.

Essentially, the formalized apparatus is intended to achieve two objectives: *optimization* (linear programming) and *balancing* (input-output analysis). These methods, however, are no substitutes for detailed partial analyses, particularly for the sectoral and intra-sectoral analyses. On the contrary, the optimizing and balancing methods and detailed sector analyses are rather complementary[5].

[3] Some elements of the proposed model have been described in a paper on investment and input-output analysis, written in 1958. A summary was published in [2].

[4] Some elements of this procedure are referred to as "discrete programming". See e.g. [3].

[5] This fact has been properly stressed by Chenery already in the first applications of the

Another characteristic feature of the model is the inference of alternative means from long-term objectives; these means are taken as short-term objectives, and instantaneous means are inferred from these short-term objectives. Such permanent transformation of objectives into means is essential in growth dynamics.

Taking into account the scope of this paper, the model based on the above principles will be described in a simplified form. Some possibilities of its further development will be mentioned, however. In our model both boundary trajectories are trajectories derived by optimization. Different criteria are used, of course, in either case. One extreme criterion is *consumption*, the other *investment*. In contradiction to the usual types of optimization, maximization in this case relates to *gross* output in either sector.

Let us start from the case where the maximum volume of gross output of the industries manufacturing consumers' goods has been selected as criterion for optimization. The case where the maximum volume of the gross output of industries supplying producers' goods has been taken as criterion for optimization will be discussed subsequently. In view of the fact that this is a two-sector model there is no need explicitly to deal with the problem of the output structure within the two sectors. Similarly, the relations between gross and net outputs (effectivity of the system) may be now left out of consideration, though data on this point are indispensable if we wish to obtain information on the effect of optimization on consumption or on the process of reproduction. Finally, the effect of foreign trade is likewise disregarded, i.e. its potential impact on the original ratio between the output volumes of the two sectors.

For the basis year (t_0) we have data on the following magnitudes:

technical coefficients	a_{11}, a_{12}, a_{21}, a_{22}
investment in the two sectors	Y_1
net output of consumers' goods	Y_2
gross output of the investment sector	X_1
gross output of consumers' goods	X_2
labor-input coefficients	w_1, w_2
investment-input coefficients	k_1, k_2
labour force	Z_1, Z_2
fixed assets	F_1, F_2
available increments	ΔZ, ΔF

The index placed in the upper right part of a symbol indicates the time interval, if such a distinction is necessary. The technical coefficients, and the labour-input and investment-input coefficients are assumed to be constant in the first draft of the model.

input–output method to problems of economic growth [4]. Experience obtained so far corroborates this proposition. Of special relevance in this connection is the principle of the "sectoral" industrial organization which is typical for the present stage of development in the planned economies.

Table of basic relations

X_1	X_2	investment	consumption	total output	
a_{11}	a_{12}	Y_1		X_1	$a_{11}X_1+a_{12}X_2+Y_1 = X_1$
a_{21}	a_{22}		Y_2	X_2	$a_{21}X_1+a_{22}X_2+Y_2 = X_2$
w_1				Z_1 (ΔZ)	$w_1X_1 \leqslant Z_1$
	w_2			Z_2	$w_2X_2 \leqslant Z_2$
k_1				F_1 (ΔF_1)	$k_1X_1 \leqslant F_1$
	k_2			F_2 (ΔF_2)	$k_2X_2 \leqslant F_2$

It is easy to see that the first part of the table relates to the input-output analysis and the other part to linear programming.

Let us first consider the constraints given by the labour force.

In the base period, production can be characterized by two boundary conditions as expressed by the following inequalities:

$$w_1X_1^0 \leqslant Z_1^0$$
$$w_2X_2^0 \leqslant Z_2^0$$

In the following time-interval ($t_0 \rightarrow t_1$) the labour constraints may be expected to shift, particularly as a result of the potential labour-increment (ΔZ^1). This increment at the beginning of the period t_1 can be allocated alternatively for either sector. Compared with the base relations we thus obtain three inequalities, consisting of two boundary conditions on each variable and one substitution constraint[6].

$$w_1X_1^1 \leqslant Z_1^0+\Delta Z^1 \quad \text{or} \quad X_1^1 \leqslant X_1^0+\frac{\Delta Z^1}{w_1}$$

$$w_2X_2^1 \leqslant Z_2^0+\Delta Z^1 \quad \text{or} \quad X_2^1 \leqslant X_2^0+\frac{\Delta Z^1}{w_2}$$

$$\frac{w_1X_1^1}{Z_1^0+Z_2^0+\Delta Z^1}+\frac{w_2X_2^1}{Z_1^0+Z_2^0+\Delta Z^1} \leqslant 1 \quad \text{or} \quad w_1X_1^1+w_2X_2^1 \leqslant X_1^0+w_1+X_2^0w_2+\Delta Z^1$$

Similarly, having determined X_1^1 and X_2^1 (by some optimization or by *ex post* observation) we can further develop the relations for subsequent periods. In this model we shall assume that the allocation of the new labour force, once it is carried through, is not changed any more.

Let us now consider the objective function. As mentioned above, the object of maximization is the gross output of consumers' goods. Such optimization deter-

[6] This freedom to substitute is in this case reserved only for the potential increment. Naturally, the range of choice may be narrowed or widened according to the substitutibility and mobility of labour.

mines at the same time the output ratio for the two sectors ($X_1 : X_2 = \lambda_1 : \lambda_2$). In view of the fact that the optimization will result in a number of solutions giving identical values of the given objective function, we must introduce into the problem a secondary optimizing criterion in addition to the principal maximizing function. The volume of the gross output of the sector producing investment goods will serve as such a criterion. In the combined objective function we shall thus maximize X_2, and, for equal values of X_2, we shall maximize X_1 $\left(\text{maximizing the ratio } R = \frac{\lambda_1}{\lambda_2}\right)$[7].

Below there is an example with numerical data. It is also presented in the following diagrams.

Initial data

$Z_1^0 = 12$	$X_1^0 = 6$	$w_1 = 2$
$Z_2^0 = 6$	$X_2^0 = 2$	$w_2 = 3$
$R^0 = \frac{6}{2} = 3$	$\lambda_1^0 = 0.75$	$\lambda_2^0 = 0.25$

Additional data

$\Delta Z^1 = 12$ (total addition of labour force in period 1)

$\Delta Z^2 = 18$ (total addition of labour force in period 2)

Calculations

At the beginning of period t_1

$$X_1^1 \leqslant 6+\frac{12}{2} = 12$$

$$X_2^1 \leqslant 2+\frac{12}{3} = 6$$

$$2X_1^1+3X_2^1 \leqslant 12+6+12 = 30$$

Solution

$$X_1^1 = 6 \quad X_2^1 = 6 \quad R^1 = \frac{6}{6} = 1 \quad \lambda_1^1 = 0.5 \quad \lambda_2^1 = 0.5$$

At the beginning of period t_2

$$X_1^2 \leqslant 6+\frac{18}{2} = 15$$

---

[7] It is obvious that this optimizing procedure is based on a proposition contrary to that which is implied in the wellknown model of Professor Oscar Lange [5]. We shall see, however, that Lange's proposition may be taken as the basis for the design of the second boundary trajectory. In that case we shall maximize X_1, and for equal values of X_1 we shall maximize X_2 (minimizing to the ratio R).

$$X_2^2 \leqslant 6+\frac{18}{3} = 12$$

$$2X_1^2+3X_2^2 \leqslant 12+18+18$$

Solution

$$X_1^2 = 6 \quad X_2^2 = 12 \quad R^2 = \frac{6}{12} = \frac{1}{2} \quad \lambda_1^2 = 0.33 \quad \lambda_2^2 = 0.67$$

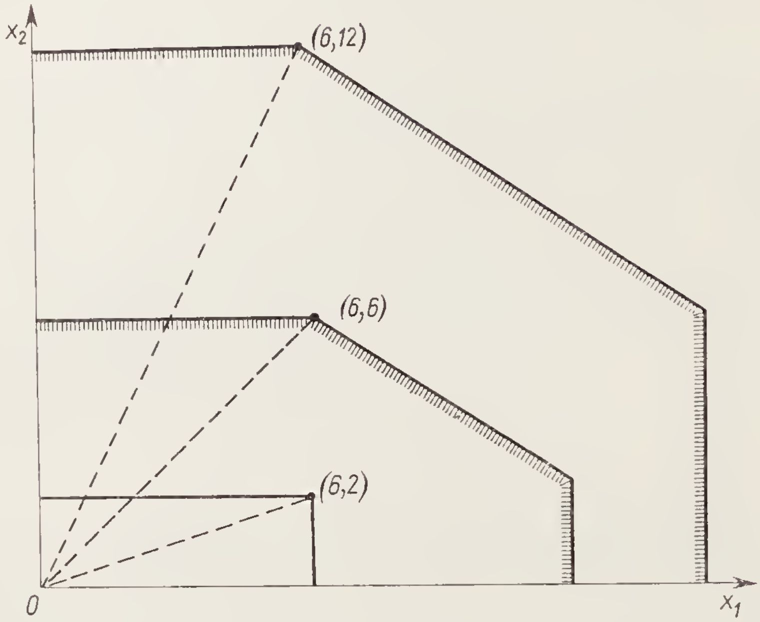


FIG. 4

Partial generalization:

If there were no other constraint, the ratio R would successively decline (and the indicator λ_2 would increase) as a result of the steady rise in output X_2, i.e. in correspondence with the potential increase of the labour force[8]. The intensity of this growth would depend on the absolute rise in employment (ΔZ) as well as on all factors which influence the coefficient w_2.

The tendency for all the increase in employment to be allocated to the second sector would continue as long as the labour force is the decisive bottleneck in this sector's production. However, as soon as capacity becomes a bottleneck in this sector $\left(\text{conf.} \frac{F_2}{k_2}\right)$ the situation for optimization will be a different one.

There will be now two cases possible[9]. If both the capacity limit and the employment limit result in the same output level in sector two, or if the capacity limit gets *below* the level which was warranted by the existing allocation of labour force

[8] It is obvious that full employment is postulated in the model.

[9] Disregarding the situation where there is an excess supply of labour for either sector.

in this sector $\left(\frac{F_2^1}{k_2^1} < \frac{Z_2^0}{w_2}\right)$, optimization will take into account the supplementary criterion, that is, the simultaneous maximization of X_1. Since the same level of X_2 may be attained with different levels of X_1, maximization will be effected by allocating the entire increment in employment to sector one (Fig. 5a).

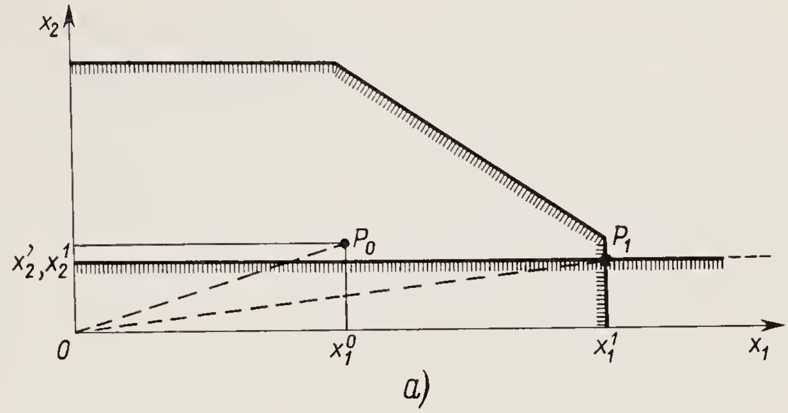


FIG. 5a

If the capacity limit is *within* the range of the potential increment $\left(\frac{Z_2^0 + \Delta Z^1}{w_2} > \right.$ $\left. > \frac{F_2^1}{k_2^1} > \frac{Z_2^0}{w_2}\right)$, optimization will result in distributing the employment increment between sector one and two. The allocation ratio will depend on the vertex of the convex feasibility surface (see point P_1 in figure 5b).

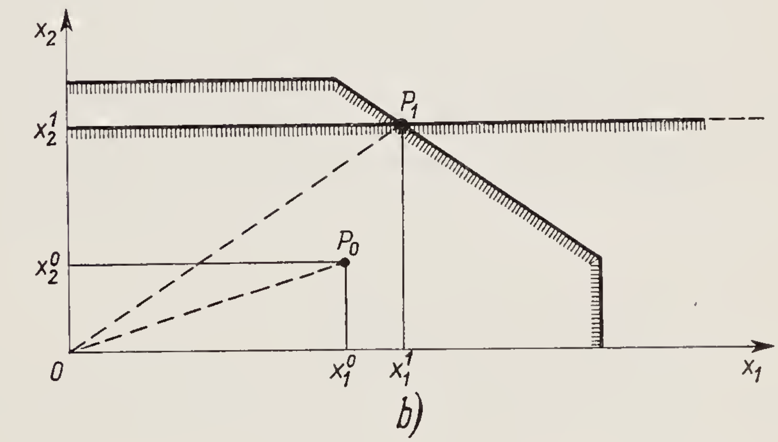


FIG. 5b

It is now possible to discuss the process of optimizing gross output for sector two for all constraint combinations which may occur in employment and in capacities (live and embodied labour) in both sectors. Let us start with a situation in which the labour force is the bottleneck in the production of sector two. In such a case all potential increment in employment will be allocated to this sector. If the allocation of subsequent increment will be effected in the same way, the ratio λ_2 will increase as will the total gross output X_2; this will continue up to the point when the delivery of investment goods from sector one will not be sufficient to warrant such further expansion of production in sector two as would correspond to the increment of employment. Thus the output capacity of sector two will become a limiting factor again. As long as further capacity increments in sector two are not sufficient to absorb the potential increments in employment, the surplus labour force will be allocated to sector one. Thus the ratio λ_2 will decline and there will be a shift in favour of the production of sector one. The rate of growth of gross output in the second sector will be slowed down until it will almost reach stagnation. On the other hand, changing the ratio R in favour sector one—and hence allocating the potential labour force increment to this sector—will result in gradual expansion of the output basis for investment goods. Depending on the length of the investment cycle and the internal ratio between the output of investment goods for the first and second sectors, the above factor will result—after a certain time-lag—in gradual rise of the capacity limit in sector two. The labour force will again turn into a limiting factor.

It can be seen that under otherwise constant conditions (theoretically) development will be automatic. The trajectories which are derived from maximizing the gross output of consumption goods will exhibit an oscillatory motion. The shape of this sinusoidal function (period, amplitude, phase) will be given by the development of bottlenecks (labour force → capacity → labour force, etc.).

This applies analogically, though in the opposite direction, to the growth and rhythm of trajectory development derived from maximizing the gross output in sector two (with subsidiary maximization of the ratio 2). This trajectory likewise would tend to oscillate steadily, the oscillation depending on whether the bottleneck in sector one is given by the labour force or by the capacity constraint.

The idea now suggests itself of making use of both the trajectories derived by maximization according to the two extremal principles and to obtain a new trajectory which might be characterized as the path of dynamic equilibrium. Economic policy would concentrate attention on this trajectory, which might serve as the basis of the plan. Deviations of actual development from planned growth might be compensated by influencing the parameters whose alteration would shift the vertex of the convex feasibility surface to the level of the planned trajectory. The planned trajectory might be corrected in accordance with the possible changes in the boundary trajectories.

3. *Further Development of the Model*

The two sector model described above may be developed in at least two directions: increasing the number of constraints, increasing the number of variables. The following modifications of the model are possible.

So far the labour force entered the optimizing process in the form of two boundary conditions on the variables and of one substitution limitation, while the capacity constraints were only introduced as boundary conditions on the outputs. Taking into consideration substitution possibilities, one could treat capacities in an analogical way as the labour force.

So far we did not consider changes in the structure of X_1 and X_2 due to the effect of forcign trade. It might be taken into account by means of additional constraints, possibly in terms of substitution.

Conceptually it might be quite easy to expand the two-sector model into a three-sector one. In contrast to conventional concepts of three-sector models[9], the third variable will be represented in this case by the sector of intermediate output.

In the two-sector model the important problem of the ratio of investment in sector one and sector two (ΔF_1, ΔF_2) is not taken into consideration. It might be possible, of course, to disaggregate the variable X_1 also in this respect.

So far we have assumed both the stability of the coefficients w and k and the independence of the limiting factors Z and F. Making the model a more realistic one certainly implies dropping these assumptions and taking into consideration of such influences as technical progress, using the results of research and development ("investment in people"), the influence of technology on the technical coefficients[10], the influence of investment coefficients on the technical coefficients as well as substitution effects[11].

In this form the model does not explicitly touch upon the problem of the central allocation of bottleneck factors and of decentralized optimal decision-making (the problem of "pyramidation"). So far this question has not been solved satisfactorily. Considering the extent of the problem there is no other way than solving it through approximations by means of simulation[12].

In conclusion we should like to stress that the method of development of alternative trajectories is universal in application to such an extent that it is not confined to the model described. Obviously, this model is only one of many possible tools that may be employed in the successive development of the trajectory spectrum.

REFERENCES

1. I. Maevskii, A. Fomin, *Nekotorye voprosy sovershenstvovania planirovania narodnogo khoziaistva*, "Voprosy ekonomiki" 12/1963.
2. J. Habr, B. Korda, *Rozbor meziodvětvových vztahů*, Praha 1960.

[9] Confer e.g. Johansen's study [6].

[10] In this context we refer to a point made by Lange [5, p. 314].

[11] Confer Frisch's concept of substitution rings and infra-effects [7].

[12] This is in agreement with the views of Frisch [7, p. 11]. Confer also the interesting approach of Hungarian economists [8]. For approximate optimizations of extensive systems the "bottleneck method" has been developed by the author of this paper [9].

3. P. Massé, *Discretionary or Formalized Planning. International Congress on Economic Development,* Vienna 1962.
4. H. B. Chenery, *Problems of Economic Development. Proceedings of an International Conference on Input-Output Analysis,* Varenna 1954.
5. O. Lange, *The Output–Investment Ratio and Input–Output Analysis,* "Econometrica", April 1960 (Vol. 28,2).
6. L. Johansen, *A Multi-Sectoral Study of Economic Growth,* Amsterdam 1960.
7. R. Frisch, *A Survey of Types of Economic Forecasting and Programming, and a Brief Description of the Oslo Channel Model,* Oslo 1961.
8. J. Kornai, T. Lipták, *Two-level Planning,* Budapest 1963.
9. J. Habr, *Approximation Methods in Linear Programming,* IFIP Congress, Munich 1962.

Roy Harrod
Great Britain

OPTIMUM INVESTMENT FOR GROWTH

The subject to be discussed is the best division of resources between the production of consumer goods and the production of capital goods; in other words, the choice between consumption now and consumption at a later date. One solution to this problem was given by F. P. Ramsey[1]. Ramsey's solution was an impracticable one, because it involved plotting out the whole path through time to a condition called Bliss, at which the return to any further investment would be zero. It may also be objected to this solution that it ignores other and more important factors responsible for growth. While the theoretically correct solution may imply measuring quantities that cannot easily be measured and assuming away uncertainties, it is claimed that what follows furnishes a correct solution in principle without reference to a long stretch of future time.

One factor of growth is the population increase. We may assume that there ought to be at least as much new investment as will maintain the existing capital/labour ratio. This will be required to prevent a decline in output per head. It may even be necessary to raise the capital/labour ratio somewhat as an off set to diminishing returns from natural resources in cases where that condition is present in significant degree. It could further be argued that some extra investment is required in cases where there is disguised unemployment, so as to set these unemployed to work. The question of disguised unemployment will be considered below.

For the rest growth consists in the growth of output per head. This is the central problem. Some writers have proceeded at once to the consideration of the growth in output per head as being determined by an increase in the capital/labour ratio. Doubtless this is one element in the situation, but I suggest that it has been greatly over stressed. It is better not to regard it as a factor of growth operating in its own right, but as a necessary corollary where other factors of growth are operating. Indeed if an attempt were made to get growth in output per head merely by increasing the capital/labour ratio, when no other factors of increase were operating, it is likely that one would run into sharply diminishing returns, and very soon a nil return, from any further rise in the capital/labour ratio.

---

[1] *A Mathematical Theory of Saving*, "The Economic Journal", December 1928.

The factor making for growth in output per head may be described by the compendious expression "technical progress". This has two aspects. (1) Technical progress may consist in the application of new discoveries in science and technology. This will be importantly so in the most mature countries. (2) In developing countries there will normally be great fields for the application of knowledge that already exists somewhere in the world. What limits the rate of technical progress is the maximum possible rate at which cadres of qualified personnel can be increased. Such personnel must include those capable of giving already existing ideas in science and technology practical application. The cadres consist also of general managers, engineers, draftsmen, accountants, maintenance men, foremen etc. A higher rate of increase of these cadres can be secured by additional education and training, which is one form of investment. But educational and training facilities alone are not enough; it is necessary that there should, at the same time, come into being modern industrial equipment at the rate required to give employment and experience to the increasing cadres. For some of them the necessary training and the progressive raising of their quality can be achieved only by the actual experience of working on the job.

It may be noted that technical progress does not *necessarily* require any fresh investment at all. It may consist merely of a more efficient utilization of existing capital resources. Or capital equipment may be modernized without the use of fresh capital, merely by the substitution of improved forms of capital when the old is due for replacement. The new equipment would not necessarily cost more than the old. It must be allowed, however, that technical progress will more usually require additional capital.

In what follows I shall discuss, first, the choice among possible uses of fresh capital, and secondly, the question of what is the optimum amount of fresh capital in total to be brought into use.

I. Direction of investment. It seems that projects should be preferred where the capital/output ratio is lower. This seems unequivocal. The problem at once comes up of how the "output" is to be valued.

Fresh capital may be used merely to reduplicate existing equipment, e.g. so as to give employment to a population increase. This is sometimes called "widening". In this case there is no difficulty in valuing the output, since one could refer to the value of the output already being produced by similar equipment. As against this, the fresh capital may be used to introduce improved methods, or to produce goods and services not hitherto produced. In this case the valuation is more complicated.

(i) The prospective output should be assigned a value equal to the unit value, or better, the unit cost of production, of the goods in question *at the time when the investment (or decision to invest) is made*. It may be contemplated that the goods produced in consequence of the fresh investment will have a lower cost than the similar goods previously produced. But the right way to measure the output resulting from the fresh investment is to value it at the cost of production prevailing prior to that investment. Where the goods are not produced in the country at all, the import price may be taken.

The value of the output of capital invested in infrastructure presents a more difficult problem. In this case we may have to fall back on rough estimates, or even guesses; unhappily in economics we are unlikely to be able to escape from the need for rough estimates in many cases.

I will note at once that the recommendation often made to developing countries, where labour is abundant and capital scarce, that they should give preference to labour intensive projects, i.e. those with low capital/labour ratio, appears to be without foundation. There is, of course, no direct relation between the capital/output ratio and the capital/labour ratio. The recommendation is mistaken, anyhow, if it is desired to get the maximum uplift of production from a given sacrifice of present consumption.

II. To determine the capital/output ratio, output must be measured net. To start with, we must take "value added" i.e. subtract the value of materials etc. used. But we also have to subtract the value of the non-capital factors. These should be valued at "opportunity cost", i.e. at their values in other occupations, as ruling *at the time when the investment* (*or decision to invest*) *is made.*

An example of this may be given in global terms, taking the average capital/output ratio of the global incremental investment in a given year. Suppose that it is hoped by investment to raise national income from £1 million to £1.05 million and that, the capital/output ratio being 3 : 1, £15 million of capital is required. Suppose that the projects use labour etc. previously otherwise employed. If the value added by this labour was previously £0.5 million, then for the fresh investment of £15 million to add £0.5 million to national income, the specific projects embodying the new investment must show value added at £0.1 million.

If there is initial disguised unemployment, the true "opportunity cost" of labour might be zero. This point is often made, but there is danger of exaggeration. Those previously unemployed are not likely to be of so good a quality as those taken into employment on the new projects. There is likely to be a multilateral swopping of jobs.

The case where the opportunity cost of labour is truly zero is clearly the one most favourable to labour intensive projects. But even in this extreme case it does not follow that labour intensive projects are always to be preferred to capital intensive projects.

An example may be given. In the following table C stands for a unit of capital arbitrarily defined; x_1 and x_2 show the amount of value added by each project, y is the opportunity cost of labour.

TABLE 1

	Capital/Labour ratio	Capital/Output ratio
Capital Intensive Project	$C:1$	$C:x_1-1y$
Labour Intensive Project	$C:2$	$C:x_2-2y$

Next consider two countries. The two projects which each country has to compare are identical. It is assumed that the opportunity cost of labour in the capital abundant country is one and that in the labour abundant country zero.

TABLE 2

Capital/Output ratios

	Labour abundant country	Capital abundant country
C. I. Project	$C:x_1$	$C:x_1-1$
L. I. Project	$C:x_2$	$C:x_2-2$

In the labour abundant country nothing is subtracted from value added since the opportunity cost of labour is zero. It follows from this table that in the labour abundant country the labour intensive project has a lower capital/output ratio relatively to the capital intensive project than it has in the capital abundant country. It follows from this again that the labour abundant country is likely to be in a position in which it ought to choose more labour intensive projects than the capital abundant country. It does *not* follow that the labour abundant country ought normally to prefer labour intensive projects to capital intensive projects. It all depends on the capital/output ratios of the projects. Let us suppose that the value of x_1 in the above table happened to be 10 and that of x_2 to be 5; we get the following result.

TABLE 3

Capital/Output ratios

	L. A. Country	C. A. Country
C. I. Project	$C:10$	$C:9$
L. I. Project	$C:\ 5$	$C:3$

The question of foreign trade is not under consideration here. E.g. both projects might relate to non-tradeable goods (including infrastructure) or to goods the transport costs of which preclude international interchange.

It is evident that, if there has to be a choice between these two projects, *both* countries should choose the capital intensive project, even although in the labour abundant country labour is a free good.

The only possible modification to the rule that the project with the lowest capital output ratio is to be preferred is if giving extra employment is considered to be an end in itself, even at the sacrifice of national income.

It is doubtful whether it would ever be wise to entertain such an objective, viz. giving extra employment at the cost of a lower level of total consumption. There is a certain maximum increase of income, say ΔY, (income), if projects with the lowest capital output ratio are chosen. If there is to be continuing growth some part of ΔY has to go to ΔI (investment). What remains, namely Δ consumption, may be urgently required as an incentive for the co-operating factors. This is especially likely to be so in developing countries where the increases in the cadres of qualified personnel are by far the most important contributors to growth. It is impossible to translate unskilled labour into skilled labour, or less qualified into more highly qualified personnel without providing any incentive. The training of qualified personnel is a cumulative process. If the growth of these cadres can proceed at a higher rate than the growth of national income, the growth of the latter ought to begin to accelerate. It is accordingly very dangerous to sacrifice any of the increasing output that would otherwise be available for distribution, merely in order to mop up the disguised unemployment.

Furthermore the disguised unemployed may have certain compensations in their way of life for their very low income. It is surely unwise to disturb them, perhaps at some sacrifice to their happiness, until it is quite certain that they can be given a distinctly higher material standard of living. In regard to the disguised unemployment some patience may be necessary. The first task surely is to get the maximum possible increase of national income.

There may, on the other hand, sometimes be a case for preferring a project with a higher capital/output ratio, if this is one which is especially favourable to the training of qualified personnel, because that training is so important a factor in growth. We might even reach the paradox of preferring a more capital intensive project, where this had a rich output in the way of training personnel, even if its capital/output ratio was higher than those of some investment projects to which it was preferred.

(2) How much investment? In my *Second Essay*[2] and also in my *Presidential Address* to the Royal Economic Society[3] I supplied the following equation

$$r_n = \frac{pcG(\text{con})n^{4}}{e} \tag{1}$$

r_n is the welfare optimum rate of interest. As such it might be thought to be relevant only capitalist societies. I would claim, on the contrary, that this equation is absolutely fundamental to growth theory and applicable to socialist and

---

[2] "Economic Journal", June 1960.

[3] "Economic Journal", September 1963.

[4] This was given as *pcGn* in my *Second Essay*, but corrected to *pcG*(con)*n* in my *Presidential.*

capitalist economies alike. It can be re-stated without any reference to "interest" in thc capitalist sense. In capitalist value theory the actual rate of interest is, in equilibrium, equal to the marginal rate of return on investment. In the above equation r_n is not an actual rate of interest but a welfare optimum rate. The welfare optimum rate of interest is equal to the welfare optimum marginal rate of return on investment. We may call this the minimum acceptable rate of return on investment, assessing what is "acceptable" by the welfare optimum criterion. (The precise meaning of "rate of return" will be discussed shortly).

Thus the left hand tcrm (r_n) of the above equation may be interpreted as meaning "the minimum acceptable rate of return on investment".

I hope that the fundamental character of the equation is becoming apparent. What it is showing is that all investment projects should be undertaken, the prospective rate of return on which is not less than $\frac{pcG(\text{con})n}{e}$, and that no investment project should be undertaken the prospective rate of return on which is less than $\frac{pcG(\text{con})n}{e}$. In fine, it gives the criterion for whether investment projects should be undertaken or not. The equation may be called simply "the investment criterion".

pc Gn (from which *pcG*(con)*n* may be derived in a manner shown in my *Presidential*; see also below) is the "natural" or welfare optimum rate of growth of output *per caput*. I take this to be determined primarily by technical progress, assuming that all new knowledge is applied as soon as convenient. In developing countries technical progress depends almost entirely on the maximum feasible rate of increase of cadres of qualified personnel.

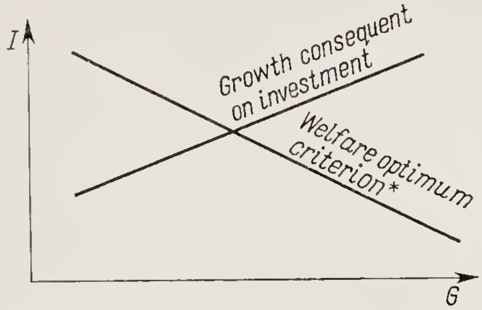
pcGn also assumes that all and no more than all investment projects warranted by the investment criterion are undertaken. If less are undertaken then presumably *pcG* (actual) will be less than *pcGn*. Thus I (investment) will be less than I_0 (optimum investment) on two counts: (1) I is less than I_0 supposing *pcGn* and (2) if more, not less, I is required to get the optimum.

If more investment is undertaken than required by the criterion, this will presumably push *pcG* above *pcGn*, but at the expense of the welfare optimum. The loss of utility from present consumption will not be compensated for by the increased consumption latcr due to higher growth. These relations my be shown by a simple diagram.

e in the investment criterion is the elasticity of the curve showing the marginal utility of income over the relevant range. The curve for the whole economy must be found by weighting individuals appropriately. u (Con) is the marginal utility of consumption.

$$e = \frac{\text{Con}_r - \text{Con}_1}{\text{Con}_1} X - \frac{(u\text{Con}_1)}{u(\text{Con}_r) - u(\text{Con}_1)}$$

The evaluation of e is of course a difficult matter. It is interesting to note that the independent investigations by Dr N. P. Barten[5] and Dr L. Johansen[6] give it values of 0.5 and 0.53 respectively. Further enquiries are needed. I confess that these values seem to me to be rather low, but their close similarity is striking.



* The higher the growth, the greater the rate of return there must be on a marginal investment, to justify it from a welfare optimum point of view; therefore the higher the growth, the lower *ceteris paribus* the welfare optimum amount of investment.

If we took e as $\frac{1}{2}$ and $pcG(\text{Con})n$ as 5% we should get a minimum acceptable rate of return of 10%.

The capital/output ratio may be thought of as a global statistical fact relating to the whole existing corpus of capital in the country. An approximate figure for this in a number of countries is said to be 3 : 1, giving a rate of return of 33,3%. If this global ratio remains constant through time, then the incremental capital/output ratio must be also 3 : 1. Then $I = 3(\Delta Y)$.

I may add, by way of digression, that I feel sure that in the course of development from a very low level the capital/output ratio rises. If during the course of such development the capital/output ratio rises from 2 : 1 to 3 : 1, then the incremental capital/output ratio during the course of this period must be higher than 3 : 1.

In an example previously given, it was suggest that the capital required to effectuate an increase of income of £0.5 million might be £15 million.

Two difficulties must now be faced.

1. The return on an investment is usually achieved over a number of years. It may be measured by a "present value" method; if, when future returns are discounted backwards at x% a year (compound), the present value is the greater than the value of the initial investment, then the rate of return is greater than x%. Or other methods of measurement may be used. Such methods imply a constancy of r over the term of years in which the investment is being productive. This restriction is involved by the equation. But there is no reason why the result given

[5] *Bayesian Estimated Consumer Demand Functions under Conditions of almost additive preferences*, Rotterdam, Economic Institute, 1962, p. 27.

[6] *Multi-sectoral Study of Economic Growth*, p. 107.

by the equation for the minimum acceptable rate of return (r_n) should not be modified, if changes in the growth rate in the future, or in ε, should seem probable.

2. In capitalist economics it is assumed that the value involved in any investment project that comes up for decision is small in relation to the total economy. (A more important consequence of this assumption is discussed below). If an investment plan (or projcct) is large, it will affect $u(\text{Con}_1)$ by a substantial amount. The right assumption here seems to be that investment decisions will in the following years reduce consumption and thereby elevate $u(\text{Con})$ by a proportionate amount. One could, on a rigid postulate, assume that national income would in future grow at the same rate as consumption; that would totally eliminate the problem discussed in this paragraph, since on that assumption $pcGn$ would be equal to $pcG(\text{con})n$. But one might, on the contrary, wish to assume that technical progress was continually biasing productive methods towards a higher capital/output ratio. One can be rough and ready only in these approximative evaluations. If the assessment is that the capital/output ratio will be rising—not an assumption to be taken as the one most usually applicable—then we must suppose that consumption will be rising at a somewhat slower pace than total output. The "investment criterion" should be modified to allow for this.

It is now necessary to consider how the "return" that is an ingredient of the "rate of return" should be measured, and the relation of the "rate of return" to the capital/output ratio, or, better, to the output/capital ratio. This is indeed the crux of the problem.

This might be true in an economy where the overall capital/output ratio remained constant at 3 : 1. But it might well be the case that output might rise somewhat even if there was no increase of capital at all. Even in the absence of fresh investment, technical progress might secure a better re-deployment of labour or greater efficiency; cadres of qualified personnel might be increasing. The forms taken by existing capital might be modernized on occasions of replacement without the total corpus of capital increasing. It might be that for these reasons income could rise by £0.25 million without fresh investment. Next let us suppose that the greater rise, viz. of £0.5 million, occurs, there bcing a net investment of £.15 million. In this case only £0.25 million is to be attributed to the fresh investment and the capital/output ratio on this fresh investmcnt will be 6 : 1 (rate of return 16.6%). All specific projects undertaken may have this high capital/output ratio, while the overall capital/output ratio of the whole corpus of capital in the country remains constant at 3 : 1. The reason why the higher capital/output ratio of the specific new projects does not gradually cause a rise in the average capital/output ratio of the whole corpus of capital is that output is all the while growing for other reasons.

The next point to be noted is the distinction between the average incremental capital/output ratio and the marginal incremental capital/output ratio. The average ratio may be, say, 6 : 1, while the marginal ratio, representing the minimum acceptable rate of return, is as high as 10 : 1.

We next come to a point of the utmost importance. Much of ΔY is distributed among the various factors of production, including the population increase.

It may now begin to become plain why I asked that, in computing the capital/output ratios, we should take the opportunity costs of other factors *at the moment that the investment (or the decision to invest) is made.* This includes the case where the opportunity cost of labour is zero owing to disguised unemployment. The net return to the agency looking after an investment project will be lower than the output (capital ratio, because its costs will rise, as, owing to general progress, what it has to pay to its factors rises. The output, in the sense used in the expression output/capital ratio, includes this, since the higher pay to factors of production is found out of the value of the additional output.

There is a further point of importance. It is likely that that part of ΔY which is dependent on I will be produced by a small fraction of the population only, viz. that part of it which is in direct contact with the new investment projects undertaken in a given year. This could conceivably be as little as 5% of the whole population. Then, if the average rise in income in the whole economy attribut able to fresh investment is $2\frac{1}{2}\%$, the output of this part of the labour force will go up by $20 \times 2\frac{1}{2}\% = 50\%$.

Agencies responsible for investment can dispose of the additional product in one more of 3 ways. 1. They can raise the wages of the factors they employ so as to absorb it. They would hardly allow the total increase of output to be so absorbed. 2. They can lower prices fully in proportion to increased output per caput (or per unit value of non-capital factors). In this case the whole of the economy shares in the benefits of the higher productivity. 3. They can maintain prices at the old level and retain for themselves the difference between costs as now reduced and prices (not reduced), so as to have a fund for further investment. If this last mentioned (3) is the sole treatment, the fund thus accruing must be used *at once* for further investment, whether by the agency itself or by transference to some other agency. If this is not done, unemployment will occur. I think that this holds of socialist, as well as capitalist, economies.

It is to be noted that if, under 1 above, the agencies raise wages, there may be sympathetic wage increases in sectors where there has been no increase in productivity (or not so much as the rise in wages). If the average overall increase in wages is more than 5% (on my numerical example), then there will be a cost inflation. If it is exactly 5%, inflation can be avoided, provided that the agencies responsible for increased productivity translate the whole of this into lower prices and hold nothing back for a capital fund. Otherwise the average rise of wages must be less than 5%.

The rate of return on a particular investment project that is relevant to the investment criterion is net output valued on the basis that the price or prices of output are frozen from the moment that the investment is made and that the wages etc. that it has to pay to factors are also frozen. The rate of return thus defined appears to me to be identical with the output/capital ratio of the investment project in question.

In a capitalist economy the rate of return, the forecasting of which influences investment decisions, is simply the expected flow of receipts from sales minus all expenses; in other words, the profit.

In general it appears to me that this expected capitalistic rate of return will be lower than the output/capital ratio. The capitalistic firm will emphatically not forecast its profit on the basis that wages will continue to be the same as at the time of the investment, if it believes that wages will be levered up by general forces in the economy. A very tightly monopolistic firm might push on any wage increases into prices, thus safeguarding its net receipts per unit; but the circumstances in which this would be most conducive to profit are very unlikely to be realized. Wages do in fact normally increase relatively to prices and in this way part of the ΔY due to I is distributed among the main mass of the population.

What justified the classical economists, stemming from Adam Smith, in regarding the rate of profit rather than the output/capital ratio as the right criterion for investment was their atomistic assumption that any one particular investment project was *so small* in relation to the total economy that in fact its rate of profit was equal to its output/capital ratio. There was an implicit assumption that neither wages nor prices could be at all affected by one particular investment project.

I have suggested that the criterion for an investment decision is that its rate of return or its output/capital ratio (r_n), *as measured above*, shall not be less than $\frac{pcG(\text{con})n}{e}$. This could be a matter for evaluation and action by a planning agency.

In a capitalist system the projected rate of return (profit) that actuates investment is, I submit, normally *less* than the output/capital ratio; and the criterion for decision is the relation of this projected rate of return to the current rate of interest. Accordingly it is expedient for the current rate of interest in capitalist economies to be as much below $\frac{pcG(\text{con})n}{e}$ as the projected profit rate is below the output/capital ratio as measured in this paper.

When I gave my *Presidential Address* I observed that, taking r_n as the optimum rate of interest, the equation

$$r_n = \frac{pcG(con)n}{e}$$

seemed to require a rate of interest much above the level that I felt in my bones to be right. I gave an explanation of the discrepancy, which may have some validity. I now add this further explanation of why the optimum rate of interest in a capitalist economy is likely to be *below* the r_n given in the equation.

It has been assumed in the foregoing that the amount of capital disposal entailed by the "investment criterion" can be forthcoming. This is normally the case. But, when, owing to a central plan for expedited development, there is an upward kink in the potential growth curve, it may not be possible to supply such capital disposal (i.e. productive resources to be devoted to investment projects) without a depression of the pre-existing level of consumption. I would urge that no such depression should normally be contemplated as an ingredient of a plan for optimum development; if there is a plan for an expedited rate of investment and growth, it should be assumed that an incentive, constituting some fraction of ΔY, should be provided for the increasing cadres of qualified personnel, with possibly something over for the main masses of labour.

Without these incentives the growth just will not occur, since, as stated earlier, an increase of investment, unmatched by growing cadres of qualified personnel, is likely soon to run into the realm of zero net return. Developing countries, if and when actuated by a new plan for expedited growth, may have to forego some investment projects that are justified by the "investment criterion". If an expedited growth is achieved, the amount of capital disposal available, after the minimum allowance required, out of the increased output, for incentives to the increasing cadres of qualified personnel etc., should grow and approach in due course the amount sufficient to implement all investments required by the "investment criterion".

What I have offered in this article, as a contribution to dynamic economics, does not, alas, give complete solutions. M. Kalecki has made seminal contributions to this branch of economics. Although a festschrift is now being compiled, I have confidence that he will still continue to make contributions in this field, which is so important to the thousands of millions of men and women who do not have enough income.

L. R. Klein
U.S.A.

THE ROLE OF ECONOMETRICS IN SOCIALIST ECONOMICS

1. INTRODUCTION

The Econometric Society has been an international body since its founding in 1930, and the society functions today on an international plane. Our subject is truly international and transcends political or national boundaries. It is true that much of econometric work has been centered in the United States and Western Europe, but the predominant position of these areas is changing, and surely developments of the future will come in good measure from other parts of the world and will be relevant to the framework of other kinds of economic systems, especially socialist systems. As it is some of the most celebrated contributions in econometrics emanated from Eastern Europe and the Soviet Union. Although the quantity was limited, the quality was high. The works of Kalecki and Lange, together with those of Slutsky, Konius, and others have stood the test of time, a rigorous test in scholarly work, and we have hopes of seeing a great flow from a new generation of Eastern econometricians as a result of renewed interest in our subject.

Of what does econometrics consist? It is the statistical description of a mathematical formulation of economic behavior. Economic behavior of households, production units, and government bureaus goes on whether we are dealing with capitalist or socialist systems. The analysis of this kind of behavior has proved to be amenable to econometric treatment in the capitalist environment, and there is no reason why it is not equally amenable to such treatment in the socialist environment. To a large extent we have dealt with economic decisions under the price or market system in econometric analysis of the capitalist economies, but in the socialist economies decisions are interrelated and connected by dynamic leads, lags, or accumulations and should be subjected to the same kind of treatment. Econometric tools are powerful, giving subtle insights into human economic behavior if skillfully applied, and there is no reason why this application should not now be made in the case of socialist behavior. The essence of the matter is the unraveling and understanding of the chains of interrelatedness among economic decisions. This is a problem for application of mathematical statistical methods regardless of the environment. Econometrics is not necessarily geared to the price system, the market economy, the maximization of private profits or any of the other peculiar characteristics of the capitalist system. It is merely the quantification of human behavior of a type that is fundamental in any environment.

It is true that much of econometric work has been linked to the characteristics of the capitalist economies, but if it were not so these methods would have failed in their analysis of capitalist economic behavior. Even within the capitalist group of nations a great deal of econometric analysis varies with the environment. In the field of macroeconomic model construction, crude imitative attempts to construct models for one country that are exact duplicates of models previously constructed for other countries have not been successful. Better models result when they are properly adapted to show the institutional environment of the economy they are trying to describe. Also, within a country, different periods with more or less of a market economy (war and peace, e.g.) must be differently treated in an econometric model. The essential thing is to capture the main economic problems and processes of the environment being studied. The methods, and not purely the form, of econometrics that have fruitfully been applied to capitalist problems are now in need of transplantation to socialist problems. If our methods work well in such diverse places as the United States, Japan, and India, it would be strange if they did not also work well, upon proper adaptation, in Poland and other socialist economies.

2. ECONOMETRICS APPLIED TO ECONOMIC PROBLEMS OF A SOCIALIST ECONOMY

Econometrics is developing along two main routes at this time. A more traditional type is based on the inferential methods of mathematical statistics. These studies include such things as demand analysis, estimation of cost or production functions, graduation of income distributions, time series analysis, index construction, macroeconomic model building, and applications to problems of forecasting or public policy formation. The main characteristic of these studies is that they are based on statistical samples of data from non-experimental observations and attempt to make inferences about population parameters from these samples. They are essentially applications of the standard methods of mathematical statistics to hypotheses suggested by economic analysis. The main problem in applying standard methods of mathematical statistics arise because the sample data are non-experimental and often few in number.

The other main route of development in econometrics is in linear programming, input–output analysis, and similar planning models. It is not impossible that this kind of analysis could also be based on samples and make use of the methods of statistical inference, but there are good reasons why this is implausible. A distinguishing feature of programming and input–output analysis is that finely detailed sectors and processes are treated. In order to accomodate a fine degree of detail, simple linear relationships are posited and coefficients are estimated from single observations or controlled variables. Simplicity in parametric structure and in estimation of unknown coefficients is necessary because there are so many sectors and interrelationships to be considered. Other computation problems besides those associated with parameter estimation are formidable here; therefore the problems of statistical inference tend to be overlooked or set aside for the present.

Professor Lange's book (*Introduction to Econometrics*) is roughly divided along these lines. The first half covers time series, demand, and income distribution analysis, while the second half is devoted to programming methods. Programming and input–output methods have recently been applied and developed on a large scale in socialist economies; therefore, I shall concentrate now in trying to amplify the possible uses of these methods in Socialist economics. It is the main purpose of this paper to try to explore the possibilities for application of the other methods of econometrics to economic problems of socialism.

Demand Analysis: First, let us consider demand analysis. The problem here is to study the relationship between consumer demand for individual goods, on the one hand, and relative prices, consumer incomes, and similar explanatory variables on the other. In capitalist market economies, consumer purchasing decisions are largely left to individual choice, and it is a challenging problem to analyze the factors responsible for such choices and indeed to predict them—whether for private gain or for public policy formation. What is the corresponding demand problem for socialism?

In a socialist economy based on free consumer decisions; where choice may be made for the composition of expenditures, or the total of expenditures, or both; there must be forecasts of future consumer demand. Given the purchasing power that will be distributed to wage earners and the prices of consumer goods, the flow of goods that is being currently produced must be matched against consumer demand for this flow when the goods reach final markets. Econometric estimates of demand functions, as mathematical equations associating quantities demanded with incomes, prices, and other variables will be indispensable tools for making intelligent judgments about the magnitude of consumer demand. The cost of not making such an analysis will be to have excessive inventory build-up or queuing. Alternatively, if consumer markets are not left with a large measure of freedom of choice, there is a cost in planning the population's consumption of individual commodities. It seems to me that it would be more efficient to allow consumer choice to a large extent, predict the magnitude of this choice using data on market prices and incomes together with econometric methods, and adjust production plans accordingly. This does not mean that people will get what they want, for they may very well desire more than available resources permit, but it would be extremely useful for production planners to know the magnitude of the unsatisfied demands that will exist and the overall inflationary pressure. Armed with the econometric tools, it will also be possible to make *conditional* predictions of what would happen to demand if incomes and prices were changed—changed in such a way as to make overall plans more feasible in terms of available resources. In this way, demand analysis, which has become an important tool in capitalist economies, whether used by public authorities or private business, can be made to serve socialist production plans.

The overall level of demand is represented in macroeconomic models by aggregate savings or consumption functions. These two are part of demand analysis, and they can be derived as the summation, over commodities, of individual demand functions. Even though a socialist economic plan will often allocate the amount of goods to be left for personal consumption, it is still valuable to have econometric estimates of the aggregative consumption function, for the plan may be close to or far from what consumers would select if left to their own choices. It is important to know how much inflationary pressure a given production—investment plan is going to generate. Then counter measures can be more efficiently devised. It is for these reasons that I feel that demand analysis is just as significant for the socialist as for the capitalist market economy.

Production and Cost Analysis: Input–output analysis shows the flows of intermediate goods through the various sectors of the economy and the way in which they are transformed into final production. In a sense, these are a set of highly inter-related production functions. Production analysis, however, as it has developed in econometrics is a more detailed study of the processes by which original and derived factors of production convert their powers into a flow of output. This could be either at a global level for the economy as a whole or for individual production units. I have estimated production functions for individual railway systems and have seen studies for individual farms, electric power plants, and other sectors of the economy. These studies have revealed important characteristics of labor productivity, economy in the use of fuels or materials (intermediate inputs), and the course of technological progress over time. These are technical or engineering relationships and exist in any kind of economy as laws of nature, quite apart from the form of market organization. It is true that hypotheses of cost minimization or profit maximization according to bourgeois tenets of neoclassical economics have been used to assist in parameter estimation for these studies, but these assists were really conveniences and not always essential to the estimation process. In any event, the implications of cost minimization or some other optimization objective would seem to be justified for the description of behavior patterns of plant managers under socialism, and estimation procedures that are analogous to those used in capitalist studies would seem to be quite applicable to the case of socialist production.

Cost function are equally valuable. They might be used to assist in cost accounting, for inferential statistical methods help to allocate joint costs that are not ordinarily separable by standard accounting procedures. They can be used for setting of cost standards. They can also be used to estimate capacity outputs, where capacity is understood to be constrained by economic considerations; i.e. capacity output is defined within the framework of associated economic costs and not as a pure physical engineering concept. Cost functions can also be estimated from individual plant data as well as from industry or economy wide totals.

Macro Models: Demand, production or cost analysis can be developed, econometrically, for individual commodities or individual plants, as well as for broad aggregates covering the economy as a whole. Indeed, these studies are usually understood as part of microeconomics. Their relevance to problems of socialism should be evident, as argued above.

In capitalist countries we originally estimated macroeconomic models for the purpose of studying business cycles. This was a major contribution of Kalecki's original macro model, which became a fountainhead of later development in this line of research. But if business cycles are eliminated from the socialist economy, do socialist economists have use for econometric business cycle research in the form of macroeconomic model building? I shall argue that there are other uses for macro models besides pure business-cycle analysis, and these uses make such models of great value for socialist planning. I shall also argue that while the capitalist business cycle is not present in the socialist economy, there are dynamic movements (or possibly fluctuations) that can and should be studied from such models.

Aggregative models are extremely valuable for the study of economic growth. They show how broad decisions about the division between consumption and capital formation, between the personal and public sectors, or between the foreign and domestic sectors will affect the rate of economic growth. Such models help to answer questions about the degree of "returns to scale" at different stages of development in the life of an economy. They may be useful, in developing countries, in showing the extent to which fast growth rates in the early stages, when there is so much scope for expansion, can be maintained and for how long[1]. If such models are programmed to *simulate* growth of an economy with an assumed time path of external factors, either as systematic variables or as random shocks, they can be used to predict the future growth of a planned economy under alternative policies. Such growth models are aggregative and deal with global variables.

They do not show the fine industrial detail of output divided, say, into several hundred sectors but they can feasibly be constructed to show growth paths in a few sectors. Beyond the ultimate aggregation into such totals as national prouct or an index of industrial output, growth models can be constructed to display the following outline of variables:

1. Production and employment
   a. Agriculture
   b. Manufacturing
   c. Distribution, trade, and other services

[1] L. R. Klein, *A Model of Japanese Economic Growth, 1878–1937*, "Econometrica", Vol. 29 (July 1961), 277–92.

2. Expenditure
   a. Consumption
   b. Fixed capital formation
   c. Stock changes
   d. Public administration
3. Foreign trade
   a. Exports
   b. Imports
   c. Shipping, tourism, and other services
   d. Capital flows
4. Prices
   a. Prices of goods and services by production sector
   b. Prices of goods and services by consumption sector
   c. Wage rate
5. Miscellaneous
   a. Money
   b. Surpluses
   c. Population
   d. Taxes[2]

This is not a unique classification but indicative of the kind of detail that could readily be shown in an aggregative growth model. Systems of 30–40 equations could probably explain movements in these variables. Through consolidation and aggregation, explanation could be obtained in systems of approximately 10 equations, and possibly more detail could be included by going to systems of 50 or more equations. I am, however, speaking of an order of magnitude considerably below that of large input–output systems.

As was mentioned already, the original development of macro models in econometrics was for the study of capitalist business cycles[3]. The cycle mechanism of capitalism will not carry over to the socialist economy, but that does not mean that analogous dynamic systems have no place in the study of a socialist economy. Such an economy does not stand still. It is an interesting and useful study to predict where it will be at any point of time in the near future under alternative assumptions. In fact, much of the emphasis in the use of dynamic macro models in capitalist economies has shifted from business cycle analysis to prediction of alternative positions of the system under varying short-run conditions. One of the main sources of movement in such systems in the post-war capitalist economy has been inventory fluctuations. These are short-cycles that might be distinguished from the traditional capitalist business cycle mechanism. It seems to me that so-

[2] Taxes are here understood to play quite different roles in socialist and capitalist economies; nevertheless they define important money flows that are relevant to the understanding of either kind of economy.

[3] J. Tinbergen, *Statistical Testing of Business Cycle Theories, Vol. II, Business Cycles in the United States of America, 1919–1932*, (Geneva, League of Nations, 1939).

cialist economies will also have inventory fluctuations. Plans are not always perfect. Random errors may give rise to these short fluctuations or temporary pauses in the path of growth and development. In fact, the most celebrated simulation study of a capitalist business cycle model has shown that the mechanism responsible for the generation of short cycles (approximately four years' periodicity) has been the *propagation* of random shocks through a dynamic model of the U.S.A.[4].

The effect of *propagated* shocks on the dynamics of an economy were brought to the attention of economists by Slutsky and Frisch[5]. There are numerous sources of shocks in a socialist economy. There may be years of exceptionally good or exceptionally poor harvests. Foreign trade may swing widely as a result of events far beyond plan control, halting the flow of strategic materials or capital goods. Such swings may cause shifts and realignments in domestic production. Or domestic planners may make mistakes. They may supply too much of one kind of good or too little of another. Inventory fluctuations cannot be avoided in the best of circumstances. There will be degrees of inflationary or deflationary pressure. With all these kinds of disturbances piling up on even the most efficiently managed socialist economy, it would seem useful to have an econometric model of the lead-lag interrelationships among the various sectors of the economy on the sides of production and consumption. Whatever the resulting fluctuations or movements are called, it is important to have econometric methods for proper understanding of them.

Problems of Prediction: Some positive applications of econometric investigations in a socialist economy would be in the prediction of overseas activity in the capitalist economies, predictions of trade flows, predictions of the degree of inflationary pressure, predictions of future growth rates, or predictions of specific lines of demand.

Macro model construction, demand analysis, and production analysis have been established as accepted means of investigation in Western Europe, North America, India, Japan, and other places of the capitalist world. Passive description of the market economy should be accepted without debate as a field for econometric analysis. It has proved extremely valuable in forecasting aggregate activity, in predicting agricultural demand, and in many other applications. Since the socialist countries are extremely anxious to predict capitalist developments it would seem logical to develop econometric methods for this job. Socialist economists would gain better insight into the functioning of the market economy if they were to employ econometric methods of inference from non-experimental observational samples according to principles of mathematical statistics. There have

---

[4] I. Adelman and F. Adelman, *The Dynamic Properties of the Klein–Goldberger Model*, "Econometrica", Vol. 27, (Oct., 1959), 596–625.

[5] E. Slutsky, *The Summation of Random Causes as the Source of Cyclic Processes*, "Econometrica", Vol. 5 (April 1937), 105–46. R. Frisch, *Propagation Problems and Impulse Problems in Dynamic Economics, Economic Essays in Honor of Gustav Cassel*, London 1933.

been many ill founded predictions of capitalist aggregate economic activity in the socialist countries, and much of this can be traced to faulty analysis that would be rectified by turning to the more powerful techniques of econometrics.

Where trade agreements are negotiated bilaterally, there may be little scope for statistical inference to reveal more about underlying behavior patterns. But a segment of international trade in socialist countries will follow the world market economy. Here the proper approach would be to estimate propensities to export and import for the socialist nations and their trading partners.

In the domestic economies of socialist countries, if input–output schemes are to be used, there is the remaining matter of determining the bill of final demand. Some of these cannot be fixed by planning authorities and will have to be estimated from family budget expenditure functions or time series demand functions. The aggregation of these estimated functions will determine the equation for overall final demand, and this will enable planners to determine the force of inflationary pressure. The more precisely one knows this pressure in advance, the more intelligently and efficiently can the planners implement their programs.

Estimates of behavioral and technological relationships from non-experimental data require samples of moderate size, say 20 or more annual observations or 10 years or more of quarterly and monthly observation. In many newly developing countries there is not an adequate historical record from a homogeneous environment. In such cases, much progress can be made through the use of microeconomic data contained in cross-section samples of households (family budget) or producing plants. It would be possible to estimate most or perhaps the whole of a macro model from cross section samples if properly treated by modern econometric methods.

3. A RECONSIDERATION OF THE KALECKI MACRO MODEL

In macroeconomic model building there are two distinct branches of the subject. One deals with matters of principle and shows the logical mathematical structure of a self contained system that brings out the essence of economic dynamics, by focusing attention on very specific aspects of behavior. From very few main principles of behavior, systems that are not wholly realistic or practical are built to exhibit movement like that of the actual dynamic economy. These are *pedagogical* models that are intended to emphasize some main points of economic dynamics. This exercise belongs largely to *mathematical economics*.

In the other branch of the subject much larger, more detailed, and more cumbersome models are prepared that are designed to be fitted to data of the economy and trace out actual movements of economic variables over time. These are part of econometrics. They are pieced together by combining several *a priori* hypotheses from the pedagogical models of mathematical economics and adding equations to bring out the institutional nature of the economy being studied—its tax laws, banking practices, degree of market imperfection, dependence on external trading relationships, exchange controls, etc.

The econometric models that I have constructed as practical tools for analyzing or predicting the economies of the United States, Canada, United Kingdom, and Japan have been based on combinations from the theoretical models of Marx, Kalecki, Keynes, Lange, Hicks, Kaldor, Metzler, Goodwin, and others[6]. It is fitting at this time to re-examine the position of the Kalecki model in this hierarchy and its relevance to model building of the present day.

The Marxian schemes of reproduction and accumulation and the Keynesian models of effective demand are the forerunners of the present theoretical model building. It is often not adequately appreciated how the Kalecki model, constructed in the Marxian spirit, actually pre-shadowed all the essential ingredients of the Keynesian system that have made the latter system so popular among the present generation of Western economists. It is usually thought that the recent rapid development of macroeconomic model building in the econometric branch of the subject is an outgrowth of the neo Keynesian development. Actually most models in existence today could be decomposed into ideas first found in the models of Kalecki, Kaldor, Metzler, and Goodwin. The latter three could have been developed as natural extensions of the Kalecki theory. The mathematical interpretations of Keynes by Lange and Hicks undoubtedly reinforced the development and certainly enhanced it, but the basic ingredients of the Keynesian development were already available in Kalecki's model.

Although the aggregative econometric models now being used in practical work look fairly complicated they can actually be seen to consist of the following basic endogenous components[7]:

1. Aggregate demand—consumption and capital goods.
2. Stock adjustment processes—celled flexible accelerators.
3. Production adjustment.
4. Theory of interest wage, and price formation.
5. Shifts between sectors of the economy.

---

[6] K. Marx, *Capital*, Vol. II, (Chicago 1909). M. Kalecki, *A Macrodynamic Theory of Business Cycles*, "Econometrica", Vol. 3 (July 1935), 327–44 [This is the best mathematical statement, but the model was available in published form elsewhere at an earlier date]. J. M. Keynes, *The General Theory of Employment, Interest and Money*, (London 1936). O. Lange, *The Rate of Interest and the Optimum Propensity to Consume*, "Economica" Vol. V, N.S. (Feb. 1938), 12–32. J. R. Hicks, *Mr. Keynes and the Classics: A Suggested Interpretation*, "Econometrica" Vol. 5 (April 1937), 147–59. N. Kaldor, *A Model of the Trade Cycle*, "Economic Journal", Vol. L (March 1940) 78–92, L. Metzler, *The Nature and Stability of Inventory Cycles*, "The Review of Economic Statistics". Vol. XXIII (Aug. 1941), 113–29. R. M. Goodwin, *The Nonlinear Accelerator and the Persistence of Business Cycles*, "Econometrica", Vol. 19, (January 1951), 1–17.

[7] All the systems contain important exogenous effects such as exports, population trends, technical improvement, and growth of the public sector. These can readily be added to any theoretical model.

Let us now examine the Kalecki model from this point of view, which attributes modern development to a narrow chain of ideas. The model is:

$$B = C + A \tag{1}$$

$$C = C_1 + \lambda B \tag{2}$$

$$I/K = \alpha B/K + \beta \tag{3}$$

$$L(t) = I(t - \Theta) \tag{4}$$

$$A(t) = \frac{1}{\Theta} \int_{t-\Theta}^{t} I(\tau) d\tau \tag{5}$$

$$\frac{dk(t)}{dt} = L(t) - V(t) \tag{6}$$

B = gross profits accruing to the capitalist sector
C = consumption by capitalists
A = saving by capitalists
I = orders of capital goods
L = deliveries of capital goods
K = stock of capital
V = depreciation

If V is a known function of time, this forms a closed system of 6 equations in 6 variables.

Equation (2) is a familiar linear consumption function. Like Marx, Kalecki assumes that workers have no savings, consume all their income, and that all savings are done by capitalists. The principal results would not be greatly changed if this consumption function were to be modified by allowing wage earners to make a small contribution (at the margin) to saving. In fact, we have excellent modern econometric evidence from family budgets that show the marginal savings coefficient out of wage income to be greater than zero.

In addition to allowing wage earner savings, equation (2) is modified in econometric models today by introducing distributed lag effects and institutional tax-transfer variables. These are the usual steps that are taken to adapt a theoretical model for use in applied econometrics.

The investment function (3) is a close foreshadowing of many modern developments in econometrics. In general terms, it makes investment orders a function of capitalist income and the stock of capital, though it combines all these variables in ratio forms. It is an open question under debate among econometricians whether investment should depend on output or on non-wage income. Plausible arguments can be made for both sides, but empirically a decisive choice cannot be made. In this sens, Kalecki's investment function is one that is still in use and not refuted by data available today. It is also very popular to include the stock of capital in the investment function. Although Kalecki, and later Kaldor, did not argue explicitly as many modern theorists do, their investment equations are, in fact, expressions for the flexible accelerator, written as:

$$I = \alpha_1 Q + \alpha_2 K,$$

where Q = aggregate real output. This is the Goodwin function, except for the nonlinearities at the capacity ceiling or depreciation floor.

Together equations (2) and (3) and identities make up the total of effective demand if we consider a closed economy without government demand. Therefore Kalecki's model covers points 1 and 2 above, as basic endogenous components of modern econometric models. The relationship between orders and deliveries of capital goods is also remarkable in anticipating a later development. Today, much work on capital formation in econometric models is based on data for housing starts, capital appropriations, and orders for capital goods. These are then averaged over time to be "phased in" to actual effective demand components of aggregate output (equations (4) and (5)). This is precisely what Kalecki did in his model thirty years ago.

The other equations of the model are definitions that find a place in practically all modern systems. At this stage of Kalecki's work, there is not an explicit treatment of prices, wages, or interest rates in his system. He did, however, introduce very soon after publication of this model a theory of interest, in which velocity of circulation is made to depend on the rate of interest. This is an alternative way of looking at the Keynesian theory of liquidity preference. Both theories are alike in assuming that velocity is not a constant and is affected by interest rate movements[8].

The theory of production adjustment and inventory behavior came later in the writings of other authors. While it should not be said that all the basic ingredients of modern econometric systems stemmed from Kalecki's model, it can be said that all the components of Kalecki's model are finding their way into strategic places in modern econometric models. His theories of the early 1930's are seen to be intellectual *tours de force* in the light of modern developments.

---

[8] Cf. J. N. Behrman, *The Short-term Interest Rate and the Velocity of Circulation*, "Econometrica", Vol. 16 (April 1948), 185–90.

Zygmunt Knyziak
Poland

THE EFFICIENCY OF INTERNATIONAL SPECIALIZATION IN PRODUCTION AMONG SOCIALIST COUNTRIES

(*Economic criteria and the method of analisis of efficiency*)

One of the methods of developing the international division of labour among socialist countries is specialization in production achieved among particular countries. It may appear in the form of inter-sector or within-sector specialization. The subject of our considerations is specialization within-sector and only those aspects of inter-sector specialization which stem from certain relationships and elements common to both these forms of specialization.

Within-sector specialization on an international scale, similarly as co-operation in production within one country, is warranted by the economic superiority of large-scale over small-scale production, the superiority of long production runs over short runs. Thus its development is confined mainly to manufacturing. The range of goods produced increases in consequence of growing and more varied needs, but production runs are relatively short due to the limited domestic demand. This makes it impossible to develop large-scale production and consequently to lower the cost of production and to raise the productivity of labour. It is not always possible to expand export considerably with the help of classical forms of expansion on foreign markets and thus to secure the desired amount of foreign currencies for import requirements. Economic co-operation among sccialist countries create the possibilities of planning for increased foreign trade which should result in longer production runs and lead in consequence to the development of within-sector specialization in production among different countries. By narrowing the range of goods produced by a given industry or sector and by making production runs longer for a smaller number of products each of the countries participating in this kind of specialization obtains economic advantages consisting in a general reduction in social labour outlays necessary for satisfying the demand for a given amount of goods needed for investment and consumption purposes.

For the determination of rational and, at the same time, realistic criteria for the appraisal of these advantages it is necessary to take into account the specific conditions of the co-operation among the socialist countries. The major observations that could be made in this respect are as follows:

1. The co-operating socialist countries have different levels of the productivity of labour in their corresponding manufacturing industries. These differences are, as a rule, greater in older industrial establishment and smaller in newly designed ones. There may be cases, of course, in which in a given sector or industry there are no differences between labour productivities in particular countries. This applies particularly to some sectors of the machine industry developed in recent times in all socialist countries at a very rapid rate and on the basis of similar levels of technique. Economic advantages from specialization will depend upon whether or not in a given sector of production included in the process of specialization there are differences in the levels of the productivity of labour among the countries concerned.

2. There is no perfect mobility of labour and investment means among the co-operating countries. This causes that the equalization of the levels of the productivity of social labour among these countries must be achieved on the basis of their own domestic resources. The sovereign nature of the national economy of each country necessitates the preservation of balanced foreign trade and payments.

3. The exchange of the products included in specialization among the countries concerned is based on world prices which are uniform and independent of differences in labour outlays needed for the production of a given commodity in particular countries[1].

A simplified diagram of within-sector specialization corresponding to the above conditions will look as follows:

— let us assume that in the three countries A, B and C there is the same level of the productivity of labour in a given sector and that each of them produces 15 commodities in the same amount of 20 units. Since world prices are the same for particular commodities then each country will obtain, say, 150 units of international currency for its products. The cost of production measured by the length of working time is also the same and amounts to 40 thousand hours in each country;

— let us assume that no investment outlays are required to change the range of products;

— let us assume that as a result of specialization each country limits the range to $\frac{1}{3}$ of the number of commodities and increases the production of each of the retained commodities threefold. Thus each country will produce 5 commodities, each in the amount of 60 units. Since we have already assumed that world prices, regardles of the product, are the same, then the value of production expressed in the international currency will not change and will amount to 150 units in each country. Each country will now exchange its products to the value of 100 international monetary units, in order to satisfy by imports the domestic demand for the goods that it has ceased to produce.

[1] Studies have been embarked upon by CMEA to develop a common cost basis for unit prices used in trade among the CMEA countries in accordance with the conditions of production and exchange prevailing in the socialist countries.

— let us assume that the threefold increase in the production of each commodity was accompanied by a rise in the productivity of labour by 20 per cent which will result in the lowering of the cost of production in each country by 8 thousand hours.

As a result, the advantage accrued to each country in consequence of specialization was a lowering of the social labour outlays needed for meeting the domestic demand of the same magnitude and structure. The advantage has accrued as a result of enlarging the market for particular products.

In our example the advantages were the same in each country and were proportional to the volume of foreign trade turnover that has resulted from specialization. In actual practice the advantages achieved by specialization in production vary, as a rule, from country to country. This is due to the fact that:

— the level of the productivity of labour in a given industry (branch) may be different in particular countries;

— the sensitivity of production costs to production increases may vary. In those products in which the share of raw and other materials is large, even a substantial increase in production may only slightly lower the cost of production. As a rule, the costs expressing the outlays of living labour are more sensitive to increases. Thus the cost structure in production affected by specialization will influence a rise in the productivity of labour;

— an increase in production will be due, as a rule, to new investments. The size of investment outlays and consequently the technical level of investment projects may be different in particular countries. It will depend upon the human and capital resources of the country concerned. In effect, countries possessing more capital will be able to apply better technique, and vice versa. This does not necessarily predetermine the volume of savings in the cost of production thus achieved, because these savings depend also upon the level of real wages in the country. However, the impact of the level of technique on new investments may cause differences in the magnitude of advantages achieved in particular countries.

Let us now consider a situation in which, in consequence of the factors described above, the unit costs of production without specialization are different and are the lowest in country *A*. In this situation, if we wanted to achieve the absolutely lowest costs of production for all three countries we would have to produce all commodities in one country. The remaining countries *B* and *C* could satisfy their demand by importing these commodities from country *A*. To balance their foreign trade countries *B* and *C* would have to export those commodities which they could produce more efficiently than country *A*. It could be surmised, with a high degree of probability, that under these conditions country *A* would turn out to be more economically developed than countries *B* and *C*. In this case their export potential could conceivably be in agriculture or in the extracting industry providing that they were endowed with the resources of raw materials that would be easy to exploit. It is easy to show that this would lead to a further increase in the economic inequalities among these countries because the export of raw materials and agricultural products is generally less profitable than the export of manufactured goods.

Of course, not every inter-sector division of production among economically sovereign countries leads to the widening of economic gap. The effect may be positive and may even lead to a greater equality in the economic levels of the countries concerned if, under the conditions of different levels of productivity among different countries, we are guided not by the criterion of the absolute minimum of labour outlays in all countries, but by the criterion of a relative minimum. Then each country has an equal chance of participating in inter-sector specialization and in reaping the advantages thus accrued. The second condition is the determination of type of specialization and of its scope from the point of view of balancing foreign trade. With this approach the distinction between within-sector and inter-sector specialization becomes, really, a formality. The differences appearing in this case are those of scale rather than of the nature and essence of the advantages achieved by specialization in particular countries[2]. The smaller the differences in the level of the productivity of labour among the countries concerned, the smaller the part played by these differences.

In the above example we could have proved just as easily the rationality of inter-sector specialization assuming that the division of production applies not to the products of the same industry (sector) but to the products of different sectors. As a result, the countries involved would reap the advantages of larged markets for this products. The greater the differences in lowering production costs by increasing production and the greater the differences in the material and value structures of costs, the greater would be the differences in the advantages accrued.

The idea of co-operation and economic assistance could, in this situation, be expanded by the acceptance of the rule of dividing joint advantages resultant from specialization among all the participants in proportion to their participation in specialization. In other words, the advantages accruing to each country in proportion to increased trade resulting from specialization should be on the same level. The equalization of the rate of gain among the countries concerned could be attained by applying appropriate equalization subsidies in international clearing, or in some other way.

It follows from the above considerations that when capital and human resources are in short supply the rational criterion of within-sector specialization among particular countries is the criterion of relative and not of absolute advantages. This corresponds to viewing specialization from the standpoint of the interests of particular countries. It is in the interest of all countries to maximize joint advantages. The criterion of an absolute increase in advantages can be fully applied only

[2] This applies to manufacturing. The international devision of labour between manufacturing and the extracting (raw materials) industry is a more complex problem, outside the scope of this paper.

when the economies of particular countries constitute one economic entity in which capital and human resources can circulate freely[3].

Let us now deal with the problem of determining the advantages of within-sector specialization. The magnitude of advantages for one country depends upon the difference between the cost of production before and after specialization. The calculation is based on the hypothesis that as production increases labour outlays per unit of product decline. This hypothesis is not always true in practice. In most cases production increases depend upon the introduction of more efficient technique which raises the productivity of labour and lowers the cost of production.

An analysis of the efficiency of specialization cannot be limited to studying the current costs (K) of an enterprise. Specialization should by analysed from the point of view of the criterion of general economic efficiency. And then efficiency depends not only upon the effect of a decrease in cost as production increases but also upon its relationship to the size of investment outlays required for introducing new technique which raised the productivity of labour. In order to take into account the general economic criterion we have to add the investment factor $\frac{1}{T}J$ to the cost of production factor in our calculations $\left(E=\frac{1}{T}J+K\right)$. J denotes the size of the investment outlays necessary for increasing production and T is a limiting period of the profitability of investment outlays for more efficient technique $\left(\text{thus } \frac{1}{T} \text{ is a normative coefficient od efficiency}\right)$. The extent to which the investment factor in the calculation of efficiency affects the result depends upon changes in the capital intensity of production and the limiting period of profitability—T.

T is the same for the whole country. There are differences in T in particular countries. Where capital resources are relatively high and manpower low the marginal period of profitability is greater, and *vice versa*. Thus T determines the level of investment acceptable from the point of view of the internal economic conditions of the country and of the internal criterion of the efficiency of investments.

The effect of increasing production on the cost of production and on the capital intensity of production together with quantity T affect decisively the efficiency of specialization and the optimal level of specialized production (e.g. production runs) determined by the point in which $E=\frac{1}{T}J+K=\min.$

---

[3] This criterion can be applied at present to the co-operation among the socialist countries with respect to the extracting industries. If the output of a raw material is most efficient in country A then an increase in production in country A to meet the requirements of other countries may take place providing that assistance is obtained from other countries for investment and perhaps even operating purposes. Investment assistance for the manufacturing industries is not warranted because in this field the chances of efficient production are the same in all countries in the near or more distant future. The only permanent limiting condition may turn out to be the cost of transportation.

It is not always possible to achieve the optimal volume of the production (run) of a given commodity in the course of the co-ordination of production among the countries concerned. This is due to the simple reason that the scale of production does not depend upon one country, but it depends upon all countries participating in specialization and their demand for a given product may vary. And secondly, it is in the interest of all the countries involved to maximize joint advantages (i.e. the sum of the advantages accrued to particular countries) and not to maximize the advantages for each country separately.

Within-sector specialization leads to matched sales in foreign trade. The increased production of certain commodities with the elimination of others produced before is accompanied by increased exports and imports. As mentioned above, the rule is that payments resulting from increased foreign trade are balanced. Therefore, the notion of the programme of specialization in production for the whole industry or branch should be used and the advantages due to specialization should be calculated with respect to this programme.

Let us assume that in consequence of specialization agreements country A has the following production programme before und after specialization:

Production Programme

Product	Before specialization					After specialization				
	Quantity	Unit cost	Total cost	$J\cdot\frac{1}{T}$	Total $J\frac{1}{T}+K$	Quantity	Unit cost	Total cost	$J\cdot\frac{1}{T}$	Total $J\frac{1}{T}+K$
X_1	100	25	2,500	700	3,200	200	20	4,000	1,300	5,300
X_2	180	23	4,200	1,017	5,217	250	20	5,000	1,400	6,200
X_3	60	30	1,800	417	2,217	—	—	—	—	—
X_4	40	25	1,000	200	1,200	—	—	—	—	—
X_5	20	25	500	166	666	—	—	—	—	—
$\sum$	—	—	10,000	2,500	12,500	—	—	9,000	2,700	11,700

The programme of production after specialization will be accompanied by the following programme of foreign trade:

Export-Import Programme after Specialization

Product	World unit prices in international currency	Export		Import	
		Quantity	Value	Quantity	Value
X_1	0.4	100	40	—	—
X_2	0.5	70	35	—	—
X_3	0.6	—	—	60	36
X_4	0.4	—	—	40	16
X_5	0.4	—	—	20	8
$\sum$	—	—	75	—	60

It follows from the above tables that the variant of the production programme after specialization is more efficient from the point of view of production costs and investment outlays than the variant before specialization. Hence, it follows that $\sum J_0 \frac{1}{T} + K_0 > \left[\sum J_s \frac{1}{T} + K_s\right] \pm 0_{\text{bal}} \cdot E_{D_n}$, where: J_0 denotes investments before specialization, K_0—the cost of production before specialization, J_s—investments after specialization, K_s—the cost of production after specialization, 0_{bal}—the balance of foreign trade after specialization, E_{D_n}—the index of the average efficiency of investments in export production[4].

This inequality must be satisfied by the variant after specialization if it is to be considered as more efficient in comparison with the variant before specialization. In the numerical example abovc the fact that investment outlays after specialization are higher than in the variant before specialization by $2700 \cdot T - 2500 \cdot T = 200 \cdot T$, has been outweighed by the lowering of the cost of production and by the additional advantages from the favorable balance of trade. This reflects the index of the efficiency of the specialization programme. In our example it is $\left[\sum J_s \frac{1}{T} + K_s\right] \pm 0_{\text{bal}} \cdot E_{D_n} = 11700 - 15 \cdot E_{D_n}$, whereas for the programme before specialization it is $\sum J_0 \frac{1}{T} + K_0 = 12500$. Thus $11{,}700 - 15 \cdot E_{D_n} < 12{,}500$. In consequence country A has saved $12{,}500 - 11{,}700 = 800$ units (in the international currency), having satisfied its domestic demand to the same extent as before. Moreover its balance of foreign trade is now positive and amounts to $75 - 60 = 15$ units in the international currency which after converting into the domestic currency is $15\ E_{D_n}$. Thus the combined advantages of specialization in production are $800 + 15\ E_{D_n}$ units in the domestic currency.

It should be stressed that in the above example exports and imports have not been balanced and therefore country A has gained an additional advantage in the amount of its positive balance of foreign trade. In the example this has been emphasized on purpose, because cases when foreign trade is fully balanced after specialization are rare and not typical. It is extremely difficult to achieve the ideal state of the full equilibrium of foreign trade in the co-ordination of the plans of particular countries. In fact, this is impossible to achieve in practice and certain deviation from equilibrium have to be expected. This, of course, does not undermine that

---

[4] The index of the investments in export production (or import prevention production) is calculated in the following way: $E_{D_n} = \dfrac{\frac{1}{T} J + K_p}{D_n}$ where: J denotes investment outlays, $\frac{1}{T}$—the normative coefficient of efficiency, K_p—the cost of manufacturing after deducting the cost of raw materials in foreign currencies from the cost of production, D_n—net revenue in foreign currencies after deducting from gross revenue the foreign currency value of raw materials. The numerator is expressed in domestic currency and the denominator in the international currency.

whole principle of striving for a full equilibrium. It remains an important goal by which we should be guided in co-ordinating the plans of the countries undertaking within-sector specialization of production.

We can now determine the general conditions and formulae for calculating the relative advantages of within-sector specialization.

The basic condition of the greater efficiency of the programme of production allowing for specialization and of the possible variants of this programme is that the following inequality should be satisfied:

$$\sum J_0 \frac{1}{T} + K_0 > \left[\sum J_{s1} \frac{1}{T} + K_{s1}\right] \pm 0_{\mathrm{bal}\cdot 1} E_{D_n} > \left[\sum J_{s2} \frac{1}{T} + K_{s2}\right] \pm$$

$$\pm 0_{\mathrm{bal}\cdot 2} \cdot E_{D_n} > \left[\sum J_{sn} \frac{1}{T} + K_{sn}\right] \pm 0_{\mathrm{bal}\cdot n} \cdot E_{D_n}$$

where the notations are as above and $s1$, $s2$... sn stand for different variations of the specialization programme.

Thus, the main formula for defining the amount of the economic advantages of within-sector specialization is as follows[5]:

$$W_s = \left[\sum \frac{1}{T} J_0 + x_0 \cdot w_0 - \sum \frac{1}{T} J_s + x_s \cdot w_s\right] \pm 0_{\mathrm{bal}} \cdot E_{D_n}$$

where

W_s denotes the economic advantages of specialization in terms of the domestic currency,

$\frac{1}{T}$ denotes the normative coefficient of efficiency,

J_0 denotes the investment outlays in particular product lines without specialization,

x_0 denotes the number of commodities produced before specialization

w_0 denotes the unit cost of the commodities produced before specialization (in the domestic currency),

J_s denotes the investment outlays in particular product lines after specialization

x_s denotes the number of commodities produced after specialization,

w_s denotes the unit cost of the commodities produced after specialization (in the domestic currency),

0_{bal} denotes the balance of foreign trade in the commodities included in specialization (in the international currency),

E_{D_n} denotes the index of the average foreign currency efficiency of investments related to the export production of a given country.

[5] This technique—only a little modified—was put before the Economic Commission of CMEA as the most suitable for interest of various countries. It was based upon S. Góra's, Z. Knyziak's M. Rakowski's works.

The amount of advantages should be comparable to the advantages accrued in other countries participing in specialization. Therefore, these advantages should also be calculated in the international currency. This can be achieved by dividing W_s by the index E_{D_n}.

$$W_{sd} = \left[\frac{\sum \frac{1}{T} J_0 + x_0 \cdot w_0 - \sum \frac{1}{T} J_s + x_s \cdot w_s}{E_{D_n}}\right] \pm 0_{\text{bal}}.$$

where W_{sd} denotes the economic advantage of specialization in terms of the international currency; the other notations are as above.

Thus the combined advantages of specializations are the sum of the advantages W_{sd} of each of the countries participating in specialization. Striving for the maximization of combined advantages should constitute a basis for the allocation of specialization targets among the particular countries.

It is easy to see that in the above formulae the advantages of specialization are expressed in the units of the index of the investment efficiency of the production programme. They combine the real quantities of the cost of production (K) and the conventional investment factor $\frac{1}{T} J$, thus providing a basis for a full determination of the economic effects of specialization. Apart from this basic computational approach to the advantages of specialization additional calculations can be made to separate the cost effect from the investment effect arising from specialization.

In our example within-sector specialization resulted in the lowering of production costs by 10000–9000 = 1000 units in the domestic currency. To these advantages should be added the advantage of the positive trade balance in the amount of 15 units in the international currency which after converting into the domestic currency amounts to $15 \cdot e_{ex}$, where e_{ex} is the index of the average foreign currency efficiency of the export of a given country[6].

Thus the overall real savings in the cost of production are: $1000+15 \cdot e_{ex}$. Similarly we can calculate the investment effect. In our example the investment outlays on production without specialization were $2500 \cdot T$, and after specialization $2700 \cdot T$. However, the excess of investment outlays after specialization amounting to 200 T should be reduced by the additional advantage of having a positive balance of payments. In this case the investment gain is $15 \cdot m_{ex}$ where m_{ex} is the average capital intensity of export production in a given country[7]. The final real effect of

---

[6] The index of the foreign currency efficiency of exports differs from the index E_{D_n} in that the former does not allow for the investment factor.

$e_{ex} = \frac{K_p}{D_n}$, the notations being as in E_{D_n}. The index of the average foreign currency efficiency of exports determines the cost incurred on the average by the national economy for obtaining a foreign unit for the exported goods.

[7] The index $m_{ex} = \frac{J}{D_n}$, where J denotes investment outlays and D_n—as above.

within-sector specialization is in our example $1000+15 \cdot e_{ex}$ in savings in the cost of production in the domestic currency and $200-15\ m_{ex}$ in the surplus of investment outlays. (The total of the real effect may assume positive or negative values depending upon the costs and investment outlays in the programmes compared and upon the balance of foreign trade).

The above method of calculating the efficiency and the scale of advantages accruing to each country from within-sector specialization can be used also for estimating the advantages of inter-sector specialization in manufacturing. As the productivity of labour in manufacturing approaches the same level in particular countries, and the facts seem to confirm this assumption, the inter-sector specialization of production may become a natural consequence of the development of within-sector specialization, not excluding the latter, of course. International specialization in production amongst the socialist countries does not exhaust the possibilities of economic co-operation within CMEA. In contrast to specialization in production, however, other forms usually require financial assistance in current operations, both in the form of material and human resources. Economic criteria of these forms of co-operation and the corresponding methods of calculation are now being studied in the socialist countries.

Tadeusz Kowalik
Poland

R. LUXEMBURG'S THEORY OF ACCUMULATION AND IMPERIALISM

(An Attempted Interpretation)

I

When in January 1912 R. Luxemburg set about completing a popular outline of her lectures in economics she encountered an unexpected difficulty in presenting capitalist production in its specific interrelations and the objective historic limits of capitalism. She then concluded that—as she wrote in the Foreword to *The Accumulation of Capital*—what matters is not only the method of exposition, but also the solution of an important theoretical problem, only touched upon by Marx in the second volume of *Capital*. While arranging the first chapter of the popular outline she was still convinced that in Marx's theory economics has found its "crowning achievement" and can be developed by his disciples only in detalis. Working on the last chapter (*Trends in the Development of a Capitalist Economy*) she comes to the conclusion that it is necessary to embark upon a new analysis of the basic problems of the capitalist system as a whole and to study the new problems that have emerged in the new stage of capitalism.

Her changed attitude toward Marx's theoretical achievements manifested itself in a changed attitude toward *Capital*. In the *Introduction to Political Economy* she uses the method that Marx applied in the first colume of *Capital*. Similarly as in this part of Marx's work and in the first two parts of the second volume of *Capital*, the starting point for R. Luxemburg was an analysis of individual capital. It is true that this method differs considerably from the micro-economic method of analysis developed toward the end of the 19th century, since both Marx and R. Luxemburg always remembered that individual capital is only a relatively independent part of social capital. However, as long as this analysis has not been crowned by a study of total social capital and of its internal interdependence, their method also could not be called macro-economic. The very essence of the turning point in the views of R. Luxemburg in the course of the year 1912 consists in grasping the importance of this kind of (macro-economic) analysis. Fascinated both by Marx's method used by him in the third part of the second volume of *Capital* (*The Reproduction and Circulation of Total Social Capital*) and by the Marxian reproduction schemata contained in that part she embarks upon writing her opus magnus.

The genealogy of problems and methods used in *The Accumulation of Capital* stems, then, from the last part of the second volume of the work of Marx. It is through the prism of this method that she evaluates the achievements of economic thought in literature, exposing now Quesney as the author of *Economic Tables* and a forerunner of the Marxian theory of reproduction. In Marx's construction she saw the most perfect embodiment of the Marxian method of abstraction (dialectic deduction) and the most important tool of economic analysis.

Quesney advanced in her eyes to the rank of a founder of economics as an exact science and the same criterion of evaluation—the maturity of the approach to the problem of reproduction—compells her to criticize Adam Smith not for losing in universal categories the specific features of a bourgeois society, but, on the contrary, for the fact that "the specifically capitalist function of wage labour in the productive process completely obscured for him the eternal and universal function of the means of production within the labour process[1].

Answering the question why in later studies the treatment of this problem was less advanced than the approach taken by Quesnay, and why Marx could essentially push forward the theory of reproduction, she emphasizes the discovery by Marx of the dual nature of labour creating value. "This inspired fundamental law of Marx's theory of value (...) led him (...) to distinguish and to integrate those two aspects in the total reproductive process: the aspect of value and that of actual material connections"[2].

R. Luxemburg attached a great deal of importance to the fact that Marx distinguished between, and presented schematically the two basic divisions of social production—the division producing the means of production and the division producing the means of consumption. Thanks to this fundamental idea it has become possible to make a precise analysis of the problem of social reproduction. Pre-Marxian economics could not reconcile an analysis of the process of labour from the material angle with the value—approach to production and could not reconcile the forms of the movement of individual capital with the movement of total social capital. Creating the schemata of simple reproduction Marx threw on this problem a ray of light, comprising in two surprisingly simple series of numbers all these points of view with their interdependencies and contradictions[3].

R. Luxemburg was not the only one fascinated by the Marxian construction of the schemata of reproduction. Under the influence of this construction was, for instance, a great part of the economic writings of the Russian legal Marxists (particularly Boulghakov and Tougan-Baranovsky), and also the first economic works by Lenin, amongst others his *Development of Capitalism in Russia*. They valued this construction, however, for providing a solution to the problem of reproduction and realization in the conditions of a capitalist economy. Luxemburg was the only known

[1] R. Luxemburg, *The Accumulation of Capital*, London 1951, p. 73.

[2] Ibid., p. 105

[3] Ibid., pp. 264–265.

economist who had noticed, even before World War I, the universal, supracapitalistic nature and significance of this theoretical construction and who had realized that these schemata will also be of great importance in a socialist economy.

In the work of R. Luxemburg there is a fairly detailed description of the application of these schemata (of both simple and expanded reproduction) to the conditions prevailing in a socialist economy. The comments of R. Luxemburg on this subject may sound commonplace today. However, viewed historically they were an important event. Let us investigate this matter more closely.

Criticizing the classical and vulgar bourgeois economics Marx waged for decades a battle for the recognition of the historical nature of capitalism and of the categories reflecting its features and of the economic laws governing it, and thus also for the recognition of the historical character of political economy. Under these circumstances it was easy to fall into polemic extremes, to disregard the importance of formulating categories and laws of universal application, that is those pertaining to all material production by man. Even Engels whose interest more often went beyond the problems of the capitalist system (even in his work in which he postulated the creation of political economy in a broader sense, having as its subject-matter also the economy of pre-capitalistic formations) wrote: "Who would like to apply to the economy of *Tierra del Fuego* the same laws as to the economy of the contemporary England he would produce nothing but most commonplace clichés"[4]. This conviction haunted Marxian writers for a long time and can explain the great importance attached to one sentence written by Lenin with reference to Bukharin's book on the period of transition and published only toward the end of the nineteen twenties. In this sentence Lenin pointed out the necessity of analysing the proportions between Division I and Division II and the share of accumulation also in the conditions of communism[5]. A general realization of the importance of the Marxian reproduction schema for the theory of the political economy of socialism is a relatively recent development[6]. Later still, partly due to the influence of Leontieff's input-output tables, the scope of its general theoretical application, regardless of the political system, and its real significance have come to be generally recognized. The first systematic outline of "the general theory of reproduction" was given by Oscar Lange[7]. He also anounced that his presentation of the more basic problem of political economy will begin

[4] F. Engels, *Anty Dühring*, Warsaw 1948, pp. 174–176. I discussed this matter in greater detail in my paper *Z prehistorii ekonomii politycznej socjalizmu* (From the Prehistory of the Political Economy of Socialism), "Ekonomista" No. 4/63.

[5] *Leninskij Sbornik*, vol. XI, published for the first time in 1929. An interesting comment on this utterance by Lenin—see G. Tyemkin *Karola Marksa obraz gospodarki* (Marx's Picture of Economy), Warsaw 1962, p. 244–5.

[6] J. Stalin enumerates a number of theses of the Marxian theory of reproduction which are applicable also to a socialist economy (*Ekonomiczne problemy socjalizmu ZSRR*, Warsaw 1952, p. 87)—which can be regarded as an adequate summary of the views of R. Luxemburg.

[7] O. Lange: *Teoria reprodukcji i akumulacji* (The Theory of Reproduction and Accumulation). Warsaw 1961.

with the theory of reproduction and accumulation thus interpreted[8]. A return to the theory of reproduction of Quesney—Marx manifests itself in contemporary economic literature not only in direct references to the schemata, but in an increasing application of the analysis of total quantities, first of all, in the theory of growth, and particularly in the analysis of the dynamics and the factors of growth of national income and its distribution with respect to developed and underdeveloped countries both in the socialist and in the capitalist system. It is interesting to note, however, that all this happened as if irrespective of the conclusions reached by R. Luxemburg, without any reference to her achievements in this field.

II

R. Luxemburg appraised very highly the cognitive values of the Marxian schemata of reproduction primarily as a tool of macro-economic analysis and she thought that for Marx it would have only been a matter of time to take advantage of this tool of analysis for studying the specific features of reproduction and the accumulation of total social capital. From this point of view this is perhaps the least elaborate part of *Capital* (the third part of the second volume) and the analysis of accumulation is only just begun. Without further transformations the Marxian schema of accumulation (expaned reproduction) cannot be used for an analysis of the accumulation of capital because it contains a number of assumptions which make it difficult to understand the movements of total capital.

According to R. Luxemburg the fault of the schema of accumulation stems from the following four incorrect assumptions:

1. The schema assumes that capitalist production creates a sufficiently large sales market for itself and that, therefore, an indentity between the conditions of production and realization can be assumed. This is at odds not only with the spirit of Marx's theory but also with many utterances in the first and third volume of *Capital* in which it is stressed that there is a tendency on the part of total demand to lag behind rapidly increasing production.

2. In the Marxian schema the monetary form and the monetary phase of capital in the process of the accumulation of capital are disregarded. No conclusions are drawn from the fact of rejecting by Marx (in the first volume of *Capital*) the Say's Law. It is assumed that all savings are somehow transformed by the capitalists into real accumulation. The disregard of the circulation of money in the schema in her opinion, "has great disadvantages of its own"[9].

---

[8] O. Lange, *Political Economy*, Vol. I, Warsaw 1963, p. XI.

[9] Because it is forgotten that "Even if the transformation of the surplus value is not essential to real reproduction it is the economic *sine qua non* of capitalist accumulation" (ibid., p. 139). Although Marx raised this problem many times in the second volume of *Capital*, but in her opinion, the shortcoming of his analysis was that "he attempted to solve the problem formulating it incorrectly as the matter of the sourees of money. In fact, what matters is the actual demand, the consumption, of commodities and not the sources of money used for paying for these commodities". In the light of

3. Marx analyses the accumulation of capital within the framework of society composed exclusively of the capitalist class and the working class, that is within the framework of pure capitalism. This assumption rendered impossible, in her opinion, the discovery for whose benefit the expansion of production takes place. Approached from this angle the schema can only be interpreted as a vision of production for production[10]. R. Luxemburg keeps asking: what is the starting point for accumulation, what provides incentives for expanding production and consequently, who is the buyer of steadily expanding production, or more strictly, of the capitalized surplus value? According to R. Luxemburg no answer can be given to these questions on the basis of Marx's assumptions.

4. Finally, R. Luxemburg criticizes the Marxian schema for disregarding the increasing productivity of labour, for assuming an unchanging organic composition of capital. Similarly as was the case with many Marxists of her days, she knew then only one type of technical progress which is now called "capital-consuming". She was convinced that technical progress must manifest itself in an increasing share of fixed capital in the value of the product, i.c. in the increasing organic composition of capital, or, what for her was only another way of expressing the same phenomenon, in an increasing share of Division' (the production of the means of production) in the total social product. Thus the fact of allowing for technical progress was supposed to introduce into the problem of reproduction numerous difficulties in maintaining the proportions both value-wise and material-wise.

---

these contentions by R. Luxemburg it is surprising that Paul M. Sweezy accused her that: "We leave out of account altogether purely monetary problems of capital accumulation though she devotes a great deal of attention to them, frequently even confusing the question, where does the demand come from with the question, where does the money come from. It is in discussing the later question that she shows to least advantage; but it is, after all, a minor problem which is essentially irrelevant to her main thesis". (P. M. Sweezy, *Theory of Capitalist Development*, New York 1942, p. 204). It can be argued, of course, that R. Luxemburg did not quite understand Marx in this respect that her reasoning is not correct. But the misunderstanding consists in the fact that she has been accused of the very thing against which she so consistently fought.

[10] Ibid, pp. 333–334. R. Luxemburg saw, of courses, that expanded reproduction is expressed in an increase in the consumption fund of the capitalists and in the wage fund (m and v). She thought, however, that this does not solve the problem of equilibrium between production and consumption in a capitalist economy. Thus the known joke by Bucharin, allegedly exposing the main error of R. Luxemburg's theory of accumulation, that "if someone excludes expanded reproduction at the beginning of a logical proof, it is naturally easy to make it disappear at the end; it is simply a question of the simple reproduction of a simple logical error" (*Der Imperialismus und die Akkumulation des Kapitals*, Vienna-Berlin, 1926, p. 20)—misses the point completely. Bucharin's joke stemmed from the extreme disregard of the problem of underconsumption in the capitalist system. It is surprising that this joke provides the basis for the criticism of Luxemburg's theory, written by Sweezy in the foreward to the Italian edition of her work which was published in 1960, at the same time when the author of the foreword worked together with P. Baran on the book on the contemporary American capitalism, whose central part is devoted to the Problem of Surplus Absorption (compare the titles of Chapters 3–7 given in the "Monthly Review", No. 3–4, 1962) which is the central theme of R. Luxemburg's work.

III

At different stages of her thinking R. Luxemburg tried to reject Marx's assumption which she criticized. However, with one exception (which shall be elaborated upon later on) this has not led her to an appropriate change in the construction of the schemata of reproduction. After a long consideration of the role of money in capitalist reproduction contained in the first part of the book R. Luxemburg does not return to this problem in Part III in which she develops her own approach to the problem of capitalist accumulation. The situation is similar with regard to the next problem. Luxemburg did not confine herself to attacking the one-sidedness of the view that capitalist expanded reproduction determines the size of the market, and emphasized the reverse relationship—that of the dependence of the process of expanding production on the size of the market, seeing in the insufficiency of total demand a bottleneck limiting production. The problem of insufficient demand is somewhat of an obsession of her book. It would not be an exaggeration to say that she succeeded in coming close to the contemporary problem of equilibrium between savings and investments. This is the way in which should be interpreted her contention that the conditions of the realization and the conditions of the capitalization of surplus value differ from each other both with respect to time and with respect to place[11]. But nowhere has she succeeded in presenting her views on this subject in the form of an improved schema.

The situation is not different with respect to the Marxian abstraction of pure capitalism. Although the criticism of this abstraction runs throughout the whole book, whenewer she resorts to the schemata of reproduction she uses the Marxian schemata based on this assumption. In this sense, that is in the sense of *her* understanding the scientific precision of an analysis following on the lines of Quesney's Tables, Rosa Luxemburg did not even make an attempt at "a strictly scientific ap-

---

[11] Ibid., p. 421 Cf. also p. 342. A similar interpretation of the work of R. Luxemburg was given in 1930 by M. Kalecki. Strassing certain basic similarities between his approach to investment and national income and the theory of reproduction developed by Marx and Luxemburg he noted that in the second volume of *Capital* Marx showed interest in the ideal conditions of equilibrium but did not ponder over what happens when investment do not ensure equilibrium and therefore failed to realize the key role of investments in determining the volume of production and employment. "Exactly the reverse attitude—wrote Kalecki—is represented by one of his eminent pupils, Rosa Luxemburg. In her *Akkumulation des Kapitals* she stressed the point that if capitalists are saving, their profits can be "realized" only if a corresponding amount is spent by them on investment. She, however, considered impossible the persistence of net investment (at least in the long run) in a closed capitalist economy; thus, according to her, it is only the existence of exports to the non-capitalist countries which allows for the expansion of capitalist system. The theory cannot be accepted as a whole, but the necessity of covering the "gap of saving" by home investment or exports was outlined by her perhaps more clearly, than anywhere else before the publication of Mr. Keynes's General Theory" (M. Kalecki, *Essays in the Theory of Economic Fluctuations*, London 1939, p. 45–46). Joan Robinson's interpretation in her foreword to the English translation of the work of R. Luxemburg goes along the same general line.

proach" to the problem of the accumulation of capital, to say nothing of the hope expressed by her in the foreward that her approach will succeed.

The only correction made by Luxemburg in the schemata was that she allowed for an increase in the productivity of labour. She did it by introducing the increasing organic composition of capital to the Marxian second schema of expanded reproduction. The ratio of fixed capital (c) to variable capital (v) is in her interpretation 5:1 in the first year, 6:1 in the second year, 7:1 in the third year and 8:1 in the fourth year. In this way she obtained for these four years (periods) the figures whose ratios indicate an increasing deficit of the means of production (16 for the second year, 45 for the third and 88 for the fourth) and an increasing corresponding surplus of the means of consumption[12].

The schema thus constructed becomes for R. Luxemburg a basis for reaching conclusions whose main theme is that no change "in the method of production in the course of accumulation" can be accomplished without undermining the basic assumptions of the Marxian schema. She also contended that disproportions arising because of that in the process of capitalist accumulation can be liquidated or dampened only outside the framework of pure capitalism by exchange between capitalist and pre-capitalist systems.

It is easy to see that all this reasoning is based on two dubious assumptions. One of them has not survided the test of confrontation with reality. There is little doubt today that technical progress does not have to manifest itself in an increase in the organic composition of capital. From this point of view progress may be neutral or even capital-saving, that is it can result in a decrease in the organic composition of capital. The second error consist in accepting as consistent with reality an acceptable simplifying assumption, when in fact accumulation does not have to be allocated to the same division in which it had been obtained[13,14].

---

[12] Ibid., pp. 337–339.

[13] O. Lange was right in writting: "In the Marxian schemata and in later considerations by Lenin there was the simplifying assumption that accumulation is allocated to the same division in which it had been obtained. In real life, however, there are flows of accumulation between the divisions (...). In a planned economy accumulation comes primarily from Division II and is allocated mostly in Division I". O. Lange, *Teoria reprodukcji i akumulacji* (The Theory of Reproduction and Accumulation) Warsaw, 1961, p. 41. In this work Lange also proved that it is not true that changes in the proportions between Division I and Division II are directly affected by changes in the organic composition of capital (cf. ibid., p. 48–49).

[14] Both these erroneous theses by Rosa Luxemburg had become a theoretical starting point for two different books by Marxist authors. The book by Fritz Sternberg: *Der Imperalismus* was published in 1926. The author's contention (not a simplifying assumption, but a contention) that accumulation cannot be transferred from one division to anather plays an important part in the theoretical construction. Cf. also *Der Imperialismus und seine Kritiker* by the same author, 1929, p. 29. The basic theoretical design aspounded by Henryk Grossmann in his work: *Das Akkumulations-und Zusammenbrucksgesetz des kapitalistischen Systems*, 1929, is based on the law of more rapid growth of Division I. Stenberg was at that time a follower of Rosa Luxemburg's economic theory and Grossmann was her ardent epponent. These two books were the subject of the heated

IV

Thus in her only attempt to introduce corrections in the Marxian schemata of expanded reproduction R. Luxemburg cannot claim any visible theoretical achievements. Her attempt deserves attention as the first effort in literature to introduce technical progress to the general formula of economic growth[15], but it has not contributed to advancing the problem of the capitalist accumulation of capital. What is more, no conclusions resulted from this attempt concerning her main contention that the tendency to underconsumption is the main source of basic difficulties in capitalism. In relation to other faults noticed by her in the Marxian schemata of expanded reproduction as tools for solving this problem, she did not even make such an attempt. In this sense the book by Rosa Luxemburg is disappointing. However, even in its purely theoretical aspect this book was an important event in the development of economic thought, and not only because it drew attention to the universal value of inter-branch proportions in the schemata.

Its importance consists in expounding the following theses or postulates:

1) The stressing of the necessity of analysing the accumulation of capital (growth of a capitalist economy) in terms of aggregates. Rosa Luxemburg suggested that the problem of the movements of total social capital and the laws of its accumulation should be studied and presented in a mathematical form and should be analysed in terms as precise as those in which Marx presented his proportions and inter-branch relationships in social reproduction. Her error consisted in thinking that the schemata of reproduction are suitable for this purpose after a few improvements. Thus she overestimated Marx's theoretical construction, rather than underestimated it.[16]

2) An attempt at a theoretical formulation of the known Marxian statement that the conditions of production are not identical with the conditions of realization. In consequence of rejecting Say's Law she tried to prove that also as far as the capitalist class is concerned savings do not have to be equal to actual accumulation (investments), that accumulation is affected to a large extent by the prospect of a growing sales market which, in turn, is determined primarily the existing sales situation, that pure capitalism provides too weak a basis for rapid economic growth.

This was the direction of the development of political economy in the following decades. The analysis of capitalism and of its development in aggregative terms and the emphasis on the deficiency of demand are the characteristic features of the Key-

---

controversy that went on in the twenties and at the beginning of the thirties concerning the question of the collapse of capitalism.

[15] This was done earlier by Lenin in his work: *With Reference to the So-called Question of the Markets* (1893). This work was published for the first time only in 1937.

[16] Writing about the thirty-years long discussion on the importance of the Marxian schemata of reproduction for determining the prospects of the capitalist mode of production Oscar Lange concluded: "This discussion led nowhere because, as it has turned out, the schemata of equilibrium of reproduction do not suffice for solving the problem involved in this discussion". O. Lange, *Teoria reprodukcji i akumulacji* (The Theory of Reproduction and Accumulation) op. cit. p. 61.

nesian revolution. The same applies to the pre-Kcynesian theory of business cycles based on the Marxian principles and of the theory of the dynamics of capitalism developed later by M. Kalecki.

3) For similar reasons well deserving notice is the attempt by Rosa Luxemburg to include the monetary system in the theory of capitalist reproduction and accumulation. In traditional economic literature this problem was neglected. Either the problem of dynamic equilibrium under the conditions of capitalist accumulation was considered on the assumption of neutral money (see, for instancc, works by Tougan-Baranovsky, Boulghakov, Grossman and others) or the problem of economic equilibrium was limited to narrowly conceived market-money processes, in isolation from the problems of reproduction. It can be seen from numerous passages of the second volume of *Capital* that Marx's aim was to include the monetary system to the analysis of the problem of capitalist reproduction. However, his death prevented him from accomplishing this. It is true that neither had Rose Luxemburg succeeded in solving this tremendously difficult question. But in contrast to many other disciples of Marx she considered the solving of this problem as one of the most important tasks and she formulated it in a much more lucid and precise way than her predecessors. Similarly as in previously mentioned problems intuition had not failed her. She sensed correctly that without the inclusion of the monetary system the problem of equilibrium in the process of capitalist accumulation cannot be solved[17].

V

Why did Rosa Luxemburg raise again the problem of incentives to accumulation, investments and technical progress? Marx analysed this problem exclusively with reference to an individual capitalist and only with this approach was it possible to be satisfied with the explanation of investment incentives given in the first volume of *Capital*. However, this explanation, consisting in saying that the capitalist strives incessantly to maximize his profits and that this striving becomes for each individual capitalist the "external law of compulsion" determined by competition, does not suffice when one wants to explain this problem in terms of total social capital. This answer, however, seems incomplete. It may be surmised that there was also another reason that prompted her to deal with this problem again. Reading her works one can detect the ripening understanding of changed investment incentives in the period of monopolistic capitalism. The promoter of technical progress in capitalism was, in her

[17] One of the ways of solving this problem was given by Lange in his work: *Price Flexibility and Employment*, Bloomington 1944. From the point of view of interest to us Lange' analyses in his book the mechanism of cumulative processes aiming at the transformation of the distortion of equilibrium on the market for one commodity into the state of disequlilibrium of the whole national economy and the conditions of restoring equilibrium. It seems that the book by Lange referred to here and the studies published by Kalecki taken together provide a sufficient solution to the problems raised by Rosa Luxemburg. This problem, however, cannot be elaborated upon here.

opinion, the small and medium entrepreneur. For the conditions in which this type of entrepreneur predominates it would suffice to analyse the incentives determined by free competition. But the entrepreneur of the period of free competition and with him the mechanism of competition are begining to fade away into the past. We are entering the era of industrial giants which R. Luxemburg analysed in her *Social Reform*. Therefore, it was necessary to go back to incentives and forces determining the dimensions of the accumulation of capital.

In her discussion with Berstein Rosa Luxemburg expressed the following view: "In the general development of capitalism small capital, according to Marx, plays the part of the pioneer of technical revolution (...). If small capital is the champion of technical progress and if technical progress is the pulse of a capitalist economy then small capital is a phenomenon inseparable from capitalist development (...). The gradual disappearance of medium sized firms would not mean, as Bernstein seems to think, that the development of capitalism is revolutionary, but on the contrary, it would indicate, that it is stagnant and drowsy"[18].

In the light of this view *The Accumulation of Capital* can be regarded as R. Luxemburg's attempt to answer the following question: Why does capitalism not show signs of stagnation despite the fact that in the period of the maturity of capitalism the role played by small and medium sized enterprises is becoming less and less important[19].

The general tenor of her answer is that because of a tendency to limit the consumption fund of the working class and because consumption expenditures of the capitalists have their natural limits, pure capitalism is not in a position to provide sufficiently strong incentives to accumulation. An analysis in those terms cannot give an explanation for a rapid development of productive resources and the growth of production so far. A very large part of incentives to accumulation capitalism owes to a steady and uninterrupted economic exchange between the capitalist and non-capitalist environments. In this way Luxemburg combined an analysis of the economic dynamics of capitalism with changes in the economic and social structure of the world economy. In her opinion this should lead to a change of the view on the mutual relationship of what Marx called primitive accumulation and what he defined as the (proper) accumulation of capital. She argued that the processes included in the first category cannot be treated exclusively as an historical annex presenting the genesis of capital, the time of its birth. Seeing over more clearly the importance of the underdeveloped countries for the growth and future of the capitalist system

[18] Although the subtitle in *The Accumulation* is: *A Contribution to the Economic Explanation of Imperialism*, Rose Luxemburg does not deal in this work with an analysis of capitalistic associations. In one of the footnotes she explained that "It would go beyond the scope of the present treatise to deal with cartels and trusts as specific phenomena of the imperialist phase". R. Luxemburg, *The Accumulation of Capital*, p. 457.

[19] R. Luxemburg, *Reforma socjalna czy rewolucja*? (Social Reform or Revolution?) in *Wybór pism* (Selected Papers) vol. I, Warsaw 1959, pp. 161 and 163.

she pressed for a broader interpretation of the proeess of the development of eapitalism than the interpretation given by Marx in *Capital.* Economic exchange between the eapitalist and pre-capitalist environments is so important a part of this proeess that the term "primitive aeeumulation" does not seem to her to be a fortunate one[20]. Capital, in her opinion, not only is born "soaked in blood and dirt", but grows later in very much the same way, until the moment of its collapse.

Marx analysed the aeeumulation of capital on the assumption of the general and exclusive rules of eapitalist production treating it exelusively as a relationship between the bourgeoisie and the proletariat. Rosa Luxemburg considers this approaeh as too narrow, because accumulation, as a real historical proeess, has two aspeets. "One coneerns the eommodity market and the place where surplus value is produeed—the faetory, the mine, the agricultural estate. Regarded in this hight accumulation is a purely economic proeess (...). Here, in form at any rate, peace property and equality prevail and the keen didaetics of scientific analysis were required in the eourse of aeeumulation into appropriation of other people's property' how eommodity exehange into exploitation and equality becomes class-rule.

The other aspeet of the aceumulation of capital coneerns the relations between eapitalism and the non-capitalist modes of produetion which start making their appearance on the international stage. Its predominant methods are colonial poliey, an international loan system—a poliey of spheres of interest—and war. Forees, fraud, oppresion, looting are openly displayed without any attempt at eoneealment and it requires an effort to discover within this tangle of politieal violenee and contests of power the stern laws of the economie process (...). The conditions for the reproduetion of eapital provide. The organie link between these two aspeets of the accumulation of capital. The historieal eareer of capitalism can only be appreeiated by taking them together"[21].

The authoress of *Accumulation* aeeuses liberal economics of analysing only one side of the process of aecumulation: "the area of peaeeful coexistence, wonders of technique and pure commodity trade". The second aspect of accumulation is left by liberal economies outside the seope of its interest, and the acts of violence aeeompanying accumulation are treated as a separate matter of foreign policy. But politieal violence is "nothing but a vehiele for the economic process"[22]. Starting from this assumption R. Luxemburg devotes almost one third of her book to the analysis of historieal eonditions of aecumulation and argues not only with bourgeois eeonomics but also with the then prevailing trends of theoretical Marsian economics.

The idea of a peaeeful development of aeeumulation on a world scale is eonsidered by her as an expression of the ideology and harmony of the interest of capital and labour. Free trade, in her opinion never expressed the needs of accumulation

[20] R. Luxemburg, *The Accumulation of Capital*, London 1951, p. 364.
[21] Ibid., p. 452.
[22] Ibid., p. 452.

of capital so broadly interpreted and could not be in the history of capitalism anything else but an episode which in Europe happened to occur in 1860' and 1870'. For this reason it is erroneous, in her opinion (contrary to Engels) "attributing the general reversion to protective tariffs after the seventies simply to a defensive reaction against English Free Trade[23]".

Including in the analysis of the process of the accumulation of capital the problem of precapitalist environment and its role in the development of capitalism R. Luxemburg attempted to develop a theory of the development of capitalism much broader than the Marxian theory, she tried to enrich the content of the scientific abstraction: "capital", or "total capital", because from the point of view of pure capitalism it is impossible to understand the antagonisms of the world economy arising in the process of accumulation. When for Marx the term "capital" meant only the relationship of exploitation between the bourgeoisie and the proletariat, for her it was the term expressing the relationship of power over both the proletariat and the precapitalist environment being drawn into the cogwheel of the capitalist world economy (in today's terms: over the underdeveloped countries and areas).

Rosa Luxemburg deemed it necessary to abandon the assumption of the absolute hegemony of capital over the whole world not only because this assumption excluded a priori the process of imperialism. She thought, in accordance with the spirit of Marxian theory, that the experience of imperialism is so important and essential that it is necessary to reformulate in its light the Marxian theory of the development of capitalism (as being based on too narrow historical foundations). And the newest phase of capitalism system—imperialism—should be explained by this general theory. She thought that there is no "Chinese Wall" between classical capitalism and the phase of imperialism. Neither is there such a wall between the processes contributing to the rise of capitalism by subjugating the traditional forms of production and the process of the collapse of capitalism whose main sources lie also in the contradictions arising against the background of economic and political relationship between these two worlds. In her interpretation imperialism is a period of wars and revolutions due to the exhaustion of the non-capitalist environment providing for capitalist accumulation outside markets, areas for the profitable investment of capital, and basic raw materials. Without this environment as a feeding ground accumulation would be impossible.

And again in the purely theoretical field of endeavour in which she anounces a formulation of these processes in precise laws of accumulation Luxemburg's efforts rather end in a failure. Her important achievement, however, is that she put the problem of underdeveloped countries in the centre of interest and that she approached this problem from the point of view of the prospect of the further development (collapse) of capitalism sensing that here is the key to the problem of the disintegration of the international capitalist system heralding the collapse of the capitalist mode of production.

[23] Ibid., p. 449.

In analysing the mutual relationship between capital and its historical environment developed against the background of international loans she perceives the problem of national liberation revolutions in the epoch of imperialism and points out that this epoch comprises both the division of colonies among the colonial powers and the process of the emancipation of these colonies from the imperialist yoke[24].

She then shows that the emancipation of "hinterlands" is accomplished by revolution whose object is to abolish the obsolete forms of government and to establish a new system of goverment suitable for the purposes of capitalist production. This was the sense of the then revolutions in Russia, Turkey and China. In the Russian and Chinese revolutions she notices new features consisting in drawing to the surface not only "precapitalistic reckoning" but also new antagonisms directed against the rule of capitalism. This, in her opinion, makes these revolutions more vigorous but also delays the final victory and makes it more difficult to achieve.

A young capitalist nation usually resorts to war as a means of rejecting the yoke of imperialist control. National liberation wars become a baptism of fire and a test of independence of a young state. The first step to economic independence is a military and financial reform[25].

Thus Rosa Luxemburg's interpretation of the phase of imperialism is very broad. It contains both the period (and the phenomena) of the division of colonies among the capitalist powers and the period (and the process) of the liberation of colonies and dependencies observed on such a large scale after World War II. In its political aspect this process is almost complete. However, in other respects Luxemburg's prognostication has not been confirmed by historical developments. Together with many other economists, she overestimated the ability of international capital to industrialize backward countries. The awakening of these countries from their lethargy and their striving for economic and political independence is born in them at a relatively low level of economic development and is related to a more and more often voiced conviction that monopolistic capitalism cannot industralize those "hinterlands". This conviction, however, is of a relatively recent date and is connected with the establishment of the socialist system.

Sometimes Rosa Luxemburg defines imperialism very narrowly. The chapter on protective custom duties and accumulation opens with the following sentence:

---

[24] The chapter on international loans (interpreted broadly—both as loans in the strict sence and the export of productive capital) begins with the following interesting theses: "The imperialist phase of capitalist accumulation which implies universal competition comprises industrialization and capitalist emancipation of the *hinterland* where capital formerly realised its surplus value. Characteristic of this phase are: lending abroad, railroad constructions, revolutions and wars (...). Just as the substitution of commodity economy for a natural economy and that of capitalist production for a simple commodity production was achieved by wars, social crises and the destination of entire social systems, so at present the achievement of capitalist autonomy in the hinterland and backward colonies is attained amidst wars and revolutions". (*The Accumulation of Capital*, p. 419).

[25] Ibid. p. 420.

"Imperialism is the political expression of the accumulation of capital in its competitive struggle for what remains still open of the non-capitalist environment"[26]. It does not seem right, however, to treat this and similar sentences, as it had been done by her numerous critics, as an exhaustive definition of imperialism. In the above sentence she showed one side of the phenomenon, as can be seen from the sentences following it. In the *Accumulation of Capital* there are several partial definitions which—even though they may not form a classical and comprehensive definition—indicate the main features of the epoch of imperialism, noticed by her[27].

Stressing the importance of underdeveloped countries for the accumulation of capital and the resultant contradictions between capital and its environment she noticed also another "contradictory phenomena that the old capitalist countries provide ever larger markets for... one another"[28].

The description of imperialism given in the *Accumulation of Capital* is completed by her numerous remarks on this subject written in the first year of the war, entitled: *The Crisis of Social Democracy*. Writing that the maturing of imperialism can best be seen on the example of Germany where this process lasted longer than elsewhere she stressed two specific forms of capital accumulation: the rapid process of cartelization and the centralization of banking. "Heavy industry, the very branch of capital directly interested in government contracts, armaments and such imperialistic enterprises as railways, ore mines etc., became in the hands of cartels the most influential factor in the state. And banks moulded capital into a monolithic force of accumulated energy"[29]. She stresses further that "this young imperialism bursting with power, which entered the world arena with a tremendous apetite at the moment when the whole world was actually divided, had to become very quickly an incalculable factor of general anxiety"[30].

These opinions seem to form a sufficiently strong basis for the following conclusions. The authoress of *The Accumulation of Capital* has not created a uniform and internaly solid and comprehensive definition of imperialism. Although she dealt in her book mainly with international aspects of imperialism she was fully aware of the phenomena of the concentration and centralization of production and banking, institutionalized in the form of cartels and trusts, and characteristic of the new phase of capitalism. We find in her writings all features of imperialism enumerated by Lenin in his very precise definition, although she did not attempt to arrange them in the order of importance.

---

[26] Ibid., p. 446.

[27] R. Luxemburg, *Akumulacja kapitału, Antykrytyka—Akumulacja kapitału czyli co epigoni zrobili z teorii Marksa* (Accumulation of Capital, Anticritique—Accumulation of Capital or what the Epigones Did with the Marx's Theory), Warsaw 1963, pp. 615, 634 etc.

[28] R. Luxemburg, *Accumulation of Capital*, London 1951, p. 367; cf. also R. Luxemburg, *Akumulacja Kapitału, Antykrytyka* (Accumulation of Capital, Anticritique). Warsaw 1963, p. 730.

[29] R. Luxemburg, *Kryzys socjaldemokracji* (The Crisis of Social Democracy), Warsaw 1951, p. 62.

[30] Ibid. p. 63.

A specific feature of Rosa Luxemburg's approach to the problem under consideration is an attempt to explain imperialism as a phenomenon conected primarily with the peculiarly capitalistic tendency for demand to lag behind potential and actual production. Her specific interpretation of the market (the problem of realization) has become a basis for a very interesting and original analysis of the armament industry as a spherc of accumulation.

VI

Militarism as a sphere of the accumulation of capital—this is the title of the last chapter of R. Luxemburg's work. She makes an attempt to approach from a theoretical angle the importance of armament production (as production and not as a tool of external expansion) for stimulating economic growth in capitalism[31]. The pioneering analysis of this thoroughly contemporary problem was undertaken by her very early and contained, of course, some loopholes in reasoning and certain inconsistencies. It is interesting to note, however, that the basic direction of the solution proposed by her can be considered as an antecedent of the contemporary Marxian and Keynesian attitude to this problem.

She attacked the conviction prevailing at that time that the bourgeois state can only redistribute profits and incomes without changing antyhing in the conditions of reproduction of total social capital. This conviction also applied to government expenditures for armament production and to the belief that the state acting in this capacity creates "by the sleight of hand" new demand, new purchasing power, thus influencing the magnitude of the total accumulation of capital. The demand created in this way by the state "has the same effects as a newly opened market"[32]. In the era of imperialism armament production becomes one of the important ways of solving difficulties in the realization of growing production. The attractiveness of expanding this sphere of the accumulation for capital consists, in addition, in the fact that this form of the purchassing power of the state for military equipment is "free of the vagaries subjective fluctuations of personal consumption, it achieves almost automatic regularity and rythmic grows"[33]. Thus from the purely economic point of view the armament sector would tend the dampen business fluctuations in capitalism which is also related to other features of this sector. "Finally, the lever of this automatic and rhytmical movement of armament production in capitalism is in the hands of capital itself... That is why, this particular province of capitalist accumulation at first seems capable of infinite expansion[34].

[31] In the notes prepared for her lectures there is the following, very meaningful, passage: "Artificial stimulation of consumption: militarism, colonial policy, railways in Africa, sea expansion". R. Luxemburg, *Wstęp do ekonomii politycznej* (Introduction to Political Economy), Warsaw 1958, p. 325.

[32] Rosa Luxemburg, *The Accumulation of Capital*, p. 460.

[33] Ibid., p. 466.

[34] Ibid., p. 466.

Of course, from today's point of view Rosa Luxemburg's approach is still rather narrow. The role of the state is limited to being a protector of private capital (subsidies, custom exemptions, etc. were discussed by her in the preceding chapters) and the institution levying taxes and procuring orders for this capital. Thus the state has not yet assumed directly the role of an entrepreneur. She also did not perceive the possibility of credit creation by budget deficits. The multiplier effecct of the armament sector has hardly been noticed. Not included in the analysis was the problem of unused productive capacity. Too much stress was laid by her on wages and individual income of small producers, as a main source of government revenue.

Attempting to define precisely the role of the military sector in the accumulation of capital on the basis of the scheme of expanded reproduction Luxemburg separates this sector as the third division of social production (which in itself, was a commendable idea). But this attempt does not bring the expected results primarily because in the scheme itself Rosa Luxemburg has not succeeded in coping with the problem of the identity of the conditions of production and sales.

But the mere fact of raising this problem, considered very important today, and of showing the fundamentally correct direction in which its solution should go elevates her to the ranks of the precursors of contemporary economics.

VII

The question of the collapse of capitalism plays an important part in R. Luxemburg's considerations. The desire to grasp theoretically the objective historical limits of this mode of production was one of her motives of dealing with the problem of accumulation. In *The Accumulation* and *Anticritique* she often returns to this problem. The most theoretically abstract approach can be reduced to the same thesis as was to provide solution to the problem of accumulation. As an historical process the accumulation of capital is, according to her, "depends in every respect upon the non-capitalist social strata and forms of social organization"[35]. In this way, the solution to the problem that had been a subject of controversy since the time of Sismondi according to whom the accumulation of capital is altogether impossible, and the naive optimism of Ricardo, Say and Tougan-Baranovsky, in whose opinion capitalism can fertilize itself ad infinitum, is in dialectical contradiction which is expressed in the fact that the environment of non-capitalistic social formations is essential for the accumulation of capital and that only by the exchange of matter with them it can progress and last as long as this environment exists[36].

This last thought, emphasized by us, and the contention that accumulation internationalizes the rule of capitalism eliminating the traditional modes of production and at the same time, cannot survive in pure capitalism, is repeated by her several times[37].

[35] Ibid., p. 366.
[36] Ibid., p. 365–366.
[37] Ibid., pp. 416, 466–467 and other.

This, however, is in her approach only an abstract starting point for the analysis of the problem and not a comprehensive concept of the collapse of capitalism. She stresses that it is only a "theoretical formulation"[38] showing a tendency in the development of capitalism—and nothing else.

When her critics have simplified their task interpreting this design as a concept of automatic or mechanical crash explaining the collapse of capitalism exclusively by the impossibility of the realization of surplus value after the disappcarance of the non-capitalist environment, she called her design "a theoretical fiction"[39]. This, however, has not prevented her work from being treated for half a century as a book on the automatic collapse of capitalism.

It seems, however, that Rosa Luxemburg herself outlined primarily a much more interesting and historically correct analysis of contradictions leading to the collapse of capitalism and to the socialist revolution. She made her abstract thesis on the impossibility of the existence of capitalism without the pre-capitalist environment more specific by her analysis of the role of the armament sector in the process of total accumulation. It follows from this analysis that capitalism can create its own internal market which plays in accumulation the same function as an external market. From this point of view, however, most important is her analysis of economic and socio-political conflict of interests between the imperialist countries and the dependent countries as well as the conflict amongst the imperialist powers. The manifestations of these conflicts are imperialist wars, national liberation revolutions, catastrophies facilitating revolutionary struggles of the international proletariat and its final victory. In this sense primarily imperialism was, in her opinion, a preparatory stage to socialism. Ewerything seems to indicate that Rosa Luxemburg was close to the prognostication developed later by Lenin.

---

[38] Ibid., p. 418

[39] R. Luxemburg, *Akumulacja kapitalu, Antykrytyka* (Accumulation of Capital, Anticritique), Warsaw 1963, p. 728.

JÜRGEN KUCZYNSKI
D.D.R.

EIN NEUER WEG FÜR DIE SCHWACH ENTWICKELTEN LÄNDER?

BISHER hat man zwei Entwicklungswege für die ehemals kolonial unterdrückten und in ihrer Entwicklung zurückgehaltenen Länder gesehen: den „kapitalistischen Textilweg" und den „sozialistischen Eisen- und Stahlweg".

Der „Textilweg"[1] wird im allgemeinen von den Experten der kapitalistischen Länder empfohlen. Er ist identisch mit dem Weg ihrer eigenen Länder. In England, den USA, Frankreich und Deutschland entstand zuerst cine mechanisierte Textilindustrie, die teils für die erweiterte Reproduktion des Textilkapitals, teils für die Textilmaschinenindustrie, teils für andere Industrien eine bedeutende Quelle der Kapitalakkumulation war. Der Textilweg hat den großen Vorteil, daß eine moderne Industrie entwickelt wird, deren organische Zusammensetzung relativ niedrig ist, die bei weitem nicht der Kapitalmengen bedarf wie etwa die Schwerindustrie.

Der „Textilweg", den zum Beispiel schon vor bald einem Jahrhundert Indien gegangen war, hat einen entscheidenden Nachteil. Er kann, und muß unter den Bedingungen der Herrschaft einer Kolonialmacht, die Entwicklung der Wirtschaft vereinseitigen, das Land in Abhängigkeit halten und so den gesamten gesellschaftlichen Fortschritt hemmen und stören — wie es eben in Indien der Fall war, das nicht einmal eine der Textilindustrie komplementäre Industrie wie den Textilmaschinenbau schaffen durfte.

Mit Recht wird deswegen der „Textilweg" in den politisch vom Kolonialjoch befreiten Ländern mit Mißtrauen betrachtet.

Das bedeutet natürlich nicht, daß der „Textilweg" an sich und immer verfehlt ist. Man kann sich kleinere Staaten vorstellen, in denen Baumwolle eines von mehreren landwirtschaftlichen Hauptprodukten ist und in denen es durchaus nützlich sein kann, mit dem Aufbau einer Textilindustrie zu beginnen. Auch ohne nachfolgende Errichtung einer Textilmaschinenindustrie — denn wozu sollte jedes kleinc Land eine Textilmaschinenindustrie haben, wenn andere Industrien seinen Produktivkräften gemäßer sind und dem weltwirtschaftlichen Bedarf entsprechen.

Der „Eisen- und Stahlweg" wurde zuerst in der Sowjetunion beschritten. Die

---

[1] An die Stelle der oder neben die Textilindustrie tritt bisweilen auch die Lebensmittelindustrie — zum Beispiel in Argentinien und in einigen Ländern Osteuropas.

Sowjetunion mußte ihn aus zweierlei Gründen gehen. Erstens einmal braucht jedes große Industrieland eine Schwerindustrie als Grundlage der Wirtschaft. Zweitens mußte das damals einzige sozialistische Land so schnell wie möglich eine vom kapitalistischen Ausland völlig unabhängige starke Industrie aufbauen — es hatte historisch keine Zeit für langsame Kapitalakkumulation zuerst in einer Industrie ohne hohe organische Zusammensetzung wie der Textilindustrie. Dieser Weg war heroisch, war opferreich — geopfert wurde einmal ein Großteil der möglichen Verbesserung des individuellen Konsums, der Lebenshaltung der werktätigen Massen, und sodann ein Großteil des möglichen technischen Fortschritts in der Landwirtschaft.

Statt diesen Weg als den Weg des ersten sozialistischen Landes, den für die Sowjetunion einzig richtigen, historisch in jeder Beziehung gerechtfertigten und notwendigen Weg zu bezeichnen, hat man ihn fälschlicherweise allgemein den sozialistischen Weg genannt und eine Theorie entwickelt, die besagt, daß alle sozialistischen Länder, ob groß oder klein, ihn unter allen Umständen gehen müßten, und ist auch nach 1945 entsprechend dieser Theorie verfahren.

Faktisch ist es so, daß ebensowenig wie der „Textilweg" an sich schädlich, der „Eisen- und Stahlweg" an sich nützlich ist.

Faktisch ist unter den Verhältnissen der Existenz eines sozialistischen Lagers, also der Unmöglichkeit des Neuaufbaus eines Kolonialreiches durch imperialistische Mächte, der Weg der beste, der die zeitlich und materialmäßig optimale Entfaltung der Produktivkräfte eines Landes erlaubt.

Nun erfordert heute jede Entfaltung von Produktivkräften Kapital oder sein sozialistisches Äquivalent. Materiell ausgedrückt sind erforderlich:

Arbeitskräfte und sie erhaltende Nahrungsmittel, etc.,
Rohstoffe,
Maschinen,
Fabrikgebäude u. ä.,
Verkehrswege und Verkehrsmittel,

um die wichtigsten Elemente zu nennen.

Dabei können einige dieser Elemente bzw. Teile von ihnen im Austausch erworben werden — wofür dann ein relativer Überschuß von anderen Elementen für den Außenhandel notwendig ist: also zum Beispiel Nahrungsmittel oder Rohstoffe im Überschuß, um Maschinen einzutauschen.

Entscheidend für ein modernes Land ist nicht, was „an sich" es produziert, sondern daß es seine Produktivkräfte optimal entwickelt und dabei politisch wie ökonomisch frei ist — wobei die ökonomische Freiheit miteinschließt, daß es durchmechanisiert bzw. durchchemisiert ist.

Die vom Kolonialjoch bzw. Halbkolonialjoch befreiten bzw. sich befreienden Länder sind alle infolge imperialistischer Unterdrückung ökonomisch schwach und einseitig entwickelt. Sie alle bedürfen vor allem Maschinen und Chemikalien und technisch geschulter Kräfte.

Wie aber sollen sie Maschinen und Chemikalien erhalten? Natürlich können sie sie auf Kredit von sozialistischen oder kapitalistischen Ländern beziehen, und das tun sie auch. Aber Kredite reichen nicht aus und tragen auch nicht zur Unabhängigmachung dieser Länder bei — auch wenn die sozialistischen Länder im Gegensatz zu den kapitalistischen auf jede politische Bindung verzichten, so bleibt doch die objektive ökonomische Bindung, die jeder Kredit beinhaltet.

Es kommt also darauf an, auf Kredit erhaltene Maschinen und Chemikalien dort einzusetzen, wo sie am schnellsten ein marktgängiges Produkt herzustellen helfen, dessen Export sowohl die Kreditzinsen zahlt wie auch die Einfuhr von Maschinen und Chemikalien erlaubt, wie auch andere Einfuhren gestattet, um möglichst bald mit der eigenen Produktion der entsprechenden Maschinen und Chemikalien beginnen zu können.

Die Sequenz dieser Zwecke scheint unter allen Umständen anzudeuten, daß irgendeine lohnende Produktion dieser Art auf Kosten des eigenen Konsums dieser Produkte gehen muß. Denn die Produkte sollen ja ausgeführt werden, um Maschinen und Chemikalien einzuführen, da Mechanisierung und Chemisierung des gesamten Produktionsprozesses eine absolute Voraussetzung der Überwindung des Zurückbleibens dieser Länder ist.

Das heißt, abgesehen von Krediten, scheint es so, daß diese Länder sich gewissermaßen nicht nur am eigenen Schopfe sondern zunächst auch auf Kosten irgendeiner ernsthaften Besserung ihres laufenden Lebensstandards aus dem Sumpf ihrer kolonialen Vergangenheit ziehen müssen.

*

Betrachten wir die Produktivkräfte dieser Länder, dann finden wir, daß in den meisten von ihnen die Mehrheit, soweit sie direkt nutzbar sind, in der Landwirtschaft lokalisiert sind.

Die Landwirtschaft aber hat in der Geschichte auch der fortgeschrittensten Länder eine Vergangenheit äußerst langsam steigender Produktivität.

Wenn wir die Entwicklung in den Vereinigten Staaten nach Wirtschaftszyklen der Industrie zusammenfassen, dann entwickelte sich in den rund 100 Jahren von 1843 bis 1941 die Produktivität (Leistung pro Beschäftigten) so:

Entwicklung der Arbeitsleistung im 19. Jahrhundert

Zyklus	Industrie	Landwirtschaft
1843–1848	100	100
1849–1858	110	108
1859–1868	121	96
1868–1878	171	119
1878–1885	212	148
1885–1897	253	162

Entwicklung der Arbeitsleistung im 20. Jahrhundert

Zyklus	Industrie	Landwirtschaft
1885–1897	100	100
1897–1908	121	119
1908–1914	135	115
1915–1921	148	126
1922–1933	217	148
1933–1941	276	168

In der hier betrachteten Zeit des 19. Jahrhunderts ist die Arbeitsleistung der Industrie um 153 Prozent, in der Landwirtschaft nur um 62 Prozent gestiegen. In der Zeit von 1885/97 bis 1933/41 betrug die Steigerung in der Industrie 176 Prozent, in der Landwirtschaft nur 68 Prozent.

Das heißt, die Geschichte des Kapitalismus erwies, daß die Arbeitsleistung in der Industrie um wesentlich mehr als doppelt so schnell stieg wie in der Landwirtschaft. Es schien offenbar, daß Anlagen in der Industrie wesentlich größeren Nutzeffekt haben würden als in der Landwirtschaft.

Daraus ergab sich, daß Länder in ihrer Entwicklung zurückbleiben mußten, wenn sie, wie alle Kolonialgebiete, vor allem Landwirtschaft trieben. Und umgekehrt: dadurch, daß man die Kolonialgebiete zwang, vor allem Landwirtschaft zu treiben, hielt man ihre Entwicklung zurück, verhinderte man ihre Konkurrenz mit den fortgeschrittenen kapitalistischen Ländern auf dem Weltmarkt.

Solche Überlegungen und Gedankengänge haben sich bis heute vielfach gehalten. Aber sie haben an Gültigkeit verloren.

Betrachten wir die folgenden Zahlen, wieder für die USA:

Entwicklung der Arbeitsleistung im zweiten Drittel des 20. Jahrhunderts

Zeitraum	Industrie	Landwirtschaft
1933–1941	100	100
1942–1945	118	127
1945–1954	129	159
1955–1959	157	248
1960	173	287

Die Arbeitsleistung in der Industrie ist in den USA um 73 Prozent, die in der Landwirtschaft um 187 Prozent gestiegen. Die Verhältnisse haben sich umgekehrt, die Landwirtschaft ist wesentlich schneller vorangekommen.

Das heißt, heute, auf Grund der enormen Entwicklung gerade dieser Produktivkräfte, auf Grund der Mechanisierung und Chemisierung, besitzen wir in der Landwirtschaft einen Wirtschaftszweig, in dem Investitionen einen seher großen Nutzeffekt haben können. Der Grundwirtschaftszweig, die Gewinnung von Nahrung, der in allen Gesellschaftsordnungen vor der kapitalistischen eine schneller steigende Produktivität aufwies als die „industriellen" Gewerbe, ist heute wieder zu einem modernen Betätigungsfeld geworden. Und zwar dadurch, daß dieser Grundwirtschaftszweig zu einer Industrie geworden ist, in der (in den USA) die Investitionen pro Kopf des Beschäftigten doppelt so hoch sind wie in dem, was wir bisher Industrie nannten.

Diese Entwicklung eröffnet völlig neue Perspektiven für alle schwach entwickelten Länder.

Nehmen wir nur folgendes an: In den USA ist in einem Vierteljahrhundert die Arbeitsleistung in der Landwirtschaft um rund 190 Prozent gestiegen. Obgleich in einer zurückgebliebenen Landwirtschaft die Steigerung unter Anwendung der modernen Produktivkräfte natürlich größer sein könnte, nehmen wir nur reichlich die halbe Steigerung, also 100 Prozent an. Wenn wir nun annehmen, daß die bebaute Fläche mit der Größe der Bevölkerung steigt, dann könnte die Hälfte der Produktionssteigerung — 50 Prozent — zu einer ständigen und im Laufe der Zeit erheblichen Steigerung des Konsums der einheimischen Bevölkerung verwandt werden, und gleichzeitig ein erheblicher und ständig steigender Betrag zur „Kapitalakkumulation" verwandt werden.

Selbstverständlich erfordert die Mechanisierung und Chemisierung der Landwirtschaft in den schwachentwickelten Ländern eine Bodenreform. Es ist unmöglich, unter den heutigen Eigentumsverhältnissen eine Mechanisierung und Chemisierung der Landwirtschaft durchzuführen. Eine solche Bodenreform dürfte sich auch nicht damit begnügen, kleine Pachtparzellen oder kleines verschuldetes Bauernland den Bauern als schuldenfreies Eigentum zu übergeben. Das Land müßte zu großen Flächen zusammengefaßt werden, die entweder kollektives Eigentum der Bauern oder Staatseigentum sind. Jeder andere Weg der Hebung der Produktivkräfte in der Landwirtschaft mittels Mechanisierung und Chemisierung wäre nicht nur kostspielig, sondern faktisch ergebnislos. Darum muß man kurz und knapp feststellen: Eine Hebung der Arbeitsproduktivität in der Landwirtschaft der schwachentwickelten Länder mit den Mitteln moderner Mechanisierung und Chemisierung setzt eine Bodenreform mit dem Ergebnis der Verstaatlichung in Großwirtschaften und/oder der Kollektivierung voraus.

Ein solcher Ausbau der Landwirtschaft wird natürlich einen bedeutenden Einfluß auf die Profilierung der Gesamtwirtschaft, insbesondere der Industrie haben. Landwirtschaft und Industrie verwachsen mehr denn je zu einem komplexen Organismus.

Selbstverständlich wäre es unsinnig, die These aufzustellen, daß jedes Land, das Landwirtschaft auf moderne Weise betreibt, auch alle dafür notwendigen Maschinen und Chemikalien selbst produzieren müsse. Das wäre nichts anderes als primitiver Autarkismus, der nichts mit ökonomischer Unabhängigkeit zu tun hat. Es sei denn, es handelt sich um große Länder wie zum Beispiel Indien, die selbstverständlich und mit Recht danach streben, einen umfassende, eine „komplette" Wirtschaft zu besitzen.

Auf der anderen Seite erscheint es nur natürlich, daß Länder mit einer hochentwickelten Landwirtschaft auch eine hochentwickelte Landmaschinen- und Landchemikalien-Industrie besitzen.

Das heißt, es sollte zum Beispiel von einer Reihe afrikanischer Staaten ein gemeinsamer Plan für eine allumfassende Landmaschinen- und Landchemikalien-Industrie aufgestellt werden, und dann sollten sich die einzelnen Länder auf bestimmte Produkte spezialisieren, etwa auf Traktoren oder auf Kleinmaschinen wie Melkmaschinen, auf künstliche Düngemittel oder auf Insekten vernichtende Chemikalien.

Das heißt, jedes Land müßte über eine Industrie verfügen, die zur Mechanisierung und Chemisierung der Landwirtschaft beiträgt, ohne daß sämtliche Maschinen und Chemikalien von jedem Land hergestellt werden.

*

Wie akut diese Problematik ist und wie genau die führenden imperialistischen Länder sie zu sehen beginnen, ergibt sich nur allzudeutlich aus folgenden Angaben über die Stickstoffdünger-Situation.

Die Stickstoffdüngerproduktion in der kapitalistischen Welt entwickelte sich wie folgt:

Produktion in 1000 t Reinstickstoff

	1938*	1954/55	1956/57	1959/60	1960/61
Westeuropa	1 101	2 732	3 169	3 990	4 660
davon:					
Westdeutschland	354	745	895	1 051	1 180
Frankreich	196	360	451	574	670
Italien	109	313	366	590	654
Großbritannien	124	305	334	400	449
Niederlande	99	293	330	406	417
Belgien	93	241	232	305	278
Norwegen	90	192	214	245	276
Österreich	2	113	137	162	162
Amerika	530	2 296	2 567	3 066	3 170
davon:					
USA	240	1 812	2 059	2 605	2 680
Asien	370	737	898	1 100	1 230
davon:					
Japan	356	633	781	932	1 030
Indien	5	83	81	81**	96
Afrika u. Ozeanien	5	46	66	84	100
Kapitalistischer Teil der Welt	2 006	5 811	6 700	8 240	9 160

* Ohne die heute sozialistischen Länder; alle Angaben beziehen sich auf das jeweilige Düngejahr.

** 1958/59.

Quellen: *Fertilizers, An Annual Review of World Production, Consumption and Trade* (FAO), Rom, 1960 (für 1954–1960) id. 1953 (für 1951/52). *UN Food and Agriculture Organization* (FAO), *Yearbook of Food and Agricultural Statistics* 1951, Rom, S. 173 (für 1938). — *FAO Production Yearbook* 1961, S. 257 (für 1960/61).

Der Verbrauch nahm die folgende Entwicklung:

Verbrauch in 1000 t Reinstickstoff

	1938	1954/55	1956/57	1959/60	1960/61
Westeuropa	1 089	2 167	2 525	3 167	3 060
davon:					
Westdeutschland	343	452	527	624	618
Frankreich	218	348	403	505	565
Italien	129	238	273	361	332
Großbritannien	60	252	307	430	460
Niederlande	95	187	194	212	224
Belgien	63	93	88	99	100
Norwegen	11	36	45	49	48
Österreich	5	30	38	45	47
Amerika	390	2 049	2 310	3 013	3 280
davon:					
USA	346	1 779	1 937	2 533	2 734
Asien	500	958	1 168	1 269	1 690
davon:					
Japan	253	521	590	594	753
Indien	22	117	165	224	376
Afrika u. Ozeanien	107	218	238	263	340
Kapitalistischer Teil der Welt	2 086	5 392	6 241	7 712	8 370

Verbrauch under (−) oder über (+) Produktion 1960/61
in 1000 t Reinstickstoff

Westeuropa	−1 600
USA	+ 54
Asien	+ 460
Japan	− 277
Indien	+ 280
Afrika und Ozeanien	+ 240

In den schwachentwickelten Ländern liegt der Verbrauch wesentlich über der Produktion.

Der Verbrauch von Stickstoffdünger in den schwachentwickelten Ländern ist minimal:

Kilogramm Verbrauch von Stickstoffdünger
pro Hektar landwirtschaftlicher Nutzfläche

Südamerika	0,37
Afrika	0,35
Asien	2,28
Westeuropa	20,70

Man erkennt sofort, welch ungeheure Mengen Stickstoffdünger gebraucht werden, um die Chemisierung in Süd- und Mittelamerika, in Afrika und in Asien auf ein modernes Niveau zu bringen. Eine Verhundertfachung der Produktion würde gerade erst einen ernsthaften Anfang der Stickstoffdünger-Chemisierung der schwachentwickelten Länder bringen.

Das ist gewissermaßen die eine Tatsachengruppe, die die hier behandelte Problematik so aktuell macht.

Und nun zu einer anderen Tatsachengruppe.

Zuerst einige historische Fakten. Vor dem zweiten Weltkrieg gab es ein Internationales Stickstoffkartel. In dieser Convention Internationale de l'Azote spielte die sogenannte DEN-Gruppe, die aus den Konzernen Deutschlands, Englands und Norwegens bestand, die Hauptrolle. Sie kontrollierte 73,6 Prozent des europäischen Exports, während das ganze Kartell 79,6 Prozent des Weltexports beherrschte. Der Rest stand unter der Kontrolle der US-amerikanischen und der zu einem Teil von diesen kontrollierten chilenischen Monopole[2], soweit es sich um Produktion und Export des chilenischen Natursalpeters handelte. Dem Kartell gehörten außerdem die Stickstoffkonzerne bzw. die nationalen Kartelle Belgiens, Frankreichs, der Niederlande, Italiens sowie Polens und der Tschechoslowakei an. Die chilenischen und japanischen Konzerne beteiligten sich durch besondere Vereinbarungen. Im Jahre 1934 sicherte sich Chile vertraglich Einfuhrquoten auf den europäischen Märkten. Japan schloß 1936 mit dem Kartell eine Vereinbarung über Preise und Quoten[3]. Dieses internationale Kartell monopolisierte also praktisch den gesamten Welktmarkt für Stickstoffdünger. Es setzte einheitliche Preise fest, sicherte den gegenseitigen Schutz der heimischen Märkte und verteilte Exportquoten. Mit Produktionseinschränkungen unter Entschädigung der betroffenen Konzerne aus einem gemeinsamen Fonds wurde der Markt ,,reguliert". Das Kartell funktionierte bis zum Beginn des zweiten Weltkrieges. Es unterhielt in Basel unter der Firmierung ,,Internationale Gesellschaft der Stickstoff-Industrie AG" eine auch heute noch nicht gelöschte Abwicklungsstelle, deren Verwaltungsrat bis weit in den zweiten Weltkrieg hinein ununterbrochen vom Generaldirektor der I. G. Farbenindustrie AG, Herman Schmitz, geleitet wurde, obwohl der Chemietrust mit Kriegsbeginn den Vertrag mit dem Kartell gelöst hatte[4].

Sodann einige Fakten aus der Gegenwart. Im Juli 1962 wurde in Zürich die Nitrex AG gegründet. Als Geschäftszweck wurde der gemeinsame Export von Stickstoffdünger genannt. Es handelt sich bei dieser Gründung um nicht weniger als die Wiederbelebung des ehemaligen Internationalen Stickstoffkartells.

Die Gründung der Nitrex AG ist ein weiteres Beispiel dafür, wie die ehemaligen internationalen Kartelle, von denen einige schon bald nach 1945 ihre Tätigkeit insgeheim wieder aufgenommen haben, jetzt ihre Tarnung abwerfen und wieder an die Öffentlichkeit treten. Das neue Kartell ist noch nicht so mächtig wie das alte. Die im Vorkriegskartell führende DEN-Gruppe, die durch Kartellbindungen

---

[2] Stocking, George W., Watkins, Myron W., *Cartels in Action*, New York 1946, S. 145.
[3] *La Documentation Française*, Paris 1950, Nr. 1305, Annexes, S. 12.
[4] VGL. ,,DWI-Berichte", 14. Jg. Nr. 8, 2. April Heft, Berlin 1963, S. 13 f.

und Lizenzverträge zusammengehalten worden war, ist zerfallen. Der britische Konzern Imperial Chemical Industries (ICI), der vor dem zweiten Weltkrieg mit der I. G. Farbenindustrie, Du Pont de Nemours und anderen internationalen Konzernen in zahlreichen Chemiekartellen zusammenarbeitete, ist zwar bisher noch nicht Mitglied des neuen Kartells geworden, führt jedoch seit längerem „informatorische Gespräche".

Das Zögern des britischen Konzerns drückt keine grundsätzliche Opposition gegen das Kartell aus. Die geschäftlichen Interessen der Düngemittelsparte der ICI waren bisher vorwiegend auf den britischen Binnenmarkt gerichtet, den sie mit ihren Tochtergesellschaften zu etwa 75 Prozent monopolisiert. Die abwartende Haltung der ICI ist einmal von der Entwicklung ihrer Exportinteressen beeinflußt, zum anderen spielte auch eine vorsichtige Einschätzung der Brüsseler Verhandlungen über den Beitritt Großbritanniens zur EWG eine Rolle. Das britische Monopol zeigte sich zunächst nur an einer Vereinbarung über den gegenseitigen Schutz der Binnenmärkte interessiert.

Im Gegensatz zu ICI hat der größte britische Produzent von Mischdünger, Fisons Ltd., der infolge seiner internationalen Verzweigung sehr bedeutend ist, durch seine belgischen und holländischen Niederlassungen und Beteiligungen (z.B. an dem bedeutendsten belgischen Produzenten Union Chimique Belge) eine Verbindung zur Nitrex. Die Tochtergesellschaften sind den jeweiligen nationalen Kartellen angeschlossen.

Ein bedeutender Außenseiter ist schließlich auch die zum italienischen Staatskonzern ENI gehörende ANIC (Azienda Nazionale Idrogenazione Combustibili). Es ist bekannt, daß der Staatskonzern — nicht zuletzt unter dem Druck der fortschrittlichen Kräfte in Italien — auf vielen Gebieten seiner Produktion, zum Beispiel Erdöl und synthetischen Fasern, eine autonome Geschäftspolitik betreibt, die ihn in Gegensatz zu den großen internationalen Monopolgruppen gebracht hat. Seine Preise für Stickstoffdünger lagen bisher unter dem Niveau der Nitrex-Konzerne.

Alle übrigen italienischen Produzenten, einschließlich des Montecatini-Konzerns, der mit einem Anteil von 50 Prozent der Gesamtproduktion am bedeutendsten ist, gehören als Mitglieder ihres nationalen Kartells der Nitrex an[5].

Die Nitrex hat ein Ziel, das im alten internationalen Kartell nur eine ganz untergeordnete Rolle gespielt hat, und dessen Angabe die zweite Tatsachengruppe beschließen soll. Das Deutsche Wirtschaftsinstitut umreißt das Ziel so:

„Die Nitrex wird ihre Tätigkeit vor allem auf die ökonomisch schwachen Länder konzentrieren. Sie hat in ihr Programm die Ausnutzung der von der FAO, der Ernährungs- und Landwirtschafts-Organisation der Vereinten Nationen, proklamierten Kampagne 'Kampf gegen den Hunger' aufgenommen. In dieser Kampagne, die am 1. Juli 1960 offiziell begann und bis Ende 1965 laufen soll, ist unter

---

[5] Vgl. ebendort, S. 11 und 14.

anderem eine Verstärkung des technischen Beratungsdienstes vorgesehen, um die Landwirtschaft in ökonomisch schwachen Ländern mit neuzeitlichen Produktionsmethoden vertraut zu machen. In diesem Punkte des Programms sehen die Düngemittelkonzerne gewisse Möglichkeiten, die Tätigkeit der FAO ihren Profitinteressen unterzuordnen. Schon vor dem Beginn der Kampagne, am 26. und 27. April 1960, versammelten sich am Sitz der FAO 41 Vertreter der Düngemittelindustrie aus verschiedenen Ländern. Sie beschlossen eine Unterstützung des Programms und stellten der FAO einen Beitrag von einer Million Dollar zur Ausarbeitung eines internationalen Düngemittelprogramms in Aussicht[6]. Wie die Nitrex an diese Aufgabe herangehen will, erläuterte ihr Direktor Hawlik in einer Pressekonferenz:

‚Von Zürich aus sollen Agronomen vor allem in Entwicklungsländern eingesetzt werden, die in die Lage zu versetzen sind, den Kampf gegen den Hunger selbst in die Hand zu nehmen. In der notwendigen Kreditgewährung glaubt die Nitrex leistungsfähiger zu sein, als ein einzelnes Mitglied. Auch bei Kompensationsgeschäften für devisenschwache Länder will die Nitrex behilflich sein, eventuell durch Einschaltung selbständiger »barter«-Firmen.'[7]

Die Düngemittelkonzerne versuchen also, eine fortschrittliche Aktion der UNO für ihre profitsüchtigen Zwecke auszunutzen. Sie können so einen Teil ihrer Kosten für die Auslandswerbung als Beitrag zu einer humanitären Aktion verbuchen, sofern nicht sogar die Kartell-Propaganda unmittelbar durch die FAO finanziert wird".

Das heißt, das internationale Monopolkapital begreift sehr wohl, welche Rolle sowohl für die Profitanhäufung wie auch für die ökonomische Fesselung die Chemisierung der schwachentwickelten Länder in einer Periode rapider Entwicklung der landwirtschaftlichen Produktivkräfte in den fortgeschrittenen Ländern spielen kann.

*

Mechanisierung und Chemisierung der Landwirtschaft und Profilierung der aufzubauenden Industrie nicht zum wenigsten unter dem Gesichtspunkt der selbständigen Modernisierung der Landwirtschaft sind also ein wichtiger Faktor in der Wirtschaftspolitik, ja im gesamten Befreiungsprozeß der schwachentwickelten Länder. Mit dieser Mechanisierung und Chemisierung der Landwirtschaft wird auch eine moderne das heißt gelernte und gebildete Arbeiterklasse in Stadt und Land heranwachsen.

Es kann, meiner Ansicht nach, sehr wohl sein, daß dieser „Landwirtschaftsweg", der Weg der Mechanisierung und Chemisierung, für eine Reihe von schwachentwickelten Ländern geeigneter ist als der „Textilweg" oder der „Eisen- und Stahlweg".

---

[6] „United Nations Review", New York 1960, August, S. 13.
[7] „Deutsche Zeitung", Köln, Nr. 205 vom 4. 9. 1962.

Jurgen Kuczynski

A NEW WAY FOR UNDERDEVELOPED COUNTRIES?

Summary

So far two ways of development have been recommended to the underdeveloped countries: a "capitalist textile way" and a "socialist iron and steel way".

The first is based on the historical experience of the present-day developed capitalist countries. Its main advantage is that a modern industry can be built up with relatively low capital inputs. However, when applied to an underveloped country under colonial rule, it could not but lead to a unilateral development of its economy and maintain its dependence upon the colonial power, thus hampering the whole social progress. It is, therefore, mistrusted by the countries concerned.

The "iron and steel way" was followed, first, by the Soviet Union under specific circumstances which imposed such a way of industrialization at the expense of potential improvement in consumption and the technological level of agriculture. Notwithstanding, it has been called amiss a "socialist way" and was followed by other socialist countries.

Neither of them is actually bad or good per se. But, given the existence of the socialist camp and the impossibility of rebuilding a colonial system, the best way to be followed is actually that enabling the country to develop its productive forces at an optimum rate and in optimum directions. What matters for a modern economy is not what the country produces, but the optimum way in which its productive forces are being developed. This involves a political and economic freedom, the latter including a high level of mechanization and chemization of production.

It follows that the underdeveloped countries need, first of all, machines and chemicals. These may be obtained on credit which, however, does not suffice to cover their needs nor contributes to their independence. They should be used, therefore, so as to expand the marketable exports of the country as rapidly as possible, thus providing the means to increase the inflow of those products and to develop their production at home. Since mechanization and chemization of the whole production process is an absolute condition of overcoming backwardness, it seems that, apart from credits, this cannot be obtained except at the expense of any significant improvement in living standards in the near future.

Most of the productive forces of the underdeveloped countries are located in agriculture. Comparative developments in industry and agriculture during a century before the World War II were in favour of the former: this may be seen in the data for the United States, where the productivity *per capita* employed in industry rose twiee as rapidly as in agriculture. This is just the reason for the present underdevelopment of these countries. But since that time the trend has been reversed: the rate of growth of productivity in American agriculture has been much higher than in industry, owing to its mechanization and chemization. Agriculture has thus become a branch of the economy, where investment can bring about considerable advantages.

These developments open completely new prospectives to all underdeveloped countries. Their agriculture, if mechanization and chemization is used, can provide for both a steadily rising consumption and an increasing rate of capital accumulation. The effective use of these methods requires an agrarian reform involving the creation of big enterprises nationalised or collectively owned. This will lead to a complex development of agriculture and industry which will have to supply the

necessary machines and chemicals. Of course, a specialization in particular lines of industrial production for these purposes will be necessary so far as small countries are concerned.

The importance of the problem is exemplified in the data relative to the production and consumption of nitrogen fertilizers in developed and underdeveloped countries which show enormous differences in this respect. Any serious step towards a chemization of agriculture in the underdeveloped part of the world would require a multiplication of the present production of nitrogen fertilizers. On the other hand, the activities of big monopolies aiming at drawing profits from forthcoming developments in this field should be pointed out. This is the case of Nitrex, an international trust, which has recently been founded by the main producers of these fertilizers, apparently to support the FAO Freedom from Hunger campaign: their true aim is to intercept that humane action and to use it to make profits.

The conclusion of the author is that an 'agriculture way', a way of mechanization and chemization, may be more suitable for a number of underdeveloped countries than either of the ways described at the beginning.

Oskar Lange
Poland

QUANTITIVE RELATIONS IN PRODUCTION

I

Production is a combination of co-ordinated processes of labour in which conscious and purposeful human activity, i.e. labour, transforms objects of labour using for this purpose means of labour. In other words: in production combination of human labour and means of production takes place and the result of this union is the product.

Labour and means of production are factors in the production process or briefly: *factors of production.* Such factors are concrete kinds of labour as, spinning, processing metals, ploughing, transportation of goods and so on; and various concrete means of production like wool, pig iron sulphuric acid, spades, machine-tools, electric motors, locomotives, etc. Individual concrete kinds of labour constitute the so-called *personal factors of production*; they require not only the existence of people capable of labour but also the possession by these people of adequate knowledge, and skill, i.e. proper qualification. The various means of production constitute, the *physical factors of production* and their concrete character and form are an expression of the technique of production, i.e. material technique applied in various labour processes[1].

For a product to be created, a combination of personal and physical factors of production must take place. The way in which this combination takes place is different in various social processes of production and what is more, is the feature which characterises a definite historical mode of production. Moreover, the fact of the combination itself is an indispensible condition of production in any of its social forms. Marx expressly emphasised this: "Whatever the social form of production, labourers and means of production always remain factors of it. But in a state of separation from each other either of these factors can be such only potentially. For production to go on at all they must unite. The specific manner in which this union is accomplished distinguishes the different economic epocks of the structure of society from one another"[2]. Moreover in this combination, the role of personal and physical factors of production is different. Personal factors, or human labour in its various concrete shapes, are the active and creative factor in the production process,

---

[1] Marx uses the terms "personal" and "material" factors of production. *Capital*, Moscow 1961, vol. II, p. 35. In German the words *persönlich* and *sachlih* are used.

[2] Marx, *Capital*, Vol. II, Moscow 1961, p. 34.

while means of production, or physical production factors, constitute the material objects which human labour sets in motion and transforms.

Therefore, we define human labour which constitutes the active factor in the production process, as *living labour*, as opposed to means of production which are the result of earlier labour, in some way *objectified* in them[3].

The nature of the production process as a combination of labour and means of production leading to the creation of a product, may be expressed by the schematic formula:

$$\begin{bmatrix} \text{labour} \\ \text{means of production} \end{bmatrix} \to \text{product.}$$

By using the symbol L to signify labour, Q for means of production and P to denote the product, the formula may be written thus:

$$\begin{bmatrix} L \\ Q \end{bmatrix} \to P.$$

In this formula L stands for the aggregate of various kinds of concrete labour, e.g. spinning, weaving, metal processing, assembling, bricklaying, sowing, ploughing etc. Similarly, Q stands for the aggregate of various concrete means of production, e.g. coal, pig iron, definite kinds of tools and machines, definite kinds of land (arable land, meadows, pastures, forested land) etc. Individual concrete kinds of labour or means of production entering into the composition of the aggregates L or Q, we call *components* of the given aggregate[4].

---

[3] K. Marx, *Capital*, Vol. I, Moscow 1961, p. 181: "Though an use-value in the form of a *product* issues from the labour-process, yet other use-values, products of previous labour, enter into it as means of production". The failure to distinguish between the active role of personal factors of production, or living labour, and the auxiliary role of physical factors of production which constitute the material means of human activity, is the basis of thc so-called theory of factors of production. This theory, which goes back to J. B. Say (*Traité d'économie politique*, Paris 1803) was widely accepted among the proponents of the so-called Austrian school and the Neoclassical school. It treats human labour and physical means of production as production factors of equal rank which jointly create the product. Each of the factors of production contributes equally to thc making of the product, "has a share" in the product, which is their composite creation. This theory fetishises the production process, interpreting it in a naturalistic way, as a natural proeess of automatic transformation of factors of production into the product. By placing on the same level human labour and physical faetors of production this theory ignores the human character of the production processes as conscious and purposeful human activity. In practicc, this theory serves as a basis for apologetic conclusions justifying the categories of distribution of the social product peculiar to the capitalist mode of production, as being, allegedly, the result of the natural character of the production process. See O. Lange, *Political Economy*, Vol. I, London 1963, Part III.

[4] Wc can denote the components of the aggregate L, or individual concrete kinds of labour by $L_1, L_2 \ldots L_m$, and the components of the aggregate Q, or individual concrete means of production, by $Q_1, Q_2 \ldots Q_n$. Then thesc aggregates can be presented symbolically, as follows:

$$L = \begin{bmatrix} L_1 \\ L_2 \\ \cdot \\ \cdot \\ L_m \end{bmatrix} \qquad Q = \begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ Q_n \end{bmatrix}.$$

The product P appears on the right hand side of the formula may be a single good produced in the production process. Frequently, however, a number of various products is created at the same time by the same production process. In such cases, we speak of *joint production*. Sometimes in joint production, one distinguishes, between the main product for the sake of which production is carried on and secondary products, usually called by-products. However, it is often difficult to distinguish between the main product and by-products; all products appearing in the joint-production process may be to an equal degree the purpose of productive activity. This is so in the case of joint production of radio and TV sets, or motor-cycles, bicycles and motor-boats. If the production process has the character of joint production, then the symbol, P, appearing in the above schematic formula, stands for an aggregate of individual products created by a joint process: individual products are the components of this aggregate[5].

Concrete kinds of labour or means of production and the product (or products in the case of joint production) are quantities which are expressed by denominated numbers indicating the units in which these quantities are measured. As far as means of production are concerned, its is necessary to distinguish between the case of circulating and the case of fixed means. Circulating means are completely used up in the course of one production period. This use-up is measured in the same units as products, i.e., weight volume, length, energy, pieces, etc. Fixed means, however, preserve their natural shape through more than one period of production. In the

---

It is possible, then, to write the above schematic formula presenting the production process, in a developed form:

$$\begin{bmatrix} L_1 \\ L_2 \\ \cdot \\ \cdot \\ L_m \\ Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ Q_n \end{bmatrix} \rightarrow P.$$

The individual components of the aggregates L and Q are shown in this form of the formula. Thus the aggregate character of labour and means of production is clearly marked.

[5] In such a case, we may denote the individual components of the aggregate of products, P, by P_1, P_2 and P_k, and write:

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_k \end{bmatrix}.$$

It is possible to put this expression into the formula mentioned in the earlier note, on the right hand side of the formula.

course of a production period, they are utilised, but still preserve their usefulness. Thus a measure of use-up is not suitable here, as in the case of circulating means, but a measure of utilisation is required. Utilisation is measured in units of time, for instance, in hours of utilisation of a definite machine, building or truck (e.g. in machine-hours, building-hours and truck-hours) etc. Similarly labour is measured in units of time (e.g. man-hours) of various kinds of labour. The units of measurement for individual kinds of labour, means of production and products mentioned here, we call *natural* or *physical units.*

The amount of labour applied in the production process, the quantity of circulating means of production used up in the process and the quantity of fixed means of production utilised, we indicate by the general namc of *outlay* or *input*[6]. The quantity of the product obtained, however, we call the *return* or *output*[7] of the given production process. Clearly, both the input (or inputs) and output are measured in natural units. Input and output are defined for a given period of time, e.g. for the production period, or for a calendar unit of time (a month, quarter, year etc.). Thus they have, the dimension of "that much of natural (physical) units in such time", or a given quantity of natural (physical) units in a given period of time. If the period of time we are considering changes, then the amount of input and output changes proportionately. We express this by saying that input and output have the character of *streams*, i.e. a *flow* of a certain quantity of natural units in the period of time under consideration. However, the qunatity of fixed means of production taking part in the production process (as opposed to the outlay (input) of their utilisation) is measured in natural units without reference to time, e.g. the quantity of machines, of building, the surface of cultivated land etc. Such quantities, defined without reference to time, we call *stocks*[8]. Inventories of circulating means (as opposed to their input, i.e. their use-up in the production process, which is a stream) are also stocks.

---

[6] In French *dépenses*, in German *Aufwand*, and in Russian *zatraty*.

[7] In French *rendement*, in German *Ertrag* and in Russian *prichod*.

[8] As is well-known, all physical quantities may be expressed in units of length, mass and time i.e. centimetres, grams, seconds (denoted by *L.M.T*, or in *C.G.S.* units). The units in which a physical quantity is expressed constitutc its dimension. For instance, the dimension of velocity we write in symbols $\frac{L}{T} = LT^{-1}$, the dimension of acceleration LT^{-2}, the dimension of force MLT^{-2}, and the dimension of mechanical work ML^2T^{-2}, etc. By analogy it is possible to speak of the dimension of economic quantities. Considering for the present only quantities measured in physical units and denoting such physical (natural) units by N, stocks have the dimension N, and streams have the dimension NT^{-1}. W. S. Jevons was the first to consider systematically in economic theory the dimension of the quantities under consideration. Scc W. S. Jevons, *The Theory of Political Economy*, London, 1871. P. H. Wicksteed corrected certain errors in Jevon's treatment of the problem in *The Common Sense of Political Economy*, Vol. II, Second Edit., London 1946, appendix: *Dimensions of Economic Quantities*. This appendix is a reprint of a paper published in *Palgrave's Dictionary of Political Economy*. Sce on the subject also S. C. Evans, *Mathematical Introduction to Economics*, New York–London 1930, Chapter II and finally A. J. Boyarski *Matematiko-ekonomicheskiye ocherki*, Moscow 1962, Chapter VII. See also O. Lange, *Teoria reprodukcji i akumulacji* (Theory of Reproduction and Accumulation), Warsaw 1961, pp. 20–21.

Inputs of particular kinds of labour and means of production and the output of a given production process, may be defined for an individual productive establishment (plant) or for a group of such establishments (e.g. for a combine of plants or other industry group), or finally for a whole branch of production (e.g. the steel industry). Input and output for a group of establishments or for a whole production branch are obtained by summing the relevant inputs and outputs in individual productive establishments. This summation takes place in such a way that the separate inputs of individual concrete kinds of labour and individual concrete means of production are added (and in the case of joint production, also the quantities of individual products are added. As a result an aggregate of such sums is obtained and this expresses the input of labour or the input of means of production, or also the output of a group of productive establishments or a branch of production. These sums constitute the components of the aggregate.

Quantities expressed by means of aggregates (i.e. ordered sets) of real numbers, which have the property that it is possible to add them together by the adding the individual numbers which constitute their components, and to multiply, them by a real number through multiplication by this number of all individual components, are called *vectorial* quantities or more simply *vectors*[9].

---

[9] Let $(x_1, x_2 \dots x_n)$ and $(y_1, y_2 \dots y_n)$ be ordered sets of real numbers. We say that these sets are n dimensional vectors and the numbers x_1, x_2 and x_n and $y_1, y_2 \dots y_n$ are components of these vectors, and we write

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad \text{and } y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$

if

$$x+y = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ x_n+x_n \end{bmatrix}$$

and

$$\lambda x = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda x_n \end{bmatrix}, \ \lambda y = \begin{matrix} \lambda y_1 \\ \lambda y_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda y_n \end{matrix}$$

where λ is an arbitary real number (scalar). It is possible to write the vector components in a row instead of in a column, i.e. $x = (x_1, x_2 \dots x_n)$ and $y = (y_1, y_2 \dots y_n)$. Further we say that the vectors x and y are equal, or that $x = y$ if the corresponding components are all equal, i.e. $x_1 = y_1$, $x_2 = y_2 \dots x_n = y_n$.

The real numbers of which such an aggregate is made up, are called the *components* of the vector. As opposed to vectorial quantities, quantities which are expressed by a single real number (and, thus, not by means of an aggregate of numbers) are called *scalar quantities* or *scalars*. The output P is a scalar quantity, if only one good is produced; however, it is a vectorial quantity in the case of joint production.

The relation between the output of the product and the input of labour and means of production is the basic quantitative relation occurring in production. This ratio depends on the complex of conditions which Marx called the *productive power of labour*. The productive power of labour depends on the fact that "the same quantum of labour yields in a given time a greater or less quantum of product dependent on the degree of development in the conditions of production"[10]. The productive power of labour depends on the whole level of growth of productive forces in given historical conditions. Marx says: "The most varied circumstances, amongst others the average level of the worker's skill, the degree of development of science and its technological application, the social organisation of the production process, the extension and effectiveness of the production means and natural conditions, determine the productive power of labour"[11]. A change in the quantity of product, or output, connected with a given input of labour (i.e. living labour) and means of production is the concrete result of a change in the productive power of labour.

In order to obtain a closer insight into this relation we shall consider the input of individual concrete kinds of labour and of particular concrete means of production per unit of output obtained. We call this input, per *unit input* (or *outlay*). It is obtained by dividing all the input by the quantity of product received i.e. by the output. The quotients resulting from this division constitute the components of the vector of per unit inputs. Using the notation of the schematic formula which presents the production process, the per unit unit input vector may be written as follows[12]:

---

[10] K. Marx, *Capital*, Vol. I, Moscow 1961, p. 519. On the subject of the concept of the productive power of labour in Marx and the relationship of this concept to the concept of labour productivity about which I shall treat later, see S. G. Strumilin, *Problemy proizvoditielnosti truda* in the book *Izbrannye proizviedienya* (Selected Works), Vol. 3–*Problemy ekonomiki truda* (Problems of Economics of Labour), Moscow 1964, pp. 423 etc. and F. D. Markuzon, *Izmienienie proizwoditielnosti truda w kapitalisticheskikh gosudarstvakh, Uczeniye zapiski po statistikie*, Vol. III, Moscow 1957, p. 249. See also B. Minc, *Ekonomia polityczna socjalizmu*, (The Political Economy of Socialism), 2nd edition, Warsaw 1963, pp. 190–193.

[11] K. Marx, *Capital*, Moscow 1961, vol. I, p. 40, also p. 314. "By increase in the productiveness of labour, we mean generally an alteration in the labour-process of such a kind as to shorten the labour-time socially necessary for the production of a commodity and to endow a given quantity of labour with the power of producing a greater quantity of use-value".

[12] In this diagram $\frac{L}{P}$ and $\frac{Q}{P}$ are the products of the vector Q and the real number $\frac{1}{P}$. Let $L_1, L_2 \dots L_m$ and $Q_1, Q_2 \dots Q_n$ be the components of these vectors. Then

$$\begin{bmatrix} \dfrac{L}{P} \\ \\ \dfrac{Q}{P} \end{bmatrix}$$

In this formula $\frac{L}{P}$ represents the aggregate (vector) of per unit inputs of individual concrete kinds of labour, and $\frac{Q}{P}$ represents the aggregate (vector) of per unit unit inputs of individual concrete means of production. The components of these aggregates (vectors) or the individual per unit inputs, are also called *technical coefficients of production*[13]; for they depend on the technical condition under which the production process takes place. Coefficients of production are, thus, inputs of production factors, i.e. concrete kinds of labour and means of production which, under the given technical conditions, are necessary for obtaining a unit of output of the product[14].

The division of inputs by the output obtained is obviously possible only when output is a scalar quantity. In the case of joint production output is a vector, i.e. an aggregate of individual products. Then we choose one of the individual products arbitrarily as "reference product" and we calculate the individual per unit inputs, or coefficients of production in relation to the reference product. Beside the coefficients of production we also then have additional coefficients expressing the output of individual products obtained jointly with a unit of reference product[15].

$$\frac{L}{P} = \begin{bmatrix} \dfrac{L_1}{P} \\ \dfrac{L_2}{P} \\ \cdot \\ \cdot \\ \cdot \\ \dfrac{L_m}{P} \end{bmatrix} \quad \text{and} \quad \frac{Q}{P} = \begin{bmatrix} \dfrac{Q_1}{P} \\ \dfrac{Q_2}{Q} \\ \cdot \\ \cdot \\ \cdot \\ \dfrac{Q_n}{P} \end{bmatrix}$$

It is possible to present the above formula in a form to make these components explicit.

[13] Vide O. Lange *Introduction to Econometrics*, 2nd ed. Oxford 1962, pp. 227–228 and O. Lange, *Teoria reprodukcji i akumulacji* (Theory of Reproduction and Accumulation) pp. 72–73. The coefficients of production were introduced into economic analysis in a systematic way by Leon Walras in his *Eléments d'économie politique pure*, Paris 1874, Chapter IV. Walras used the term *coefficients de fabrication.*

[14] In the practice of economic planning coefficients of production are called *technical norms.*

[15] Let $P_1, P_2 \dots P_k$ be the components of the vector P expressing output in the process of joint production and let, for instance, P_1 be the component corresponding to the reference product. Then we obtain the coefficients $\frac{P_2}{P_1}, \frac{P_3}{P_1} \dots \frac{P_k}{P_1}$. For further details see O. Lange *Optymalne decyzje* (Optimum Decisions), Warsaw 1964.

Instead of the per unit inputs of production factors, or coefficients of production their reciprocals may be considered. The reciprocal of per unit input we call the *productivity* of a given factor of production. We thus speak of the *productivity* of labour and the productivity of the means of production. The productivity of labour is the aggregate (vector) of the productivities of individual concrete kinds of living labour applied in the production process; the productivity of means of production is the aggregate (vector) of the productivities of individual concrete means of production. Each conrete kind of labour and each concrete means of production has its own productivity[16]. This productivity depends upon the production technique applied; the technique applied establishes a relation between the productivity of labour (i.e. living labour) and the input of means of production. As a rule greater productivity of labour requires a larger amount of means of production combined with labour. An increased productivity of labour is conncected with processing of a larger quantity of raw materials and with greater equipment of living labour with means of labour (in particular with instruments of labour).

A given production technique is characterised by a definite aggregate (vector) of per unit inputs, i.e. coefficients of production, or—which amounts to the same—by a definite aggregate of productivity of the factors of production. As a rule it is possible to produce a given product by various production techniques or, as we shall say, by various *technical processes*. Each of these processes is characterised by an appropriate vactor of per unit inputs (coefficients of production). If a given product can be produced by different technical processes, the various technical possibilities can be stated in the following schematic table[17]:

$$\begin{bmatrix} \frac{L_1}{P} & \frac{L_2}{P} & \cdots & \frac{L_r}{P} \\ \frac{Q_1}{P} & \frac{Q_2}{P} & \cdots & \frac{Q_r}{P} \end{bmatrix}.$$

In this table each column stands for an aggregate (vector) of per unit inputs corresponding to a given technical process. Each row of the table indicates the per

[16] Marx speaks of the productivity of means of production in *Capital*, Vol. I, p. 42 and 654. He uses the expression "effectiveness (Wirkungsfähigkeit, Wirksamkeit)" of means of production.

[17] This table can be written in expanded form, showing the components of per unit input of labour and of means of production. In this expended form, the table looks as follows:

$$\begin{bmatrix} \frac{L_{11}}{P} & \frac{L_{12}}{P} & \cdots & \frac{L_{1r}}{P} \\ \frac{L_{21}}{P} & \frac{L_{22}}{P} & \cdots & \frac{L_{2r}}{P} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{L_{m1}}{P} & \frac{L_{m2}}{P} & & \frac{L_{mr}}{P} \end{bmatrix}.$$

unit input of a given production factor in different technical processes. We call such a table the *technical production matrix*.

Technical processes may also be differentiated with regard to the length of the production period. It is possible to reduce such a difference to a difference in per unit inputs. If, in two technical processes, the quantity of production factors used up or utilised and the output of the product obtained is the same but one process lasts longer than the other, then the output of product obtained per unit of time is smaller in the process taking up a longer production period. The per unit inputs are, then, correspondingly larger. The difference is expressed in the technical production matrix. Moreover, in various technical processes the input of factors of production can be differently allocated within time. In this case it is possible to treat input; in different periods of time as inputs of different factors of production. Thus a different allocation of inputs in time finds expression in the technical production matrix.

The technical production matrix shows the variety of technical processes by which it is possible to produce a given product. If the given product is produced by one of the technical processes to which a column of per unit inputs in the technical production matrix corresponds, we say that the product is made by a *pure* technical process. However, it is possible to make the given product in such a way that a certain quantity of the product is made by one technical process, another quantity by a second technical process, and yet another quantity by a third technical process, and so on. We say, then, that the product is produced by a *mixed* technical process. The mixed technical process consists in producing various quantities of the same product by different pure technical processes. In the utilisation of a *mixed* technical process, the per unit inputs coefficients of production of the factors of production are weighted means of the per unit inputs in the pure technical processes of which the mixed process is made up. The outputs obtained by the various pure technical processes are the weights of the mean.

In fact, let a_{i1} be the per unit input of a certain (say, the i-th according to the accepted numeration) factor of production in the first technical process and a_{i2} the per unit input of the factor in the second technical process. Let the output x_1 of the given product be produced by the first technical process and the output x_2 by the second. Then, in the first technical process, the input of the factor is $a_{i1}\,x_1$,

---

$$\begin{bmatrix} \frac{Q_{11}}{P} & \frac{Q_{12}}{P} & \cdots & \frac{Q_{1r}}{P} \\ \frac{Q_{21}}{P} & \frac{Q_{22}}{P} & \cdots & \frac{Q_{2r}}{P} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{Q_{n1}}{P} & \frac{Q_{n2}}{P} & \cdots & \frac{Q_{nr}}{P} \end{bmatrix}.$$

Here the first index stands for the input of a given factor of production and the second index for the technical process. For instance L_{ij} means the input of the i-th kind of labour in the j-th technical process.

and in the second process $a_{i2}x_2$. Jointly the input of the factor of production is $a_{i1}x_1+a_{i2}x_2$. In total in both technical processes an output equal to x_1+x_2 is produced. The per unit input of the factor of production in the mixed technical process thus is:

$$\frac{a_{i1}x_1+a_{i2}x_2}{x_1+x_2}, \tag{1}$$

i.e. it is the weighted mean of the per unit inputs in both processes. This applies to all factors of production (i.e. to all possible indices). This line of argument may be generalised to any number of technical processes. In this way it is possible to compute from the technical production matrix the per unit inputs corresponding to mixed technical processes.

In the various technical processes, it is possible, as a rule, to change (within certain limits determined by the character of the process) the output of the product i.e. the quantity of product made. Of special interest are processes in which a change in output does not imply a change in the per unit inputs of production factors (or coefficients of production). In such processes inputs are proportional to output and the coefficients of production (per unit inputs) are constant quantities, independent of the size of output. Such processes we call *divisible*[18], for it is possible to divide them up arbitrarily into processes of smaller output size preserving the same proportions between inputs and output, between various inputs and, in joint production, also between outputs of various products.

Divisible pure technical processes always can be replaced by mixed processes composed of pure processes of smaller output size, and such mixed processes are also divisible, i.e. the coefficients of production corresponding to them are not dependent on the size of output. This follows from the fact that the weighted mean of per unit inputs—according to formula (1)—depends only on the ratio $\frac{x_2}{x_1}$, i.e. on the proportion in which the output is divided between the various processes of which the mixed process is composed. It does not depend on the absolute size of output. Mixing of divisible technical processes always yields the divisible process. Thus divisible technical processes may be arbitrarily mixed, the mixture always yielding devisible processes.

III

Certain relations occur between the per unit inputs in various technical processes (pure and mixed), with are available for the making of the same product. Let us take a technical process in which the per unit inputs of all factors of production are larger than in another process, or in which the per unit input of at least one factor is larger and the per unit input of no other factor of production is smaller.

---

[18] Divisible technical processes are also called *linear*, for the quantitative relation between inputs and outputs is that of simple proportionality, i.e. a linear function.

We call such a technical process *inefficient*. An inefficient technical process will not be applied in the production process, for it requires a greater input of all factors of production than an other existing technical process, or at least a greater input of one factor without diminishing the input of some other factor. Thus inefficient technical process may be removed from the technical production matrix, by striking out the corresponding columns.

If two (or more) technical processes require precisely the same per unit inputs of the various factors of production, we shall say that they are *equivalent*. Equivalent technical processes may be treated as one process. If equivalent technical processes appear in the technical production matrix, it is sufficient to retain one of them in the matrix; the rest may be removed as superfluous by striking out the corresponding columns.

By removing inefficient technical processes and superfluous equivalent processes from the matrix, we obtain the *effective* technical production matrix. The effective technical production matrix is the result of a selection in which inefficient and superfluous equivalent processes are removed. The technical processes which are left when the selection is completed, we call *effective* technical processes. In production choices (and possibly mixing) are made only among of effective technical processes.

Effective technical processes have the following properties. In any two processes, the per unit input of at least one factor of production must be smaller in one process than in the other process. For, if the per unit input of factors of production were equal in both processes, these processes would be equivalent. And if the per unit input of a factor of production were larger or smaller in one process than in the other, and the per unit inputs of the remaining factors were equal in both processes, then one of the processes would be inefficient. Therefore, the smaller per unit input of a factor of production in one process must be compensated by a larger per unit input of at least one other factor of production in the other process. We call this property of effective technical processes the *law of input substitution of inputs*. This law must embrace the inputs of at least two factors of production, it many but need not embrace more of them.

With joint production, there appears, under certain conditions, also a *law of output substitution*, i.e. of substitution of the quantities of various products. This occurs when the per unit inputs of factors of production are equal in both technical processes. Then these processes must differ with regard to the output of different products obtained from a given input of factors of production (otherwise they would be equivalent). If, with the same inputs of factors of production, one process yields a larger (smaller) output of a product, it must yield a smaller (larger) output of at least one other product. For, otherwise, one of the processes would be inefficient: with the same inputs of factors of production one process would yield a smaller output of one product than the other process and the same output of the remaining products. As in the case of inputs of factors of production, the substitution must embrace the outputs of at least two products and may, but does not need to, embrace more of them.

In the case when the effective technical processes are divisible, besides the law of input substitution these holds also, the *law of increasing rate of substitution of inputs*. Under certain conditions also a *law of diminishing rate of substitution of outputs* holds for joint production.

Let us take three effective technical processes. Denote the per unit inputs (coefficients of production) of the i-th factor of production in the three processes, respectively, by a_{i1}, a_{i2} and a_{i3}; and denote the per unit inputs of the j-th factor of production correspondingly by a_{j1}, a_{j2} and a_{j3}. Let us assume that the inputs of these two factors of production are subject to substitution. By changing from the first technical process to the second, we increase the per unit input of the i-th factor by $a_{i2}-a_{i1}$ and decrease the per unit input of the j-th factor by $a_{j2}-a_{j1}$[19]. The absolute value of the ratio of these changes in per unit inputs, i.e.

$$\left|\frac{a_{i2}-a_{i1}}{a_{j2}-a_{j1}}\right|$$

expresses the increment of per unit input of the i-th factor per unit of decrease of the per unit unit input of the j-th factor. We call this expression the *rate of substitution*.

By changing from the second process to the third and assuming, that the per unit input of the i-th factor is increased and the per unit input of the j-th factor is decreased, we obtain the rate of substitution

$$\frac{a_{i3}-a_{i2}}{a_{j3}-a_{j2}}.$$

If (as we assume to be the case) the processes are divisible, the inequality

$$\left|\frac{a_{i2}-a_{i1}}{a_{j2}-a_{j1}}\right| < \left|\frac{a_{i3}-a_{i2}}{a_{j3}-a_{j2}}\right|, \tag{2}$$

must be satisfied. This means that in the substitution of inputs connected with a successive change in technical processes, the rate of substitution increases. This results from the following reasoning.

As we know, divisible technical processes can be mixed arbitrarily. Let us then take any process mixed of the first and third process and yielding the same output as the second process. Denote the per unit inputs of the i-th and j-th factors in the mixed process by $\bar{a}_{i2}$ and $\bar{a}_{j2}$. We have, then, according to formula (1)

$$\bar{a}_{i2} = \frac{a_{i1}x_1+a_{i3}x_3}{x_1+x_3} \quad \text{and} \quad \bar{a}_{j2} = \frac{a_{j1}x_1+a_{j3}x_3}{x_1+x_3}$$

[19] Since the numeration of the factors of production is arbitrary we may always assume that it is the i-th factor whose per unit input is increased, and the j-th factor whose per unit input is decreased.

where x_1 and x_3 stand for the outputs of the first and third process entering into the composition of the mixed process. From this, it follows that

$$\frac{\bar{a}_{i2}-\bar{a}_{i1}}{\bar{a}_{j2}-a_{ji}}=\frac{a_{i1}x_1+a_{i3}x_3-a_{i1}(x_1+x_3)}{a_{ji}+x_1+a_{j3}x_3-a_{ji}(x_1+x_3)}=\frac{a_{i3}-a_{i1}}{a_{j3}-a_{j1}}$$

and

$$\frac{a_{i3}-\bar{a}_{i2}}{a_{j3}-\bar{a}_{j2}}=\frac{a_{i3}(x_1+x_3)-a_{i1}x_1-a_{i3}x_3}{a_{j3}(x_1+x_3)-a_{j1}x_1-a_{j3}x_3}=\frac{a_{i3}-a_{i1}}{a_{j3}-a_{ji}}.$$

Consequently,

$$\frac{\bar{a}_{i2}-a_{i1}}{\bar{a}_{j2}-a_{j1}}=\frac{a_{i3}-\bar{a}_{i2}}{a_{j3}-a_{j2}},$$

i.e. both rates of substitution are equal.

If the second technical process is effective, either $a_{i2}<\bar{a}_{i2}$ and $a_{j2}\leqslant\bar{a}_{j2}$, or $a_{i2}\leqslant\bar{a}_{i2}$ and $a_{j2}<\bar{a}_{j2}$, i.e. the per unit input of one factor of production must be smaller in the second process than in the mixed process, and the per unit input of the other factor must not be larger (for otherwise the technical process would be either equivalent to the mixed process, or would be inefficient). Substituting a_{i2} and a_{j3} for $\bar{a}_{i2}$ and $\bar{a}_{j2}$ in the above expression and taking account of the inequalities, it appears that on the left hand side the numerator is decreased or the denominator is increased, or both happens; while on the right hand side the numerator is increased or the denominator is decreased, or both. As a result we obtain

$$\left|\frac{a_{i2}-a_{i1}}{a_{j2}-a_{j1}}\right|<\left|\frac{a_{i3}-a_{i2}}{a_{j3}-a_{j2}}\right|.$$

Thus the successive transition to other technical processes implies successive increase of the rate substitution of inputs. This is the law of increasing rate of substitution of inputs [20].

In a similar way it is possible to show that with joint production, under the conditions in which the law of output substitution operates, effective and divisible technical processes are subject to the law of decreasing rate of substitution of outputs[21]. The substitution of outputs becomes increasingly difficult; the decrease of output of a product by successive units is accompanied by a diminishing increase of output of the other product.

It should be noted that the law of increasing rate of substitution of inputs, and the law of decreasing rate of substitution of ouputs, concern only those factors of production or products which are subject to substitution. These laws express certain limitations to which substitution is subject: even where substitution arises, it takes place under conditions of increasing difficulty.

Another expression of the increasing difficulty of substitution peculiar to divisible and effective technical processes is the *law of increasing additional inputs.* This

---

[20] The graphic interpretation of the law of increasing rate of substitution is given in the Mathematical Notes § 6.

[21] See on this point Mathematical Notes § 6.

law appears when output is increased by means of successive transition from one technical process to another and the inputs of all factors of production except one, are kept constant. The increase of output under such conditions requires an increase of the input of the factor whose input is variable, for otherwise, the technical processes would not be effective. This is the counterpart in relations between inputs and output of the law of input substitution inputs (or of output substitution in the case of joint production)[22]. If effective technical processes are divisible, then successive unit increments of output require increasing additional inputs of the variable factor of production.

Let the input of the i-th factor be variable. Denote its per unit input in three processes respectively by a_{i1}, a_{i2}, a_{i3} and the output in these processes x_1, x_2, x_3. Let us assume that $x_1 < x_2 < x_3$. The input of the factor in the three processes is respectively $a_{i1}x_1$, $a_{i2}x_2$, $a_{i3}x_3$. With transition from the first to the second process, the input increases by $a_{i2}x_2 - a_{i1}x_1$, and with transition from the second to the third process by $a_{i3}x_3 - a_{i2}x_2$. Per unit of increment of output, the input increments, or additional inputs are

$$\frac{a_{i2}x_2 - a_{i1}x_1}{x_2 - x_1} \quad \text{and} \quad \frac{a_{i3}x_3 - a_{i2}x_2}{x_3 - x_2}.$$

Instead of the second process we shall now examine a mixed process in which the output x_2 is obtained in the following way: the quantity $\frac{x_1(x_3 - x_2)}{x_3 - x_1}$ is produced by the first process and the quantity $\frac{x_3(x_2 - x_1)}{x_3 - x_1}$ by the third process. Together there in the mixed process the output is

$$\frac{x_1(x_3 - x_2)}{x_2 - x_1} + \frac{x_3(x_2 - x_1)}{x_3 - x_1} = x_2.$$

The input of the factor in the mixed process is

$$\bar{a}_{i2}x_2 = \frac{a_{i1}x_1(x_3 - x_2) + a_{i3}x_3(x_2 - x_1)}{x_3 - x_1},$$

where a_{i2} stands for the per unit input in the mixed process. Therefore

$$\frac{\bar{a}_2 x_2 - a_{i1}x_1}{x_2 - x_1} = \frac{a_{i3}x_3 - \bar{a}_{i2}x_2}{x_3 - x_2} \quad \frac{a_{i1}(x_1 - x_2) + a_{i3}x_3(x_2 - x_1)}{x_3 - x_1}.$$

Since it is assumed that the second process is effective consequently $a_{i2} < \bar{a}_{i2}$. By substituting this in the equation obtained, we find that

$$\frac{a_{i2}x_2 - a_{i1}x_1}{x_2 - x_1} < \frac{a_{i3}x_3 - a_{i2}x_2}{x_3 - x_2}. \tag{3}$$

[22] This becomes immediately clear when output is considered as negative input. When the increase in input is connected with an increase in the output, it is possible to interpret it as an increase in input connected with a decrease of a negative input, i.e. as input substitution. In a similar way it is possible to interpret it as output substitution by treating input as negative output.

Thus in the sucessive transitions from the first to the second process and from the second to the third process, etc. the additional input of the factor needed to obtain a unit increment of output increases. This is the law of increasing additional inputs. Instead of the additional input per unit of increment of output, its reciprocal may be considered. The reciprocal indicates the increment of output yielded by a unit of additional input, or the *productivity* of an additional input. To increasing additional inputs per unit of output increment there corresponds a decreasing productivity of additional unit inputs of the factor of production. Thus the law of increasing additional inputs can be formulated also as a law of decreasing productivity of additional inputs. Both these formulations are equivalent.

The law of input substitution and—under certain conditions of joint production—and of output substitution, and—in the case of divisible technical processes—also the law of increasing (or decreasing—with regard to outputs) rate of substitution, and the law of increasing additional inputs are results of a selection performed among technical processes. These laws are expressions of *praxiological* regularities resulting from the application of a certain praxiological rule of behaviour, namely the rule of omitting from the production process inefficient and superfluous equivalent technical processes. They are not universal laws of production technique as is frequently, but wrongly, asserted. The assertion that universal laws of production technique, operate in any branch of production in a way independent of the historical level of growth of social productive forces, is a generalization going beyond the sphere of factors which can be verified empirically. Such a generalization borders on metaphisical speculation[23].

The technical processes available at a given level of historical development of productive forces are of varied kinds: some allow for substitution of inputs and of outputs others do not allow it; some allow it in one sphere and not in another; in so far as they allow substitution, they do so in various ways, under conditions

[23] The view that the law of increasing rate of substitution of inputs (and of decreasing rate of substitution of outputs) and the law of increasing additional inputs are universal laws of production technique connected with the question of the so-called law of diminishing returns (i.e. productivity) of factors of production. This law states that additional units of a factor of production successively applied in the production process while the input of the other factors remains unchanged, yield, (after a certain initial input is passed) diminishing increments of outputs. The law of increasing rate of substitution of inputs is interpreted as an inference of this law. For, by succesive substitution of one factor for another, successively withdrawn units of the substituted factor have an increasing productivity; to compensate for this ever larger successive quantities of the substituting factor are required. Moreover, the successive units of the substituting factor are characterised by diminishing productivity which, in addition, brings about the necessity of successively increasing inputs of this factor. Similarly in the case of joint production, the diminishing productivity of factors of production causes sucessive units of factors set free by the decrease of output of a product, to bring about dominishing increments of output of the other product. However, the law of diminishing productivity (returns) of factors is neither a necessary, nor a sufficient condition for the law of increasing (or decreasing—with regard to outputs) rate of substitution. A necessary condition is only that the productivity of succesive units of the substituting factor increases at a rate slower than the productivity of the units

of an increasing, decreasing or constant rate of substitution. The variety of technical processes is wide. However, not all technical processes are applied in production: selection takes place, a sifting of inefficient and superfluous equivalent processes.

The observation of the technical processes actually applied in the production process may give the impression that these properties are connected with the nature of production technique, for in actual production processes one does not meet, in general, with technical processes not possessing the properties described. But this fact is not the result of the "nature" of the technical processes but is a result of

---

of the substituted factor are withdrawn successively. This is a much wider condition than the law of diminishing productivity holding for both factors. Moreover, the law of diminishing productivity of factors is not always sufficient to secure an increasing rate of substitution of inputs. Its working may be counteracted by the dependence of the productivity of additional units of the factor of the input of another factor. It may happen that the productivity of successive units of the substituting factor increases as a result of a decrease in the input of the substituted factor. On the other hand, if the productivity of successive units of the substituting factor diminishes greatly as a result of a decrease in the input of the substituted factor, the rate of substitution may increase even without diminishing productivity of successive units of a particular factor of production. Thus the law of increasing rate of substitution of inputs (and also the law of decreasing rate of substitution of outputs) and the law of diminishing returns do not coincide. See on this point the Mathematical Notes § 7 in the appendix. The fact that the law of the increasing (or decreasing) rate of substitution does not coincide with the law of diminishing returns, is important because this alleged law, interpreted as universal, is empirically unprovable. This law was first formulated only for inputs of labour and means of production on a constant area of land in agriculture. Anne R. Turgot first formulated this in *Observations sur un mémoire de G. de Saint-Péravy*, Paris 1768. Edward West also formulated it independently in his *Essay on the Application of Capital to Land*, London 1815. Ricardo introduced the law into his theory of land rent and because of this it gained widespread renown. In the second half of the nineteenth century the outstanding chemist Justus Liebig tried to justify the law of diminishing productivity of input on land in terms of natural science. E. A. Mitscherlich (1909) tried to deduce it from empirical research into the productivity of inputs of fertilizer on soil. Marx, in contrast to Ricardo, did not connect land rent with the law of diminishing returns from land; vide *Capital*, Vol. III, pp. 40–44. Lenin criticised the law as an abstraction contrary to historical experience which fails to take into account the fact that the increase in input of labour and of means of production on a given area of land is, as a rule, connected with technical progress. Vide V. I. Lenin, *Agrarnyj vopros i "kritiki" Marksa* (The Agrarian Question and Marx's Critics) Works. Vol. 5, Moscow 1950, p. 113. The founder of the American branch of the subjectivist trend in political economy, J. B. Clark, extended the law of diminishing returns to all factors of production (The *Distribution of Wealth*, New York 1899) and based on it the theory of marginal productivity. He considered diminishing returns of factors of production to be a universal law of production technique. In this form the law of diminishing returns of factors of production was widely accepted in the neoclassical school and in kindred schools of economic thought. Its clearest formulation was given by P. H. Wicksteed, *The Common Sense of Political Economy*, Vol. I, London 1902, second edition 1933 and by K. Wicksell, *Lectures of Political Economy*, Vol. I, London 1934. This is a translation from Swedish, the first Swedish edition is of 1901). E. Schneider, *Theorie der Production*, Vienna 1934 and S. Carlson, *A Study in the Pure Theory of Production*, London 1939, gave a modern exposition of the law of diminishing returns. It should be noted that the founder of the neoclassical school Alfred Marshall in his theoretical analysis referred to the law of the diminishing returns of factors of production the fact that processes which do not have the above mentioned properties are eliminated in the sifting of the incfficient or superfluous equivalent processes. This is the result of the praxiological rule of behaviour applied in production[24].

IV

In the form given above, the technical production matrix does not yet give a full picture of the possible production techniques. It takes into account only the per unit inputs peculiar to the various technical processes. However, technical processes differ not only with regard to the per unit inputs of factors of production, but they also differ with regard to the stocks of fixed means required in a given technical process. Fixed means take part in the production process not only in the form of inputs, i.e. utilisation during a definite period of time. As we know, utilisation has the character of a stream, measured in numbers of physical units in a given period of time, e.g. in machine-hours, truck-hours during a month or a year. Fixed means take part in the production process also as stocks, independent of the extent of their utilisation.

Fixed means of production preserve their natural form and their usefulness for more than one period of production. Once introduced into the production process, they participate in it with their whole indivisible stock. A machine, a truck or a building may be utilised for several hours daily, but it is not possible to work without

---

with much circumscription and caution. He was of the opinion that substitution of factors of production is not a universal phenomenon, but takes place in a limited sphere. He also confined the activity of the law of diminishing returns mainly to agriculture and to the production of raw materials. Vide A. Marshall, *Principles of Economics*, Eighth Edition, London 1958, pp. 318 and seq. and p. 387. The law of diminishing returns of factors of production does not play any important role in the so-called Lausanne School. Walras based his theory of production on the assumption of constant coefficients of production: for the making of each product only one technical process is available and there is no possibility of substituting factors Vide L. Walras, *Eléments d'économie politique pure*, Chapter IV. Later in the fourth edition of his work (1900), Walras supplemented his analysis introducing the assumption of universal substitution of factors of production in conformity with the theory of marginal productivity (Chapter VII). Pareto treated substitution of factors of production as a special case which does not have universal validity. Vide V. Pareto *Manuel d'economie politique*, Paris 1907. J. Schumpeter presents the history of the problem in *History of Economic Analysis*, London 1954, pp. 1026–1053. The essay of the well-known mathematician K. Menger, *The Laws of Return, A Study in Mataeconomics*, in the book *Economic Activity and Analysis* edited by O. Morgenstern, New York 1954, and the study of S. Kruszczyński, *Problem kształtowania się przychodów i kosztów* (The Problem of the Formation of Returns and Costs) Poznań 1962, contain a critical analysis of the problem. As we have shown in the text, all these pseudoproblems bordering on metaphysics are irrelevant for the quantitive relations arising in the production process. The substitution of inputs and of outputs and its increasing difficulty under successive transition from one technical process to another do not result from the nature of production technique; they are a consequence eliminatory selection of technical process. The problem is one of praxiology and not of technology.

[24] Z. Bosiakowski draws attention to the praxiological character of the results of empirical research into the relations arising between inputs and output. See his review of the study of S. Kruszczyński mentioned in the preceding footnote. Vide "Ekonomista", 1963, No. 2, pp. 437–432.

the whole machine, the whole truck or the whole building. The stock of fixed means utilised in the production process, is usually called the *technical equipment* of production. For a full description of conditions of production, it is necessary to mention beside the per unit inputs of factors of production also the technical equipment consisting od stocks of fixed means. Individual technical processes differ also in the matter of the technical equipment required.

As a rule a definite stock of various fixed means—buildings, fittings, installations, machinery, transport facilities, etc. is required for the application of a given technical process; i.e. some definite technical equipment is necessary. Various technical processes serving to make a given product require different technical equipment. The size of the technical equipment, i.e. the stock of various fixed means of production is independent of the extent of its utilisation. Therefore it is also independent of the output of the product made. It is a constant quantity peculiar to any given technical process.

To obtain a full picture of the technical possibilities of production, the technical equipment necessary to the various technical processes must be introduced into the technical production matrix. For this purpose it is necessary to dinstinguish between fixed and circulating means of production. Let us—for sake of distinction—denote the input of circulating means by $Q^{(0)}$ and the input of fixed means by $Q^{(1)}$. Further, let S be the stock of fixed means. The technical production matrix then can be written as follows:

$$\begin{bmatrix} \frac{L_1}{P} & \frac{L_2}{P} & \cdots & \frac{L_r}{P} \\ \frac{Q_1^{(0)}}{P} & \frac{Q_2^{(0)}}{P} & \cdots & \frac{Q_r^{(0)}}{P} \\ \frac{Q_1^{(1)}}{P} & \frac{Q_2^{(1)}}{P} & \cdots & \frac{Q_r^{(1)}}{P} \\ S_1 & S_2 & & S_r \end{bmatrix}.$$

In this matrix each column stands for an aggregate of per unit inputs *and* of technical equipment corresponding to the given technical process. The technical equipment (stock of fixed means of production) appears at the bottom of each column; the last line of the matrix shows—the technical equipment peculiar to various technical processes[25]. This matrix we shall call the *full technical production matrix*.

[25] The technical equipment is an aggregate, i.e. vector, whose components are the stocks of individual fixed means. For a given, let us say the j-th, technical process, the vector representing the technical equipment may be written in the following form:

$$S_j = \begin{bmatrix} S_{1j} \\ S_{2j} \\ \cdot \\ \cdot \\ \cdot \\ S_{1j} \end{bmatrix}.$$

Here j is the technical process index ($j = 1, 2, \ldots, r$).

Taking into account the technical equipment, it is also possible to extent the concept of inefficient and equivalent technical processes. A technical process is inefficient when it requires a greater per unit input of one or several factors of production *or* one or several larger components of technical equipment, while no other per unit input *or* component of the technical equipment is smaller[26]. Technical processes are equivalent if they require the same per unit inputs *and* the same number and size of components of technical equipment. By eliminating inefficient and superfluous equivalent technical processes, we obtain an extended concept of effective technical processes. It is possible to show by reasoning like that above that the effective (in the extented sense) technical processes are subject to the law of substitution.

As we have seen, Marx noted that a decrease in per unit input of labour, i.e. an increase in labour productivity, is, as a rule, connected with an increase in technical equipment of the production process. The substitution of living labour by increased technical equipment is the chief motor of the historical process of growth of productivity of human labour. It is generally connected with substitution of per unit inputs of means of production for per unit inputs of labour, because the rise in output attained by a per unit input of labour requires an increase in the quantity of objects of labour (i.e. raw materials) transformed into product[27].

The technical equipment peculiar to a given technical process determines the maximum input i.e. the maximum extent of utilisation of fixed means. For the input of the fixed means can not exceed their uninterrupted utilisation in the production process. Taking account of interruptions needed for conservation and repair, to any fixed means of production has a maximum of possible utilisation in the course of a given period of time (a day, month, year). With a given stock of a fixed means it is not possible to exceed a certain maximum extent of its utilisation, i.e. a certain maximum input.

In consequence there is a certain maximum output of product which can be obtained (in the course of a given period of time) by applying a given technical process. This output is determined by the maximum utilisation of the technical equipment peculiar to the given process; we call in the *productive capacity* of the technical process. Each technical process has a productive capacity of its own. If the output

---

[26] These components are components of the technical equipment vector S_j corresponding to the given technical process. These are concrete machines, buildings, installations, means of transport, etc.

[27] In *Capital*, vol. I, pp. 671–672 (Polish edition) Marx connects the growth of labour productivity both with the substitution of technical equipment for per unit inputs of labour ("the bulk of machinery used, draught animals, mineral fertilisers, drains and so on, is a condition of labour productivity growth. The same is true of buildings, blast furnaces transport facilities") and of the substitution of per unit inputs of means of production particularly circulating means for per unit inputs of labour ("the volume of raw material transformed at the same time, increases"). It is necessary to distinguish these two kinds of substitution connected with growth in labour productivity. Much confusion has arisen in economic literature because of the lack of a clear distinction.

is less than the productive capacity of the technical process, we say that the productive capacity is not fully used. The ratio of the output and of the production capacity of a technical process, is called *degree of utilisation of productive capacity.*

It may happen that the stocks and the maximum utilisation time of the various fixed means are such that they allow the same output to be obtained. Then we say that the technical equipment has a *harmonious structure*. Often, however, various components of the technical equipment allow different outputs. Then we say that there is *disharmony* in the structure of technical equipment. With disharmony in the structure of technical equipment, the productive capacity of the technical process is determined by the component of technical equipment which allows the smallest output to be obtained. This component we call the *limiting component*. The remaining components of technical equipment are not utilised to the fullest possible extent: they are underutilised.

The structure of the technical equipment may be harmonious or disharmonious. This is a question of the particular production technique and constitutes a property of the given technical process. Individual technical process may, then, be connected with under utilisation of certain components of technical equipment even under full utilisation of their productive capacity. It is possible, however, to transform a disharmonious structure of technical equipment into a harmonious structure by a suitable pooling of technical processes.

Let us consider two technical processes utilising the same fixed means of production. Suppose that in the first process there are three components of technical equipment which allow the production (in a given period of time) of 100, 120 and 150 units of output, respectively. Here the first component is the limiting component; the two remaining components are underutilised. Suppose that in the second process, as for as their technical aspect goes, components allow the production of correspondingly 200, 180 and 150 units of output. Here the third component of technical equipment is the limiting component; the first and second components are underutilised. By pooling the two processes into one, we obtain a mixed process in which 300 units of output are produced and in which all components of technical equipment are utilised to the full. The structure of the technical equipment of this mixed process is harmonious. Such pooling of technical processes we shall call *harmonisation* of the structure of technical equipment.

Harmonisation of the structure of technical equipment by polling of technical processes is possible when various technical processes use the same—in the technical sense—fixed means of production and, when the components of technical equipment limiting in one process are underutilised in another process. The pooling of technical processes then leads to an approach to a harmonious structure of technical equipment. It does not always lead to full harmonisation of the structure of technical equipment, for the underutilisation of individual components of technical equipment in different processes is not always such that it can be eliminated by pooling of the processes.

It is, however, always possible to obtain full harmonisation by increasing the size of output to a multiple of the outputs allowed by the various components of technical equipment. With such an increase in the size of output, not all components of technical equipment are multiplied in the same ratio. E.g. if, in the first of the processes mentioned, the size of output is incresed six times, i.e. to 600 units of output it is necessary to multiply the first component of technical equipment six times, the second component five times and the third only four times. Then each component allows 600 units of output to be produced and the structute of technical equipment is harmonized.

Similarly in the second process mentioned as an example, it is possible to obtain harmonization of the structure of technical equipment with an output amounting to 1800 units. In this case it is necessary to multiply the first component of technical equipment 9 times, the second 10 times and the third 6 times. Every output which is a multiple of the output made possible by individual components of technical equipment, leads to harmonization of the structure of technical equipment. The least common multiple suffices for this purpose. The least common multiple determines the smallest output (and consequently the smallest productive capacity of the multiple process) which brings about harmonization of the structure of equipment[28].

The harmonization of structure of technical equipment is always accompanied by an increase in productive capacity. Under harmonization through pooling of technical processes, the new mixed process has a productive capacity equal to the smallest output which the individual components of the poolee technical equipment allow to be obtained. But when technical processes are increased to multiplisize the productive capacity of the multipled process is a multiple of the outputs made possible by the individual components of technical equipment, consequently it is greater than the output made possible by the limiting component which determines the productive capacity of the original process. [Sometimes harmonization of the structure of technical equipment can be achived by pooling technical processes producing different products if these processes utilise (wholly or partially) the same (in the technical sense), fixed means of production. In such cases joint production of two or more products is the result. Joint production is frequently a result of pooling different technical processes for the purpose of harmonizing the structure of technical equipment.

---

[28] *Vide* Mathematical Notes, § 3. The Danish engineer-economist Ivar Jantzen drew attention to the problem of harmony of the structure of technical equipment in his essay (*Voxende Udbytte i Industrien* "Nationalknomisk Tidsrift", Vol. 62, Copenhagen 1924). An English translation appeared in the book by the same author *Basic Principles of Business Economic*, Copenhagen 1939. A German translation appeared as the appendix to the book of E. Schneider *Theorie der Produktion*, edition cited. See also Ivar Jantzen *Laws of Production and Cost*, "Econometrica", Vol. 17, Supplement 1949 (Report on Washington Meeting). As far as we know, Jantzen was the first to state that it is possible to obtain harmonization of the structure of technical equipment by way of increasing a multiple the size of output. He called this theorem the "law of harmony". *Vide* F. Zeuthen, *Economic Theory and Method*, London 1955, pp. 117–118.

Clearly, harmonization of the structure of technical equipment either by an increase in the output of a single product made (as a result of pooling or multiplying of processes) or by a transition to joint production (possibly also by an increase in the variety of product) requires an increase in technical equipment. Thus it is connected, with an increase in technical equipment and as well as with an increase of the productive capacity of a productive establishment[29], or—as we say colloquially—with an increase of the "size" of the establishment. It may require a combination of a larger number of productive establishments into a new "larger" establishment, which has more technical equipment and greater productive capacity (or more varied productive capacity as in the case of transition to joint production or of an enlargement of the variety of products). Moreover, it may happen that the harmonization of the structure of technical equipment in one branch of production is dependent on the growth of production in another production branch. Such a dependence may be due to the fact that growth of production in one branch is a condition for the utilisation of the increased productive capacity connected with the harmonization of the structure of technical equipment. It also may be due to the fact that a second production branch supplies raw materials or is a recipient of the product made [30].

Taking account of technical equipment leads to a significant limitation of the sphere of operation of the law of increasing rate of substitution of inputs (and decreasing rate of substitution of outputs and of the law of increasing additional inputs). As we know, these laws are the result of an application in the production process of the praxiological rule eliminating inefficient processes. Processes which do not conform to these laws are inefficient if they are divisible, for then there exist one or more mixed processes, which yield the same output with a smaller input of at least one factor of production. However, technical equipment determines a definite productive capacity of the technical processes. In consequence, the mixing of two or more processes for the production of a given output may lead to underutilisation of the productive capacity of these processes and also to disharmony in the structure of technical equipment. Under these conditions, the technical processes are indivisible and the application of mixed processes does not come into consideration. Mixed processes do not "compete" for application in the production process. As a result, the processes which do not conform to the above mentioned laws concerning the rate of substitution and additional inputs, do not have to be inefficient.

In this situation, these laws do not come into play. They may come into play in the particular case when the technical equipment and productive capacity of the individual processes making up the mixed process are comparatively small. In such case the processes are nearly divisible and a mixed process does not cause underutili-

---

[29] The harmonization of technical equipment is technical basis of the law of concentration of production which operates in the capitalist as well as in the socialist mode of production. It is also connected with the well known phenomenon of *increasing returns of scale* or *economies of scale*.

[30] This constitutes the basis of the phenomenon which Marshall called *external economies*. Vide A. Marshall, *Principles of Economics*, edition quoted, p. 226.

sation of productive capacity or serious disharmony of the structure of technical equipment. Therefore, these laws operate only in branches of production where technical equipment and productive capacity of the technical processes are comparatively insignificant. They cease to operate as technical equipment and productive capacity grow in size and, in consequence, the "size" of productive establishments grows too[31].

Thus the law of increasing rate of substitution of inputs (and decreasing rate of substitution of outputs) and the law of increasing additional inputs interpreted as consequences of a praxiological rule of the selection of technical processes are not universal either. They hold only for divissible processes where technical equipment and productive capacity do not play a major role.

---

[31] It follows that these laws operate as praxiological regularities chiefly in agricultural production and in some raw material production, where technical equipment is comparatively small. They do not operate in industrial production which is characterised by large technical equipment. This is in agreement with the empirical observation made by A. Marshall in *Principles of Economics*, edition quoted, pp. 318–319 and 137–154. But as the amount of technical equipment increases in agriculture and in mining as a result of technical progress, these laws cease to operate in these spheres too. This explains Lenin's empirical results concerning the so-called law of diminishing returns from land under conditions of capitalist development of agriculture. Vide V. I. Lenin, *Kwestia agrarna a krytycy Marksa*, Works, Vol. V, pp. 112–125. L. Krzywicki's *Kwestia rolna* (The Agrarian Question), Warsaw 1903, pp. 188–190, in this connection expressly emphasises the role of technical equipment in agriculture.

§ 1. *The Technical Production Matrix.*

Let us denote, in a similar way as in the text above, the vectors of input of labour, of circulating means of production and of fixed means of production in the j-th technical process by

$$L_j = \begin{bmatrix} L_{1j} \\ L_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ L_{hj} \end{bmatrix}, \quad Q_j^{(0)} = \begin{bmatrix} Q_{1j}^{(0)} \\ Q_{2j}^{(0)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Q_{kj}^{(0)} \end{bmatrix}, \quad Q_j^{(1)} = \begin{bmatrix} Q_{1j}^{(1)} \\ Q_{2j}^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Q_{lj}^{(1)} \end{bmatrix}. \quad (j = 1,\ 2,\ \ldots,\ r).$$

Let the number of different kinds of labour be h, the number of different kinds of circulating means of production be k and the number of different kinds of fixed means be l. Let us now consider r different possible technical processes and let P denote the output. The inputs and the outputs are measured in physical units in a given period of time (i.e. they are streams). We assume also that the inputs are non-negative (and at least some of them are positive) and the outputs are positive.

The vectors defined form the *input matrix*

$$\begin{bmatrix} L_1 & L_2 & \cdots\cdots & L_r \\ Q_1^{(0)} & Q_2^{(0)} & \cdots\cdots & Q_r^{(0)} \\ Q_1^{(1)} & Q_2^{(1)} & \cdots\cdots & Q_r^{(1)} \end{bmatrix}.$$

This matrix can also be written in the expanded form

$$\begin{bmatrix} L_{11} & L_{12} & \cdots\cdots & L_{1r} \\ L_{21} & L_{22} & \cdots\cdots & L_{2r} \\ \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot \\ L_{h1} & L_{h2} & \cdots\cdots & L_{hr} \\ Q_{11}^{(0)} & Q_{12}^{(0)} & \cdots\cdots & Q_{1r}^{(0)} \\ Q_{21}^{(0)} & Q_{22}^{(0)} & \cdots\cdots & Q_{2r}^{(0)} \\ \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot \\ Q_{k1}^{(0)} & Q_{k2}^{(0)} & \cdots\cdots & Q_{kr}^{(0)} \\ Q_{11}^{(1)} & Q_{12}^{(1)} & \cdots\cdots & Q_{1r}^{(1)} \\ Q_{21}^{(1)} & Q_{22}^{(1)} & \cdots\cdots & Q_{2r}^{(1)} \\ \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot \\ Q_{l1}^{(1)} & Q_{l2}^{(1)} & \cdots\cdots & Q_{l2}^{(1)} \end{bmatrix}.$$

The columns of this matrix show the inputs of different factors of production when a given technical process is used and the rows of the matrix show the inputs of a given factor of production in different technical processes. Putting $n = h+k+1$ we find that this matrix has n rows and r columns.

For the sake of simplicity let us denote the elements of the expanded input matrix by x_{ij}, i.e. x_{ij} is the element standing in the i-th row and j-th column of the matrix ($i = 1, 2, ..., n$; $j = 1, 2, ..., r$). Then this matrix can be written in the form

$$X = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1r} \\ X_{21} & X_{22} & \dots & X_{2r} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ X_{n1} & X_{n2} & \dots & X_{nr} \end{bmatrix} \tag{1.1}$$

The per unit input of the i-th factor of production in the j-th technical process is defined as

$$a_{ij} = \frac{X_{ij}}{P} \quad (i = 1, 2, \dots n; \qquad j = 1, 2, \dots, r) \tag{1.2}$$

The per unit unit inputs will be also called *coefficients of production.* These coefficient have dimensions independent of time, i.e. the dimension of a ratio of two stocks.

The coefficiens of production form the *technical production matrix*;

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1r} \\ a_{21} & a_{22} & \dots & a_{2r} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nr} \end{bmatrix}. \tag{1.3}$$

The columns of this matrix denote the per unit unit inputs of different factors of production in a given technical process and the rows show the per unit inputs of a given production factor in different technical processes.

The stocks of different fixed means necessary in the j-th production process form the vector S_j which will be called the *technical equipment vector* of the given process:

$$S_j = \begin{bmatrix} S_{1j} \\ S_{2j} \\ \cdot \\ \cdot \\ \cdot \\ S_{lj} \end{bmatrix} \quad (j = 1, 2, \dots, r).$$

The quantities S_{1j}, S_{2j},...,S_{lj} are non-negative and represent the components of the technical equipment. The stocks have dimensions independent of time. The components of the technical equipment can be presented in form of a matrix S with l rows and r columns.

Introducing the technical equipment necessary for different production processes into the technical production matrix we obtain the *full technical production matrix*, i.e.

$$\begin{bmatrix} A \\ S \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1r} \\ a_{21} & a_{22} & \dots & a_{2r} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nr} \\ S_{11} & S_{12} & \dots & S_{1r} \\ S_{21} & S_{22} & \dots & S_{2r} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ S_{l1} & S_{l2} & \dots & S_{lr} \end{bmatrix} \tag{1.4}$$

or

$$\begin{bmatrix} A \\ S \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \dots & a_r \\ S_1 & S_2 & \dots & S_r \end{bmatrix}, \tag{1.4a}$$

where

$$a_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \cdot \\ \cdot \\ a_{nj} \end{bmatrix} \qquad (j = 1, 2, \dots, r).$$

In the full technical production matrix the columns show the per unit inputs and the technical equipment in a given technical process and the rows show the per unit unit input of a given factor of production or the stock of a given kind of fixed means of production in different technical processes.

§ 2. *Joint Production.*

Joint production can be introduced into the technical production matrix by treating one of the several products as reference product. All the other products then are treated as *sui generis* factors of production whose inputs are negative. Per unit inputs of these "factors of production" show the outputs of other products which are obtained per unit of output of the reference product[1].

Another way of accounting for joint production in the technical production matrix is that of denoting output by positive numbers and inputs by negative numbers (thus the inputs are negative outputs). Such a procedure permits to account for all the outputs and all the inputs in the technical production matrix. Because of the rule adopted for the signs of the quantities this matrix takes the form of a balance sheet in which outputs are positive and inputs are negative[2]. Such approach has

[1] See O. Lange—*Optymalne decyzje* (Optimum Decisions), Warsaw 1964.

[2] Examples of such matrices can be found in O. Pichler *Anwendung der Matrizenrechnung bei Betriebskostenüberswachung* a paper in the book published under the title *Anwendung der Matrizenrechnung auf Wirtschaftliche und Statistische Probleme*, Würzburg 1959, and in the book *Matematicheskye metody planirovania proizvodstva* edited by M. M. Fiedorovich, Moscow 1961.

some obvious advantages. However, we shall not use it since for our purpose it is more convenient to use positive numbers for inputs of different factors of production. Also the components of the technical equipment will be assumed to be positive numbers.

§ 3. *Productive Capacity and the Structure of Technical Equipment.*

As denoted S_{ij} is the stock of the i-th kind of fixed means of production in the -th technical process. Let us denote by t_{ij} the maximum time of its use within a given time period. Then, the maximum possible input of this means is

$$(X_{ij})_{\max} = t_{ij} S_{ij} \qquad (i = 1, 2, ..., l; \qquad j = 1, 2, ..., r).$$

Using (1.2) we find that the stock S_{ij} makes it possible to produce (during the given period of time) at most the amount

$$(P_{ij})_{\max} = \frac{t_{ij} S_{ij}}{a_{ij}} \ (i = 1, 2, ..., l; \ j = 1, 2, ..., r). \tag{3.1}$$

For a given, the j-th say, technical process the components of technical equipment $S_{1j}, S_{2j}, ..., S_{lj}$ determine respectively the maximum possible output of the product, $(P_{1j})_{\max}, (P_{2j})_{\max}, ..., (P_{lj})_{\mathrm{mxa}}$, say. The smallest of these outputs determines the *productive capacity* of the given technical process. This productive capacity will be denoted by $\hat{P}_j$.

Thus

$$\hat{P}_j = \min_i (P_{ij})_{\max} \quad (j = 1, 2, ..., r). \tag{3.2}$$

In the particular case when $(P_{ij})_{\max} = (P_{2j})_{\max} = ... = (P_{lj})_{\max}$ the structure of technical equipment is said to be harmonious. When such equality does not hold true, this structure is *disharmonious*. In the latter case the i-th fixed means of production, which fulfills the condition (3.2), is the *limiting component* of the technical equipment. As a measure of deviation from harmonious structure, i.e. as a measure of disharmony, the following difference may serve:

$$\max_i (P_{ij})_{\max} - \min_i (P_{ij})_{\max},$$

which also may be written

$$\max_i (P_{ij})_{\max} - \hat{P}_j \ (j = 1, 2, ..., r). \tag{3.3}$$

This is the difference between the largest potential productive capacity and the real productive capacity as determined by the limiting component.

Pooling k technical processes (where $k \leqslant r$) gives a new process whose productive capacity is

$$\min_i \sum_{j=1}^{k} (P_{ij})_{\max}.$$

This leads to a decrease of the disharmony of the structure of technical equipment if the inequalities

$$\max_i \sum_{j=1}^{k} (P_{ij})_{\max} - \min_i \sum_{j=1}^{k} (P_{ij})_{\max} \tag{3.4}$$

$$\max_i (P_{ih})_{\max} - \min_i (P_{ih})_{\max}$$

hold for all $h = 1, 2, ..., k$.

Let us denote by M_j a multiple of the numbers $P_{1j\,\max}$ $P_{2j\,\max}, \dots P_{lj\,\max}$ in the j-th technical process.

Then

$$\lambda_{ij} = \frac{M_j}{(P_{ij})_{\max}}$$

is an integer for all $i = 1, 2, ..., l$. Multiplying the component of the technical equipment S_{ij} by λ_{ij} we get a new, multipled, j-th technical process. In the multipled process the components of the technical equipment are

$$\lambda_{1j} S_{ij},\ \lambda_{2j} S_{2j},\ \dots,\ \lambda_{lj} S_{lj}.$$

Because of (3.1) these components allow to produce

$$\lambda_{1j}(P_{1j})_{\max} = \lambda_{2j}(P_{2j})_{\max} = \dots = \lambda_{lj}(P_{lj})_{\max} = M_j. \tag{3.6}$$

This shows that multipliing the technical process leads to a fully harmonious structure of its technical equipment.

§ 4. *Substitution.*

The following geometric interpretation illustrates the law of input substitution. The technical processes are represented by vectors in $w(n+l)$-dimensional Euclidean space. The components of these vectors are the elements of the columns of the full technical production matrix. Each vector has $n+l$ components, namely n per unit inputs (coefficients of production) and l components of technical equipment. Obviously, some of the components my be equal to zero when the respective factor of production is not used in a particular technical process. The ends of these vectors determine a hypersurface composed of $(n+l-1)$-dimensional simplexes. The law of input substitution implies that the hypersurface is declined towards at least one of the co-ordinates, or, which amounts to the same, that the projection of the hypersurface on at least one plane of the system of co-ordinates is a "declining" curve.

As the numeration of inputs and of components of technical equipment is arbitrary the projection can be drawn as follows (see Figure 1):

Figure 1 represents the plane of the system of co-ordinates determined by two co-ordinates representing inputs or components of technical equipment. The vectors

OA, OB, OC, OD etc. are the projections on that plane of the vectors representing different technical processes. The points A, B, C, D etc. generate the broken line $OBCD$ etc. This line represents the ends of $(n+1)$-dimensional vectors representing different technical processes.

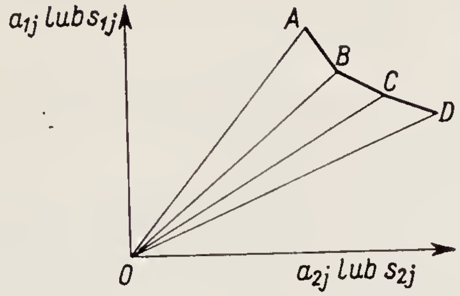


FIG. 1

The law of substitution of inputs says that this broken line is "declining". As a matter of fact, if it were "rising" then—as is seen from figure 1—technical processes corresponding to the vectors OB, OC, OD would require larger per unit inputs of the two factors of production (or larger components of technical equipment) than the process corresponding to the vector OA. Thus these processes would be in efficient. Similarly these processes would not be efficient if the line $ABCD$ were a horizontal one. The processes represented by the vectors OB, OC and OD would then require a larger per unit input a_{2j} or a larger component of technical equipment S_{2j} than the process represented by the vector OA while the per unit input a_{1j} or the size of the technical equipment component S_{1j} remain unchanged. On the other hand if two or more points A, B, C, D coincided, the corresponding technical processes would be equivalent. Consequently, the line $ABCD$ must be a "declining" one. This is the geometric interpretation of the law of input substitution.

§ 5. *Mixed Processes*

When a mixed technical process is applied the per unit inputs are weighted means of the per unit inputs in the technical processes of which the mixed process is composed. Let us assume that a given quantity of the product is produced in such a way that the quantity x_1 is made by using one process—process 1, say, and the quantity x_2 is made by using another process—process 2, say. Let a_{i1} be the per unit input of the i-th factor of production when process 1 is used and let a_{i2} be the per unit input of the factor in process 2. The input of the i-th factor in process 1 is equal to $a_{i1}x_1$ and the input in process 2 is equal to $a_{i2}x_2$. The per unit input in the mixed process (which is denoted by $\bar{a}_{ix}$) is then

$$\bar{a}_{ix} = \frac{a_{i1}x_1 + a_{i2}x_2}{x_1 + x_2}.$$

Let us denote by $x = \frac{x_1}{x_1+x_2}$ the share of the first process in the output of the mixed process. The share of the second process is then $1-x = \frac{x_2}{x_1+x_2}$. The per unit input of the i-th factor of production in the mixed process can therefore be written as

$$\bar{a}_{ix} = a_{i1}x+a_{i2}(1-x). \tag{5.1}$$

As is seen, it depends on the parametr x defining in which proportion the product is made up of outputs of the two processes.

Let us consider now the plane of the system of co-ordinates defined by the per unit inputs of the first and of the second factor of production (the order of numeration of factors of production is arbitrary) i.e., by the co-ordinates a_{1j} and a_{2j}. Figure 2 shows the projections on that plane of the vectors representing the technical process 1 and the technical process 2, respectively.

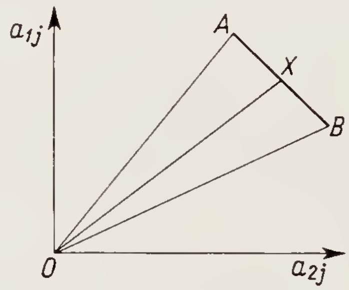


FIG. 2

These are the vectors OA and OB. The ends of the vectors, i.e. A and B have the co-ordinates (a_{11}, a_{21}) and (a_{12}, a_{22}). To the mixed process composed of the two processes there corresponds a vector the end of which has the coordinates $\bar{a}_{1x}, \bar{a}_{2x}$. This vector is denoted by OX.

Because of (5.1), we have

$$\bar{a}_{1x} = a_{11}x+a_{12}(1-x)$$

$$\bar{a}_{2x} = a_{21}x+a_{22}(1-x).$$

This is the parametric equation of a streight line passing through the points with the coordinates (a_{11}, a_{22}), i.e. through the points A and B. It shows that the point X, i.e. the end of the vector OX representing the mixed process, is situated on the segment AB. The position of the point X on this segment depends on the parameter x. If $x = 1$, the point X coincides with the point A (i.e. only process 1 is used). If $x = 0$, then the point X coincides with the point B; this means that only process 2 is used. If $0 < x < 1$, the point X is situated somewhere between A and B and its position is determined by the proportion in which the product is made up of outputs of the two processes.

§ 6. *The Properties of the Rate of Substitution.*

Let us consider three divisible technical processes which we shall denote by 1, 2, and 3 and whose vector projections on the plane of the system of coordinates determined by the first and the second factors of production are shown of Figure 3.

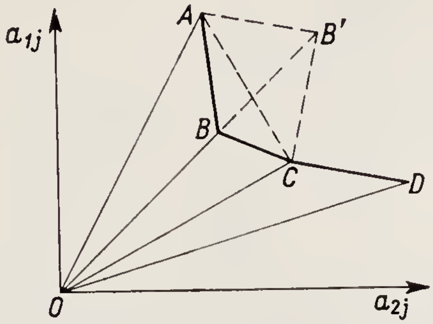


FIG. 3

These are the vectors OA, OB and OC. On the graph is shown also the vector OX corresponding to a mixed process composed of the processes 1 and 3. If the vector OB representing the technical process 2 is longer than OX (and has the length OB', say) then this process requires larger per unit inputs of the two factors of production than the mixed process mentioned above, i.e. this process is inefficient as compared with the mixed process. If the lenght of the vector OB is equal to OX then the process 2 requires the same per unit inputs as the mixed process and is therefore equivalent to the latter. It follows, hence, that the process 2 is effective only when the vector OB is shorter than the vector OX representing the mixed process.

A similar argument can be applied to the technical processes 2, 3 and 4 to the corresponding vectors OB, OC, OD. It follows that the broken line $ABCD$ generated by the ends of these vectors representing efficient technical processes is convex with respect to the origin of the system of co-ordinates. It follows also that only mixing neighbouring technical processes (for instance processes 1 and 2, 3 and 4 etc.) is efficient[3]. It is seen on the graph that any mixed process obtained from the process 1 and 3 is represented by the vector the end of which is on the segment AC. Such a mixed process requires larger per unit inputs of both factors of production than the process 2. Similarly, mixing the processes 2 and 4 leads to higher per unit inputs of both factors than the process 3 and mixing the processes 1 and 4 requires larger per unit inputs of both factors than in the case of either process 2 or 3.

The broken line $ABCD$ is "declining" and is convex with respect of the system of co-ordinates. It follows that the slope of its segments (AB, BC, CD etc.) with respect to the axis of abscissae decreases (see Figure 3).

---

[3] According to the law of substitution the broken line $ABCD$ etc. is declining. The points A, B, C, D, etc. can be monotonically arranged according to their dimensions. The vectors OA, OB, OC, OD, etc. can be similiarly arranged. The "neighbouring" technical processes we define as the processes relative to the vectors neighbouring each other in the arranged set of vectors.

The slope of the consecutive segments thus form a decreasing sequence

$$\left|\frac{a_{11}-a_{12}}{a_{21}-a_{22}}\right| > \left|\frac{a_{12}-a_{13}}{a_{22}-a_{23}}\right| > \left|\frac{a_{13}-a_{14}}{a_{23}-a_{24}}\right| > \dots \tag{6.1}$$

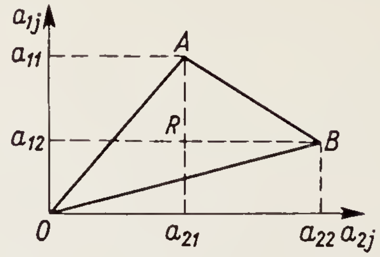


FIG. 4

This is illustrated in Figure 4. The first term of the sequence (6.1) is equal to the tangent of the angle *ABR*, i.e. to the coefficient of slope of the segment *AB*. Similarly, it can be shown that further terms of the sequence are equal to the coefficients of slope of the segments *BC, CD,* etc. These coefficients are measured in their absolute value. Otherwise they would be all negative since the segments *AB, BC, CD* etc. are "decreasing".

The reciprocals of these coefficients form an increasing sequence

$$\left|\frac{a_{21}-a_{22}}{a_{11}-a_{12}}\right| < \left|\frac{a_{22}-a_{23}}{a_{12}-a_{13}}\right| < \frac{a_{23}-a_{24}}{a_{13}-a_{14}}\left| < \dots \right. \tag{6.2}$$

These reciprocals are the rates of substitution of inputs. They express the ratio of the increment of the per unit input of one factor of production to the decrease of the per unit input of the other factor. The sequence (6.2) shows that when passing from one neighboring technical process to another one the rate of substitution of inputs increases. This property is the *law of increasing rate of substitution* of inputs.

Treating the output as negative inputs we get the *law of decreasing rate of substitution* of outputs as an immediate consequence of the law of the increasing rate of substitution of inputs. Transforming the negative inputs into positive outputs we change the signs of the terms in the sequence (6.2). The law of the decreasing rate of substitution of outputs then can be geometrically presented by means of a broken line convex to the origin of the system of co-ordinates, as shown in Figure 5. The axes of co-ordinates Oa_{2j} and Oa_{2j} represent the per unit outputs i.e. the output per unit of input.

In a similar way we get, also as a conclusion from the law of increasing rate of substitution of inputs, the law of increasing additional inputs (of their decreasing productivity). Let us consider one positive per unit input (of a factor of production) and one negative per unit input i.e. a unit output. A decrease of the per unit input is interpreted as unit increase of output i.e. a unit increase of the quantity produced. Considering the output as a positive quantity (which we denote by x) we get the geometric interpretation as shown on Figure 6.

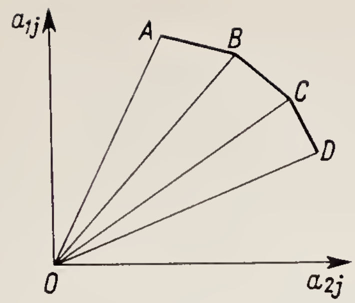


FIG. 5

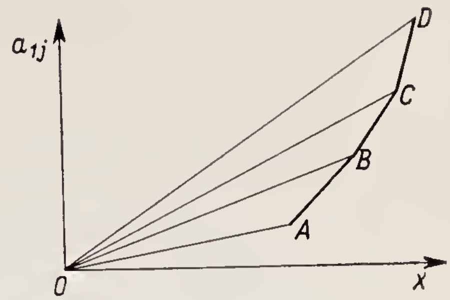


FIG. 6

§ 7. *The Neoclassical Theory of Production.*

The approach to the quantitative relations existing in the production process outlined above assumes the existence of a finite (and in practice rather small) number of technical processes by means of which the product can be obtained. Fourthemore, this approach takes account of the technical equipment and of the corresponding productive capacity of technical processes. If (1) the assumption of a definite technical equipment (and of a given productive capacity) is dropped and we assume instead that production processes are divisible and (2) their number is infinite and not enumerable (thus forming a *continuum*) and finally, (3) that the law of substitution applies to all the factors of production (and in the case of joint production to all the products) we arrive at the neoclassical theory of production.

The ends of the vectors representing different technical processes define in this case a smooth (i.e. not a broken) continuous line. Instead of the picture shown on figures 1 and 3 we get the situation as shown of Figure 7.

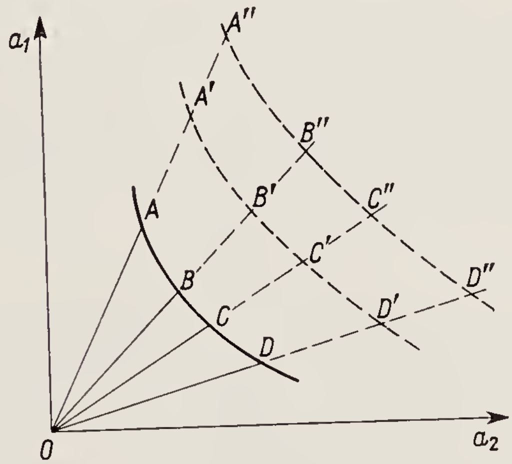


FIG. 7

The line joining the ends of the respective vectors is called *isoquants* (this is the locus of all possible technical processes yielding the same amount of product). The thick line represents the isoquant corresponding to the output of one unit for the radiuses *OA*, *OB*, *OC*, *OD* are vectors or per unit inputs. Since, acording to our assumption, there is no fixed technical equipment and no corresponding productive capacity, production can be carried on any scale. If the output is doubled then the inputs of the factors of production must also be doubled. Instead of the vectors representing per unit inputs in different technical processes we get the vectors of double lenght, i.e. OA', OB', OC', OD'—(see Figure 7) which represent the inputs of different factors of production in the processes producing 2 units of output. The ends of these vectors define a new isoquant, shown on the graph as a dashed line. This isoquant corresponds to two units of output. In a similar way it is possible to get isoquants corresponding to any amount of output or—as is often said—to different scales of output. The lengths of the vectors which generate the isoquants are proportional to the amount of product, i.e. to the scale of output. All the isoquants are hence parallel.

In consequence, there is a family of isoquants which can be represented by the equation.

$$f(a_1x,\ a_2x) = x$$

in the case of two factors of production (as in the case figure 7), or by the equation

$$f(a_1x,\ a_2x\ \ldots,\ a_n) = x \tag{7.1}$$

in the case of n factors of production. In this equation the parameter x denotes the scale of output. As in seen, the inputs of the factors of production a_1x, a_2x, ..., a_nx are proportional to the scale of output.

Writing $v_1 = a_1x$, $v_2 = a_2x_1$, ..., $v_n = a_nx$ and writing the output x on the left side of the equality sign we get

$$x = f(v_1v_2,\ \ldots,\ x_n). \tag{7.2}$$

The scale of output x is a function of the inputs of factors of production v_1, v_2, ..., v_n. In the neoclassical theory of production this function is called the *production function.* From (7.1) it follows that this function is homogenous of first degree[4].

The neoclassical theory assumes that the production function has derivatives of the first and second order. The first derivatives are called marginal productivities of the respective factors of production. It is also assumed that

$$\frac{\partial f}{\partial v_i} > 0 \text{ and } \frac{\partial^2 f}{\partial v_i^2} < 0 \tag{7.3}$$

[4] The homogeneity of the production function appears in the earlier formulations of the neoclassical theory of production as an assumption. Such an assumption was explicitly made by P. H. Wicksteed in *An Essay on the Co-ordination of the Laws of Distributions,* London 1894. In later formulations it is assumed that the production function is homogeneous only for industries but not necesserily for single plants. The homogeneity of the production function is obtained as a result of the assumption that all plants produce optimum amounts of product. In this case a k-fold expansion of output is obtained by a k-fold increase of the number of plants each of which continues producing the same optimum amount. Such a solution of the problem of homogeneity was given by K. Wicksell, *Lectures on Political Economy,* London 1935, pp. 127–131. The problem of homogeneity of the production function aroused great discussions which are related by G. J. Stigler, *Production and Distribution Theories,* New York 1941. See also E. Schneider, *Theorie der Produktion,* pp. 19–21.

for V_i exceeding some value and $t = 1, 2, \ldots, n$. The second inequality expresses the "law of diminishing returns".

The isoquants are "decreasing" lines (the law of input substitution) and are convex with respect to the origin of the co-ordinates (the law of increasing rate of substitution of inputs.) The convexity of the isoquants is expressed by the inequalities

$$\frac{\partial^2 v_1}{\partial v_2^2} > 0, \tag{7.4}$$

where v_1 and v_2 are inputs of two different factors of production (the order of numeration is arbitrary).

Between the law of increasing rate of substitution of inputs (which is expressed by the inequality (7.4) and the law of diminishing returns expressed by the second inequality (7.3) there exists the following relation.

Using the theorem on the derivatives of implicit functions we have

$$\frac{\partial v_1}{\partial v_2} = \frac{\dfrac{\partial f}{\partial v_2}}{\dfrac{\partial f}{\partial v_1}}.$$

Hence

$$\frac{\partial^2 v_1}{\partial v_2^2} = -\frac{\dfrac{\partial f}{\partial v_1}\dfrac{\partial^2 f}{\partial v_2^2} - \dfrac{\partial f}{\partial v_2}\dfrac{\partial^2 f}{\partial v_1 \partial v_2}}{\left(\dfrac{\partial f}{\partial v_1}\right)^2}. \tag{7.5}$$

However, in the case when there is $\dfrac{\partial^2 f}{\partial v_1 \partial v_2} < 0$, i.e. when an increase of the input v_2 decreases the marginal productivity input v_1 it can happen that the above expression will be negative or zero. This happens when such a decrease of the marginal productivity is very strong. The action of the law of diminishing returns is then outweighed by the mentioned decrease in marginal productivity.

Also the inverse is true: the inequality (7.4) can be fulfilled in the case when the second of the inequalities (7.3) does not hold true, i.e. when the law of diminishing returns does not operate. This can happen when

$$\frac{\partial^2 f}{\partial v_1 \partial v_2} > 0,$$

i.e. when an increase of the input v_2, increases the marginal productivity of the input v_1. If th is increase is very strong it can replace the lack of action of the law of diminishing returns.

This counteracting action of the impact of one factor of production on the marginal productivity of the second factor is, however, limited by the homogeneity of the production function. Homogeneous functions of the first degree verify the following relations between their second derivatives[5]:

$$\frac{\partial^2 f}{\partial v_1 \partial v_r} v_1 + \frac{\partial^2 f}{\partial v_2 \partial v_r} v_2 + \ldots + \frac{\partial^2 f}{\partial v_n \partial v_r} = 0 \quad (r = 1, 2, \ldots, n).$$

From these relations it follows that

$$-\frac{\partial^2 f}{\partial v_r^2} = \frac{1}{v_r} \sum_{i \neq r} \frac{\partial f}{\partial v_i \partial v_r} v_i \qquad (i = 1, 2, \ldots, n). \tag{7.6}$$

Hence, the second of the inequalities (3.3) implies some restrinction on the mixed derivatives appearing on the right side of (7.6).

In the special case when the production function is a function of only two variables the law of diminishing returns is both the necessary and sufficient condition for the law of the increasing rate of substitution of inputs. The relation (7.6) assumes then the form

$$-\frac{\partial^2 f}{\partial v_2} = \frac{v_1}{v_2} \frac{\partial^2 f}{\partial v_1 \partial v_2}.$$

Substituting this into (7.5) we get

$$\frac{\partial^2 v_1}{\partial^2 v_2} = -\frac{\dfrac{\partial^2 f}{\partial v_2}\left(\dfrac{\partial f}{\partial v_1} + \dfrac{v_2}{v_1}\dfrac{\partial f}{\partial v_2}\right)}{\left(\dfrac{\partial f}{\partial v_1}\right)^2}.$$

Taking into account that $v_1 > 0$, $v_2 > 0$ and $\frac{\partial f}{\partial v_1} > 0$, $\frac{\partial f}{\partial v_2} > 0$ we find that $\frac{\partial v_1}{\partial v_2} < 0$ if and only if $\frac{\partial^2 f}{\partial v_2^2} < 0$, i.e. if the second of the inequalities (7.3) holds true. However this does not happen in the general case when the production function is a function of more than two variables.

Joint production can be accounted for in a similar way as above, by treating products other than the reference product as factors of production with negative inputs or by assuming all the outputs to be positive and all the inputs to be negative, or *vice versa*[6]. Treating the output as negative inputs, we find that the inequality (7.4)

[5] This relation follows from the Euler's theorem for homogeneous functions. In the case of functions homogeneous of first degree the theorem states that

$$\frac{\partial f}{\partial r_1} v_1 + \frac{\partial f}{\partial r_2} v_2 + \ldots + \frac{\partial f}{\partial r_2} v_n = x.$$

By differentiating this equality with respcct to v_r we obtain the relation in the text.

[6] The approach of treating joint production by assuming inputs to be negative outputs was first introduced by J. R. Hicks, *Value and Capital*, 2nd edition, London 1946, pp. 319. Later this treatment was also applied in so-called activity analysis, i.e. in thc theory of production which assumes the existence of only a finite number of technical processes. See O. Lange *Optymalne decyzje* (Optimum Decisions), Warsaw 1964 and R. G. D. Allen, *Mathematical Economics*, cit. edition, 613.

represents also the law of diminishing rate of substitution of outputs. Assuming that v_1, denotes the input and v_2 denotes the output (with negative sign) we can interpret the inquality (7.4) as the law of increasing additional per unit inputs i.e. as the law of diminishing productivity of additional inputs.

It is to be emphasized, however, that the law of diminishing productivity of additional inputs introduced in this way has the property of being an *assumption* and is not a consequence of the law of diminishing returns interpreted as a decreasing marginal productivity or as the fulfillment of the second of the inequalities (7.3). As we know already the second of the inqualities (7.3) and the inequality (7.4) in general do not coincide.

As it is seen, the neoclassical theory of production formulates the quantitative relations in the process of production in a highly idealized way which results in a theoretical model far removed from the real production process. In reality the number of technical processes is finite (and rather small) and the respective technical processes are characterized by a given technical equipment and a given productive capacity. Besides, in the real world not all the factors of production are subject to the law of substitution and to the law of diminishing returns, understood as a property of the production function, i.e. as a technological regularity. To assume so is a generalization lacking sufficient empirical foundation.

The neoclassical theory of production must, therefore be as considered a miscarried attempt of analysis of the quantitative relations in production[7]. Its historial source is the theory of three parallel "factors of production" labour, capital and land which aimed at justification of the distribution of the social product among the owners of these factors. The neoclassical theory generalizes and modernizes this theory. Its wide acceptance can be explained by the fact that it could be used for apologetic conslusions concerning the distribution of the social product under capitalism. According to the neoclassical theory the owners of the factors of production receive the value of the marginal product of the factors they happen to own. That the owners of the factors *should* receive the value of the marginal product of the factors they own is somehow taken for granted; it is an implication which follows from the tacit assumption that they *should* own the amounts of factors of production they do happen to own, i.e. that the historically given distribution of property is right.

---

[7] About the problem of the neoclassical theory of production see also O. Lange *Optymalne decyzje*. See also the interesting remarks in the subject on H. Schultz *Marginal Productivity and the General Pricing Process*, "Journal of Political Economy", Chicago 1929.

P. C. Mahalanobis
India

STATISTICAL TOOLS AND TECHNIQUES IN PERSPECTIVE PLANNING IN INDIA

Introduction: The phrase perspective "planning" is being used in India since about 1954 or 1955 in the field of National Planning in which long range targets have to be set up 10 or 15 or 20 years in advance. The object of the present paper is to explain why perspective planning is essential in the case of under-developed countries and give some illustrative examples of the statistical information and methods which have been found useful for this purpose in India. This ist not the occasion to attempt a comprehensive discussion of techniques of perspective planning.

It is useful to make a distinction between projections and targets. The word projection, is used in the same way as in advanced countries to refer to the value of production, or of consumption or of other variates, at a specified date in future, estimated on the basis of historical records. Projections are essentially estimates obtained on the basis of analysis of time series or some kind of extrapolation in time. It is convenient to use the word target as the value of production, of consumption, or of other variates of interest which is desired to be attained on a specified date in future, through the process of implementation of an economic plan. The word target would be used consistently in this sense.

Objects of planning in India: The ultimate objects of planning are to improve the level of living, and expand facilities for education, care of health, cultural amenities etc. for all the people of the country. A spectacular improvement in the level of living of the advanced countries has been possible in the past, and a similar improvement would be possible in the less advanced areas in future, only through a continuing increase in the *per capita* production of all the people of the country. Such increase in *per capita* production can be attained only through a continuing substitution of human and animal power by machines, driven by steam or by electricity, for productive purposes of all kinds including industry, agriculture, transport and distribution.

Changes in the level of living: As our chief concern is with the improvement of the level of living, a continuing "National Sample Survey" was started in India in 1950 which is collecting comprehensive information on various aspects of the level of living in rural and urban areas with a view to assessing the change over time. The total *per capita* expenditure per month on all consumer goods and services of each household has been used as a rough indicator of the level of living of the household. The method of fractile graphical analysis[1] has been used to study the distribution by size of total *per capita* expenditure per month of households. Studies are also being made of the relationship between the total *per capita* consumer expenditure and the *per capita* consumption of individual items in terms of money and also in physical quantities where possible.

A study of the distribution of *per capita* total consumption expenditure by decile groups of households (arranged in ascending order of the total *per capita* consumer expenditure in each household), shows that for the data collected in the "National Sample Survey", round 8 (covering the period July 1954-March 1955), the percentage share of the lowest decile group was 3.01 in rural areas and 2.65 in urban areas, and of the second lowest decile group (between the tenth and the twentieth—percentiles of households ranked by *per capita* expenditure) was 4.09 and 3.90 for rural and urban areas respectively. For purposes of perspective planning, 4 per cent may be used as the share of the second lowest decile group of households[2].

Targets of planning: The average *per capita* expenditure in the second lowest decile group was a little over Rs 10 per month[3] in 1960–61. For purposes of illustration, it is possible to adopt a target of raising, over a period of 15 years, the average *per capita* consumption expenditure in the second lowest decile group of households from Rs. 10 to Rs. 20 per month (or fifty dollars *per capita* per year). This amount, at 1960–61 prices, would provide only a very modest level of living in terms of food, clothing and other essential goods or services and amenitities.

Doubling the *per capita* expenditure in fifteen years implies a rate of growth of nearly 5 per cent *per capita* per year. It is of interest to note in the present connexion that the *per capita* income in USA has increased sevenfold in the course of 120 years or at a rate a little over 1.6 per cent *per capita* per year. A reasonable target of planning in India would thus call for a rate of increase o income at a rate nearly three times greater than the actual rate of increase attained in the USA during the last 120 years. The above comparison would supply a rough idea of the dimension of the efforts required for economic development in India.

---

[1] See *A Method of Fractile Graphical Analysis*, "Econometrica", 28, 325–351, 1960; also *A Preliminary Note on the Consumption of Cereals in India*, "Bulletin of the International Statistical Institute", 39, 53–76, 1962.

[2] The lowest decile group has not been used because it may be a somewhat heterogeneous category comprising vagrants, persons living in isolation, tribal people, households in a transient income group etc. many of whom would require special ameliorative measures.

[3] One rupee = 1 shilling 6 pence = 0.21 U.S. cent approximately.

For purpose of planning it is necessary to deal with actual figures and not merely in percentages. The population of India is expected to increase to about 650 million compared with about 430 million in 1960. (This, of course, is the population projection for 1975 on plausible assumptions, and not a target; in fact, if it were possible to bring about a reduction in the rate of growth of population, Indian planners would no doubt adopt a much lower figure as a target). The number of second lowest decile group of households in 1975 would be about 12.5 million on the basis of about five persons per household. To attain a target of Rs. 1,200 (or $240) per year per household, the aggregate income of the second lowest decile group of household would have to be Rs. 15,000 million. If it is assumed that this group would still continue to have a 4 per cent share of the total expenditure of the households[4] then the aggregate national consumption expenditure of households would be 25 times greater or Rs. 375,000 million. The aggregate national income of India in 1975 would have to be somewhat larger to allow for investments and certain other items. The level of national income to be attained in 1975 would have to be somewhat more than double the target of income at the end of Third Five Year Plan in 1966. The rate of growth would have to be about 7 per cent per year.

Need of rapid industrilization: Such a rapid change (at a rate three times greater than that of USA) would call for rapid industrial expansion over a period of 15 years.

The ultimate aim is expanding continually the production of consumer goods and services. It is necessary to increase the supply of machinery and energy for this purpose. In India, and in most of the other underdeveloped countries, it is not possible continually to import machinery for production of goods or of fuel on account of shortage of foreign currency. It is essential to establish and expand industries to manufacture machinery, electricals, transport and construction equipment. To increase the capacity for the production of capital goods and energy would be thus the only sound foundation for the expansion of consumer goods and services in future.

At the same time, in all underdeveloped countries it is possible to increase the production of consumer goods with small tools by using traditional methods. This type of production is labour intensive and would give gainful employment to a large number of people who would otherwise remain idle for a good part of their time.

In India a dual strategy was adopted from 1956 in the Second Plan to expand, on one side, the strategic heavy industries for steel, metals, machinery, electricals and

[4] In India the distribution of consumption expenditure of households by size of expenditure has been found to be steady (with some small fluctuations probably due to the effect of changes in prices) over the last ten years. The pattern of distribution of income of households by size of income has also been found generally to change only very slowly over time in most countries of the world. The assumption that the share of the second decile group (or of other fractile groups) of households would remain practically the same in India in 1975 is plausible.

chemicals etc., to build up the foundations of industrial progress, and at the same time also to expand the traditional cottage industries and small scale production.

Targets of capital goods: It is, therefore, necessary to expand and set up not only targets of income or of consumer goods but also of machinery, steel and other metals, electricity, transport, etc. which would be used for the production of the desired volume of consumer goods and services.

Targets of scientific and technical personnel: To achieve the targets of production, it would be necessary rapidly to increase the technical staff to prepare and implement an increasing number of projects. Training facilities must be expanded sufficiently quickly to turn out technical and scientific personnel in adequate numbers at all levels. Scientific and technological research would have to be expanded and oriented to serve the needs of national development in an effective manner. Fundamental research as well as training in research must also be encouraged and developed at the same time to foster the accumulation of basic knowledge and to supply a sound foundation for national decisions being made increasingly on rational grounds.

Balances at the stage of production and utilization: An essential condition for successful planning is to estimate in real terms the requirements of each project to ensure that right quantities of materials, machinery and men are available at the right time at every stage of the implementation of the project. Also, products and services resulting from the completion of each project must be promptly and effectively utilised to promote the execution of other projects and for the progress of the plan as a whole.

The physical targets of production must be balanced in terms of physical quantities of raw materials, machinery, energy, transport etc., and also in terms of man power and of the flow of money. Incomes are generated in the very process of production; and supplies are utilised through market operations. Planning requires that aggregate incomes should be balanced with expenditure, savings should match investments, and the supply and demand of individual goods and services should be balanced in real terms so as to avoid any inflationary rise of prices or undesirable shifts in prices. Physical and financial planning are different aspects of the same reality.

In India a perspective view of development over a long period of years began to be taken from the end of 1954. It was recognised that the targets and the balances of materials and of man power would be only approximate partly for lack of information and partly for defects in organization and implementation. It was therefore recognised that planning would have to remain flexible and to enable necessary adjustments being made almost continuously. At the same time it was essential to keep in view a wide time horizon of 15 or 20 years or more.

The use of simple models: In 1954–55, some simple models were used to work out the basic strategy of the Second Five Year Plan. The total investment was divided into two parts, one λ_i as the fraction used for investments for the production of capital goods, and the other λ_c as the fraction used for investments for the production of consumer goods ($\lambda_i + \lambda_c = 1$). If the corresponding net output-investment ratios for the production of investment goods and for the production of consumer goods respectively are β_i and β_c then the total net output–investment ratio is $\beta = \lambda_i\beta_i + \lambda_c\beta_c$. By using the following two sector model, and using numerical values for the total investment, and estimated values of β_i and β_c, suitable values of λ_i and λ_c were selected so as to enable the economy to grow at the target rate of 5 per cent per year or so. In order to estimate the volume of employment, the capital investment required per worker, say Θ was also used.

The growth of national income Y in the two sector model is given by the following formula:

$$Y_t = Y_0\left[1 + \alpha_0 \frac{\lambda_i\beta_i + \lambda_c\beta_c}{\lambda_i\beta_i}\{(1+\lambda_i\beta_i)^t - 1\}\right]$$

in which Y_0 is the national income in the base year, Y_t the national income in the t-th year, and α_0 the rate of investment in the base year.

On this basis, a Draft Plan-frame for the Second Plan was prepared in March 1955[5]. Values of the different parameters as used in the Draft Plan-frame, the Second Plan (1956–61) as actually realised, and the Third Plan (1961–66) as estimated, are shown in the Table given below.

TABLE (1): *Investment Allocation, Capital per Worker and Net Output–Investment Ratio*

Plan	Percentage allocation of investment for		Capital per worker Rs	Net output-investment ratio		
	investment goods	consumer goods		investment goods	consumer goods	total
1	2	3	4	5	6	7
	λ_i	λ_c	Θ	β_i	β_c	β
Second Plan: Draft Plan-frame 1955	33	67	5,100	0.20	0.67	0.51
Second Plan: actual (1956–61)	36	64	5,400	0.11	0.53	0.38
Third Plan: estimate (1961–66)	39	61	6,900	0.21	0.63	0.47

Many changes were made in the targets and allocations of the Draft Plan-frame at the stage of the preparation of the Second Plan; the values of the parameters of the Second Plan as actually realized and the values given in the Draft

[5] The methods used have been described in *The Approach of Operational Research to Planning in India*, Sankhya, 16, 3–130, 1955.

Plan-frame are therefore not strictly comparable. The interesting point to note is that the estimated parameters for the Third Plan are fairly close to the parameters used in the Draft Plan-frame.

The rate of investment (α_0) in the first year of the Second Five Year Plan was 9.8 per cent, the initial national income (γ_0) was Rs. 108.0 billion and the values of the other parameters were $\lambda_i = 36\%$ $\lambda_c = 64\%$ $\beta_i = 0.11$, $\beta_c = 0.53$, as given in the second row of the above table. Using these values in the above expression, the estimated national income comes out as Rs. 129.7 billion for 1960–61 against an actual figure of Rs. 130.1 billion, both expressed at 1952–53 prices. In the case of the Third Five Year Plan, using the parameters given in the third row of the table, an initial income of Rs. 145.0 billion (at 1960–61 prices) and an initial rate of investment of 11 per cent, the estimated income for 1965–66 on the basis of the two-sector model is Rs. 188.9 billion against an estimate of Rs. 190.0 billion given in the Third Five Year Plan on the basis of detailed sector-wise calculations.

It may be concluded, therefore, that the two-sector model can supply a fairly reliable method for estimating future income. Values of the parameters used for the base period are no doubt subject to errors of estimation; but this would be true in the case of other methods also. The two-sector model gives realistic estimates presumably because it has reasonably correct structural relations between relevant variables.

Values of output–investment ratios: Output–investment ratios β_i and β_c determine, together with the chosen values of λ_i and λ_c and the total amount of investment, the rate of increase of income and have an important role in planning. These two coefficients of net output–investment ratios were calculated from technological and statistical information in respect of hundreds of enterprises combined with approppriate weights. The calculated values for manufacturing industries are given in Table (A 1).

Need of perspective planning, Steel: The need of looking a long way ahead was learnt in India through experience. I shall give one example. In 1949 when preparatory work had just started for the First Five Year Plan, a decision was practically reached to increase the capacity for the production of steel from a little less than one million ton per year to two million tons per year in the course of five years. However, a careful survey was made of the current demand as in 1949. It was found that the maximum demand would be about 1.5 million ton per year. With marginal expansion of existing steel plants, it was possible to produce about a million ton per year within the country. Owing to the wide prevalence of the views of short-range economic theory, it was, therefore, decided that it would be inadvisable to include a new million ton steel plant in the First Five Year Plan of India.

In consequence, great difficulties began to be experienced from the early years of the First Plan. Practically all the estimates for investments had been made in purely financial terms and a sizable increase in investments had been approved purely on a financial basis. As soon as the investment projects began to be implemented, there was a sharp and continuing increase in the requirements of steel and other goods and services. Very soon the demand for cement increased to nearly three times the domectic supply. There was also a continuing and large expenditure of foreign currency for the import of steel, which added up to something like 2,000 million dollars in the next ten years or so. In 1950 it would have been possible, to establish a new million ton steel plant with perhaps about 150 million dollars of imported machinery. Had this project been started at that time an additional supply of one million ton of steel (worth more than one hundred million dollars per year) would have been available from the early years of the Second Five Year Plan, and would have resulted in a very large and continuing saving of foreign exchange. The decision to drop the million ton steel project from the First Plan was due to attention being focussed only on the current demand in 1949, that is, due to a complete failure to appreciate the need of looking ahead to get ready to meet the demand for steel which was certain to increase rapidly in future.

Targets of steel in 1970: At heavy cost we had learnt the lesson of not proceeding with the building up of capacity for steel production 12 or 15 years ago. Much attention is now being given to advance planning for steel. A detailed analysis of the requirements of steel is made, where possible, by individual items of production. With a given set of production targets for, say, 1970, it is possible in this way to prepare useful estimates of the requirements of steel. Some illustrative figures for the transport equipment industry is given in the following table.

TABLE (2): *Steel Requirements for Transport Equipment Industry in 1970*

Industries	Production target in 1970	Tons of rolled steel required per unit of output	Steel requirement in 1970 (in thousand tons)
1	2	3	4
1. Steam locomotives	300	150	45.0
2. Electric locomotives	150	55	8.3
3. Diesel locomotives	200	55	11.0
4. Wagons	40,000	12	480.0
5. Passenger coaches	2,500	30	75.0
6. Automobiles	180,000	2.9	522.0
7. Motor cycles, scooters	150,000	0.1	15.0
8. Bicycles	4,000,000	0.02	80.0
9. Ships (GRT)	160,000	0.65	104.0

Source: *Demand for Steel, Special Steel and Pig Iron. India: 1960–1970*. Perspective Planning Division, Planning Commission.

The transport equipment industry would thus require about 1.34 million ton of steel per year. Requirements of other industries were estimated in the same way; the grand total for industries came to about 8 million tons of rolled metal.

In other cases a different approach is necessary. The steel requirement per rupee of net investment has been estimated for different types of activities. For example, the consumption of steel is 40 tons per investment of Rs. 100,000 in railways; the corresponding figure is so low as only 5 tons in large and medium scale irrigation. The total steel requirement for a target of investment in the Fourth Plan amounting to Rs. 170,000 million can be estimated at 20 or 21 million tons.

Also, on the basis of the investment outlay for the last year of the Fourth Plan, one can estimate the steel requirement at about 5 million tons at the end of the Fourth Plan. Adding to this the current requirement of 8 million tons for industries, the total demand would be about 13 million tons of steel in 1970–71. In the same way it has been estimated that the requirement of steel would reach 18 or 19 million tons in 1975.

Balance of electricity: It is possible in the same way to estimate the requirements of electricity from the physical targets of production for any given year. For example, the production of ferro-manganese in 1960–61 was 100,000 tons for which the electricity consumed was 500 million kwh. For a target production of 385,000 tons for ferro-manganese in 1970–71, the requirements of electricity would be 1,952 million kwh. A similar method of calculation was used for different types of industries. Table (A-3) in the Appendix gives the details. Steel and electricity are typical illustrations of the material balances which have been prepared in India for important commodities and energy for perspective planning of the economy 15 or 20 years ahead.

Perspective planning of fertilisers: The population of India is growing roughly at the rate of perhaps 9 million per year. The additional quantity of food grains required for these 9 million people would be about 1.5 million tons a year. This would add up to 22.5 million tons in the first five year period (not to speak of 60 million tons in the second five year period). At an average price of 90 dollars per ton, the cost of importing 22.5 million tons in a five year period would come to about 2,000 million dollars.

On the other hand, if imported ammonium sulphate is used, each ton on an average should increase the yield of food grains by about 2.2 tons. On this basis, roughly 10 million tons of imported ammonium sulphate would enable the domestic production of food grains being increased by about 22 million tons in a five year period. At an average price of 70 dollars per ton of fertilisers, the cost in foreign currency would be only about 700 million dollars or a third of the cost of imported food grains.

Imported food grains can be quickly distributed and it is possible to make necessary arrangements for such imports at short notice in the course of a year or so under normal conditions of easy availability of food grains in the world market. (The lack of foreign currency is the only limitation in a country like India). The import of fertilisers, however, require placing of orders a year or two or even more years in advance because the supply position is not so easy as in the case of food grains. Such a plan would, therefore, require taking a view of future needs two or three years ahead.

A third possibility would be to set up a new factory every year for the production of 750,000 tons of ammonium sulphate per year. At the cost of about 90 million dollars for each factory, thc total cxpenditure would come to 540 million dollars of which, however, only 250 million dollars would be the foreign exchange requirement. The setting up of a new fertiliser factory would require at least five or six years; the process of planning must thercfore start something like 10 years in advance.

Finally, it is also possible to manufacture in India machinery for the installation every year of a new fertiliser factore with capacity to produce 750,000 tons of ammonium sulphate per year. The foreign exchange requirement for this purpose would be less than 100 dollars, to be spent once and for all. However, the installation of a plant to manufacture machinery for the production of fertilisers would take at least five or six years. When the first batch of machinery is produced, it would take another five years or so to complete the construction of a fertiliser factory. Such a plan would require a view being taken of future requirements at least 12 or 15 years in advance.

Consumer goods: In the case of consumer goods the increase in demand is estimated on the basis of the increase of income accepted as a target. Standard methods are used to calculate the elasticity of demand from information regarding expenditure (and consumption in physical terms, where possible) of a large number of commodities and services which is being collected cvery year by the "National Sample Survey" (NSS) of India. In the NSS, the design of interpenetrating net-work of sub-samples (IPNS) is always used providing at least two independent estimates of each variatc. It is, therefore, possible to estimate the elasticity of demand on the basis of each sub-sample and also on the basis of the combined sample of the two sub-samples pooled together. Table (A-2) in the Appendix gives estimates of percentage increases in demand over the five-year period of the Third Plan. The two independent sub-sample estimates supply useful information on the margin of uncertainty of the estimates.

In a planned economy it is not possible to allow the supply to increase with the demand without any restriction. In is necessary to increase domestic savings by restricting the consumption of non-essential or luxury goods. It is, therefore, necessary to impose excise and sales tax or controls on imports or on production to bring about a balance between the planned supply and the estimated demand.

Recently the method of fractile graphical analysis is being used for estimating elasticities of demand for households having different values of total *per capita* consumer expenditure (which is a rough indicator of the level of living). This approach has the great advantage of showing, in a very simple way, the pattern of change of the elasticity of demand with a change in the level of living. Analysis by fractile groups is particularly useful in studying the effect of excise and sales tax in balancing supply and demand.

Perspective planning of man power: It is only with the help of skilled workers, technicians, technologists and engineers that raw materials can be converted into machinery, electricity and power which can then be used for the production of both capital and consumer goods. A rapidly increasing supply of engineers and technical personnel is essential for economic development. It is necessary to establish and broaden the base of primary and secondary education and to establish technical and scientific institutions and increase their number rapidly. The most serious difficulty is the lack of trained and experienced teachers at all levels. To build up a sound foundation for the outturn of technical personnel would take a great deal of time; it is a much more slowly maturing process than establishing heavy machine building, steel, heavy electrical or heavy chemical industries. Perspective planning is indispensable, and it is necessary to have targets twenty years or more in advance.

Scientific and technical man power: From about 1955 a great deal of attention is being given in India to the question of technical man power. The method used for estimating the requirements of technical personnel is simple and straightforward. Information relating to manufacturing industries for the reference period 1956 was collected as a part of the "National Sample Surveys" and was analysed in detail to ascertain the number of professional and technical workers (including engineers and scientists) employed in manufacturing industries. Estimates for a number of selected industries are given in Table 3 in the form of percentages of total employment (that is, number of engaged persons) in different industries. Separate figures are given in col. (2) for the proportion of professional, technical and associated workers taken together, in col. (3) for the proportion of engineers, architects and surveyors, and in col. (4) for the proportion of scientists including chemists, physicists, geologists and other physical scientists.

There are wide variations in requirements of professional and technical personnel or of engineers or scientists from one industry to another. In chemicals, and aircraft assembling and repair, the proportion of professional and technical staff is about 10 per cent. The chemical industries, naturally, require 5 per cent of scientists (no doubt, mostly chemists) and only 0.6 per cent of engineers. In contrast, aircraft assembling and repair requires a high proportion of about 5.5 per cent of engineers but practically no scientists.

With any assumed target of production for any particular industry in any given year, it is possible to estimate the total number of engaged persons and hence the number of professional were estimated in this way for purposes of perspective planning.

Expansion of technical staff: Appropriate action was taken, to expand the capacity of existing scientific and technological institutions and to establish new institutions all over the country to ensure a sufficiently rapid expansion of scientific and technical personnel. The following table shows the new admissions into universities and higher educational institutions of the university standard in science and technology.

TABLE (3): *Technical Personnel in Selected Industries: Sample Survey of Manufacturing Industries, 1956*

Industries	Percentage of total employment		
	professional	engineers	scientists
(1)	(2)	(3)	(4)
1. Rice milling	0.87	0.08	0.00
2. Cotton textiles	0.90	0.12	0.51
3. Glass and glassware	0.99	0.19	0.18
4. Tea manufacturing	2.39	0.31	0.03
5. Aluminium, copper, brass: secondary products	2.49	1.58	0.05
6. Sugar	2.65	0.51	0.71
7. General engineering and electrical engineering	4.27	2.02	0.01
8. Paints and varnishes	5.44	0.31	3.47
9. Cement	5.53	0.89	1.12
10. Petroleum refining	5.56	1.55	2.40
11. Electricity generation and transmission	6.50	4.79	0.04
12. Iron and steel: primary products	6.70	2.86	0.58
13. Railway wagon manufacturing	8.46	3.02	0.21
14. Aircraft assembling and repair	9.93	5.47	0.00
15. Chemicals (including drugs)	9.99	0.62	5.06

Source: *Occupational Pattern in Manufacturing Industries, India 1956* by Pitambar Pant and M. Vasudevan with a foreword by P. C. Mahalanobis. Planning Commission, Government of India, 1959. In col. (2) "professional" stands for all professional, technical and related workers. In col. (3) "engineers" cover architects and surveyors. In col. (4) "scientists" stand for chemists, physicists, geologists and other physical scientists.

TABLE (4): *Admissions into Higher Degree Level Institutions in Science and Technology*

Subject	1950–51	1960–61	1965–66	1975–76	1950–51	1960–61	1965–66	1975–76
1	2	3	4	5	6	7	8	9
		in thousands			as percentages of 1950–51 figures			
1. Science	51	116	199	264	100	227	390	518
2. Engineering	4	14	25	70	100	350	625	1,750
3. Medicine	2.5	6	8	20	100	240	320	800
4. Agriculture	2.0	5	9	15	100	250	450	750
5. Total	59.5	141	241	369	100	237	405	620

On the whole the planning for scientific and technical man power, particularly for engineering, has been quite satisfactory in India. For example, the new admissions in engineering increased from 4,000 a year in 1950–51 to 14,000 a year in 1960–61. Also the target is about 25,000 new admissions in 1965–66, and 70,000 in 1975–76.

Outturn of engineers: The Appendix Table A-4 gives the outturn of scientists and engineers in India from 1915 to 1960. It would be seen from Table A-4, line 8 and col. (4), that the number of degree level engineers turned out between 1915 and 1947 was 14,984 in 33 years before independence. This was practically matched by a turnout of 14,385 in five years during the period of the First Plan 1951–56. The outturn increased much further to 24,166 during the five-year period of the Second Plan.

The outturn for individual years between 1951 and 1960 also shows a very rapid increase. The outturn of degree level engineers was 1,700 in 1951 which was nearly doubled in three or four years. Perspective planning of technical personnel was seriously started from 1955; the effect became visible after four years in 1959 when the outturn rose to 6,779 against 3,689 in the previous year, that is, an increase of more than three thousand in one year.

Scientific Research. Although the intake and outturn of scientists also has been increasing fairly rapidly, I am sorry to say that perspective planning of scientific research has not yet started seriously. The emergence into the modern age of any underdeveloped country would be possible only with the building of the base of science education and scientific research. Certain compelling reasons can be appreciated very easily. Natural resources are not identical everywhere; there are wide variations from one country to another. Resources available within any country can be used most effectively only through continuing applied scientific and technological research in which use is made of basic scientific knowledge to solve practical problems. It is also necessary to provide facilities for fundamental research not only for the accumulation of scientific knowledge but also to supply scientists who would be able to diagnose problems properly and identify how such problems should be handled or what kind of help should be obtained from abroad. There is also a deeper need of replacing the traditional pattern of making decisions on the basis of authority by decisions to be made increasingly in objective grounds based on scientific and rational thinking.

Perspective planning is indispensable. The need of perspective planning, especially in underdeveloped countries, may be stated very briefly in conclusion. It is necessary to increase the supply of consumer goods. To do this it is necessary to expand continually the production of capital goods. Both would require an increasing supply of engineers and technicians. Industrial and technological developments would call for a rapid expansion of applied research which, in its turn, would require a sound foundation of basic research.

The factor of time may be next considered. Factories for the production of practically any kind of consumer goods can be established in a year or two with the help of imported machinery or fuel. To develop the production of capital goods and energy would take it at least 10 or 15 years. To secure an adequate supply of engineering and technical personnel would require 20 or 25 years. To have enough scientists of ability for both applied and basic research would take at least a generation or even more. It is clear that perspective planning, looking 15 or 20 or 30 years ahead, is indispensable for all underdeveloped countries.

APPENDIX

TABLE (A—1): *Estimates of β and Θ for Major Grups of Manufacturing Industries with 1957 and 1960–61 weights*

	β		Θ (thousand Rs.)	
	1957 weights	1960–61 weights	1957 weights	1060–61 weights
(1)	(2)	(3)	(4)	(5)
1. Metallurgical industries:	0.19	0.20	178.9	172.3
2. ,, semi manf.	0.47	0.45	17.1	19.0
3. Mechanical and general engineering	0.66	0.65	11.4	10.9
4. Transport equipment	0.45	0.45	15.4	15.3
5. Electrical equipment	0.50	0.49	16.6	18.5
6. Industrial machinery (I)	0.62	0.61	24.9	22.7
7. ,, ,, (II)	0.47	0.43	17.4	20.1
8. Chemicals	0.35	0.32	29.1	30.3
9. Textiles	0.38	0.38	10.6	10.5
10. Rubber and leather products	0.62	0.61	14.5	14.8
11. Food industries	0.30	0.30	13.0	12.9
12. Mining industries	0.33	0.35	17.6	20.5
13. Timber and cellulose industries	0.33	0.31	11.2	12.3
14. Mining and oil industry	0.43	0.39	9.5	11.1
15. All industries	0.36	0.35	13.2	15.3

Note: The coefficients are obtained from detailed industry-wise information compiled by the Perspective Planning Division of the Planning Commission in collaboration with the Planning Unit of the Indian Statistical Institute.

TABLE (A-2): *Estimates of Percentage Increases in Demand over the Third Five Year Plan Period 1961–1966*

	Percentage increase in demand								
	Urban India			Rural India			All-India		
	ss. 1	ss. 2	combined	ss. 1	ss. 2	combined	ss. 1	ss. 2	combined
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Pulses	32	32	32	19	21	20	21	23	22
2. Vegetables	35	38	36	22	23	22	26	27	26
3. Spices	30	31	30	18	17	17	19	20	19
4. Edible oil	35	35	35	23	20	22	26	24	25
5. Sugar	38	39	38	29	28	29	31	31	31
6. Milk and milk products	46	43	44	33	40	36	37	40	38
7. Meat, fish, eggs	38	40	39	21	21	21	26	28	27
8. Fruits and nuts	46	45	45	23	25	24	32	33	32
9. Beverage and refreshments	42	41	41	28	27	27	34	34	34
10. Tabacco	36	35	36	22	19	21	25	23	24
11. Kerosene	32	33	32	19	18	19	22	22	22
12. Fuel and light	33	33	33	19	17	18	22	21	21
13. Cotton clothing (mill-made)	49	43	48	36	33	34	39	35	37
14. Washing soap	38	39	39	30	27	30	33	32	33
15. Toilets	45	41	41	22	22	22	32	30	30
16. Railway	53	49	53	46	33	35	49	40	43
17. Conveyance	53	55	53	38	28	35	44	39	43
18. Cinema	45	53	49	32	30	31	41	44	42
19. Domestic utensils	31	46	37	37	23	36	37	26	36

Note: Based on elasticities calculated in the Indian Statistical Institute, Calcutta from 10th round NSS data relating to December 1955–May 1956.

TABLE (A-3): *Electricity Balance*

Consuming industry	Unit	Volume of production: 1960–61 production	Volume of production: 1965–66 capacity	Volume of production: 1970–71 capacity	Electricity consumption in m.kwh: 1960–61	Electricity consumption in m.kwh: 1965–66	Electricity consumption in m.kwh: 1970–71	Electricity consumption per unit of production
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1. Iron and steel	m.ton							
1 Finished steel	,,	2.2	7.5	13.5	1,100	3,750	6,750	500 kwh/ton
2 Pig iron	,,	0.9	1.5	3.0	18	30	60	20 ,,
3 Steel re-rolling					200	550		
2. Ferro-manganese	000 tons	100.0	220.0	385.0	500	1,100	1,925	5,000 ,,
3. Ferro-silicon	,,	6.0	40.0	60.0	48	320	480	8,000 ,,
4. Alloy steel	,,	—	200.0	500.0	—	250	625	1,250 ,,
5. Aluminium	,,	18.5	125.0	250.0	370	2,500	5,000	20,000 ,,
6. Copper	,,	8.9	22.0	50.0	27	66	150	3,000 ,,
7. Zinc	,,	3.2	15.0	30.0	13	63	126	4,200 ,,
8. Coal	,,							
1 Bituminous	m.ton	53.0	97.0	180.0	106	1,940	3.600	20 ,,
2 Brown	,,	—	6.0	12.0	—	120	240	20 ,,
9. Petroleum (refining)	,,	4.6	17.0	30.0	161	596	1,050	35 ,,
10. Fertilizers	,,							
1 Nitrogeneous, electrolytic process	000 tons	110.0	80.0	80.0	462	1,280	1,280	16,000 ,,
2 Nitrogeneous, rest	,,	—	920.0	1,920.0	—	3,864	8,064	4,200 ,,
3 Phosphatic	,,	55.0	500.0	1,250.0	25	225	563	450 ,,
11. Heavy chemicals								
1 Sulfuric acid	,,	363.0	1,750.0	3,700.0	91	438	925	250 ,,
2 Soda ash	,,	145.0	530.0	860.0	22	80	129	150 ,,
3 Caustic soda, chemical process	,,	—	50.0	90.0	—	25	45	500 ,,
4 Caustic soda, electrolytic process	,,	100.0	350.0	610.0	420	1,470	2,562	4,200 ,,

TABLE (A-3): *Electricity Balance—Contd.*

Consuming industry	Unit	Volume of production			Electricity consumption in m. kwh			Electricity consumption per unit of production
		1960–61 production	1965–66 capacity	1970–71 capacity	1960–61	1965–66	1970–71	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
12. Plastics	,,	10.0	85.0	250.0	1	5	15	60 ,,
13. Soap	,,	150.0	500.0	700.0	30	100	140	200 ,,
14. Synthetic rubber	,,	—	50.0	140.0	—	35	98	700 ,,
15. Paper and paper board	,,	350.0	820.0	1,500.0	630	1,476	2,775	1,800 ,,
16. Newsprint and security paper	,,	25.0	151.5	240.0	16	98	156	650 ,,
17. Cement	m.tons	8.0	15.0	26.0	960	1,800	3,120	120 kwh/ton
18. Cotton textiles	m.metres	4,572.0	5,300.0	6,400.0	1,998	2,316	2,797	437 kwh/000 metres
19. Jute	000 tons	1,065.0	1,200.0	1,600.0	415	468	624	390 kwh/ton
20. Rayon and staple fibre								
1 Rayon filament	m.lbs.	47.0	140.0	250.0	147	438	781	3,125 kwh/000 lbs.
2 Staple fibre	,,	47.8	75.0	120.0	40	62	99	825 ,,
3 Chemical pulp	000 tons	—	120.0	250.0	—	60	125	500 kwh/ton
21. Woollen fabrics-yearn	m.lbs.	28.0	67.0	100.0	84	201	300	3,000 kwh/000 lbs.
22. Silk	m.yds.	350.0	550.0	800.0	105	165	240	300 kwh/000 yards
23. Sugar	m. tons	2.7	3.5	4.5	162	210	270	60 kwh/ton
24. Vegetable oil	,,	0.8	1.2	2.2	100	150	275	125 ,,
25. Vanaspati ghee	000 tons	330.0	550.0	730.0	73	121	161	220 ,,
26. Bicycles	m.nos.	1.0	2.2	3.5	15	33	53	15 kwh/nos.
27. Sewing machines	000 nos.	300.0	700.0	1,200.0	18	42	72	60 kwh/nos.
28. Electric fans	m.nos.	0.9	2.8	4.5	18	56	90	20 kwh/nos.
29. Electric lamps	,,	39.5	83.0	120.0	6	12	18	150 kwg/000 nos.
30. Matches	m.gross boxes	33.0	45.0	52.0	20	27	31	600 kwh/000 gross boxes
31. Plywood	m.sq.metre	15.5	27.0	45.0	14	25	42	930 kwh/000 sq. metres
32. Calcium carbide	000 tons	10.0	67.0	110.0	35	235	385	3,500 kwh/ton

TABLE (A-3): *Electricity Balance — Contd.*

Consuming industry	Unit	Volume of production			Electricity consumption in m. kwh			Electricity consumption per unit of production
		1960—61 production	1965—66 capacity	1970—71 capacity	1960—61	1965—66	1970—71	
(1)	(2)	(3)	(4)	(4)	(6)	(7)	(8)	(9)
33. Automobile tyres	m.nos.	1.4	3.7	7.5	150	396	803	107 kwh/nos.
34. Automobiles	000 nos.	53.5	100.0	200.0	54	100	200	1,000 kwh/nos.
35. Coke	m.tons	5.0	15.0	25.0	125	375	625	25 kwh/ton
36. Total					8,779	27,672	47,870	

Notes: Source: Perspective Planning Division paper: Demand for Electricity, India 1960–1970. The table covers all industrial uses of electricity. The norms of electricity requirement in different industries, given in col. (9), have been used to work out the consumption of electricity in 1960–61, 1965–66 and 1970–71 given respectively in cols. (6), (7) and (8).

TABLE (A-4): *Outturn of Scientists and Engineers in India*

		Number of persons graduating					
		Master's degree in natural science		Engineering			
		total	average per year	degree	diploma	total	average per year
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.	1915–19	832	166	568	1,703	2,271	454
2.	1920–24	917	183	771	1,902	2,673	535
3.	1925–29	1,923	385	1,619	4,322	5,941	1,188
4.	1930–34	2,784	557	2,190	5,397	7,587	1,517
5.	1935–39	2,938	588	2,901	5,331	8,232	1,646
6.	1940–44	3,378	676	3,765	6,280	10,045	2,009
7.	1945–47	2,511	837	3,170	4,538	7,708	2,569
8.	1915–47	15,283	463	14,984	29,473	44,457	1,347
9.	1948–50	2,947	982	4,691	4,623	9,314	3,105
10.	1951–55 (1st Plan)	9,062	1,812	14,385	11,629	26,014	5,203
11.	1956–61 (2nd Plan)	15,799	3,160	24,166	27,037	51,203	10,241
12.	1951	1,409	1,409	2,301	1,700	4,001	4,001
13.	1952	1,680	1,680	2,559	2,049	4,608	4,608
14.	1953	1,694	1,694	2,926	1,693	4,619	4,619
15.	1954	2,068	2,068	3,238	2,833	6,071	6,071
16.	1955	2,211	2,211	3,361	3,354	6,715	6,715
17.	1956	2,456	2,456	3,456	4,131	7,587	7,587
18.	1957	2,832	2,832	3,507	4,413	7,920	7,920
19.	1958	2,982	2,982	3,689	5,944	9,633	9,633
20.	1959	3,558	3,558	6,779	6,182	12,961	12,961
21.	1960	3,971	3,971	6,735	6,367	13,102	13,102

Note: Figures are taken from *Recent developments in the organization of science in India* by P. C. Mahalanobis; *Engineers in India* by Scientific and Technical Man Power Division, Planning Commission; *Education in India* by Ministry of Education, and also direct information from the Resources and Scientific Research Division of the Planning Commission.

TABLE (A-5): Average *per capita* Consumer Expenditure in Rupees per Month (30 Days), Percentage Share of Total Consumer Exp
March 1955, A

Fractile group (percentage)	Average *per capita* consumer expenditure (Rs.)						
	rural			urban			
	ss. 1	ss. 2	pooled	ss. 1	ss. 2	pooled	ss. 1
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1. lowest	—	—	—	—	—	—	—
2. 0— 10	4.21	4.64	4.48	6.20	6.68	6.54	2.75
3. 10— 20	6.25	6.56	0.42	9.30	9.96	9.59	4.07
4. 10— 30	7.72	8.24	7.99	11.62	12.83	12.06	5.05
5. 30— 40	9.26	9.52	9.37	13.75	15.19	14.28	6.11
6. 40— 50	10.91	10.89	10.90	16.00	18.56	16.94	7.18
7. 50— 60	12.61	12.65	12.63	19.00	21.62	20.11	8.27
8. 60— 70	14.82	15.12	14.94	22.68	26.82	23.86	9.51
9. 70— 80	17.72	18.54	18.17	27.20	33.52	29.52	11.78
10. 80— 90	22.42	23.80	23.04	37.56	43.54	39.00	14.78
11. 90—100	46.44	39.00	42.16	65.20	88.22	76.78	39.50
12. 0—100	14.93	14.98	14.96	22.44	27.69	25.24	100.00
13. number of villages or blocks	353	353	706	238	228	466	353
14. number of households	931	938	1869	963	892	1855	931

Source: Indian Statistical Institute, Calcuta.

e and Limiting Values of Consumer Expenditure by Fractile Groups for the 8th Round of the "National Sample Survey", July 1954—
: Rural and Urban

Percentage share				Limiting values (Rs.)					
	urban			rural			urban		
po oled	ss. 1	ss. 2	pooled	ss. 1	ss. 2	pooled	ss. 1	ss. 2	pooled
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
—	—	—	—	2.19	2.20	2.19	2.76	2.43	2.43
3.01	2.96	2.46	2.65	5.48	5.89	5.70	8.14	8.52	8.30
4.09	4.27	3.48	3.90	7.01	7.39	7.18	10.22	11.24	10.82
5.33	4.56	4.72	4.86	8.26	8.93	8.67	12.68	14.02	13.29
6.18	6.73	5.34	5.70	10.15	10.18	10.17	14.65	16.91	15.30
7.27	7.05	6.86	6.78	11.74	11.75	11.74	17.42	20.17	18.50
8.35	8.29	7.82	8.01	13.73	13.70	13.73	20.91	23.79	21.73
9.74	9.98	9.71	9.64	16.23	16.69	16.42	24.45	29.65	26.08
11.95	11.65	12.16	11.86	19.72	20.83	20.22	30.92	37.20	33,48
15.53	15.43	15.78	15.61	26.79	28.03	27.55	46.71	53.61	46.65
28.55	29.08	31.67	30.90	239.25	112.96	239.25	525.07	333.92	525.07
100.00	100.00	100.00	100.00	239.25	112.96	239.25	525.07	333.92	525.07
706	238	228	466	353	353	706	238	228	466
1869	963	892	1855	931	938	1869	963	892	1855

В.С. НЕМЧИНОВ
СССР

ОСНОВНЫЕ КОНТУРЫ МОДЕЛИ ПЛАНОВОГО ЦЕНООБРАЗОВАНИЯ

1. *Концепция цен.* Цена — один из самых замечательных экономических феноменов. Без цены нет хозяйства. Единственная объективная база формирования общего уровня цен — это стоимость. Стоимость создается в процессе производственной трудовой деятельности людей в условиях общественного разделения труда. Современное общественное производство целиком базируется на общественном разделении труда. Члены общества и его производственно-потребительские ячейки, а также производственно-территориальные комплексы и государственные национальные образования постоянно обмениваются друг с другом результатами труда своих членов. При таком обмене соблюдается принцип эквивалентности. Только обмен по стоимости есть эквивалентный обмен результатами труда.

Стоимость создается на народнохозяйственном уровне. Затем происходит процесс ее дифференциации и индивидуализации. Стоимость, дифференцируясь, одновременно распадается на свои составные части. Сначала обособляется необходимый и прибавочный труд, а затем выделяются составные элементы денежной формы стоимости в виде перенесенной стоимости овеществленного труда (материальные затраты), стоимости оплаченного труда (заработная плата и оплата труда колхозников), стоимости прибавочного труда (фонд накопления, фонд содержания непроизводственной сферы и сальдо внешних связей). Дифференциация стоимости протекает в виде процесса превращения формы стоимости. В процессе своего превращения стоимость приобретает такую форму, которая позволяет беспрепятственно протекать процессу индивидуализации стоимости, т.е. процессу сведения общественной стоимости к индивидуальной стоимости. Одновременно это есть и процесс реализации стоимости. Реализовать стоимость можно лишь в количестве не большем, чем ее создано. Поэтому сумма реализованной стоимости равна (или меньше), чем сумма созданной стоимости. Известно, что К. Маркс не раз указывал на образование в определенных условиях нереализованной стоимости.

В процессе превращения формы стоимости, неразрывно связанной с дифференциацией и индивидуализацией стоимости, определяются народнохозяйственные коэффициенты рентабельности функционирующих производственных основных и оборотных фондов, а также рентные оценки эксплуатируемых природных ресурсов.

Система оплаты труда, система коэффициентов рентабельности основных и оборотных фондов, система рентных оценок природных ресурсов позволяет беспрепятственно переходить от народнохозяйственного стоимостного уровня к индивидуальной стоимости, все время строго находясь в рамках созданной и реализуемой стоимости.

Одновременно с процессом превращения формы стоимости происходит также несколько иного типа процесс индивидуализации общественной стоимости, который связан с взаимодействием трудовой стоимости и потребительной стоимости. В этом процессе и формируются отклонения цен от стоимости, тесно связанные с потребительскими свойствами продуктов труда, принявших форму товаров. Потребительная стоимость — носитель стоимости и в качестве таковой она воздействует на цены, отклоняя их от стоимости.

Отклонения цен ограничены: они всегда происходят в пределах стоимости той совокупности взаимозаменяемых товаров, которая в состоянии так или иначе удовлетворить данную общественную потребность.

В пределах такой совокупности взаимозаменяемых товаров отклонения цен от стоимости имеют тенденцию взаимно погашаться и сумма цен всегда стремится быть равной сумме стоимости. Этот процесс тяготения цен к стоимости следует рассматривать, идя от общего к индивидуальному. Начинать сопоставление цен со стоимостью следует с таких генеральных совокупностей товаров, как фонд потребления населения, фонд накопления, фонд возмещения.

В пределах фонда потребления населения цены на отдельные товары хотя и отклоняются от стоимости, но в пределах всего фонда потребления сумма их цен стремится быть равной сумме стоимостей. Отклонения цен происходят, прежде всего, под влиянием потребительской оценки каждой товарной группы со стороны населения, проявляемой в актах предпочтительной покупки или предпочтительного потребления. Потребительная оценка связана с мерой насыщения данной потребности, мерой ее настоятельности (насущности) и эластичностью спроса.

В процессе отклонения цен от стоимости каждая укрупненная группа товаров детализируется в свою очередь, причем более детальные наименования товаров получают свои потребительные оценки, отражающие потребительские свойства товаров и их качество. В пределах старой укрупненной товарной группы сумма цен стремится быть равной стоимости всей укрупненной группы, умноженной на групповой коэффициент пропорционального отклонения цен. При каждой последующей детализации товарной номенклатуры коэффициент пропорционального отклонения дополняется новым множителем, но цены будут все время оставаться привязанными к стоимости.

В своей совокупности цены не выходят за пределы созданной в производстве стоимости. Цены в целом все время стремятся оставаться пропорциональными стоимости.

2. *Исходная модель производства и общественного разделения труда.* В процессе формирования стоимости и цен можно выделить ряд отдельных контуров последовательного математического и экономического описания отдельных этапов процессов образования стоимости и формирования цен. Оба эти процесса объективно протекают в реальной экономической жизни и нам следует лишь имитировать их в математической форме.

Первый контур описывает процесс формирования физической (вещественной) структуры общественного производства и общественного разделения труда.

При конструировании модели общественного производства и модели общественного разделения труда особое внимание обращается на наиболее рациональный выбор номенклатуры товаров и продуктов. В этой номенклатуре воспроизводится перечень важнейших ключевых товаров и продуктов, выделяемых при составлении текущих и перспективных планов. По этому перечню конструируется ,,несущий костяк'' модели товарных цен. Он состоит из стратегически важных для планового народного хозяйства продуктов и товаров. Товарная номенклатура включает 800–1000 товарных позиций. В нее входят важнейшие жизненные средства, составляющие основу семейного бюджета; главные виды сельскохозяйственного и минерального сырья; основные виды энергии и топлива, промышленного сырья и полуфабрикатов; главнейшие виды оборудования и строительных объектов.

Перечень товаров и продуктов дается с учетом важнейших потребительских свойств товаров (топливо — в единицах условного топлива; двигатели и турбины — в условных единицах мощности; минеральное сырье — в переводе на содержание металла; удобрения — с учетом содержания в них усвояемых веществ; корма — в переводе на кормовые единицы; металлоизделия — в нормативных трудо-часах и т.д.).

Товарная номенклатура базисной модели производства и общественного разделения труда имеет своей главной целью выделить основные виды потребительных стоимостей, как носителей стоимости, и тем самым в дальнейшем обеспечить возможность определения стоимостного уровня товарных цен.

Товарно-продуктовая номенклатура включает не только важнейшие товары и продукты, но и группы ,,прочие продукты отрасли'' (80–90 таких отраслевых групп). ,,Прочие продукты отрасли'' даются в неизменных твердых ценах. Принятая в советской статистике система народнохозяйственного и низового первичного учета позволяет это сделать.

На основе такой товарной номенклатуры строится модель общественного производства и общественного разделения труда. Модель общественного производства при этом оформляется двояко: как свободный материальный баланс (в физических и денежных единицах измерения) и как межотраслевой баланс производства и распределения продукции (в денежных единицах измерения).

Сводный материальный баланс, моделирующий вещественную структуру общественного производства, состоит из двух блоков, оформленных в виде двух продуктово-отраслевых матриц:

А) промежуточные продукты;

Б) конечные продукты.

Модель общественного разделения труда включает эти два блока и дополняется третьим блоком:

В) затраты труда (по категориям трудящихся — инженерно-технические работники, младший обслуживающий персонал, рабочие по тарифным группам). Этот блок дается в единицах рабочего времени, причем принимаются меры к тому, чтобы затраты рабочего времени включали в себя не только прямые, но и косвенные затраты труда, произведенные во вспомогательных цехах и на общезаводских работах. Косвенные затраты труда разносятся по продуктам и товарам в соответствии с заводской практикой разнесения по товарам и продуктам накладных и косвенных расходов. Итоги затрат рабочего времени по столбцам дают вектор-строку ,,затраты труда''.

Третий блок (В) ,,затраты труда'' располагается под первыми двумя блоками. Блоки А и Б вместе образуют свободный материальный баланс.

В первом блоке промежуточных продуктов по строкам предусматривается общая товарно-продуктовая номенклатура (с выделением по неконкурирующему импорту товарных групп импорта), а по столбцам выделяются чистые отрасли производства, сгруппированные по видам выпускаемых товаров, а не по предприятиям.

Таким обазом первый блок (А) дает распределение продуктов и товаров одновременно по производящим их отраслям (строки) и по потребляющим (столбцы).

Во втором блоке (Б) выделены по строкам соответствующие позиции всей товарно-продуктовой номенклатуры, а по столбцам выделены общественные фонды (фонд валовых накоплений, фонд потребления, фонд внешних связей). Между первым и вторым блоками предусматривается столбец ,,потери и выбытия''.

Фонд валовых накоплений во втором блоке, в свою очередь, подразделяется на капитальный ремонт, на обновление основных фондов (реновация), на капитальные вложения и прирост оборотных фондов. По каждой из этих подгрупп, кроме того, выделяется производственная и непроизводственная сфера. В блоке конечных продуктов по фонду потребления населения обязательно выделяется потребление семей работников сферы материального производства, а также потребление других групп населения и эксплуатационные расходы социально-культурных и административных учреждений.

Если иметь в виду как модель общественного производства, так и модель общественного разделения труда, то первый и второй блоки даются в трех единицах измерения (в физических единицах, единицах рабочего времени по трудоемкости и в текущих ценах), а третий блок (затраты труда) дается в двух единицах измерения (рабочее время, оплата труда в денежной оценке).

Модель общественного производства в текущих ценах дополняется (по сравнению со сводным материальным балансом) еще одним — четвертым блоком, характеризующим в денежном измерении так называемый „условно чистый доход" (блок Д). В столбцах блока (Д) дается та же номенклатура чистых отраслей производства, а в строках показаны: амортизационные отчисления, виды оплаты труда (основная и дополнительная заработная плата, начисления по социальному страхованию, оплата труда колхозников с оценкой в деньгах натуральных выдач по трудодням, прочие денежные доходы населения, общественный фонд потребления), а также виды чистого дохода общества (прибыли, налог с оборота, рента).

Совокупность первого и второго блоков описывает структуру производства, потребления и распределения общественного продукта в трех единицах измерения. Совокупность же первого блока (в денежных единицах измерения) и блока (Д) показывает стоимостную структуру общественного производства и позволяет выделить три основных стоимостных элемента продукции, а именно: материальные затраты (блок первый, плюс строка амортизационных отчислений из блока Д), оплата труда, стоимость прибавочного продукта.

Модель общественного продукта в денежных единицах измерения дополняется, кроме того, пятым блоком „основные и оборотные фонды". В строках пятого блока приводится более укрупненная вещественная номенклатура фондов, а в столбцах показывается корреспондирующая ей номенклатура отраслей производства, то есть отраслей соответствующих укрупненным агрегатам, составленным из чистых отраслей производства.

Совокупность первого, второго и третьего блоков, оцененная в затратах рабочего времени, составляет модель общественного разделения труда.

Предполагается, что плановая модель общественного продукта выражает оптимальный план, а эта оптимальность определена на основе соответствующих экономических критериев. Предполагается также, что при отборе оптимального варианта плана применялись как общеэкономические, так и экономико-математические критерии. В соответствии с общеэкономическими критериями план производства, например, должен удовлетворять: общим директивам руководящих органов, непреложному закону хозяйственного строительства (максимальный результат при минимальных затратах) и закону экономии общественного труда. Следовательно, он должен обеспечивать минимальные потери круга при выборе как районного размещения производства, так и технологических способов производства, а также организационных способов выполнения плана. При этом предполагается, что план физической структуры производства основан на отборе технологических способов путем оценки их не только с точки зрения соответствия требованиям технического прогресса, но и с точки зрения их соответствия математическим критериям оптимального плана, например, оптимальному вектору интенсивности технологических способов производства и интенсивности взаимозаменяемых производств.

Такова структура исходной базисной плановой модели общественного производства и общественного разделения труда.

3. *Процесс образования стоимости.* Исходная базисная модель производства и общественного разделения труда позволяет математически описать и, тем самым, в известной мере имитировать сложный процесс образования общественной (трудовой и меновой) стоимости, протекающий общественным путем в реальной действительности.

Первый контур такого математического описания состоит в определении на основе абсолютных величин исходной базисной модели соответствующих параметров и коэффициентов модели планового ценообразования.

На стадии первого контура моделирования плановых цен определяется матрица расходных технологических коэффициентов (по первому блоку исходной модели), матрица трудоемкости и оплаты труда (по третьему блоку), матрица капитальных коэффициентов и коэффициентов потребления (по второму блоку), матрица фондоемкости (по пятому блоку).

Пусть физический объем выпуска продукции принят равным по строкам X_i, а по столбцам X_j, затраты i-го продукта труда в j-ом производстве — X_{ij}. Тогда технологические расходные коэффициенты будут равны:

$$a_{ij} = \frac{X_{ij}}{X_j} \qquad (1)$$

По импортным предметам труда они равны:

$$\tilde{a}_{mj} = \frac{\tilde{X}_{mj} \cdot \tilde{P}_m}{X_j} \qquad (1a)$$

где $\tilde{P}_m$ — импортный валютный коэффициент, переводящий валюту иностранную в отечественную.

Пусть T_{ej} есть рабочее время l-ой категории работников в j-ом производстве. Тогда коэффициенты трудоемкости будут равны:

$$t_{ej} = \frac{t_{ej}}{X_j} \text{ (по отдельным категориям работников).} \qquad (2)$$

$$t_{ij} = \frac{T_{ij}}{X_j}$$ (по всем категориям работников, работающих в j-ом производстве и потребляющих i-ый продукт). (2a)

Пусть $\hat{F}_{kj}$ есть основные фонды k-го вида, функционирующие в j-ом производстве, а F_{0j} аналогично оборотные фонды. Тогда коэффициенты фондоемкости равны:

$$\hat{f}_{kj} = \frac{\hat{F}_{kj}}{X_j} \quad \text{и} \quad f_{0j} = \frac{F_{0j}}{X_j} \tag{3}$$

На этом заканчивается первый контур вычислений, связанных с моделированием плановых цен.

Второй контур математического описания планового ценообразования воспроизводит процесс образования стоимости. Его моделирование состоит из трех стадий:

а) определения полных затрат труда;

б) определения общественно необходимых затрат труда;

в) определения стоимости.

Для определения полных затрат труда используется матрица технологических коэффициентов (включая импортные коэффициенты и коэффициент амортизационных отчислений) и вектор трудоемкости в единицах простого труда. Матрица коэффициентов и векторов трудоемкости была определена в процессе плановых расчетов, входящих в первый контур. Но, кроме того, подлежат определению коэффициенты трудоемкости по расчету на один рубль амортизационных отчислений и коэффициента трудоемкости по расчету на один рубль, затраченный при импорте товаров.

Во втором блоке базисной модели показан состав затрат на капитальный ремонт и реновацию и выявлена товарная структура экспорта, причем и то и другое дано как в физических единицах измерения, так и в текущих ценах. Трудоемкость рубля амортизационных отчислений получается путем деления трудовых затрат на денежные затраты на капитальный ремонт и реновацию по ним же.

Сопоставляя таким же образом оценку вектора-столбца „Экспорт" в двух измерениях (в денежных единицах отечественной валюты и в трудоемкости), получаем трудоемкость экспортного рубля. При оценке экспорта в денежных единицах отечественной валюты уже был принят во внимание импортный валютный коэффициент, позволивший перевести экспортную выручку иностранной валюты во внутреннюю отечественную валюту.

Имея это в виду и исходя из того, что экспорт призван обеспечить валютной выручкой импорт товаров, приравниваем трудоемкость импортного рубля к трудоемкости экспортного рубля.

Поэтому матрица технологических коэффициентов первого блока дополняется двумя векторами-строками, выраженными в единицах трудоемкости, а именно: вектором амортизационных коэффициентов и вектором импортных коэффициентов.

Полную трудоемкость определяем на основе этой дополненной матрицы, применяя следующие итерационные уравнения Дмитриева–Леонтьева[1]

$$\tau_j = \sum a_{ij}\tau_i + t_j \qquad \text{где} \qquad i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n \tag{4}$$

Здесь a_{ij} — расходные технологические (а также амортизационные и импортные) коэффициенты, t_j — строка коэффициента трудоемкости, τ_j — строка коэффициента полной трудоемкости, τ_i — транспонированная в колонку строка полных коэффициентов трудоемкости.

В символах матричной алгебры имеем:

$$t(E-A)^{-1} = \tau. \tag{4a}$$

Здесь $(E-A)^{-1}$ обратная матрица, A — матрица коэффициента $\| a_y \|$, E — единичная матрица, t — вектор трудоемкости, τ — вектор полной трудоемкости.

Аналогичным образом могут быть определены полные затраты труда в единицах простого труда. Для этого необходимо итерационную формулу Дмитриева–Леонтьева (или обратную матрицу) применить не в отношении общего вектора трудоемкости, а отдельно по каждому вектору трудоемкости разных категорий работников. Полученные коэффициенты полной трудоемкости (τ_{ej}) умножаются на тарифный коэффициент. Суммируя произведения, получаем вектор-строку полных затрат труда, измеренных в единицах простого труда $| \tau'_j |$.

Затем переходим к определению общественно необходимых затрат труда. Эта категория в реальной экономической действительности возникает в связи с тем, что затраты живого труда распадаются на необходимый и прибавочный труд, причем прибавочный труд начинает распределяться в условиях эквивалентного обмена пропорционально необходимому труду.

Для моделирования этого экономического процесса следует предварительно определить коэффициенты потребления жизненных средств семьями работников сферы материального производства. Во втором блоке модели из состава конечного продукта (см. базисную модель) выделяется отдельно вещественный состав фонда потребления семей работников материального труда.

На основе данных бюджетных обследований и плана повышения материально-культурного уровня жизни трудящихся строим матрицу для фонда потребления семей работников сферы материального производства в виде $\| Q_{ij} \|$, где Q_{ij} — объем продуктов и товаров i-го вида, потребленных семьями работников материального труда, занятых в j-ом производстве.

Пусть T_j — количество рабочего времени, затраченное в j-ом производстве, а T'_j — количество простого труда, израсходованного в том же произ-

[1] В. Дмитриев, *Экономические очерки*, СПБ, 1904, стр. 7–8. Об идентичности результатов, полученных по уравнению Дмитриева и Леонтьева (1938) см. "Bulletin of the Oxford University Statistics" November 1962, (статья А. Zauberman), а также в сб. *Применение математики в экономике* под ред. Аганбегяна ст. Белкина *Применение вычислительной техники и математики в экономике*.

водстве. Тогда коэффициенты потребления по расчету на единицу труда будут равны:

$$h_{ij} = \frac{Q_{ij}}{T'_j}, \quad \text{а} \quad h'_{ij} = \frac{Q_{ij}}{T'_j}. \tag{5}$$

Этот коэффициент нам будет нужен для определения объема необходимого труда.

На основе коэффициентов полной трудоемкости (τ_j), коэффициентов полных затрат простого труда (τ'_j) строим модель общественного разделения труда уже не в единицах трудоемкости (как это было сделано в базисной модели общественного разделения труда), а в единицах полной трудоемкости и в единицах полных затрат простого труда.

Для этого транспонируем векторы-строки коэффициентов полной трудоемкости и коэффициентов полных затрат простого труда и умножаем каждую строку первого блока модели общественного производства (X_{ij}) на соответствующую ей строку транспонированных коэффициентов. При этом мы исходим из допущения, что оценка товара в единицах полных затрат труда определяется только технологическими условиями производства и что эти оценки не меняются в зависимости от того, в какой отрасли потреблены товары данного вида. В результате получаем модель общественного разделения труда в единицах полной трудоемкости $(X_{i_j}\tau'_i)$ и в единицах простого труда $(X_{ij}T_i)$. Аналогичным образом получаем матрицу величин $\| a_{ij}\tau_i \|$ и матрицу величин $\| a_{ij}\tau_i \|$. Умножая справа матрицу коэффициентов h_{ij} на диагональную матрицу коэффициентов затрат простого труда (τ'_i), получаем матрицу величин $h_{ij}\tau'_i$, характеризующих объем жизненных средств, потребленных в процессе воспроизводства рабочей силы и оцененных в единицах простого труда. Сумма этих величин равна необходимому труду.

Очень важно все время помнить, что в условиях общественного разделения труда происходит не только обмен товаров на товары, но и обмен затрат рабочей силы на жизненные средства. При последнего рода обмене происходит распад затрат живого труда на затраты необходимого труда и на затраты прибавочного труда.

При обмене затрат рабочей силы на жизненные средства, количество жизненных средств i-го вида, потребленное семьями работников i-го производства (Q_{ij}) будет оцениваться отношением полных затрат простого труда на производство i-го продукта (τ') к трудоемкости в единицах простого труда j-го производства (t'_j). По расчету же на единицу выпуска продукции (X_j) будем иметь величины типа $h_{ij}\tau_i$.

Действительно, при обмене жизненных средств $(Q_{ij}\tau_i)$ на затраты рабочей силы $(X_j t_j = T_j)$ имеем следующее соотношение:

$$\frac{Q_{ij}}{X_j} \cdot \frac{\tau_i}{t_j} = h_{i_j}\tau_i \tag{6}$$

Такая оценка позволяет учесть относительный уровень производительности труда, как соотношение полных затрат труда (τ_i) и трудоемкости производства потребляемых жизненных средств (t_j).

При обмене жизненных средств на затраты рабочей силы происходит отделение прибавочного труда от необходимого труда, так как производительность труда выше, чем затраты труда на жизненные средства, необходимые для воспроизводства рабочей силы. Соотношение этих величин и определяет коэффициент прибавочного труда (e_0). Этот коэффициент есть отношение вещественного состава национального дохода ($\sum_i Y_i\tau_i'$) к вещественному составу фонда потребления семей работников сферы материального производства ($\sum_i \sum_i Q_{ij}\,\tau_i'$), то есть:

$$e_0 = \frac{\sum\limits_i Y_i\tau_i'}{\sum\limits_i\sum\limits_j Q_{ij}\tau_i'}$$

Этот коэффициент всегда больше единицы. При переходе от измерения в единицах рабочего времени (t_j) к полным затратам труда (τ_i'), а также в ходе итерационного процесса коэффициент прибавочного труда все время меняется.

В условиях общественного разделения труда весь процесс обмена товаров на товары и обмена затрат рабочей силы на жизненные средства, принявших форму товаров, может быть математически описан и воспроизведен на основе следующей итерационной формулы[2]:

$$\sum_i a_{ij}\tau_i' + e_0\sum_i h_{ij}\tau_i' = \overline{T}_j$$

В ходе итерации коэффициент e_0 меняется. Итерационный процесс заканчивается, когда этот коэффициент перестает меняться.

В символах матричной алгебры итерационное уравнение имеет вид

$$\left(E - A - \frac{1}{e_0}H\right)^{-1}\tau = \bar{\tau} \tag{8}$$

здесь H — есть матрица коэффициента h_{ij}.

В результате получаем вектор полных затрат простого труда на единицу выпуска продукции при условии, что прибавочный труд распределен пропорционально необходимому труду. Они будут общественно-необходимыми затратами, если прибавочный труд будет распределен по отраслям оптимально.

Для этого несколько видоизменим условие эквивалентного обмена результатами труда: вместо требования, имеющего характер постулата (норма прибавочного труда одинакова по отраслям) потребуем, чтобы вектор приба-

---

[2] Аналогичная процедура примененау M. Morisnima, F. Seton и L. Johansen в "Econometrica", № 2, 1961. Она использована ими однако для определения стоимости по ценам, т.е. в задаче, обратной нашей.

вочного труда ($Ш_j$) был оптимален. Для этого введем вектор оценки (E_j) живого простого труда ($T_j' = t_j'X_j$), произведенной с точки зрения прибавочного труда. Эта оценка равна:

$$\mathcal{E}_j = \frac{Ш_j}{T_j} = \frac{\mathcal{T}_j}{X_j t_j'} \cdot \text{Здесь } Ш_j = \tau_j' - \sum_i h_{ij}\tau_i'$$

Кроме того введем понятие экономического избытка (N_i), положив его равным разнице вещественных элементов национального дохода (Y_i) и фонда потребления жизненных работников сферы материального труда (Q_i).

Положим, что общественно-необходимые затраты труда (τ_i^*) равны оптимальным полным затратам простого труда, т.е. $\tau_i^* = \sigma_i\tau_i'$. Теперь определим компоненты вектора математического множителя $|\sigma_i|$ на основе решения двойственной задачи.

В прямой задаче будем минимизировать объем прибавочного труда ($Ш = \mathcal{E}_j t_j X_j$), причем варьировать будет валовый выпуск (X_j), так как величины $\mathcal{E}_i t_j'$ заданы.

Тогда прямая задача будет такой:

$$\sum_i \mathcal{E}_j t_j' X_j = \text{minim}\,; \; X_j \geqslant 0 \tag{9a}$$

при двух условиях:

а) выход общественного продукта из экономической системы не меньше экономического излишка, или в символах математической алгебры:

$$(E-A)X - tHX \geqslant N;$$

б) количество затраченного простого труда при производстве валового продукта не меньше объема прямых затрат живого простого труда:

$$\hat{t}X + T \geqslant 0; \; \hat{t} = \begin{pmatrix} t_1'00 \ldots 0 \\ 0t_2'0 \ldots 0 \\ 000 \ldots t_n' \end{pmatrix}$$

В сопряженной задаче будем максимизировать экономический излишек, оцененный в общественно-необходимых затратах труда ($\tau_i^* = \sigma_i\tau_i'$):

$$\sum_i \sigma_i \tau_i' N_i = \text{maxim}\,; \;\; \sigma_i \geqslant 0 \tag{9}$$

при двух условиях:

а) Выход из системы (за вычетом потребленных в производственном процессе средств труда и жизненных средств работников материального труда), оцененный в единицах полных затрат простого труда на единицу изделия будет не больше прибавочного труда

$$(E - A^T - H^T)\tau' - \mathcal{E}t' \leqslant 0$$

Здесь верхний значек обозначает транспонированные матрицы.

б) Общественно-необходимые затраты труда ($\sigma_i\tau_i'$) на единицу изделия будут не больше суммы необходимого ($H\tau'$) и прибавочного ($\mathcal{E}t'$) труда

$$-H^T\tau'+\sigma_i\tau_i \leqslant \mathcal{E}t'$$

Решив сопряженную задачу, находим математические множители, а следовательно, и общественно необходимые затраты труда ($\tau_i^* = \sigma_i\tau_i'$)

Затем переходим к определению величины денежной формы стоимости (ω_i). Для этого умножим общественно необходимые затраты труда на денежный масштаб стоимости (Ω_0), равный соотношению двух объемов национального дохода, измеренных в текущих ценах ($\sum\limits_i Y_iP_i$) и в общественно необходимых затратах труда ($\sum\limits_i Y_i\tau_i^*$). Следовательно, имеем:

$$\omega_i = \Omega_0\tau^*, \text{ где } \Omega_0 = \frac{\sum Y_iP_i}{\sum Y\tau_i^*} \tag{10}$$

В будущем, кроме того, следует специально исследовать вопрос об оптимизации вектора „прибавочный труд" во времени (в динамике) с точки зрения нужд расширенного воспроизводства.

4. *Процесс превращения формы стоимости.* Третий контур вычислений, производимых при планировании цен, ставит своей задачей математически описать процесс трансформации формы стоимости. В этом процессе величина стоимости не изменяется, а меняется только форма стоимости.

Изменение формы стоимости, то есть образование превращенной формы стоимости объективно необходимо для того, чтобы процесс реализации стоимости мог протекать свободно. Общественная стоимость создается на народнохозяйственном уровне, а реализуется на индивидуальном уровне отдельных производственно-потребительских ячеек общества (предприятия, семьи, производственно-территориальные комплексы).

В своей первичной форме общественная стоимость не может быть индивидуализирована, т.е. не может быть сведена к индивидуальной стоимости. Общественная стоимость должна прежде всего распасться на свои составные элементы (перенесенная стоимость, стоимость оплаченного труда, стоимость прибавочного продукта), чтобы ее можно было реализовать на индивидуальном уровне.

Появление денег в процессе общественного разделения труда приводит к очень большой модификации формы стоимости. Весь процесс создания и реализации стоимости тогда начинает протекать не в своей непосредственной трудовой форме, а в денежной форме. Оплата труда приобретает форму заработной платы; износ основных средств — форму амортизационных отчислений; затраты промежуточных продуктов — форму материальных издержек; прибавочный продукт — форму прибыли, ренты, налога с оборота и т.д.

Процесс сведения общественной стоимости к индивидуальной стоимости, определяющий процесс реализации стоимости, может протекать только в этой превращенной форме стоимости. Поэтому необходимо математически описать процесс формирования тех основных элементов стоимости, на которые общественная стоимость распадается в процессе своей реализации.

Прежде всего, обособляется заработная плата и вообще оплата труда. На основе матрицы технологических коэффициентов (A) можно определить полные выплаты по фонду оплаты труда по формуле:

$$W^* = W(E-A)^{-1} \tag{11}$$

где E — единичная матрица, а $(E-A)^{-1}$ — обратная матрица коэффициентов a_{ij}; W — вектор заработной платы по расчету на единицу продукта; W^* — вектор полных выплат по фонду оплаты труда.

В условиях социализма, кроме индивидуальной оплаты труда, зависящей от количества и качества труда, возникает общественная форма оплаты труда, состоящая из выплат, производимых из фонда общественного потребления (народное образование, здравоохранение, льготы по квартирной плате и по коммунальным услугам, бесплатные обеды на производстве, социальное страхование и т.д.). Хотя общественный фонд потребления и распределяется бесплатно, по потребностям, но для всего общества в целом он стоит больших затрат трудовых и материальных ресурсов. Затраты на этот общественный фонд подлежат воспроизводству. Эти затраты при определении стоимости должны быть прибавлены к заработной плате или непосредственно (на основе данных прямого учета или данных специальных обследований) или косвенно, то есть пропорционально фонду индивидуальной оплаты труда. Тогда вектор $|W|$ будет характеризовать все выплаты и выдачи как из индивидуального, так и из общественного фонда потребления. На основе этого вектора и определяется вектор полных выдач из фонда оплаты труда.

Аналогично определяется вектор полных выплат по амортизационным отчислениям (d_j^*). Вычитая по каждому в отдельности производству из общественной стоимости (W_j) полные выплаты по фонду оплаты труда (W_j^*) и полные амортизационные отчисления (d_j^*), получаем чистый доход общества, то есть стоимость прибавочного продукта:

$$\Pi_j = \omega_j - W_j^* - d_j^* \tag{12}$$

Если отнести чистый доход общества к использованным производственным и природным ресурсам, то получим показатели рентабельности использования этих ресурсов. В условиях социализма производственные ресурсы имеют денежную форму в виде балансовой стоимости основных и оборотных производственных фондов. Однако природные ресурсы (земля, недра земли, леса, воды) в наших условиях такой стоимости не имеют.

Поэтому сначала определяются показатели общей рентабельности использования производственных ресурсов, включающие в свой состав обе формы чистого дохода (прибыль и дифференциальную ренту).

При определении коэффициентов рентабельности использования ресурсов очень важно избежать повторного дублирования рентабельности ресурсов. Следует принять специальные меры к тому, чтобы в материальных затратах рентабельность ресурсов повторно не дублировалась. На каждой стадии производства должна учитываться рентабельность только тех ресурсов, которые задолжены на данной стадии производственного процесса. Объем перенесенной стоимости прибавочного продукта не должен зависеть от числа стадий, на которые разбито производство.

Такого повторного дублирования можно избежать, если определять коэффициенты рентабельности по отношению к полной фондоемкости.

Если $\hat{f}_{kj}$ — фондоемкость j-го производства по k-ому вещественному виду основных фондов, а f_{0j} — фондоемкость j-ого производства по оборотным фондам 0-го их вещественного вида, то тогда вектор полной фондоемкости будет равен:

$$\begin{aligned} &1) \text{ по основным фондам } \varphi_k = f_k (E-A)^{-1} \\ &2) \text{ по оборотным фондам } \psi = f_0 (E-A)^{-1} \end{aligned} \tag{13}$$

Следовательно, общая фондоемкость (по всем вещественным видам фондов) будет представлена вектором φ и ψ соответственно.

Обозначим компоненты этих векторов для r-го производства через φ_{kj} и ψ_{0j}. Их совокупность образует матрицу коэффициентов полной фондоемкости основных и оборотных фондов с учетом вещественного их состава по отраслям производства, где они функционируют.

При определении коэффициентов общей рентабельности основных фондов k-го вида (α_k) и оборотных фондов 0-го вида (β_0) должна быть по каждому вещественному виду фондов принята во внимание их воспроизводственная стоимость и срок их воспроизводства (в долях календарного года).

Пусть годовые капитальные вложения в основные фонды данного вещественного вида равны $\Delta\hat{F}$, прирост вещественных элементов оборотных фондов соответственно — ΔF_0. Тогда коэффициенты их воспроизводства соответственно равны $(1+\hat{ч}_k)$ и $(1+ч_0)$, где

$$ч_k = \frac{\Delta\hat{F}_k}{\hat{F}_k}, \quad \text{а} ч_0 = \frac{\Delta F_0}{F_k} \tag{14}$$

Пусть срок воспроизводства фондов в долях годового периода равен по основным фондам m_k, а по оборотным фондам m_0. Тогда коэффициенты общей рентабельности могут быть определены из системы уравнений:

$$\pi = \sum_k \alpha_k \varphi_{kj} (1+\hat{ч}_k)^{m_k} + \sum \beta_0 \psi_{0j} (1+ч_0)^{m_0} \tag{15}$$

Эти уравнения составляются для каждой отрасли производства в отдельности и по ним определяются коэффициенты общей рентабельности фондов данного вида (α_k и β_0).

Величины Π_j (чистый доход j-ой отрасли) уже были определены на стадии второго контура математического описания процесса образования стоимости.

Матрица параметров полной фондоемкости φ_{kj} и ψ_{0j} получается на основе матрицы коэффициентов $\hat{f}_k$ и f_0 исходной плановой модели общественного продукта, а сроки оборота фондов устанавливаются по данным технико-экономической экспертизы.

Особо следует подчеркнуть, что в этих уравнениях коэффициенты рентабельности фондов состоят из двух множителей, а именно:

1) по основным фондам из α_k и $(1+\hat{ч}_k)^{m_k}$

2) по оборотным фондам из β_0 и $(1+ч_0)m_0$

Весьма важно, что множители $(1+\hat{ч}_k)$ и $(1+ч_0)m_0$ в этих уравнениях есть величины известные, так как они определяются на основе данных исходной базисной модели ($\Delta\hat{F}$ и ΔF по данным второго блока, а $\hat{F}_k$ и F_0 по данным пятого блока). В наших уравнениях сроки воспроизводства фондов также рассматриваются как известные величины (они определяются по данным технико-экономической экспертизы).

Коэффициенты же α_k и β_0 получаются на основе решения написанной выше системы уравнений. Как видим, оба множителя, входящие в состав коэффициента рентабельности фондов данного вещественного вида, не зависят от того, в какой отрасли производства функционирует данный вещественный вид фондов. Они в этом смысле едины для всего народного хозяйства.

Практика народнохозяйственного учета обычно предусматривает для основных и оборотных фондов более укрупненную номенклатуру, чем для отраслей производства.

Для того, чтобы число уравнений равнялось числу неизвестных, образуем укрупненные агрегаты отраслей производства в соответствии с номенклатурой фондов. Наша задача — определить *средние* для j-ой отрасли коэффициенты рентабельности основных и оборотных фондов отдельно. Коэффициенты α_k и β_0, определяемые по вещественным видам фондов, играют у нас лишь вспомогательную роль при установлении таких средних отраслевых коэффициентов. Хотя коэффициенты α_k и β_0 меняются только по видам фондов, но средние отраслевые коэффициенты рентабельности ($\bar{\varphi}_j$ и $\bar{\psi}_j$)[3] изменяются и по отраслям производства, так как они как средние величины зависят от отраслевой структуры вещественного состава функционирующих в них фондов. Такое свойство отраслевых коэффициентов рентабельности фондов позволяет, в случае необходимости, индивидуализировать коэффициенты

[3] При вычислении средних отраслевых коэффициентов рентабельности фондов в качестве весов должны быть приняты во внимание не только фондоемкость, но и коэффициенты их воспроизводства и сроки их воспроизводства, т.е. величины $\hat{F}_k\,(1+\hat{ч}_k)^{m_k}$ и $F_0\,(1+ч)^{m_0}$.

рентабельности вплоть до отдельного предприятия, учитывая вещественную структуру функционирующих в них фондов.

Следует иметь в виду, что в ряде стран практикуется ускоренная амортизация, не соответствующая срокам службы основных фондов. Тогда в амортизационные отчисления попадает часть чистого дохода общества. В подобном случае экономически целесообразно определять не общую, а валовую рентабельность основных фондов, включающую в свой состав и амортизационные отчисления, а не только прибыль и дифференциальную ренту.

Тогда в предыдущей системе уравнений будем иметь на левой стороне сумму, равную $П_j+\alpha_j^*$, а на правой стороне валовые (а не общие) коэффициенты рентабельности основных фондов (α'_k). Затем, вычитая из средней отраслевой *валовой* рентабельности фондов j-ой отрасли (α'_j) средний для этой отрасли амортизационный коэффициент, получаем средний отраслевой *общий* коэффициент рентабельности, скорректированный в отношении амортизационных отчислений.

Однако при доведении коэффициентов рентабельности фондов до отдельного производственно-территориального комплекса и до отдельных предприятий, следует учитывать не общие (а тем более не валовые) коэффициенты рентабельности фондов, а чистые нормы рентабельности, из которых уже вычтена рентабельность природных ресурсов. Для этого нужно определить дифференциальную ренту. Она должна быть учтена в процессе производственного оборота также только один раз (на стадии добывающих отраслей производства).

Для определения дифференциальной ренты по каждой отрасли производства нужно иметь зональные коэффициенты ресурсоемкости (по используемым в производстве видам природных ресурсов). Они определяются как обратные показатели съема продукции с единицы природного ресурса.

В качестве единицы ресурса для сельскохозяйственного производства принимается гектар пашни (для пастбищного животноводства и овцеводства — гектар сельскохозяйственной площади, для поливного хлопководства — гектар орошаемой земли). Единицей ресурсов для горных добывающих отраслей может служить 1м^3 вынутой горной породы.

Дифференциальная рента может быть определена методами математического программирования путем использования математических множителей типа теневых цен и объективно обусловленных оценок.

При определении дифференциальной ренты соответствующая добывающая отрасль производства подразделяется на S зон. В каждой зоне выделяется по продуктивности $ч$ категорий природного ресурса, используемого в данном производстве.

Прежде всего определяется матрица коэффициентов продуктивности единицы природного ресурса $||Z_{чs}||$. Эти коэффициенты обратны коэффициентам ресурсоемкости. Далее определяется матрица зональных индивидуально-групповых стоимостей $\|\lambda_{чs}\|$.

Под зональными индивидуально-групповыми стоимостями понимается превращенная форма стоимости, определяемая отдельно для каждой данной зоны и для каждой группы земельных массивов или группы месторождений полезных ископаемых. Для каждой такой группы определяется также свой вектор расходных технологических коэффициентов $\|a_{ч}^{(чs)}\|$, а также вектор средней оплаты труда на единицу выпуска продукции $W^{(чs)}$, средней фондоемкости единицы продукции по основным фондам $\hat{f}^{(чs)}$ и по оборотным фондам $f^{(чs)}$; коэффициенты фондоемкости определяются как средневзвешенные величины с учетом вещественного состава фондов (по каждой группе земельных массивов или по каждому месторождению в пределах каждой зоны).

Зональная индивидуальная групповая стоимость $(\lambda_{чs})$ определяется по формуле превращенной стоимости:

$$\lambda_{чs} = \sum_i \alpha_i^{(чs)} \omega_i + W^{(чs)} + \overline{\alpha}_j \overline{\varphi}_j^{(чs)} + \overline{\psi}_j \overline{f}^{(чs)} \tag{16}$$

где ω_j — общественная стоимость используемых i-го вида промежуточных продуктов; $\overline{\alpha}_j$ — валовой коэффициент рентабельности основных фондов (средний по отрасли коэффициент, включая амортизационные отчисления). Затем определяется расчетная зональная оценка $(\hat{C}_s)$ единицы продукции, производимой в наихудших природных условиях, путем приравнивания зональной оценки $(\hat{C}_s)$ к максимальной зональной индивидуально-групповой стоимости, т.е. $\lambda^0_{чs}$.

Следовательно, $\hat{C}_j = \lambda^0_{чs}$. Затем по остальным категориям данного природного ресурса определяется экономия стоимости $(\mu_{\bar{чs}})$ приведенная к уровню зональной стоимости единицы данного вида продукции на основе уравнения:

$$\frac{\omega_s}{\hat{C}_s}\,(\lambda^0_{чs} - \lambda_{чs}) = \mu_{чs} \tag{17}$$

Это и будет дифференциальная рента с единицы продукции, образующаяся в условиях S-ой зоны и $ч$-ой категории ресурса.

Дифференциальная рента с единицы природного ресурса равна:

$$R_{чs} = Z_{чs} \cdot \mu_{чs} \tag{17a}$$

Общий объем дифференциальной ренты по каждому данному производству зависит от объема товарной продукции S-го района, полученной при использовании в S-ой зоне $ч$-ой категории природных ресурсов $(\Pi_{чs})$. Поставки продукции добывающих отраслей для ее дальнейшей промышленной обработки и для нужд строительства, всегда и везде рассматриваются как товарная продукция. Предприятия добывающих отраслей всегда считаются самостоятельными, даже если они и не имеют самостоятельного финансового баланса.

Дифференциальная рента с единицы валового выпуска продукции данной добывающей отрасли (ϱ_j) будет равна:

$$\varrho_j = \frac{\Sigma \Pi_{чs} \mu_{чs}}{X_j}. \tag{18}$$

Здесь X_j — валовый выпуск j-ой продукции.

Теперь могут быть определены нормы рентабельности фондов, т.е. чистые коэффициенты рентабельности (α'_k и β'_0), тем же способом, как это было сделано при определении валовых и общих коэффициентов рентабельности фондов. Только в данном случае чистый доход с единицы продукции (π_j) следует уменьшить на дифференциальную ренту. Следовательно система уравнений будет такой:

$$\pi_j - \varrho_j = \sum_k \alpha'_k \varphi_{kj} (1+\hat{ч}_k)^{m_k} + \sum_0 \beta'_0 \psi_{0j} (1+ч_0)^{m_0} \tag{18a}$$

Средняя отраслевая норма рентабельности фондов при производстве единицы j-ой продукции тогда по основным фондам будет равна:

$$\hat{\xi}_j = \frac{\sum_k \alpha'_k \varphi'_{kj} (1+ч)^{m_k}}{\sum_k \varphi'_{kj}}. \tag{19}$$

Средняя рентабельность оборотных фондов (ξ_j) установлена таким же образом на основе величин $\beta'_0(1+ч_0)^{m_0}$.

Полная формула для определения превращенной формы стоимости теперь получит следующий вид:

$$\omega_j = \sum_i a_{ij}\omega_i + W_j + (d_j + \hat{\xi}_j)\hat{f}_j + \xi_j f_j + \varrho_j \tag{20}$$

Здесь: ω_j — стоимость единицы j-ой продукции; ω_i — стоимость единицы i-го вида предметов труда (топливо, сырье и т.д.); a_{ij} — технологические расходные коэффициенты (по предметам труда); W_j — заработная плата на единицу j-ой продукции; d_j — средние отраслевые амортизационные отчисления с единицы основных фондов; $\hat{\xi}_j$ — чистая норма рентабельности основных фондов; $\hat{f}_j$ — фондоемкость единицы j-ой продукции; ξ_j — норма рентабельности оборотных фондов; f_j — фондоемкость по оборотным фондам; ϱ_j — рентная ставка с единицы продукции.

В своей превращенной форме стоимость обслуживает процесс реализации и индивидуализации стоимости. По приведенной выше формуле стоимость поддается определению на любом уровне (Союз, союзная республика, совнархоз, отрасль, предприятие). Следовательно, по этой формуле может быть определена также и индивидуальная стоимость.

Наступило время, когда настоятельно необходимо планировать по отраслям и предприятиям не только себестоимость, но и стоимость. Себестоимость (C) есть часть стоимости, равная:

$$C_j = \Sigma a_{ij}\omega_j + W_j + d_j \hat{f}_j. \tag{21}$$

Следует еще раз подчеркнуть, что процесс реализации и индивидуализации стоимости может быть воспроизведен лишь в виде превращенной формы стоимости. Только в этом случае заводская себестоимость есть определенная часть индивидуальной стоимости.

5. *Процесс отклонения розничных цен от стоимости.* Четвертый контур математического описания процесса планового ценообразования воспроизводит объективный процесс отклонения цен от стоимости, протекающей под воздействием реально складывающихся соотношений между спросом и предложением товаров или, в более общем смысле, под влиянием соотношений, реально возникающих между производительными силами общества и его потребительной способностью.

Спрос и предложение воздействуют, прежде всего, на розничные цены. Всегда важно обеспечить равновесие между платежеспособным спросом населения и товарной рыночной массой. Проверка такого равновесия при планировании цен и товарооборота обычно проводится на основе баланса денежных доходов населения и розничного товарооборота.

Балансирование основано на следующем равенстве:

$$\sum_i P_i Q_i = W_0 + D_k + C_p + D_p - Y_0 - H + O_y + \Phi_0, \tag{22}$$

где: Q_i — объем рыночного потребления населением товаров i-го вида; P_i — розничная цена товаров; W_0 — фонд заработной платы; D_k — общая сумма денежных доходов колхозников; C_p — общая сумма пенсий и стипендий; D_p — прочие денежные доходы населения; Y_0 — стоимость не товарных платных услуг; H — сбережения населения; O_y — рыночный спрос общественных учреждений для бесплатного снабжения некоторых категорий населения (армия, больницы); Φ_0 — рыночный спрос со стороны общественного фонда потребления (бесплатный или сумма льгот).

Если равновесие денежных доходов населения и розничного товарооборота нарушено, то оно должно быть восстановлено или за счет увеличения рыночной товарной массы, или, если это невозможно, — за счет соответствующего отклонения цен от стоимости на некоторые товары. При планировании таких отклонений следует учитывать эластичность спроса в зависимости от цены и семейного дохода населения.

Эластичность спроса определяется по данным семейных бюджетов.

Показатель эластичности количественного спроса (q_i) на i-ый товар при его цене (P_i) таков:

$$E_{ii} = \frac{\Delta l_n q_i}{\Delta l_n P_i} = \frac{P_i}{q_i} \cdot \frac{\Delta q_i}{\Delta P_i}. \tag{23}$$

Это уравнение описывает состояние равновесия спроса и цен на уровне семейного бюджета.

Аналогичным образом определяется перекрестная эластичность спроса на i-ый товар по цене на другой k-ый товар (E_{ik}) и эластичность спроса по доходу семьи (S).

Исходя из определения эластичности спроса (полагая $\Delta P_i = P'_i - P_i$; $\Delta q_i = q'_i - q_i$), можно установить связь между двумя равновесными состояниями спроса и цен:

$$\left(1 = \frac{P'_i}{P_i}\right) = \frac{1}{E_i}\left(1 - \frac{q'_i}{q_i}\right). \tag{23а}$$

Следовательно, соотношение двух равновесных состояний количественного спроса и цен такое, что дополнение до единицы соотношения двух цен пропорционально дополнению до единицы соотношений двух количественных характеристик спроса, причем коэффициент пропорциональности равен обратной величине коэффициента эластичности.

Из этого соотношения можно определить новую систему цен товаров (P'_i), отвечающих новому состоянию спроса (q'_i), исходя из уравнения:

$$P'_i = P_i^0\left[\frac{1}{E_{ii}}\left(\frac{q'_i}{q_i^0} - 1\right) + 1\right]. \tag{23б}$$

Здесь P_i^0 и q_i^0 — величины, относящиеся к системе бюджетных данных, по которым определяется коэффициент эластичности спроса.

Стоимостный уровень цен находится в соответствии с общим равновесным состоянием системы общественного разделения труда. Равновесное состояние цен и спроса на уровне семейных бюджетов, конечно, иное, чем на общем народнохозяйственном уровне. В этом основная причина отклонения розничных цен от стоимости.

Устанавливая на основе состояния рыночной товарной массы фактически возможное удовлетворение спроса на товары (q'_i) и исходя из величин, определенных по бюджетным данным (E_{ii}, q_i^0 и P_i^0) можно определить уровни цен, соответствующие ожидаемому уровню и структуре розничного товарооборота ($\overline{P}_i$). Разность между такими равновесными ценами и стоимостью товаров и будет отклонением цен от стоимости ($\Delta_i = \overline{P}_i - \omega_i$).

Оптимальная структура производства предметов народного потребления может быть определена из условия:

$$\sum_i \chi_i \Delta_i^2 = \text{minim}. \tag{24}$$

где χ_i — некоторые весовые количества.

Для определения отклонений цен от стоимости, в которых учитывались бы перекрестная эластичность и эластичность спроса по семейному доходу предложена формула:

$$\Delta_j = \overline{P}_j - \omega_j = P_j^0 \prod_i \left(\frac{q'_i}{q_i}\right)^{\xi_{ij}} \left(\frac{S'}{S_0}\right)^{\xi_j} - \omega_j \tag{25}$$

где ξ — величины, обратные коэффициентам эластичности, а $\underset{i}{\Pi}$ — символ перекрестных произведений двух числовых характеристик бюджетных равновесных состояний количественного спроса и семейных доходов.

Система розничных цен играет огромную роль в экономике страны, вследствие чего отклонения розничных цен от стоимости должны по возможности получить всестороннее обоснование. Розничные цены зависят не только от эластичности спроса, но и от меры насыщения и меры настоятельности отдельных потребностей, удовлетворяемых определенными товарами. От меры насыщения и от меры настоятельности потребности, в частности, зависят общие потребительные оценки товаров.

Величины меры насыщения и меры настоятельности потребности поддаются определению по следующим соотношениям:

а) мера насыщения потребностей $\eta_i = \dfrac{\log H_i}{\log H_i^0}$,

б) мера настоятельности потребности $\mu_i = \dfrac{H_i^0 \omega_i}{\overline{D}_0}$ (26)

где H_i^0 — норма потребления, соответствующая состоянию насыщения (в качестве таковых обычно принимаются перспективные нормы потребления); H_i — фактическая норма потребления; ω_i — стоимость единицы продукта; $\overline{D}_0$ — средний денежный доход по расчету на потребительскую единицу.

В качестве потребительной оценки единицы i-го товара целесообразно взять значения показательной функции от соответствующей величины меры:

$$\text{а)}\quad U(\eta_i) = e^{1-\eta_i}\,; \tag{27}$$

$$\text{б)}\quad U(\mu i) = e^{\mu i - 1}$$

На основе потребительной оценки для каждого вида товаров, входящих в баланс денежных доходов и розничного товарооборота, определим средневзвешенное значение потребительных оценок, а затем соотношение отдельных потоварных оценок со средней их величиной.

Пусть
$$k_i(\eta) = \frac{U(\eta_i)}{U(\eta)} \quad \text{и} \quad k_i(\mu) = \frac{U(\mu_i)}{U(\mu)}$$

Допуская, что розничные цены соответствуют стоимости только в среднем для всей рыночной массы потребительских товаров, а по отдельным товарным группам розничные цены лишь пропорциональны стоимости, имеем:

$$P_i = k_i(\eta)\omega_i \quad \text{и} \quad P_i' = k_i(\mu)\omega_i. \tag{28}$$

На основе расчетов такого типа органы, планирующие цены, получают в свое распоряжение систему объективных данных, характеризующих пропорциональные отклонения розничных цен от стоимости, т.е. систему отклонений, зависящих как от эластичности спроса, так и от меры насыщения и меры настоятельности потребности.

При планировании розничных цен, кроме указанных объективных данных, приходится принимать во внимание много других соображений, относящихся уже к политике цен. В частности, приходится учитывать стимулирующую и перераспределительную функцию цен. Сумма же цен в целом по товарам розничного товарооборота всегда стремится быть равной сумме их стоимости.

Для того, чтобы розничная цена не отрывалась от оптовых цен, отклонения розничных цен от стоимости следует покрывать за счет особого фонда регулирования розничных цен, из которого финансируются отклонения розничных цен вниз от стоимости и который пополняется за счет отклонений цен вверх от стоимости.

Следующий, пятый контур модели планового ценообразования состоит из обоснования отклонений оптовых цен от стоимости.

6. *Процесс отклонения и индивидуализации оптовых цен.* Оптовые цены нельзя отрывать от стоимостной базы. Обычно оптовые цены должны быть равны розничным ценам стоимостного уровня (без отклонения их от стоимости), минус торговая скидка в пользу розничной торговой сети.

Самостоятельные отклонения оптовых цен от стоимости зависят от соотношения количества товаров, фактически произведенных (X_i) и общественно необходимых с точки зрения потребительной силы общества (Z_i). Коэффициент пропорционального отклонения оптовых цен от стоимости зависит от меры насыщения потребительной силы общества, определяемой по уравнению:

$$\eta_i^{(0)} = \frac{\log Z_i}{\log X_i} \tag{26a}$$

Этот коэффициент равен

$$k^0(\eta) = e^{1-\eta_i^0} \tag{27a}$$

Тогда оптовые цены будут равны:

$$P_i^0 = k_i^0(\eta)\omega_i \tag{28a}$$

Кроме указанных отклонений цен от стоимости, при моделировании оптовых цен особенно важно отразить в оптовой цене конкретных товаров также и процесс индивидуализации цен, состоящий в переходе от укрупненных товарных позиций к конкретным товарам, т.е. к такой номенклатуре, которая учитывает отдельные виды товаров, их сорта, марки и т.д.

При планировании оптовых цен главная задача состоит в том, чтобы отразить в цене товароведческие качества выпускаемой продукции, то есть установить соотношение цен на различные товары по их маркам, сортам, профилям, моделям, фасонам, типоразмерам и т.д. Дело в том, что стоимостный уровень цен, так же как и отклонения от стоимости, поддаются определению только по укрупненным товарным группам (агрегатам).

Планирование оптовой (так же как и розничной) цены тесно связано с составлением прейскурантов цен, учитывающих потребительские качества товаров в полном соответствии с существующей системой государственных стандартов (ГОСТов, технических указаний, типажей, норм).

Практически задача планирования оптовых цен заключается в индивидуализации стоимостного уровня цен в зависимости от потребительских свойств товаров. В этом и реализуются отклонения (второго типа) оптовых цен от стоимости, связанные с качеством товара. Следовательно, характер процесса индивидуализации цен иной, чем рассмотренный выше тип пропорциональных отклонений розничных и оптовых цен от стоимости.

Природа индивидуализации стоимостного уровня оптовых и розничных цен одна и та же, так как эти отклонения складываются под влиянием потребительной стоимости, отраженной в потребительных оценках товаров. Прейскуранты оптовых и розничных цен содержат оба типа отклонений (пропорциональные отклонения цен от стоимости и отклонения цен от стоимостного уровня при индивидуализации цен).

Для товаров оптового рынка кроме того важно учитывать временную дефицитность одних товаров и неходовой характер других. Но эту сторону товарного оборота целесообразно принимать во внимание не в виде отклонений цен от стоимости, а лишь меняя норму отчислений прибыли в фонд предприятия в зависимости от временной дефицитности или залежалости товаров.

Проиллюстрируем процесс индивидуализации оптовых цен. Индивидуализация цен на пищевые товары отражает содержание в них разнообразных питательных элементов, а индивидуализация цен на топливо отражает их топливные качества (теплотворность, зольность, влажность и т.д.), на железную руду, флюсы — их металлургическую ценность (содержание элементов, облегчающих или тормозящих плавку и меняющих качество выпускаемой продукции), на машины и станки — производственную мощность и экономию, получаемую потребителем в процессе эксплуатации машин. При индивидуализации цен, т.е. при отражении потребительских свойств товаров приходится учитывать их нормативную трудоемкость или плановую себестоимость. В последнем предполагается, что цены, используемые при калькуляции плановой себестоимости соответствуют стоимостному уровню.

При индивидуализации цен широкое применение могут иметь экономико-математические методы. Так, например, по пищевым товарам математическая процедура такова. Пусть имеется $м$-ый агрегат продуктов (по которым определена стоимость), включающий в себя совокупность j-го вида конкретных продуктов в количествах $q_i^{(m)}$. Пусть каждый конкретный $г$-ый вид пищевых товаров содержит k-ые питательные элементы в количествах $Э_{kj}$, а содержание этих питательных элементов в нормальном дневном рационе равно $\breve{Э}_k$. Тогда содержание k-ых питательных элементов для весовой единицы товара в долях дневного пищевого рациона равно:

$$b_{kj} = \frac{Э_{kj}}{\breve{Э}_k}, \text{ т.е. } Э_{kj} = b_{kj}\breve{Э}_k \quad (29)$$

Для данного состава конкретных товаров, в соответствии с учитываемыми питательными элементами, строим матрицу величин $b_{kj}\breve{Э}_k$.

Пусть $\hat{C}_j$ означает нормативную или плановую себестоимость конкретных товаров, а ω_m — стоимость их m-го агрегата (с теми весовыми коэффициентами, которые заложены в базисной плановой модели общественного продукта). В этих условиях при индивидуализации оптовых цен (то есть при определении конкретных цен, исходя из стоимости единицы их агрегата) необходимо определить неизвестные потребительные оценки отдельных товаров (V_k) и норму рентабельности их агрегатов ($ч_m$), определенных в процентах к себестоимости.

Величины V_k определяем, решая следующую экстремальную задачу:

$$\sum \breve{Э}_k V_k = \text{maxim.} \quad (30)$$

При условии:

а) $\breve{Э}_k \geqslant 0$; $Э_{kj} \geqslant 0$

б) $\sum\limits_k b_{kj}\breve{Э}_k V_k \leqslant \hat{C}_j$

Согласно второму условию масштаб потребительных оценок (V_k) выбирается на уровне плановой себестоимости ($\hat{C}_j$).

Тогда цена конкретного товара (P_j), входящего в m-ый агрегат, определяется путем решения следующей системы уравнений:

$$P_{ij}^{(m)} = \sum_k b_{kj}\breve{Э}_k V_k (1+ч_m) \quad (31)$$

При условии:

$$\sum_m q_{jk}^{(m)} P_{jk}^{(m)} = \sum_m q_j^{(m)} \omega_m$$

Последнее условие обеспечивает равенство суммы потоварных цен сумме стоимости по каждому в отдельности агрегату товаров и продуктов, выделенных в особые позиции в исходной базисной плановой модели общественного продукта и общественного разделения труда (например, группа продуктов овощеводства, хлебных и мучных товаров, крупяных продуктов, растительного жира, молочных и отдельно мясных продуктов).

Аналогичным образом может быть индивидуализирована цена по зернофуражу и кормам, по видам топлива, по текстильным товарам, по машинам, оборудованию и т.д. В каждом отдельном случае в основу берется своя система оценок потребительских свойств товаров, ориентирующаяся на систему государственных стандартов, а также на систему индексов, характеризующих удобства потребителей и возможную экономию в сфере потребления.

Для заготовительных сельскохозяйственных цен, кроме того, важно иметь систему страховых доплат, гарантирующих застрахованный уровень доходности гектара земли и на этой основе обеспечивающих гарантированный минимум оплаты труда колхозников. Необходимость такой дополнительной страховой системы диктуется колебаниями уровней урожайности в силу метеорологических причин (особенно по зерну). Заготовительные цены, как известно, ориентированы на многолетнюю среднюю урожайность, на нормальную производительность труда и на общественную стоимость в нормальных условиях. Система страхования призвана учесть фактические колебания по отдельным годам стоимости и производительности труда, не зависящие от сельскохозяйственных производителей.

Последний — шестой — контур вычислений, связанных с планированием цен, имеет в виду также и текущую работу по прейскурантам цен, в том случае, когда уже обеспечен стоимостный уровень цен по основным ведущим товарам, образующим „несущий костяк" цен. На этапе составления прейскуранта цен, следовательно, предусматривается дополнительная система вычислений, связанных с текущей калькуляцией себестоимости и определением индивидуальной стоимости по всем производственным ячейкам общества.

Настало время, когда каждое предприятие в текущем порядке должно определять не только себестоимость, но и свою индивидуальную стоимость, ориентированную на такие общественные нормативы, как коэффициенты рентабельности основных и оборотных фондов и на учете дифференциальной ренты по всем используемым природным ресурсам.

Хозяйственные результаты в социалистическом обществе нельзя определять только на основе одной себестоимости, представляющей лишь часть стоимости. Экономически рентабельное ведение хозяйства или убыточность отдельных хозяйств надлежит определять на основе текущего учета индивидуальной стоимости, сопоставляя таковую с зональной отраслевой и общественной стоимостью. Кроме того, необходима периодическая проверка соответствия оптовых и розничных цен стоимостному уровню. Во всяком случае, такую контрольную проверку необходимо регулярно проводить каждый раз в период составления очередного перспективного плана развития народного хозяйства.

W. S. Niemchinow

THE BASIC OUTLINES OF THE PLANNED MODEL OF PRICE FIXING

Summary

The only objective base for fixing the general level of prices is value based upon labour. This value is formed on the level of the whole national economy. Later, however, comes the process of differentiation and individualization of value, and then appear the components of value such as the transferred value (material costs), value of labour (wages and income of collective farm workers) and surplus value (the accumulation fund, the unproductive sphere fund and the balance of foreign trade).

The differentiation of value takes place in the process of its reproduction. The general economic coefficients of the functioning of the fixed and active funds and also the profitableness of the used resources are determined in the same process. The process of deviation of prices from value also takes place. These deviations are realized according to the laws of value and that is why they are in proportion to the value.

The basic model of determining the level of value is the model of the real structure of production and of the social division of labour. Only the basic goods, of course, are taken into consideration and they serve to create an "indicative skeleton". This model is created on the basis of input–output analysis and on tables of intersectoral fluctuations. The basic balance of the real structure of production consists of two blocks. The first block covers the goods under production, the other the final products. The model of the social division of labour also covers wages. The whole basic model is completed with the block of pure profits. Starting from the basic model with the help of proper matrixes the first shapes of the planned fixing of prices may be determined, and for this purpose the proper matrixes should be created. These are matrixes of technological coefficients, labour consumption coefficients, coefficients of capital and consumption. The quantity of the full labour outlay is calculated by using the matrix of technological coefficients according to the formula:

$$T = t(E-A)^{-1}$$

where

T — vector of full labour consumption
t — vector of labour consumption
E — singular matrix
A — matrix of technological coefficients.

For the determination of the socially indispensible labour outlay it is necessary to determine coefficients of the means of living of the families of workers in the productive sphere taking into consideration at the same time the planned increase in the living standard. The indispensible time is expressed in units of unqualified labour. The full outlay of unqualified labour becomes socially indispensible if the product is optimally distributed in various sectors of production. By calculating the optimalization of the product the socially indispensible time of production, which determines the value, can be found. To effect such a calculation it is necessary to divide value into components and then to conduct their quantitative analysis.

The fund of general consumption and the full accounts of amortization should be included into the calculation. Moreover the average coefficients of utilization of resources and differential

rent (different in different regions of the country) should be also included. In this way the following, transformed formula of value of goods can be reached:

$$\omega_j = \sum_i a_{ij}\, \omega_i + W_j + (d_j + \hat{\xi}_j)\hat{f}_j + \xi_j f_j + \varrho_j$$

where

ω_j — value of a unit of j production
ω_i — value of a unit of i kind of labour products
a_{ij} — technological coefficients
W_j — labour wages on a unit of j production
d_j — average amortization accounts in the sector of production calculated for a unit of fixed funds
$\hat{\xi}_j$ — pure norm of profitableness of fixed funds
$\hat{f}_j$ — capital consumption of j production
ξ_j — norm of profitableness of active funds
f_j — capital consumption of active funds
ϱ_j — rate of rent for a unit of production.

In economic reality there is no balance between the scope of production determined by the level of productive potential and social demand. The deviations from balance may be smoothed by enlargement of the supply of goods or by a properly planned difference between retail prices and value. This could be planned taking into consideration price and the income elasticity of demand. In this way the optimal structure of production could be reached. The wholesale as well as retail prices result from value based upon labour. The coefficient of deviation of wholesale prices from value may be calculated. The mathematical calculations are analogical to those connected with the problem of retail prices. A good example showing the process of individualization of prices is the example of the food industry.

Any enterprise may now define not only its costs but, upont the basis of the coefficients of the profitableness of resources, also the individual value of goods.

Maksymilian Pohorille
Poland

THE PRODUCTIVITY OF LABOUR IN AGRICULTURE AND THE PROBLEM OF THE PARITY OF INCOMES IN AGRICULTURE AND INDUSTRY

The official slogan of the agricultural policy in highly developed capitalist countries is the parity of incomes of the population employed in agriculture, and in other branches of national economy. The lack of this parity, according to many Western economists, results from lower productivity of labour in agriculture as compared with that in industry[1].

Therefore the "rationalization" of agricultural economy is supposed to settle this disparity of incomes. Yet this problem is much more complicated than it may seem at first.

The very formulation of the problem arouses some serious theoretical doubts. What is really meant by the statement that the productivity in agriculture is lower than that in industry? Statistical data usually quoted to support the statement refer to the value of production *per capita* employed in industry and in agriculture (i.e. productivité en valeur).

These comparisons imply that the newly produced value in industry is higher than that in agriculture. However, we do not, in fact, use here the criterion of value but that of prices. In other words we say that considering the given prices of agricultural and industrial products the net production *per capita* employed in industry is higher than in agriculture. It is obvious that we are here in a vicious circle. The farmer's work is estimated lower than the work of people employed in other branches of economy (German economists use a special term for it, namely, "Unterschätzung der Arbeit in der Landwirtschaft") because it is less productive, and on the other hand, it is less productive because it is estimated lower.

How does the problem appear is the light of Marxist theory. Does the lower "productivité en valeur" in agriculture result only from the unfavourable trend of prices for agricultural products or does it depend on the efficiency of work in both these branches of economy? Several writers quote here Marx' well known argument that an hour of more efficient work produces greater value than an hour of less efficient work. Yet it seems that this argument does not refer to different branches of

[1] It should be noticed that some Marxists approach the problem in the similar way.

production. It is closely connected with the basic assumption that the value of a product is defined not by the individual time but by the socially indispensable time of labour. Besides, according to that definition the social indispensable time of labour is considered as the time of labour indispensable to make a prosuct in the given, socially average conditions of production on the given technical level and on the average level of working ability and intensity.

Marx returns to this problem in the third volume of *The Capital* stressing another aspect namely, the necessity of the allotment by the society of a certain amount of the general working time for the producing of separate kinds of goods.

Yet there is no doubt that both in the first as well as in the third volume of *The Capital* homogenous products only are taken into consideration.

It is doubtful whether the efficiency in various branches of production may be compared at all. If the daily output of a miner is 5 tons of coal and of another is only 3 tons it is clear that the former is more efficient than the latter. Is it possible to compare the efficiency of a worker manufacturing X pairs of shoes with that of a farmer producing Y tons of corn? In the light of Marx' definitions to ascribe a larger amount of value to the industrial worker than to the farmer would be justified only if the work of the former were more intensive (or lasted longer) and more complex than the work of the latter. The attempt to reduce the whole problem to these two factors only (intensity and ability) would be fictitious for *a*, it is difficult to prove that today there exist any differences in the intensity of work in favour of industry and *b*, when comparing incomes, the problem of qualifications is taken into consideration (groups of similar qualifications in industry and agriculture are compared).

Therefore it seems to me that we can reasonably approach the problem of the productivity of labour in agriculture in three ways:

1. by comparing the organic composition of capital in agriculture and industry,
2. by comparing the efficiency of labour in various groups of farms in agriculture itself,
3. by referring to the theory of comparative costs and examining what kinds of goods have the most favourable conditions for production in a given country.

When discussing the first aspect of the problem we must say that in the conditions of perfect competition the pure production (given in prices) in branches that have a higher organic composition of capital will (*per capita* employed) be higher than in branches with a lower composition of capital.

If for every 100 units of capital in the first branch we have e.g.

$$C_1 - 80 \text{ and } V_1 - 20$$

and in the second branch

$$C_2 - 50 \text{ and } V_2 - 50$$

and the rate of profit (according to the assumption) is equal and let us say amounts to 20%, then in the first branch the ratio $\frac{v+m}{v}$[2] is $\frac{40}{20} = 2$

and in the second branch $\frac{70}{50} = 1.4$.

Therefore the differences in the productivity of labour in industry and agriculture could be explained by the Marxist theory if the assumptions concerning the levelling of the rate of profit and of the differences in the organic composition of capital could be accepted.

The first assumption requires the free movement of capital and work from industry to agriculture and vice versa, which, as is generally known, has never existed. On the other hand we can hardly say that the prices of agricultural products surpass at present the level of their cost of production.

The second assumption also raises many doubts. A number of calculations point out that the organic composition of capital in the agriculture of the most developed capitalist countries (as the U.S.A.)[3] is not at present lower than in industry.

Nevertheless the main problem is quite different: the above mentioned example proves that *the differences in productivity by no means exclude the equal "compensation" of factors of production* (man-power has been estimated according to its value, and capital has the same rate of profit in both branches of production).

It seems that on the basis of this simple example it may be concluded that: *differences in the amount of pure production per capita employed do not explain the differentiation of the rate of profit or the level of wages in individual branches of production.*

Let us pass on to the other way of approaching our problem. The differentiation of production conditions in agriculture in particular countries and on the world-wide scale as well has, at present, the tendency to rise. Progress in science and technology should, however, exert a levelling influence upon the differences in natural fertility of soil, but not all the groups of farms and even more so not all the countries may take advantage of this progress equally. Consequently the increase in differences in production costs is first of all connected with varying economic conditions and not with the fertility of soil.

Figures illustrating the productivity of work in various countries are shown in the follownig table.

---

[2] Where m does not stand for the additional value, but its transformed form profit. If we do not take into consideration the problem of the intensity of work and workers' qualifications and we assume that wages are identical $\frac{v+m}{v}$ will reflect the amount of pure production *per capita* emloyed in individual branches of production.

[3] Cf. M. Mieszczankowski, *The Problems of Absolute Income in Capitalism.*

Estimated Output per Adult Male Engaged in Agriculture in 36 Countries, Average for 1956–1960

	Gross output[2]	Net output[1] Converted at official exchange rates	Net output[1] Adjusted according to differences in internal purchasing power of currencies[3]
		Indices: Italy = 100	
New Zealand	1 380	—	—
Australia	941	—	—
United States	869	412	286
Canada	567	286	199
Belgium-Luxembourg	379	290	247
Denmark	367	228	211
United Kingdom	346	256	231
Netherlands	319	212	228
Germany, Fed. Rep.	291	219[4]	218[4]
Sweden	244	—	—
Argentina	224	87	91
France	204	158[4]	132[4]
Switzerland	173	—	—
Israel	172	200[5]	195[5]
Ireland	162	92	86
Austria	153	—	—
Cuba	109	—	—
Finland	107	157	156
Norway	103	176	158
Italy	100	100	100
Greece	72	66	74
Yugoslavia	53	—	—
Colombia	49	50	45
China (Taiwan)	47	27	33
Algeria	43	32[4]	33[4]
Japan	41	54[5]	60[5]
Tunisia	40	—	—
Venezuela	39	47[4]	32[4]
Panama	39	60	54
Iran	38	—	—

	Gross output[2]	Net output[1]	
		Converted at official exchange rates	Adjusted according to differences in internal purchasing power of currencies[3]
Morocco	35	—	—
Philippines	25	—	—
Thailand	21	15[4]	21[4]
Guatemala	21	—	—
India	21	18[5]	25[5]
Korea, Rep. of	17	20	21

[1] Total output less feed, seed, and waste, aggregated with regional average producers' prices relative to wheat.

[2] Unless otherwise specified, gross domestic product in agriculture at factor cost.

[3] Adjustment factors calculated by P. N. Rosenstein-Rodan, *International aid for under-developed countries*, "The Review of Economics and Statistics", Vol. 43, No. 2, May 1961.

[4] Gross domestic product at market prices, cxpressed as relative to the corresponding figure for the United States.

[5] Net domestic product at factor cost, expressed as relative to the corresponding figure for the United States.

Range of Hours of Work Required in 1950 *to Produce* 100 *kg of Farm Products in Different States of the United States*[4]

	Hours of work per 100 kg					Ratio of largest to smallest
	National average	Largest labor requirement		Smallest labor requirement		
	1	2		3		4
Wheat	0.96	Texas	1.6	Washington	0.4	4.0
Barley	0.96	Nebraska	1.6	Washington	0.6	2.7
Oats	1.24	S. Carolina	2.3	Illinois	0.6	3.8
Potatoes	0.99	N. Carolina	1.9	Idaho	0.7	2.7
Soybeans	0.99	N. Carolina	2.7	Illionois	0.8	3.4
Tobacco	81.57	Florida	123.5	Wisconsin	30.9	4.0
Cotton	57.32	N. Carolina	110.2	California	28.7	3.8
Sugar beets	0.53	Utah	0.7	Oregon	0.4	1.8

The market value of agricultural products is not defined today by the least favourable productive conditions. Two other aspects have a special importance: 1. Big capitalist farms and "commercial" family farms supply the market with the greatest part of production.

[4] R. W. Hecht and K. R. Vice, *Labour used for field crops*, Washington, D. C., U.S. Departament of Agriculture, Agricultural Research Service, Statistical Bulletin No. 144, June 1954.

2. The relation between demand and supply is unfavourable for agriculture. As result the majority of farms are compelled to sell agricultural products below their individual value. These farms have low profits and it is clear that this phenomenon exerts a definite influence upon the level of the average agricultural income.

Drawing an analogy between the situation of less productive farms and the situation of the whole agricultural economy may be misleading. It is obvious that particular farms attain higher profits by increasing their efficiency. It does not mean, however, that this is the situation in agriculture as a whole.

Some farmers decrease their own expenses and increase differential income II by introducing technical progress in production. It is possible to assume that the short time changes in the productivity and in expanses will not exert a strong influence on the relationship between demand and supply and also on the level of prices connected with it. Therefore the decrease of production costs in some farms will lead to an increase of their net income and to an increase of the average (net) income in agriculture as a whole.

In a free competition an increase in agricultural production must be the results of lower costs (with a given price), however if demand is not adequately elastic it will be followed by the decrease of (gross) incomes in agriculture. Probably most of the net incomes will be lowered as well.

Therefore inter-sector competition is of the first importance for the level of incomes in agriculture[5]. Parity of incomes cannot be reached if the sale of agricultural products faces some difficulties and if the mechanism of monopolistic competition is an action and by thus a part of the surplus value produced in agriculture is intercepted by financial capital.

The difference in productivity of labour between individual countries exerts a more indirect influence upon the realization of the above rule. We shall discuss this problem below. Here we may only say one obvious thing: if farmers work in some countries is more productive than in others therefore—*caeteris paribus*—the former will have higher incomes than the latter. There is no evidence, however, that their relative situation (in relation to those employed in other sectors of production) will be more favourable.

Finally, the third way of approaching the problem still remains to be made clear: i.e. the connection between incomes and comparative costs. Let us consider the example given by Ricardo, in which we have two countries making two identical products. The country *A* and the country *B* have, let us assume, the same amount of labour at their disposal, but their productive abilities are different as a result of different natural conditions.

The country *A* is able to produce 100 l. of wine or 50 m of linen with a given amount of labour, but the country *B*—80 l. of wine or 20 m of linen. So, as we see, the country *A* has the advantage over the country *B* in both fields of production.

[5] Cf. K. Marx and F. Engels, *Letters on The Capital*, Warsaw 1957, p. 106.

Nevertheless, the relation of productive costs of wine and linen is more favourable in the country *B* than in the country *A*. As far as the production of linen is concerned, however, this relation is more favourable for the country *A*. Consequently, if the country *A* and the country *B* specialize in the branch, in which they have lower comparative costs and if they take advantage of international exchange, both these countries will obtain a larger quantity products than by producing everything on their own.

This statement may be illustrated in the following way. Let us assume that both these countries, before starting the exchange, alloted $\frac{2}{5}$ of the labour at their disposal to the production of wine, and $\frac{3}{5}$ to the production of linen.

The country *A* would then produce 40 l. of wine + 30 m of linen.
The country *B* 32 l. ,, +12 m ,,

In that case the production in both these countries together would be 72 l. of wine and 42 m of linen. Yet if each of the two countries decided to specialize in the field of production whose comparative costs are the lowest (the country *A* in linen and the country *B* in wine) their joint production would amount to 80 l. of wine and 50 m of linen. So, owing to the international division of labour, production would increase by 8 l. of wine and 8 m of linen. This conclusion closes Ricardo's argument.

Historical experience has proved that this reasoning though logically sound, is still incomplete. It is necessary to take terms of trade into consideration and to abandon the static approach in favour of a dynamic one. These problems are widely discussed in the theory of foreign trade. Here we shall limit ourselves to a brief presentation of the central problem interesting from our point of view. For this reason we shall continue our discussion of Ricardo's example.

In the country *A* 1 l. of wine requires as much labour as $\frac{1}{2}$ m of linen. The terms of exchaning wine for linen must be, therefore, enclosed in the following limits

$$\frac{1}{4} \text{ m of linen} < 1 \text{ l. of wine} < \frac{1}{2} \text{ m of linen}$$

or 2 l. of wine < 1 m of linen < 4 l. of wine.

Within these limits only the exchange will be favourable for both sides. Yet it is obvious that the exchange will not, in a certain sense, be equivalent; by this we mean that the product of an hour of labour in the country *A* will not be exchanged for a product that takes an hour to produce in the country *B*. This may easily be proved.

If we assume that the complete cost of labour in each of the two countries is 100 units, it follows that 1 m of linen in the country *A* costs 2 units of labour and 1 l. of wine in the country *B*—1.25 unit. (100/80).

Therefore the equivalent exchange (in the above mentioned sense) would be 1 m of linen = 1.6 l. of wine (2 : 1.25 = 1.6). Yet this ratio would not be favourable for the country *A*, because with the labour required to make 1 m of linen it could produce 2 l. of wine itself.

Accordingly, the price of 1 m of linen expressed in wine must be kept within the limits mentioned above. Let us assume that it will be formed on the level of *1 m of linen = 3.2 l. of wine* and that the *A* country will allot 35 m. of linen to its home market and 15 m of linen for exchange for wine. The situation in the country *A* and *B* will then be as follows:

A 48 l. of wine+35 m of linen

B 32 l. of wine+15 m of linen

(15 m of linen exchanged for 15×3.2 l. of wine = 48 l. of wine). In this way both countries will gain something through the exchange, *A* 8 l. of wine+5 m of linen, and *B*—3 m of linen. *Yet these gains are not equal.* Therefore, if this situation continues, the disparity of levels between the country *A* and country *B* will have the tendency to increase.

This is not only a theoretical possibility, but a reality. Numerous facts show that the exchange between highly developed capitalist countries and countries that are economically backward is not favourable to the latter. This explains objections which are raised to the suggestions of basing everything exclusively on the theory of comparative costs when planning the directions of progress for economically underdeveloped countries.

However we are interested here in another aspect of the problem. First of all, *the more the productivity of labour in the production of wine in the country A increases (or the less a relative "cheapness" of linen in the more developed country is evident) the less favourable the terms of exchange become for the country B.*

To illustrate this point let us assume that the country *A* is able to produce not 100 l. of wine, but 150 l. with the amount of labour at its disposal.

The ratio of comparative costs still imposes specialization and exchange. Yet the limits within which the price of wine expressed in linen must be comprised, will be changed in the following way:

$$\frac{1}{4}\text{ m of linen} < 1\text{ l. of wine} < \frac{1}{3}\text{ m of linen}$$

or 3 l. of wine < 1 m of linen < 4 l. of wine.

The second conclusion refers to the price of wine in the country *A*. Let us assume that the country *A*, basing itself on the principle of comparative costs, specializes in the production of linen; but for various reasons it cannot completely give up the production of wine.

The situation is as follows: *A* 10 l. of wine+45 m of linen (the original assumption that the production of 1 l. of winc requires 1 unit of labour still being vaild).

If in foreign trade 1 m of linen is exchanged for 3.2 l. of wine, it follows that 10 l. of wine may be obtained for 3.1 m of linen, which represents 6.2 labour units in the country *A*. Yet the production of the same amount of wine in that country requires 10 labour units.

Therefore, *apart from the fact that the productivity of labour in the production of wine is greater in the country A than in the B, its prices will be below its value*[6].

Simplifying to a large extent, we can draw an analogy between the situation of industrial and agricultural countries on the one hand and the situation of the countries *A* and *B* on the other. Industrial countries enjoy a decided superiority in the field of industrial as well as agricultural production. Yet comparative costs in most of these countries are in favour of industrial products. In recent years the productivity of labour in agriculture of developed capitalist countries has quickly increased. What impact may it have on the field that we are interested in, namely on that of comparative incomes in agriculture and industry?

In the light of the above reasoning we can, it seems, come to the following conclusions:
a. the technological progress that is taking place in the agriculture of developed capitalist countries must exert an unfavourable influence on the terms of trade of economically underdeveloped countries. This progress is not only the cause of a decreasing demand for agricultural products in developed countries, but also changes the conditions of the profitableness of agricultural import for these countries; b. in some highly industrialized countries agricultural production may (in spite of a relatively high productivity of labour) be too "expensive" and need subsidizing; c. the realization of the parity of incomes in agriculture and industry requires the consideration of the principle of comparative profits when selecting the directions for agricultural production. The tendency towards autarchy in agriculture (this especially refers to the economic region of European Economic Community) must act (in spite of a general rise in the productivity of labour in agriculture) as an incentive to maintain the disparity of incomes between agriculture and industry.

The above observations are not directed against the parity of incomes in agriculture and in other sectors of economy or against the rationalisation of agricultural economy either. We wish to point out only, that the rationalisation of agricultural economy is not sufficient to bring with it parity of incomes.

---

[6] As a classical example we may consider Great Britain.

Theodor Prager
Austria

THE POLITICAL ELEMENT IN POST-WAR ECONOMIC GROWTH

"It is imperative", wrote Otto Bauer, the foremost thinker of Austrian Social Democracy, in the depths of the economic crisis in 1933, "that we should explore new avenues. No economic revival can be hoped for from the free play of market forces. Great public works and orders alone can set the economy moving again. There is no lack of work to be done; the problem is how to find the money". But this, he added at once, should prove no unsurmountable obstacle: Let the Government raise a loan on the capital market or, if necessary, from the banks and pursue "a wise policy of credit expansion... unperturbed by credit and currency doctrines from the past which are of hardly any relevance to the entirely different situation of today"; let the money thus raised be spent on projects of economic and social import, preferably of a labour-intensive kind which also make the maximum use of domestic raw materials; we may be sure that the purchasing power thus created will fructify and multiply as the formerly unemployed begin to re-appear as buyers on the market for consumer goods and as the employers, encouraged by new and large-scale orders, begin to put their plants into better shape... Returning to his plea before a national Trade Union conference especially convened to deal with the emergency, Otto Bauer enumerated the projects that might usefully be undertaken—such as the construction of water power stations and railway electrification—and stressed their long-term as well as their short-term beneficial effects. But the main emphasis was on job creation: on "Work for 200,000!" (Some 60,000 workers, Bauer claimed, might find work if Austria were to declare her "complete international neutrality" and thereby improve her standing with her natural trading partners in South Eastern Europe). The conference adopted a resolution on these lines and shortly after the Social Democrat Parliamentary fraction followed suit. Later during the same month Bauer commented on attacks launched on his proposals by one of the conservative economists: "The anti-Marxist professor knows no better than to reply to our concrete questions with the most trivial generalities on capital and credit—generalities, moreover, which do not show the least trace of the spirit emanating from modern capital and credit theory as espoused even by completely un-Marxist or anti-Marxist authors"[1].

[1] "Arbeiter-Zeitung", Vienna, 8., 16., 22. and 26. July 1933. All this happened three years before J. M. Keynes' *General Theory* and ten years before M. Kalecki's observation that, once the

The Professor quoted by Bauer (he is still the leading light of Vienna's Institute of World Trade) was at least arguing; but the government of the day was not even listening; it was too busy preparing the overthrow of Parliamentary democracy and the forcible suppression of the working class movement. And although Austria later recovered a little as a result of reviving exports to Germany and Italy (who were practicing a perverse kind of Keynesian economics through their self-sufficiency and armament drives) there were still about half a million unemployed by the time Hitler took over.

Nowadays, it is considered rather shocking when unemployment rises to some 130,000 or 140,000 in the worst weeks of winter. In Britain, with seven times Austria's population, last February's unemployment figure of some nine hundred thousand was felt to be quite scandalous and gave rise to unemployed workers marches as had not been seen since the thirties (when the corresponding figures had been two to three times as large). In West Germany, France and even in Italy there is a shortage of skilled, and in the former two even of unskilled, labour after a decade or so of unprecedented economic growth and industrial expansion.

Post-war reconstruction? It has long since been completed; pre-war levels of production have been doubled or trebled. The development of new power resources, technical innovation, new patterns of mass production and distribution? Certainly, but surely these were at least as much called forth by, as well as helping to further, the process of expansion. Consumer credit? It has helped; but neither lenders nor borrowers are very willing to engage in this unless they feel fairly sure, on past experience as on likely prospects, that they are not overextending themselves. Government policy, aiming at full employment, expansion, growth? This certainly appears to be the main answer and it is one frequently given. But by itself it does not explain a great deal. For the further question arises, why was today's policy not also yesterday's? Was it because the governments of the thirties had no inkling of what to do about recessions and crises? Perhaps some of them really did not know, though Otto Bauer in Austria, Gunnar Myrdal in Sweden and others made helpful suggestions. But the main reason why they proved hard of hearing was that they (and all those who made or represented "public opinion") were either still under the spell of the dire experience of early post-war hyperinflation and therefore reluctant to experiment in cheap money, public works and the like, or under the influence of schools of thought that regarded crises in the nature of a purifying if somewhat rough tempest. At any rate they still gave the maintenance of a "sound" currency, of competitiveness in the world market, of foreign "confidence" etc. much higher

economic trade cycle was overcome, a political trade cycle would take its place when—the point of full or "brimful" employment reached—some economists would be certain to come forward, ready to support the reversion to "sound" fiscal and credit policies designed to restore discipline in the factories. (cf. Joan Robinson's *Beyond Full Employment* in "Arbeit und Wirtschaft", Vienna, November 1960, where reference is made to Michal Kalecki's *Political Aspects of Full Employment* in "Political Quarterly", October–December 1943).

priority and moreover considered the crisis a unique opportunity for breaking trade union power and for getting rid of the burden of so-called "social inflation" as inherited from the immediate post-war years.

What has happened since to make them change their minds?

First, the shock of 1944/1945 when Fascism was smashed in Western Europe (and Japan) and when the working class which had headed the patriotic resistance struggle emerged in many countries as the morally leading force of the nation while the old oligarchies were thoroughly discredited as a result of their collaboration with Nazi Germany and also greatly weakened materially. Secondly, the continuing shock of the emergence of a whole Socialist camp, which has not only spelled, visibly for all, the end of the USSR's isolation as a single Socialist country (up to then it was considered as a more or less unique "accident of history") and the final end of capitalism's near-monopoly as the world's dominant social system; but which has also presented capitalism with a powerful competitor in the development of the forces of production and in rates of growth.

In 1936, J. M. Keynes wrote that "it is certain that the world will not much longer tolerate the unemployment which, apart from brief intervals of excitement, is associated—and, in my opinion, inevitably associated—with present-day capitalistic individualism"[2].

*

The world, unfortunately, has proved more tolerant than one was willing to believe of a great many evils but it is certainly true that it has proved very allergic indeed to mass unemployment. The election of a Labour Government in 1945, the formation of coalition governments including, at first, the extreme left wing, in almost all post-war Western Europe was an expression of the universal feeling that it was "time for a change"—and more particularly of the insistent demand for policies (including far-reaching measures of nationalisation and planning) which would ensure full employment, social security, rising living standards. There were plenty of attempts to brow-beat the workers back into an acceptance of the evil old ways and it took long-drawn out and sometimes dramatic struggles to force the oligarchies (who, mainly with American aid, had re-assured their political sway) to adopt such expansionist policies as would in fact ensure something like full employment; from the fight for early post-war dismissals stops and the rather haphazard supply of funds for *ad hoc* public works to more long-term public investment plans and development schemes. In some cases, notably Britain, it is only in pre-election years (such as 1955, 1959 and 1963, "the magic years", Mr. Harold Wilson has called them recently[3]) that the government is willing to incur large-scale deficits to set the economy moving energetically. But by and

---

[2] J. M. Keynes, *The General Theory of Employment, Interest and Money*, London 1936, p. 381.

[3] "The Times", London, April, 4 1963..

large expansion, growth and full employment have become the key slogans, and by and large the promise has been fulfilled. For people in Western Europe simply will not stand for large-scale unemployment and stagnation, and no government can afford the kind of drift that will lead to such results.

One reason, as has been said, has been the change in the relation of class forces immediately following on the defeat of Fascism. The case has been more fully argued elsewhere[4], but just to illustrate the point: American experts then suggested that six out of Italy's 15 shipyards should be closed down, on the grounds of inefficiency, surplus capacity and so on. But "this is of course a very difficult task since, in the present economic and political condition of the country, no government has the strength to close down industrial plants giving employment to tens of thousands of workers. Only within the framework of an expanding economy could such a decision become politically possible"[5]. Since then, the economy *has* expanded enormously—yet it is still found pretty difficult to close down plants, and even more, to dismiss workers in a high-handed fashion. This applies perhaps most obviously in Austria with its large measure of nationalisation[6], strong trade unions, long-standing coalition government and—above all—its close vicinity to those "enslaved" countries to whom it is meant to shine as a kind of Western shop window (hence the almost unique amount of Marshall-aid given her); it applies even to efficiency-conscious Gaullist France: "Even in France", it is being said, "it is not easy to consider the closing down of a deficit-ridden coal mine. A growth-oriented, selective policy of investment aid would indeed require as its logical concomitant that capital should be directed away from shrinking industries. But if it proves politically impossible to pursue a line of 'laisser mourir', on account of the danger of strikes, how much more so a policy of 'faire mourir!'"[7]

Reference has been made, above, to the immediate neighbourhood to the Socialist countries, and this brings un to the other major political element in post-war economic growth in Western Europe. (The case of the USA is a little different, if only because of the fact that the ruling oligarchy has never been discredited there as it has in Europe or in Japan, by collaboration with Fascism; and also because of the comparative absence of a Socialist tradition in the working class movement.)

[4] Cf. Theodor Prager, *Wirtschaftswunder — oder keines?*, Vienna 1963.

[5] Mario Einaudi, Maurice Bye, Ernesto Rossi, *Nationalization in France and Italy*, Ithaca-New York 1955, p. 217.

[6] "The nationalised enterprises are keeping on their pay-rolls at least 10.000 workers for purely political reasons—whether these political motives are obvious (as in the case of the State-owned Oil Combine where the veto of the Communist-ridden shop-steward committee against dismissals was taken lying down, or as in the case of the hard-coal mine of Grünbach which is still being worked although it should have been closed down long ago—only to stop the Communists from gaining their key Parliamentary seat in that area); or whether there are underlying motives such as the doctrine that no nationalised enterprise must be allowed to close down". ("Finanznachrichten", Vienna, February 2, 1962).

[7] "Die Presse", Vienna, April 18, 1963.

It is obvious, and has been pointed out innumerable times, that the mere existence of this *alternative* to capitalism, and of the rapid rates of growth here, have acted as a major spur to expansionist policies in the West.

"Today", writes Professor George N. Halm, "capitalism is only one of several economic systems... Capitalist countries are endeavouring, by means of economic policy instruments, to maintain a high level of employment... If it is true that the centralised command economies can guarantee full employment, the free market economies cannot afford the luxury of mass unemployment as this would no longer be tolerated"[8].

"To-day", runs the economic commentary of one of Austria's leading commercial banks, "when the world is governed by the economic and ideological antagonism between East and West, it is imperative for the West, on these grounds alone, to avoid a real economic crisis at all costs... Any incipient reduction of employment following from a slackening in economic activity will at once call forth such political forces as would immediately press for a policy of reflation, and they would certainly prove successful"[9].

"Expansion", emphasises the leading theoretician of France's 'économie concertée', "became a political necessity from the moment that the conflict between East and West took on the form of economic competition"[10].

"The influence of Soviet planning on Western thought has been negligible", asserts Prof. Mario Einaudi; but he adds at once that this "does not, of course, imply that the massive increase in Soviet economic strength is casting no shadows across the world to-day and is not prompting nearly every nation to a reconsideration of its future outlook and plans"[11].

"The principal underlying fact for US economic and financial policy of the Sixties", states Under-Secretary of the US Treasury, Henry H. Fowler bluntly, "is the Soviet challenge and its threat to security and freedom"[12].

Here the accent is foremost on "security" (as indeed US public expenditure goes primarily into armaments) but military considerations are only one element in Western reaction to the "Soviet challenge". "When the heads of government met in (NATO) council in December 1957, after Russian achievements in rocketry had had such a tremendous impact on world opinion", records M. Margaret Ball in her "NATO and the European Union Movement", "they were faced among other things with the problem of trying to offset this victory on the minds of the world's uncommitted nations. The Council... affirmed its interest in 'an enlargement

---

[8] George N. Halm, *Wirtschaftssysteme. Eine vergleichende Darstellung*, Berlin 1960, p. 1.

[9] "Wirtschaftliche Nachrichten" der Österreichischen Länderbank, Vienna, November 1962.

[10] Albin Chalandon, "Le Monde", June 8, 1960.

[11] Mario Einaudi, op. cit., p. 13 et seq.

[12] "Commercial and Financial Chronicle", January 1, 1962 (quoted by Victor Perlo in "International Affairs", Moscow, June 1962).

of the resources, both public and private, available for the purposes of accelerating the economic advancement of the less developed areas of the free world' ". Moreover, "it is presumably also sound policy for NATO members to continue to help each other to improve the conditions of life which are regarded as an important part of the showcase of Western civilisation..."[13].

The "Soviet challenge", and competition between the systems, has made the West growth conscious. "Growth theory, too, has a background in real life", states Prof. Gottfried Bombach, "i.e. the growth race between the old industrial countries of the West and the countries of the East"[14].

The point needs no further labouring: the most important "new element" in the situation is the rise of a whole group of countries with a fundamentally different and at least potentially superior social order, impinging on old-style capitalism and prompting it to adopt new policies and techniques aimed at developing and streamlining its economies. This has even led to the adoption of remarkably far-reaching planning techniques of which France's "planification" is the outstanding example. Originally introduced to meet the urgent requirements of post-war reconstruction and of such modernisation as would enable the country to hold its own against the more advanced capitalist rival nations, the 'économie concertée' is now said to have become "the only way which permits the Western nations to develop a sufficient degree of economic dynamism to withstand the Communist countries"[15].

Perhaps we should beware of overstating the case. Not with regard to the successes achieved (these can hardly be overstated, in view of the widespread reluctance in the Marxist wing of Western Europe's labour movement to face up to the "challenge" of new-style capitalism by which we are now, in turn, faced) but with regard to the seemingly simple causal chain indicated. Certainly, the pressure of circumstances has caused the Western oligarchies to change their priorities to some extent; to put near-full employment, development and growth before, say, price stability and "sound" fiscal policies etc.[16], to adopt a more dynamic strategy and, in the all-important interest of preserving social and political power, to restrain on occasion even quite weighty vested interests and short-term profit considerations. But this pressure of circumstance has been many-sided and has encountered powerful

---

[13] M. Margaret Ball, *NATO and the European Union Movement*. London Institute of World Affairs, London 1959, p. 127 et seq.

[14] Gottfried Bombach in *Wachstum und Konjunktur*. Darmstadt u. Opladen 1960, p. 7.

[15] Albin Chalandon, op. cit.

[16] "Why was Europe's Expansion so prolonged?... Nearly everywhere in the region a profound attitudinal change appears to have taken place over the past several decades with respect to the responsibilities of governments in economic affairs. To put the matter perhaps too simply, modern governments in the democracies of Western Europe appear more fearful of the prospect of unemployment than of inflation... Post-war governments, whatever their political coloration, almost universally preferred to err on the side of inflation rather than deflation—and they have acted on that preference most of the time since the war..." (*Europe's Needs and Resources. Trends and Prospects in 18 Countries*. Twentieth Century Fund, New York 1961, p. 28).

counter-pressures, both of a material kind (such as the insistence of the typical banking and "City" interests on "prudence" and "restraint") and of an ideological nature (such as the strongly-held beliefs in those conventional doctrines that were encountered by Otto Bauer). In actual historical fact, it required tremendous struggles throughout most of the post-war years to ensure that the governments of the day did not return to the perverse restrictionist and "axe-swinging" policies of the thirties. Sometimes these battles were fought in the streets (as they have been, time and again, in Italy and France but also in Belgium), more often on the floor of the workshop or by balloting and picketing (as in Austria in 1957 when the shop stewards of the electrical engineering and vehicles industries succeeded in getting the Government to raise funds and place orders to implement its own long-term investment plans for railway electrification etc.—themselves the outcome of the electoral battles and promises of 1953/1954). In all such cases the "threat of Communism" within *and* without played its part; and neither would, by itself, have had half the effect it has, in fact, had. It was the conjunction and interaction of these two major political elements—the workers of Western Europe determined to secure jobs, social security, a "share in the fruits of the boom" on the one hand, and the rise and strengthening of the new democracies in Eastern Europe on the other—which got the oligarchies on their toes, seeking ways and means of realising higher rates of growth, a more or less continuous process of expansion, and thereby the wherewithal for what has become a mixture of welfare state and warfare state. It is in this light, surely, that the Marshall-plan must be primarily viewed, as well as all the efforts at "European integration" down to the shaping of the European Common Market (in which the specific aims of the Franco-German oligarchies play their special part, of course); the drives at raising productivity as well as those for "freer trade" etc.; the spread of "human relations" within the combines as well as of "public relations" without (designed to ensure the undisputed sway of the monopolies and the "integration" of the decisive sectors of the working class into the system).

Joan Robinson's tag, originally applied to the economically underdeveloped countries, that there is one thing worse than being exploited, i.e. not to be exploited, applies equally to the industrially developed countries. Having more or less full employment, often with the "chance" of making extra money by over-time, week-end jobbing and the like, the mass of Western European's workers have settled down to cultivating their garden plots, watching their television sets and tinkering with their motorbicycles or second-hand cars. Beneath the surface, the old militancy still remains and occasionally it breaks out into powerful upsurges: as in Italy, in July 1960, when the Tambroni Government attempted an "apertura a destra" and was swept out of office; as in Belgium in the winter of 1960/1961, when the post-Congo retrenchment laws provoked a general strike; as in March 1963, when the great miners' strike in France has "helped", as "The Times" put it, "by challenging the personal authority of General de Gaulle and winning such widespread popular support, to define the nature and limits of his power"[17]. However, the challenge to "personal" authority or rather to the authority of the oligarchies in general has not extended to their armament drives; nor has it yet decisively reverted those authoritarian trends which have come to dominate the political landscape of Germany, France and, in more subtle fashion, even Holland and Britain. But at any rate, it seems fairly certain that stagnation, in Western Europe at least, will not be quietly accepted as an act of God or even of impersonal market forces too majestic or subtle to challenge. True, the cure that is increasingly applied—ever-extending arms expenditure—is worse than the disease and there is the danger that it will smash the world to bits even before people have woken up to it. If they do, and succeed in pushing the oligarchies away from this particular pump, they will sorely also find the strength and resourcefulness to keep the wheels turning for more useful purposes.

[17] "The Times", March 16, 1963.

Joan Robinson
Great Britain

KALECKI AND KEYNES

It is difficult now to recapture the state of orthodox opinion in the capitalist world in the early years of the great depression.

There was heavy unemployment in England even before the world slump set in. In 1929 Lloyd George was campaigning for a programme of public works. In reply, British Officials propounded the "Treasury View" that if the Government borrowed, say, a hundred million pounds to set men to work on road building and so forth, foreign investment would be reduced by an equal sum and no overall increase in employment would occur.

In 1931 the British Labour Government was led to distruction through the belief that it was necessary to balance the budget in order to save the exchange value of sterling.

Academic opinion was sereenly oblivious to the problems of reality. Professor Robbins, surrounded by unemployed labour and idle plant, defined economics as "the science which studies human behaviour as a relation between ends and scarce means which have alternative uses"[1].

According to accepted theory, the price level was determined by the quantity of money. But the suggestion that the depression might therefore be relieved by increasing the quantity of money was confined to cranks. In the orthodox view it would create a dangerous inflation.

The Marxists abused the academics, but they shared their belief in the principles of sound finance.

In this fog Keynes was groping for a theory of employment. He had backed up Lloyd George with a rather vague and half-baked argument that an increase in investment would generate an increase in saving (so that borrowing in one form need not be substracted from borrowing in another[2] and he set a young pupil, R. F. Kahn, to work it out properly. During the sessions of the Macmillan Committee on currency and banking he was coming to the view that there was a fallacy in the accepted argument that a cut in money wage rates would restore profitability to enterprise, by lowering costs relatively to prices, because prices would come down more or less in proportion. But in his great theoretical *Treatise*

[1] *Essay on the Nature and Significance of Economic Science*, 1932.
[2] *Can Lloyd George Do It*? by J. M. Keynes and H. D. Henderson.

his mind was working on a different plane and it failed to produce a theory of employment, though it contained the highly significant conception that an increase of investment without (as we should now put it) a corresponding increase in the propensity to save raises profits, while an increase in propensity to save without a corresponding increase in investment reduces them.

Over the continent, no doubt including Poland, the fog of orthodoxy was even thicker than in England. Only in Sweden Wicksell's pupils were puzzling out a new line. In *Monetary Equilibrium*, published in Swedish in 1931, Gunnar Myrdal twitted Keynes upon his "attractive Anglo-Saxon kind of unnecessary originality" but he was not altogether clear of the fog himself.

The *Treatise on Money* was passed for the last time to the printers in September 1930 and Kahn's article appeared in the "Economic Journal" of June, 1931, setting out the analysis of the multiplier—the relation of an increase in employment in investment to the total increase in employment that it generates—and showing how the rise in incomes that accompanies an increase in investment brings about a rise in savings of an equal amount.

There followed a great bout of argument that churned over these ideas for three years.

In 1933 I published a kind of interim report, which clears the ground for the new theory but does not supply it[3]. It was not till the summer of 1934 that Keynes succeeded in getting his theory of money, his theory of wages and Kahn's multiplier into a coherent system.

In January 1935 he wrote to Bernard Shaw: "I believe myself to be writing a book on economic theory which will largely revolutionize—not, I suppose at once but in the course of the next ten years—the way the world thinks about economic problems[4].

The General Theory of Employment, Interest and Money was published in January 1936.

Meanwhile, without any contact either way, Michal Kalecki had found the same solution.

His book *Essays in the Theory of Business Cycle* published in Polish in 1933 clearly states the principle of effective demand in mathematical form. At the same time he was already exploring the implications of the analysis for the problem of a country's balance of trade, along the same lines that I followed in drawing riders from the *General Theory* in essays published in 1937.

The version of his theory set out in prose (published in "Polska Gospodarcza" No. 43 X 1935) could very well be used today as an introduction to the theory of employment.

---

[3] *The Theory of Money and the Analysis of Output*. In the first number of the "Review of Economic Studies" reprinted in *Collected Economic Papers*, Vol. I.

[4] R. F. Harrod, *Life of Keynes*, p. 462.

He opens by attacking the orthodox theory at the most vital point—the view that unemployment could be reduced by cutting money wage rates. And he shows (a point that the Keynesians came to much later, and under his influence) that, if monopolistic influences prevent prices from falling when wage costs are lowered, the situation is still worse, because reduced purchasing power causes a fall off in sales of consumption goods, so that higher profit margins do not result in higher profits.

Having demolished the case for the orthodox remedy for a depression, he shows how an increase of investment, coming about, for instance, as the result of a great new invention, would increase employment, and then points out that if a spontaneous increase in investment is possible, it must be possible also by deliberate government policy to carry out schemes of investment that would not otherwise be undertaken and so relieve unemployment and increase consumption as well.

Kalecki's statement of the theory avoids the problem of the equality of saving and investment, which plagued us so much, by relying simply on the fact that the equivalent of investment outlay is added to profits. He cuts through another passage where Keynes made heavy weather by taking it for granted that the rate of interest is a monetary phenomenon. When investment, income and saving increase, it is necessary for the supply of the medium of exchange to be increased also; otherwise the rate of interest would rise and a drag be set upon investment.

Kalecki did not approach the theory of employment through the multiplier, which makes his version in a way less rich than Keynes', though no less forceful. On the other hand, he went straight to a theory of the trade cycle, on which Keynes was very weak. In this essay there is a clear statement in a few lines of the capital-stock-adjustment mechanism which is now recognized as the basis for all modern trade-cycle models.

Michal Kalecki's claim to priority of publication is indisputable. With proper scholarly dignity (which, however, is unfortunately rather rare among scholars) he never mentioned this fact. And, indeed, except for the authors concerned, it is not particularly interesting to know who first got into print. The interesting thing is that two thinkers, from completely different political and intellectual starting points, should come to the same conclusion. For us in Cambridge it was a great comfort. Surrounded by blank misunderstanding, there were moments when we almost began to wonder if it was we who were mad or the others. In the serious sciences, original work is *discovery*—finding connections that were always there, waiting to be seen. That this could happen in economics was a reassurance that what we had discovered was really there.

I well remember my first meeting with Michal Kalecki—a strange visitor who was not only already familiar with our brand-new theories, but had even invented some of our private jokes. It gave me a kind of Pirandello feeling—was it he who was speaking or I? Reading his article of 1935 (now for the first time available in

English) gives me the same feeling. Several times, in those old days, I wrote that very article—though with less concentrated force—trying to explain Keynes' theory in simple words.

Kalecki had one great advantage over Keynes—he had never learned orthodox economics. The preface to the *General Theory* ends thus: "The ideas which are here expressed so ·laboriously are extremely simple and should be obvious. The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds".

Kalecki was not brought up so. The only economics he had studied was in Marx. Keynes could never make head or tail of Marx. In the letter to Shaw, quoted above, he maintains that his new theory is going to cut the ground from under the feet of the Marxists. But starting from Marx would have been saved him a lot of trouble. Kahn, at the "circus" where we discussed the *Treatise* in 1931, explained the problem of saving and investment by imagining a cordon round the capital-good industries and then studying the trade between them and the consumption-good industries; he was struggling to rediscover Marx's schema. Kalecki began at that point.

In his *Essays in the Theory of Economic Fluctuations* published after he had been a little while in England, he filled in several gaps in Keynes' formulation of the theory of employment.

In Keynes' scheme, the concept of *marginal efficiency of capital* means that, at any moment, there is in existence a schedule of possible investment projects, listed in descending order of their prospective profitability (allowing for risk). The schedule is cut off at the point where the prospective rate of net profit is equal to the rate of interest to be paid for finance. This determines the total value of investment to be undertaken. Kalecki asked the pertinent question: If there are schemes which promise a rate of profit greater than the rate of interest, would not each individual enterprise be willing and anxious to carry out an indefinitely large amount of investment? It was no use to reply that a faster rate of investment would raise the cost of capital goods and so reduce the prospective rate of profit; for the rise in costs would come about as a result of actual investment, *ex post*, while the marginal efficiency of capital concerns investment plans *ex ante.*

Kalecki supplied an answer, (drawing upon his model of 1933) first, by making clear the separation between investment decisions and actual investment; and second, by introducing into the argument the obvious fact that no individual enterprise can command an indefinitely large amount of finance at a given rate of interest. He took risk over from the demand side (where it lies rather uneasily in Keynes scheme) to the supply side, and postulated that the amount of finance that each individual enterprise will commit to investment is an increasing function of the prospective rate of profit, depending upon the ratio of borrowing to its own capital. Then, with any given distribution of capital amongst enterprises, there is a particular relation between the total amount of investment plans being drawn up at any moment and the level of prospective profits. The second difficulty was that, though Keynes himself attached great importance to the idea that the present is always over-weighted in forming a view about the future, he treated his schedule of prospective profits as though it was independent of the actual rate of investment. Kalecki showed how a higher level of investment this year than last, means a higher level of current profits, therefore a higher expected rate of profit, therefore enlarged investment plans, therefore a higher rate of investment next year.

A rise in the actual rate of investment cannot go on indefinitely. When the rate of investment ceases to rise, the level of current profit ceases to rise. But the amount of productive capacity competing for sales is steadily growing. The rate of profit is therefore declining, and so the boom will break. Thus prosperity can never last. "The tragedy of investment is that it causes crisis because it is useful". He ended the argument with the poignant saying: "Doubtless many people will consider this theory paradoxical. But it is not the theory which is paradoxical, but its subject—the capitalist economy".

The third point at which Kalecki tightened up the slack in the *General Theory* was in connection with the relation of prices to wage rates. Keynes relied upon a rather vague sort of Marshallian concept of competition, with short-period diminishing returns, so that an increase in employment is accompanied by a fall in real wages for workers already employed. Kalecki elaborated his original insight into the relation of monopolistic price policy to employment with the analysis of imperfect competition (then in its heyday) to produce his famous short-period theory of distribution—the share of wages in the value of output is determined by the degree of monopoly.

This formulation has been attacked as being merely circular, since the degree of monopoly is defined as the ratio of gross margins to the value of output, and so is identically equal (on the stated assumptions) to one minus the share of wages. The apparent circularity lies only in the way the argument is set out. When by degree monopoly we mean, not the *ex post* level of gross margins, but the price policy of firms, then, in slumpy conditions, when all plants are working under capacity, it is clearly true to say that if firms pursue a competitive policy, cutting prices in an attempt to sell more, real wage rates will be higher, and the utilisation of existing plant greater, than if they pursue a monopolistic policy, maintaining or even raising gross margins.

These amendments have been incorporated into "Keynesian" thought; few of the present generation of "Keynesians" stop to inquire how much they owe to Kalecki and how indeed to Keynes. All the same, as Michal Kalecki is the first to admit, the "Keynesian Revolution" in Western academic economics is rightly so called. For without Keynes' wide sweep, his brilliant polemic, and, above all, his position within the orthodox citadel, in which he was brought up, the walls of obscurantism would have taken much longer to break.

The political interpretation of the new theory for Kalecki, was very different from the "moderately conservative" implications that Keynes saw in it.

Keynes was throughly disgusted with latter-day capitalism for moral and aesthetic reasons, but he was by no means a socialist. After proving that building pyramids or digging holes in the ground and filling them up again would maintain effective demand and so prevent a fall in useful production, he adds "It is not reasonable, however, that a sensible community should be content to remain dependent on such fortuitous and often wasteful mitigations when once we understand the influences upon which effective demand depends". He believed, or at least allowed himself to hope, that once the new theory was understood, capitalism would reform itself. If full employment could be maintained for a generation by useful investment (without much growth of population) poverty would melt away, and the rate of interest would fall so low that unearned income would cease to be a burden upon the economy. Only honest toil and imaginative speculation would be rewarded by society. (We have seen near-full employment maintained in the Western world since the war, not by useful investment, but less harmlessly foolish than digging holes, by piling up armaments. Keynes analysis has proved correct, but his pleasant day-dream has turned into a nightmare).

Kalecki saw a less agreable vision. In an article written during the War[5], he predicted that now that the causes of the commercial trade cycle are understood, we shall have instead a political trade cycle. The Government will make a full-employment policy by means of a budget deficit. When full employment prevails, prices will be rising and the bargaining position of workers will be strong.

"In this situation a powerful block is likely to be formed between big business and *rentier* interests, and they would probably find more than one economist to declare that the situation was manifestly unsound". A return to "sound finance" will create unemployment again. But as the next election looms up, the Government returns to the vote-getting policy of full employment.

"The regime of the 'political business cycle' would be an artificial restoration of the position as it existed in nineteenth century capitalism. Full employment would be reached only at the top of the boom, but slumps would be relatively mild and short lived". This is a remarkably exact prediction of life in the Western world since the war. (But now that even a Conservative Government in England admits the need for planning, we may be entering a new phase).

After the war Kalecki was mainly occupied with applications of theory to the diagnosis of current developments in the capitalist world and to the problems of planning in the socialist world. But in the new wave of theory in Cambridge, concerned with long-run growth, his influence is still at work.

As well as the short-run theory of distribution connected with the "degree of monopoly" his *Essays* contained a long-run theory based on the principle that

[5] *Political Aspects of Full Employment*, "Political Quarterly", Oct. 1943.

"the workers spend what they get and the capitalists get what they spend". From this is derived the conception that the rate of profit on capital is determined by the rate of accumulation and the propensity to save of capitalists. Kaldor has called this the Keynesian theory of distribution, since it is adumbrated in the *Treatise*, but, like the *General Theory* itself, it has a separate source in Kalecki.

There is still a great deal of work to be done on the plane of theory as well as of applications. I hope that Michal Kalecki will not think that, because he has reached the age to receive a *festschrift*, he need do no more.

IGNACY SACHS
POLAND

LEVELS OF SATIETY AND RATES OF GROWTH

1. IN HIS LATEST essay on *Problems of Financing Economic Development in a Mixed Economy*[1], Professor M. Kalecki uses the following equation of the rate of growth of demand for necessities:

$$c_n = q + e(r - q) \tag{1}$$

where: c_n stands for the rate of growth of demand for necessities;
r stands for the rate of growth of the economy, and, the rate of growth of total consumption, if we assume the share of consumption in the national income to be constant;
e stands for the income-elasticity of demand for necessities, and
q for the rate of increase of the population (no price fluctuations are forseen).

The same relation holds true for food alone, which accounts for the bulk of consumption in less developed countries:

$$c_f = q + e(r - q) \tag{2}$$

where c_f denotes the rate of growth of demand for food.

If we discard foreign trade and movements of stocks, and agree to deal only with the part or population which purchases its food on the market, and to equate the food production with the marketed surplus[2], the rate of growth of food output and of food consumption become identical. Let us denote the maximum rate of growth of the agricultural output, by $a_{\max}$ and the rate of growth *per capita* of the national income and of consumption for sake of simplicity by y; ($y = r - q$). From (2) we get the following maximum rate of growth without inflation of national economy $y_{\max}$, warranted by the maximum rate of growth of agricultural output $a_{\max}$

$$y_{\max} = \frac{1}{e}(a_{\max} - q) \tag{3}$$

Let us observe, that to have any growth of *per capita* income and consumption, at all, the rate of expansion of agricultural output must be higher than the

---

[1] See *Essays on Planning and Economic Development*, vol. II (Papers of the Center of Research on Underdeveloped Economies) in preparation.

[2] This is, of course, a far-fetched simplification. We shall discuss the problem later on.

rate of increase of population. Though this condition is not always fulfilled in developing countries, we shall assume throughout this paper that $a_{max} > q$, and we shall introduce the symbols a_q to denote the rate of growth of agricultural output *per capita* and a_{qmax} to denote the maximum rate of growth of this output. Equation (3) reads, therefore, as follows:

$$y_{max} = \frac{1}{e} a_{qmax} \tag{3'}$$

The higher the income-elasticity of demand, the lower the maximum rate of growth of national income, warranted by the performance of the agricultural sector. Figure (1) illustrates the point:

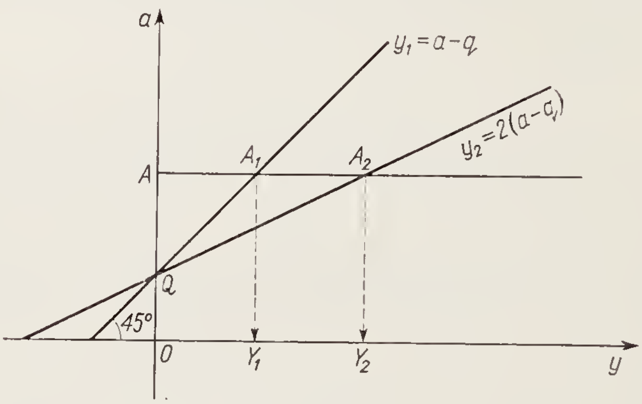


FIG. 1

We have $OQ = q$, $OA = a_{max}$, and of course, $QA = a_{qmax}$.

For the curve y_1, in which $e = 1$, we get: $y_{1max} = a_{qmax}$. Graphically $y_{1max} = OY_1 = AA_1 = AQ$, as y_1 makes with both the axis an angle of 45 degrees and it passes through the point Q.

For the curve y_2, in which $e = 0.5$, we get: $y_{2max} = 2a_{qmax}$. Thus, in the latter case, the economy develops twice as fast as in the former (in *per capita* terms), with the same agricultural output. Were the income-elasticity of demand for food to decrease to zero, the rate of growth of the economy would become independent of the rate of growth of agricultural output, given $a \geqslant q$.

In these conditions, knowledge of the behaviour of e, both in the long and in the short run, acquires considerable importance for the planner dealing with the less developed economies. We propose to explore the problem in the subsequent sections of this paper.

2. Income-elasticity of demand is a function, both of the level of income and of the rate of its change, the latter being particularly important in the short run. We can write:

$$e = f(Y, y) \tag{4}$$

In this section we shall discuss the behaviour-pattern of e in the long run, discarding for the moment the influence of y.

Ever since Engel's pioneering studies, two propositions have usually been enunciated:

(i) the income-elasticity of demand for foodstuffs decreases with the growth of income;

(ii) the share of food in total expenditure decreases with the growth of income, and, therefore, the income-elasticity of demand for food must be lower than 1[3].

As for the second proposition empirical evidence and recent theoretical research combine to show that it holds true only after a certain level of prosperity has been achieved. The usual shape of Engel's curve does not fit the case of extremely poor countries, regions or sections of population, which actually show an income-elasticity of demand higher than 1, and, therefore, spend a higher proportion of their total income on food when they are better off.

The following data support this view:

In United States the share of food in consumption expenditure for households with a yearly net income below 1000 dollars is 33.2 per cent, while in the bracket between 1000 and 2000 dollars it goes up to 35.2 per cent. In the next group (earning from 2 to 3 thousand dollars) food still makes up 33.9 per cent of total expenditure, that is, more than in the lowest bracket.

We obtain the same picture for agricultural labourers in India (data from 1950–51). In the lowest group, with a yearly expenditure per consumption unit of less than 50 rupees, as much as 83.6 per cent of total income is spent on food. It increases to 85.5 per cent for the 50–100 rupee and 100–160 rupee brackets. Even for those earning more than 350 rupees the share of food in total expenditure (84.5 per cent) exceeds that of the lowest bracket.

Statistics on wage earners in Djakarta, in 1957, and on urban wage earners in Japan, in 1960[4], show the same pattern: the lowest groups live on a very miserable diet, because they must cover certain unavoidable extra-food expenses, such as rent, taxes, interest for loans, etc. from their meager incomes. Every increase in money income is translated almost exclusively into food purchases and at times the income-elasticity may exceed 1.

---

[3] By definition $e = \dfrac{\frac{\Delta A}{A}}{\frac{\Delta Y}{Y}} = \dfrac{\Delta A}{A} \cdot \dfrac{Y}{\Delta Y} = \dfrac{\Delta A}{\Delta Y} \cdot \dfrac{Y}{A} = \dfrac{\frac{\Delta A}{\Delta Y}}{\frac{A}{Y}}$ where $\dfrac{\Delta A}{\Delta Y}$ corresponds to the keynesian marginal propensity to consume food and $\dfrac{A}{Y}$ stands for the average share of expenses on food in total expenditure. If $e = 1$, $\dfrac{A}{Y}$ remains constant, it grows when the marginal expenditure on food is higher than it used to be, i.e. when $e > 1$ and it decreases, when the consumer devotes to food purchases a lower portion of the increment of his income than he used to do with his previous income, that is when $e < 1$.

[4] See Appendix.

Indirect evidence for the fact that income-elasticity for food (mostly for home-grown food) exceed 1 among poor pesants in poor countries comes from the study of the variations of marketed surplus. This surplus may actually dwindle when prices increase, because peasants sell the least they can. The phenomenon is well known from the economic history of feudalism[5]. On the other hand, the marketed surplus as a percentage of the gross value of output decreases when we move from small holdings to medium ones, and increases again when we pass to big agricultural enterprises[6]. Instead of speaking of marketed surplus in the case of depressed, small holdings, we should really speak of forced commercialization and consumed surplus. The "genuine" marketed surplus appears only at a higher level of agricultural income. That is why many "grow-more-food" compaigns fail to achieve their main purpose, i.e. the increase of the marketed surplus, although they contribute to the betterment of the peasant's standard of living, which is by all means a desired and commendable goal in itself. Figure 2 ilustrates the point. AA' denotes the rise of output over time, CC' shows the level of minimum commercialization, $MM'M''$ stands for the level of forced satisfactory consumption, after deduction of forced commercialization, and the area $M'M''A'$ for the genuine marketed surplus.

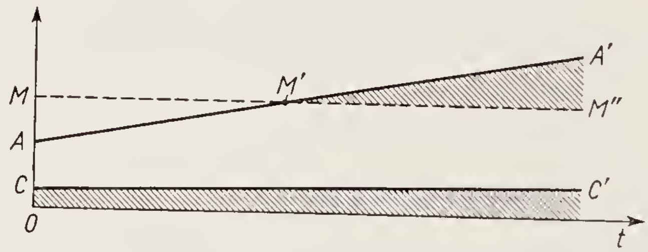


FIG. 2

The data on the countries reproduced in Table 1 are less revealing, because nation-wide averages conceal social and regional disparities in incomes, as well as the differences between rural and urban patterns of consumption. The problem is complicated furthermore by the differences between income-elasticities for marketed and home-grown food supplies of food[7] to which we referred above. They still show, however, that in many countries income-elasticities for food are quite

[5] Cf. W. Kula, *Teoria ekonomiczna ustroju feudalnego*, Warszawa 1962 and *Problemy i metody historii gospodarczej*, Warszawa 1963.

[6] According to calculations of D. Narain from the Delhi Institute of Economic Growth, in India marketed surplus as percentage of the gross value of output is as high as 20.7 per cent in holdings below 5 acres, a minimum of 9.7 per cent in holdings between 10 and 15 acres and returning to 20.4 per cent in holdings from 20–25 acres.

[7] In India, where only 25 to 30 per cent of the food grain production is marketed, the income-elasticity of home-produced foods has been estimated at 1.0 (N. V. Sovani, *Food Problem and Economic Development in Underdeveloped Countries*, in *Paths to Economic Growth*, ed. by A. Datta, Delhi 1962, p. 70).

high, especially in Asia. On the other hand they confirm that a broad inverse correlation exists between the level of income and the income-elasticity for food.

The picture is more accurate if instead of considering income elasticities for aggregate food demand as shown in col. 3, Table 1, we single out elasticities for grains (col. 5) and consider the income-elasticity of aggregate demand in terms of calories and of animal proteins (columns 4 and 6, Table 1)[8].

Let us note that countries of a relatively low level of income *per capita* reach a stage when the income-elasticity of demand for grains becomes negative, while only a few countries in the world have attained absolute satiety in terms of calories (income-elasticity for calories = 0), though the same countries did not arrive at a level guaranteeing full satisfaction of potential needs in terms of animal proteins (the income-elasticity for animal proteins is 0.14 in Oceania)[9].

On basis of the data contained in Table 1 it is possible to distinguish the following four critical levels of satiety for purposes of classification, only.

Level I: the borderline between hunger and subnutrition (the income-elasticity for aggregate food in terms of prices becomes less than 1). This is a level, which all the countries covered by international statistics have surpassed by now, although it should be noted that income-elasticity for food in Pakistan runs as high as 0.96. The income-elasticity in terms of retail prices is probably still higher.

Level II: the relative satiety in terms of calories, achieved by increase in cereal consumption which passes maximum (income-elasticity for cereals is at that point equal to 0). This is the level reached by Mexico, and, not long ago by Japan;

Level III: the absolute satiety in terms of calories (the income-elasticity for calories decreases to 0). This level has been surpassed by Oceania and North America and is about to be reached by the most advanced countries of Western Europe;

Level IV: the absolute satiety in terms of quantity and quality, expressed by the fact that the income-elasticity for animal proteins becomes equal to zero[10]. No country has reached this stage as yet but Colin Clark tried to give a quantitative expression of the asymptote to which the hyperbola of human consumption of food tends. His estimate is 114 I.U. *per capita* per year, at farm value[11].

---

[8] The coefficients of income-elasticity for calories are lower than those of income-elasticity for aggregate food in terms of prices, because the unit-value of each calorie goes up, when the incomes are increasing (in other words the quality of food improves).

[9] We are speaking, of course, of national averages, which may conceal a wide difference between an overfed minority and an undernourished section of population.

[10] The income-elasticity of aggregate demand for food may still be positive at that moment, on account of certain luxury items, which, however, do not weigh heavily on the aggregate demand for food. E.g. the income-elasticity for cocoa is about 0.1 in North America and Oceania, but the income-elasticity for coffee runs as high as 1 in Oceania, while it is 0.6 in Canada and 0.3 in the United States.

[11] Colin Clark, *The Conditions of Economic Progress*, 3rd ed., London 1957, p. 445.

TABLE 1

Country or Region	Gross National Product *per capita* in 1958 (converted in dollars according to parities of purchasing power)	Coefficients of income-elasticity for aggregate food in terms of prices paid to the farmer	Coefficients of income-elasticity for calories (in quantity)	Coefficients of income-elasticity for cereals (in quantity)	Coefficients of income-elasticity for animal proteins (in quantity)	Consumption of calories per head and per day (average for 1957–59)	Consumption of cereals per head in kgs per year (average for 1957–59)	Consumption of starchy roots per head in kgs. per year (average for 1957–59)	Consumption of animal proteins per head, in grams per day (average for 1957–59)
1	2	3	4	5	6	7	8	9	10
North America	2,190	0.16	—0.03	—0.5	0.23	3,118	67	48	66
Oceania	1,570	0.10	—0.06	—0.5	0.14	3,250	87	55	62
EEC	1,285	0.47	0.10	—0.3	0.57	2,850	109	103	41
Mediterranean Europe (Greece, Portugal, Spain, Yugoslavia)	575	0.55	0.18	—0.3	0.90	2,655	141	91	23
Japan	917(613)	0.58	0.20	—0.17	0.94	2,220	153	66	16.6
Argentine a. Uruguay	825	0.17	0.04	—0.3	0.22	3,100	118	82	65.2
Mexico	535(415)	0.58	0.30	0.0	0.81	2,420	147	10	16.0
Middle East (excluding Egypt and Turkey)	260	0.79	0.57	0.3	1.21	2,144	140	11	12.0
North Africa	315	0.64	0.37	0.2	1.08	2,603	188	10	13.1

TABLE 1 (*continued*)

Country or Region	Gross National Product *per capita* in 1958 (converted in dollars according to parities of purchasing power)	Coefficients of income-elasticity for aggregate food in terms of prices paid to the farmer	Coefficients of income-elasticity for calories (in quantity)	Coefficients of income elasticity for cereals (in quantity)	Coefficients of income-elasticity, for animal proteins (in quantity)	Consumption of calories per head and per day (average for 1957–59)	Consumption of cereals per head in kg per year (average for 1957–59)	Consumption of starchy roots per head in kg per year (average for 1957–59)	Consumption of animal proteins per head, in grams per day (average for 1957–59)
1	2	3	4	5	6	7	8	9	10
Rest of Africa (excluding Rep. of South Africa)	210	0.64	0.39	0.4	1.08	2,356	35	397	10.0
Asia and Far East (excluding Japan)	165	0.89	0.62	0.5	1.49	1,980	133	33	7.0
Ceylon	240	0.78	0.49	0.35	1.46	1,970	125	22	6.2
India	150	0.89	0.64	0.5	1.57	1,950	133	11	6.0
Indonesia	175	0.79	0.54	0.5	1.23	2,079	117,1	159	5.4
Pakistan	145	0.96	0.68	0.5	1.62	1,950	149	6	7.6

Source: FAO, *Produits Agricoles—Projections pour 1970*, Rome 1962. The estimates of GNP, arrived at by the FAO experts seem too high, mostly in case of Japan and Mexico. We indicated in brackets the estimates of P. Rosenstein-Rodan, published in the Review of Economics and Statistics 1961. Coefficients of income-elasticity for calories and for animal protein refer to the caloric content and animal protein content of aggregate demand for food and not to selected commodities, as in the case of grain.

The relevant parameters for each critical level are in Table 2. It goes without saying, that these parameters are but a rough approximation and only indicate a broad order of magnitude. Discrepancies between them and the actual performance of diverse countries should be expected due to differences in the pattern of consumption, independent of the level of income[12], historical circumstances, etc.

Figure 3, reproduced from the FAO study already quoted shows the correlation between the different income-elasticity coefficients and the level of income, converted into dollars on basis of parities of purchasing power.

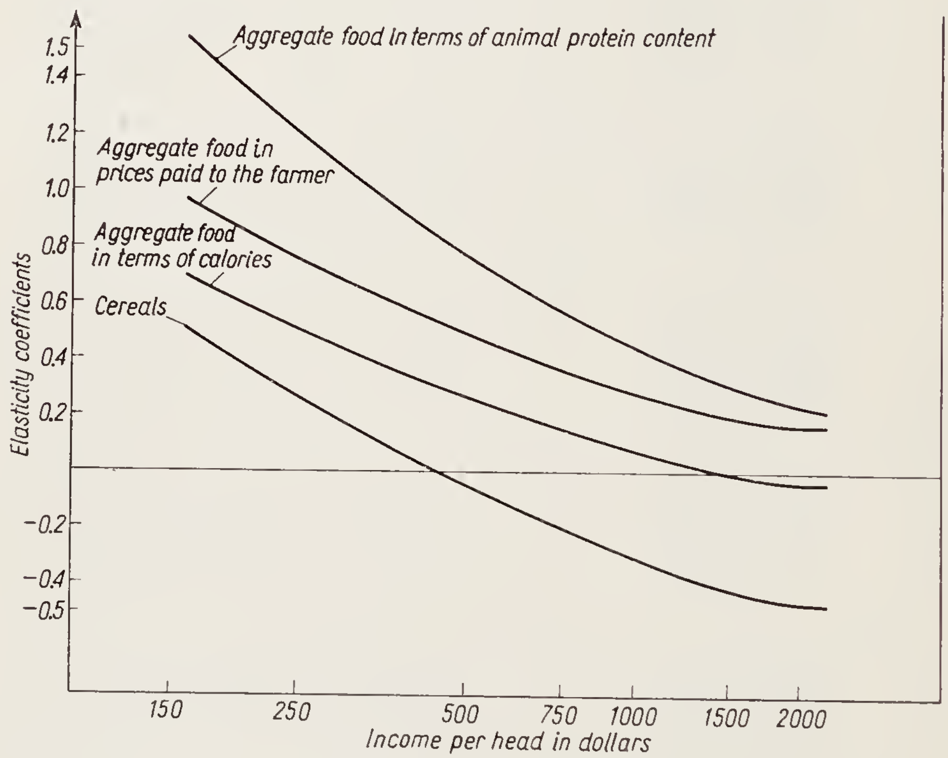


FIG. 3 *Income-elasticities and levels of income.*

The four critical levels delimit five zones (we shall call them zones *A*, *B*, *C*, *D*, *E*). Each zone poses specific problems of food supply.

In *zone A* (below the level I), income-elasticity for food is higher than 1 and the share of food in total consumption expenditure may be as high as 0.8. In such circumstances the rate of supply of food practically sets a ceiling to the overall rate of growth. If, furthermore, the rate of supply of food does not exceed the rate of population growth, the country (viz. region) is doomed to stagnation, if there is no recourse to foreign trade and/or to inflation.

---

[12] Let us mention e.g. the unusually high consumption of meat in Argentine and Uruguay on the one hand, and the vegetarian diet of Hindus on the other.

TABLE 2
Critical levels of satiety

Levels	Income-elasticity for aggregate food in terms of prices paid to the farmers	Income-elasticity for cereals	Income-elasticity for calories	Income-elasticity for animal proteins	GNP per head in 'real' dollars	Consumption of cereals (kgs per head per year)	Consumption of starchy roots (kg per head per year)	Consumption of animal proteins (grams per head per day)	Daily intake of calories per head
1	2	3	4	5	6	7	8	9	10
I	1	0.9–0.7	0.8	>1.5	<100	50–150	10–150	5	<2,000
II	0.6	0	0.3	1–0.8	400–600	150	10–100	15–20	2,200–2,400
III	0.4	–0.3	0	0.3–0.1	1,500	100	50–100	50	3,200
IV	about 0	strongly negative	negative	0	about 3,000	60–70	about 50	70–80	3,200

The agricultural barrier acts with unparalleled intensity[13], although the demand for food expresses itself mainly in terms of cereals, while starchy roots supplement the diet[14].

In *zone B* (between levels I and II), the income-elasticity for food decreases considerably, but it still runs high and the share of food in total expenditure frequently exceeds 50 per cent (in India it varies between 60 and 70 per cent). The consumption of cereals *per capita* continues to increase. At the same time important substitutions take place between roots and cereals, as well as between cereals of lower and higher grade[15]. A serious problem is posed by products of animal origin (the *per capita* intake of protein increases 3 to 4 times). On the whole the agricultural barrier is still very intense and if we take into account the complications involved in expanding the supply of animal products, it may be as intense as in zone *A*, despite the decrease of the overall coefficient of income-elasticity for food.

In *zone C* the supply of grain ceases to be a problem assuming it was tackled successfully, while the country was still moving through zone *B*. The only difficulty is the trebling of daily *per capita* intake of animal proteins.

In *zone D* the agricultural problem ceases to act as a barrier, the more so because the share of food in total consumption expenditure has been reduced considerably. The decrease of domestic consumption of cereals may even create surpluses for export, or agricultural production may be decreased.

In *zone E* the rate of supply of food needs only equal the rate of growth of population.

On the whole, a paradoxical picture emerges: Adam Smith was certainly right in saying that "the desire for food is limited in every man by the narrow capacity of the human stomach". In terms of calories, members of well to-do communities eat less than double the daily intake of the starving populations of Asia. But in terms of animal proteins the difference is fifteen-fold, and it certainly takes a very long time to reach the level of satiety, both in terms of calories and of quality. What is more, the richer a country, the less intensive is the barrier of agriculture. This barrier vanishes at the very moment when a country, thanks to its general development and, more particularly, to industrialization, can overcome it. But in the initial stages of development when it most needs to increase the rate of growth,

---

[13] Our case is unrealistic to the extent to which we do not take in account the existence of subsistence economy, based on self-consumption. We have seen, however, that the income-elasticity for home-grown food is even higher than the elasticity coefficient of the demand for marketed surplus.

[14] The level of consumption of roots depends very much on specific consumption patterns of a given country. It is extremely high in Africa, where, as a compensation, consumption of cereals is the lowest in the world.

[15] According to estimates of the Indian Statistical Organization the income-elasticity for wheat—still a "luxury" product in India—is over 1.5 in rural areas and over 0.8 in urban areas, while the aggregate income-elasticity for cereals is much lower (0.63 and 0.32). For rice it is 0.65 and less than 0.3.

no suitable conditions exist to cope with the task: institutional obstacles and the necessity of investing for a number of years before a steady rise of agricultural productivity is achieved, face the developing countries with the dilemma of either going through a radical transformation of agrarian relations or giving up ambitious rates of growth[16].

For the planner, it is extremely important to know how the income-elasticities will change over time. For a full assessment of the situation, he should know the income-elasticities for basic commodities by broad sections of population (at least separate sets of figures should be provided for rural and urban areas). Then he should evaluate the increase of population not dependent on home-produced food (migration from rural areas to towns), the rate of growth of their personal incomes, and the rate of supply of marketed surplus. As a first approximation, we shall deal with overall income-elasticities for food.

Professor Kalecki in the paper already quoted assumes the income-elasticity coefficients to be stable over a period of five years or so (the period of medium-term planning). Such an assumption is reasonable, but it should not be extended to long-term planning (10–20 years). Though we have established five zones and, we know, by and large, what the behaviour of income-elasticity coefficients is in each zone, it is quite risky to apply the figures arrived at to any single country[17], the more so because we have to assume a rate of overall growth in order to know how long it will take to reach a given level of income *per capita.*

For the sake of illustration, we have computed the compound rate of change in income-elasticity, on the assumption of a 3 per cent *per capita* rate of growth of income per year. The results are given in Table 3.

The rate of change, on the whole, is higher in the last two intervals, where, the whole matter is of much less importance than in the lower ones. A welcome, though not unexpected, result is the relatively rapid decrease of income-elasticity for cereals in the third interval[18].

In projections over a period of 12 years, the FAO experts have used inverse logarithmic functions with decreasing coefficients of elasticity. Table 4 compares their projections of annual rates of growth of demand agricultural products (the lower hypothesis) with the rate of growth of agricultural production in the fifties and the potential demand, calculated on the assumption of constant elasticities. With the exception of Japan, which has a high rate of growth and is going through a phase of rapid transformation of consumption pattern, the differences

---

[16] We abstract here from foreign credits and exclude the possibility of inflationary growth.

[17] Studies of behaviour of income-elasticities in the long run face the usual limitations: lack of long time-series, risks involved in international comparisons or in substituting the scale of incomes of different strata of population (family-budgets) for the time-series.

[18] The difference may be significant even in terms of a 5-year plan, the more so, that the income of Japan has been overestimated, and, therefore, the time-span between Latin America and Japan is shorter than indicated in the Table.

TABLE 3

Country or region	GNP per head in 'real' dollars	Time necessary to catch the level of income of next region in years	Average annual rate of decrease of income-elasticity for food	Number of years necessary for a decrease by 0.1	Average annual rate of decrease of income-elasticity for cereals	Number of years necessary for a decrease by 0.1	Average annual rate of decrease of income-elasticity for calories	Number of years necessary for a dcrease by 0.1	Average annual rate of decrease of income-elasticity for animal proteins	Number of years necessary for a decrease by 0.1
1	2	3	4	5	6	7	8	9	10	11
Asia (without Japan)	165	16	1.3%	8	3%	5	1.8%	8	1.1%	5
Near East and Africa	260	22	0.6%	22	1.1%	44	1.0%	22	1.2%	5
Latin America	500	20	0	—	6.0%	6	1.4%	20	0.6%	20
Japan	910	11	1.5%	11	5.3%	8	3.8%	11	2.6%	4
Europan Economic Community	1285	18	2.9%	5	2.9%	9	4.7%	14	2.7	5
North America	2190									

TABLE 4

Country or region	Projected rate of growth of GNP (1958–70)	Projected rate of growth of GNP *per capita*	Projected rate of growth of demand for food	Projected rate of growth of demand for food on assumption of constant elasticities	Past rate of growth of agricultural production (in the fifties)
1	2	3	4	5	6
Asia and Far East	3.6	1.3	3.4	3.5	2.9
Middle East and Africa	4.0	1.5	3.5	3.6	2.1
Latin America	4.7	2.0	3.7	3.9	3.8
Japan	6.0	5.3	2.9	3.9	4.6
EEC	4.7	3.9	1.8	2.7	2.8
North America	3.1	1.3	1.9	2.0	1.9

between columns 4 and 5 do not exceed 0.1 per cent, which, in the case of less developed countries, is within the statistical margin of error. But, if one looks at the gap between estimates of growth of demand for food set in column 4 (arrived at on the basis of a very conservative rate of overall development) and the actual rates of growth of agricultural produce, every tenth of per cent acquires significance.

3. Let us turn now to the influence of the rate of growth of income y on the short-term behaviour of income-elasticity for food. We shall discuss three problems:

(i) the difference between intensive and extensive growth of income;
(ii) the impact of the speed of the growth process;
(iii) the consequences of steady or erratic growth.

The same increment of national income ΔY may be obtained by increased productivity with no additional employment (intensive growth) or by enlarging the roll of working people (extensive growth). All other conditions being equal[19], extensive growth in an underdeveloped economy will cause a bigger pressure on the food supply, than intensive growth[20], except in drastic situations, when each employed worker has a large number of dependants and their standard of living is so depressed, that all additional income is used for additional purchases of staple food. It is clear, therefore, that the overall income-elasticity for food will be

[19] We assume in particular that no changes take place in the distribution of income between social classes, i.e. that the share of wages remains constant.

[20] Cf. W. Herer *Rolnictwo a rozwój gospodarki narodowej*, Warszawa 1962, p. 187: "A zloty spent on increasing employment goes more to purchases of agricultural consumer goods than the zloty spent on the increase of average wages".

lesser in the case of intensive growth. To look at the same problem from another angle, the overall-elasticity for food is a weighed average of elasticities of different groups of population, classified according to income. Extensive growth means increasing the number of wage-earners in the lower brackets with higher income-elasticities for food, and therefore increasing the weight of these brackets, while intensive growth means shifting people from lower to higher brackets.

If we abandon the field of economics proper and move to that of social psychology, we can make the following observations on the behaviour of consumers in periods of intensive growth: the mood of optimism creates conditions for the working of the "demonstration effect"; if people expect incomes to continue rising, they start to save for durable consumer goods[21] and the income-elasticity for food drops below the "normal" level. On the contrary a slow and imperceptible rise of incomes reflects itself in substitution process and/or in increased purchases of non-durable consumer goods, mainly food.

The most important question, however, is that of erratic changes in levels of income, both in positive and negative directions.

As a rule, poor populations try to keep the standard of nutrition obtained, even if their income shrinks. They do it by sacrificing other extra-food consumptions to the outmost limit and, if necessary, by making substitutions for lower grade cheaper food articles[22]. That is to say when personal income is decreasing the demand for food becomes highly inelastic and the income-elasticity coefficient shrinks to zero or remains very low (big decreases of income cause small decreases in demand for food). In such circumstances, the total consumption over a period of n years, in which an average rate of growth y has been achieved, will be higher if this average conceals ups and downs than in the case of steady growth. In an underdeveloped predominantly agricultural country, a fall in total income usually is a consequence of bad crops. This adds to the drama of the situation. The demand for the marketed surplus of food is at least at the level of the previous year[23], but food is in short supply, and the extraction of marketed surplus from villages quite frequently reduces the peasants to starvation. The stage is set for violent inflationary pressures in the town and social unrest throughout the country.

4. Up to now, we have assumed a non-inflationary process of growth. It is necessary to examine now the case when food is in short supply compared with

[21] A bicycle, a sewing machine or a cheap radio.

[22] An extreme case of it is the so-called "Griffen paradox," observed in Ireland; consumption of bread increasing after an increase of prices, because to keep up with the previous consumption of bread people had to renounce richer food and, therefore, compensate by eating extra rations of bread.

[23] Quite frequently a fall in agricultural product coincides with a rise of monetary incomes of urban populations, not to speak of the acceleration of the peasant exodus to towns in periods of acute shortages of food supplies because they abandon famine-ridden villages.

the effective demand. Let y_n be the rate of growth of nominal income *per capita*. We shall assume that the rate of effective supply of agricultural goods a_r is less than the required one:

$$a_r < a_n = ey_n + q \tag{5}$$

Let it be:

$$a_n - a_r = d,$$

and let us assume, for sake of simplicity, that $q = 0$.

The prices will obviously rise until

$$d = -pf \tag{6}$$

p being the percentual price rise and f the price-elasticity of demand for food (f being negative $-pf > 0$). The rate of growth of real income y_r will be:

$$y_r = y_n - pA \tag{7}$$

where A is the share of food in total expenditure. Since we are only discussing the short run, we will not take the repercussions of p on the other prices into account. For that very reason our argument does not deal with incomes of intermediaries which come from the increase in prices. Nor will we examine, at this stage, the possible shift in employment induced by the price movements and their effects. Let us assume that $y_n > pA$, that is to say $0 < y_r < y_n$.

If we now return to equation 3, and assume that $a_n - d$ is the maximum rate of supply of food, we can write:

$$y_{\max} = \frac{1}{e}(a_n - d) \tag{8}$$

Let us compare (7) and (8).

$y_{\max}$ will be equal to y_r, if the following condition holds true:

$$y_n - pA = \frac{1}{e}(a_n - d) \tag{9}$$

and after substitution of a_n and d, and simplifications:

$$A = \frac{-f}{e} \tag{10}$$

or

$$eA = -f \tag{10'}$$

Of course if $eA > -f$, then $y_{\max} > y_r$, and if $eA < -f$ then $y_{\max} < y_r$[24].

In other words, if $eA > -f$, and food is in short supply relative to the pro-

[24] In the short run we get a rate of real growth higher than the maximum rate of non-inflationary growth warranted by a given level of food supply.

spective rise of income, it is better to restrain the rate of growth of nominal incomes, than to restaure the equilibrium on the market by increasing prices because the final effect on the income will be worse: the rate of growth of real income y_r will be less than y_{max} compatible with a given rate of increase of food supply. In both cases real consumption of food will grow by 3 per cent. But the shift from other consumption will be lesser in the first case[25]. The above description implies a situation when the benefits of rising food prices accrue to intermediaries, and not to the tillers. Such a redistribution of income may increase the demand for luxury goods (unless the increase of luxury consumption is restricted by proper taxation, as assumed in Professor Kalecki's paper), but the decreased demand for non-agricultural essentials by working people, who have to pay more for food, is not compensated by a rising demand for industrial goods by agriculturists. In the long run, therefore, we are likely to get a shift of employment from the production of essentials to that of non-essentials. An alternative assumption might be that of hoarding the additional savings by intermediaries. This would lead, on the one hand, to inflation, due to the lack of a proper supply of food, and on the other hand it would lead to a reduction of employment in the consumer goods industries due to a lack of effective demand. The increasing unemployment would in turn decrease the effective demand for food. But at this stage we should definitely introduce further assumptions about the rate of investment, taxation, etc. This would complicate our case. That is why we prefer to stick to the short run.

Now, it is necessary to interpret the formal condition: $eA > -f$. It occurs in practice only for goods for which e is high, f low and which account for a considerable part of total consumption. This is precisely the case of foodgrains in an underdeveloped country[26]. The disproportionate reactions of prices of grain in the short supply were observed about three hundred years ago by Geoffrey King. We believe, that the formal condition analysed above, permits a more satisfactory interpretation of the so-called King effect. We know by empirical evidence, furthermore, that price policies, used in less developed countries to equilibrate the food market, as a rule, prove self-defeating: to offset the demand in excess it is necessary to raise the prices to such an extent that the real incomes begin to stagnate or to fall; inflation ensues with undesirable flows of incomes from working

[25] The following description by N. V. Sovani seems to apply to the case discussed in this section, namely an increase of real wages through a combination of rapid increase in nominal incomes and price increases which partly offset it: "The primacy of demand for foodgrains over all other demands leads to a peculiar situation in which a sharp rise in food-grain prices sucks up so much of the increased total demand that the residual total demand for other commodities and services smaller than previously and results in a situation of slack demand conditions in those markets" (N. V. Sovani, *Analysis of Inflation in Underdeveloped Economies, Changing India. Essays in Honour of Professor D. R. Gadgil*, Bombay 1961, p. 304).

[26] This is probably the only important instance of a rather unusual combination of e, f and A (or its equivalent).

people to capitalists and traders and the full set of well known adverse effects, of which the overall rate of growth is usually the last victim.

The short-term fluctuations, described in this section, are not as relevant for medium or long term planning, as for the framing of economic polieies.

5. While limitations imposed on the process of growth by the agricultural barrier should be duly accountens for, we should not adopt an attitude of resignation. A slightly more optimistic view is warranted, if the following circumstances are taken into account:

(i) untapped possibilities of rapid increase of agricultural output exist in many less developed countries; the real barrier, here, is the institutional one;

(ii) rational utilization of the capacity to import may help to easy the grip of the agricultural barrier;

(iii) policies of redistribution of national income, though mainly directed at limiting or banning, whatever the political case may be, the luxury consumption of the upper classes, would also justify a lower rate of growth of popular consumption that of the national income, if such a redistribution steps up the share of investments, enjoying high social priority.

Moreover certain specific policies may be recommended (apart rationing which requires certain smoothly working institutional arrangements) namely:

(i) manipulations of relative prices, aimed at inducing desirable substitutions among similar goods;

(ii) price and credit policies, aimed at inducing desirable changes in patterns of consumption, favouring e.g. a shift of the purchasing power of some clerks and workers, who are relatively better off, from higher grades of food to durable consumer goods[27], and thus reducing the effective income-elasticity coefficient for higher grade foods;

(iii) resorting to planned shortages of selected goods, while simultaneously increasing the supply of others (some kind of forced substitution). Of course, such a policy involves many grave risks—including that of creating a black market—and planners should not abuse it, nor apply it for too long a period. But certain experiences would point to the possibility of patching up the situation if selected goods which do not weigh heavily on total consumption are in short supply by a fraction of 5 per cent or so of the total demand and, therefore, occasional defaults of supply occur here and there at irregular intervals[28].

(iv) Some price increase on selected goods may be made, especially if they do not affect significantly the general price level and do not weigh on the "basket" of popular consumption.

---

[27] E.g. by selling bicycles on 24 monthly installments, payable by deduction from salaries and wages, with no interest or a symbolic one.

[28] People will be queuing and buying perhaps more that they actually need, but organized speculation will not be rewarding in such a case.

It goes without saying that measures (iii) and (iv) should not be considered a virtue, but a hard necessity, to be applied in the last instance.

On the whole, there is not much elbowroom left for manouvering and the more underdeveloped a country the lesser the amount of freedom. Still, no opportunity of easing the agricultural bottleneck should be neglected: it is here in overcoming the shortages of supply in food and other necessities (we might call it Kalecki's paradox) that the real problem of financing economic development in a less developed economy lies.

APPENDIX

Private household consumption expenditure by income level, and percentage distribution by object of expenditure

Continent, country, year coverage and income or expenditure level	Average total consumption expenditure per household	Percentage distribution of consumption expenditure						
		Food and drinks	Housing		Clothing			
			Rent	Fuel and light	Furniture and utensils	Clothes and utensils	Personal effects	Miscel-laneous
	1	2	3	4	5	6	7	8
United States 1950 Urban, all types of household								
Yearly net money income per household (in $)								
Up to 1 000	1 333	33.2	17.9	7.2	4.2	5.5	0.5	31.5
1 000– 2 000	1 822	35.2	15.2	5.9	4.8	8.2	0.8	29.9
2 000– 3 000	2 784	33.9	12.5	4.7	6.2	9.1	1.0	32.6
3 000– 4 000	3 654	32.0	11.2	4.2	6.5	9.6	1.0	35.5
4 000– 5 000	4 571	30.2	10.6	3.9	7.2	10.0	1.2	35.9
5 000– 6 000	5 423	29.3	9.9	3.6	7.0	10.7	1.2	38.3
6 000– 7 000	6 262	28.1	9.9	3.3	6.9	10.9	1.4	35.5
7 500–10 000	7 439	27.6	9.5	3.2	6.1	11.2	1.7	40.7
10 000 and over	11 869	22.3	9.7	2.5	7.6	10.9	1.9	45.1
India 1950–51	1	2	3	4		5+6+7		8
Rural, agricultural labourers								
Yearly expenditure per consumption unit (in rupees)								
Up to 50	219.00	83.6	0.5	1.8		6.8		7.3
51–100	343.00	85.5	0.6	1.2		6.0		6.7
101– 15	440.00	85.5	0.7	1.2		6.3		6.3

Continent, country, year coverage and income or expenditure level	Average total consumption expenditure per household	Percentage distribution of consumption expenditure						
		Food and drinks	Housing		Clothing			
			Rent	Fuel and light	Furniture and utensils	Clothes and utensils	Personal effects	Miscel-laneous
	1	2	3	4	5	6	7	8
151–200	528.00	84.9	0.8	1.1		6.6		6.6
201–250	584.00	85.2	1.2	1.0		6.0		6.6
251–300	628.00	84.3	1.6	1.1		5.9		7.1
301–350	644.00	83.7	1.7	1.1		6.2		7.3
351 and over	700.00	84.5	1.5	1.0		6.3		6.7
Indonesia 1957 Wage earners, Djakarta Monthly income per household (in rupiahs)	1	2	3+4+5			6	7+8	
Up to 199	190.04	65.4	13.6			11.4	9.6	
200– 299	287.46	68.3	14.0			6.5	11.2	
300– 399	378.51	66.7	12.1			7.6	13.6	
400– 499	460.49	65.0	11.6			8.4	15.0	
500– 599	537.20	67.8	8.4			7.7	16.1	
600– 699	681.76	63.6	10.4			9.3	16.7	
700– 799	748.97	63.6	10.6			8.5	17.3	
800– 899	833.45	63.1	10.4			8.1	18.4	
900– 999	910.08	61.1	10.1			10.1	18.7	
1 000–1 099	1,034.72	58.6	9.3			13.1	19.0	
1 100–1 499	1,146.35	61.9	8.5			4.6	25.0	
1 500 and over	1,781.81	52.9	14.0			9.6	23.6	

Continent, country, year coverage end income or expenditure level	Average total consumption expenditure per household	Percentage distribution of consumption expenditure						
		Food and drinks	Housing			Clothing		Miscel-laneous
			Rent	Fuel ond light	Furniture and utensils	Clothes and utensils	Personal effects	
	1	2	3	4	5	6	7	8
Japan 1960								
Urban wage and salary earners								
Monthly money income per household (in yens)								
Up to 4,999	18,802	47.8	3.7	5.4	4.4	5.8	2.9	30.0
5,000– 9,999	12,777	53.5	4.3	6.0	2.1	5.1	2.5	26.5
10,000–14,999	14,417	52.2	6.4	5.6	1.7	5.4	2.6	26.1
15,000–19,999	17,774	48.2	6.2	5.4	3.7	6.0	2.6	27.9
20,000–24,999	21,368	46.2	5.4	5.2	4.4	6.2	2.9	29.7
25,000–29,999	24,800	44.5	5.1	5.2	3.7	6.5	3.2	31.8
30,000–34,999	28,782	41.8	4.8	5.0	4.4	7.1	3.4	33.5
35,000–39,999	31,772	40.5	4.4	4.9	3.9	7.1	3.5	35.7
40,000–44,999	35,040	38.9	4.2	4.9	4.3	7.5	3.7	36.5
45,000–49,999	38,112	37,4	4.2	4.9	5.0	7.6	3.8	37.1
50,000–59,999	42,790	35.5	4.5	4.9	4.4	7.9	3.9	38.9
60,000–69,999	49,318	32.9	4.6	4.6	4.6	8.1	3.9	41.3
70,000–79,999	52,701	31.3	3.8	4.5	5.0	9.0	4.3	42.1
80,000–89,999	57,184	30.4	4.3	4.7	6.6	9.2	4.1	40.7
90,000–99,999	62,862	28.4	4.9	4.3	5.9	8.8	4.4	43.3
100,000 and over	76,741	24.6	4.2	3.7	5.8	9.2	4.6	47.9

Source: UN Compendium of Social Statistical: 1963.

Amartya Kumar Sen
India

THE EFFICIENCY OF INDIRECT TAXES*

I

The argument about the relative efficiency of direct and indirect taxes has gone through several phases. In the first phase it was discussed in partial equilibrium terms, and the superiority of direct taxation looked quite obvious[1]. The necessity of bringing the production substitutability considerations into the picture reduced the simplicity of the superiority proposition; but it was still shown that given "a fixed supply of labor and savings, a selective excise, and perfect markets", "the selective excise proved inferior to an income tax"[2]. Professor Musgrave points out, however, that the superiority extends only to "the avoidance of excess burden", which is "only one consideration among others in choosing between different texes"[3]. In particular he refers to the question of equity. Dr. I. M. D. Little, to whom we owe the rigorous proof of the necessity of assuming a fixed supply of work effort for the superiority proposition, concedes only that "the gainers could overcompensate the losers if direct taxes were substituted for indirect"[4]. Professor Milton Friedman points out, in addition, the special case that if we start from an initial situation of monopolistic deviation from a perfectly competitive equilibrium, an indirect tax might bring the economy back to an equality of the marginal rates of substitution[5]. However, he too accepts that when the initial position is one of "full competitive equilibrium", the conclusion about the superiority of direct taxation is valid.

The object of this note is to study the relative merits of direct and indirect taxation in the circumstances, and from the point of view, that are consider to be

* I have benefitted from the comments of Dr. Amiya Bagchi, Mrs. Ursula Hicks, Professor Harry Johnson, Mr. Stephen Marglin, and Professor James Meade.

[1] M. F. W. Joseph, *The Excess Burden of Indirect Taxation*, "Review of Economic Studies," June 1939; J. R. Hicks, *Value and Capital*, Oxford, 1939, p. 41; H. Wald, *The Classical Indictment of Indirect Taxation*, "Quarterly Journal of Economics", August 1945; George J. Stigler, *The Theory of Price*, New York 1946, pp. 81–2; A. Henderson, *The Case for Indirect Taxation*, "Economic Journal", December 1948.

[2] Richard A. Musgrave, *The Theory of Public Finance*, New York 1959, p. 155.

[3] Musgrave, p. 157.

[4] I. M. D. Little, *Direct vs. Indirect Taxes*, "Economic Journal", September 1951; also in *A Critique of Welfare Economics*, second edition, Oxford 1957, op. 295.

[5] Milton Friedman, *The Welfare Effects of an Income and Excise Tax*, "Journal of Political Economy", February 1952; also in *Essays in Positive Economics*, Chicago 1953, pp. 110–111.

most favorable for direct taxation. We shall assume perfect competition, fixed supplies of work-effort and savings, and we shall (reluctantly) close our eyes to considerations of equity. It appears that even under these conditions it is not clear that indirect taxation has an excess burden, and, on the contrary, it can be argued that direct taxation will not, except through an accident, achieve Paretian Optimality, whereas, given certain assumptions, a skillful use of indirect taxation is likely to ensure it. The crucial argument concerns the nature of government expenditure on which the whole question seems to depend.

II

We assume that there are m commodities $(x_1, x_2, \ldots, x_m)$ which the public buys in various amounts. Since we have left out equity considerations from the picture, it is not easy to imagine that the government will also demand these same commodities, since the main reason for the government buying these commodities will be to distribute them to the poorer section of the population. In the absence of equity considerations and the presence of perfect competition, it might seem best to leave each consumer to buy the amounts of $(x_1, \ldots, x_m)$ that he would like. However, there might be another group of commodities which the government would have to provide to the community for one reason or another. The government would have to build highways to allow the public to use motor cars, to build more cancer hospitals when the public smokes too much tobacco, and so on. We refer to these commodities provided by the government as $(y_1, y_2, \ldots, y_n)$. We assume that these commodities are of a permissive nature, i.e., they do not provide any satisfaction in themselves, but they are necessitated by other (utility-giving) consumption of the public. It is *not necessary* for proving our main contention either that the commodities x and y could be divided into water-tight compartments, or that commodities in the latter category are only "permissive," but these assumptions make the problem a good deal easier to handle.

We have the following system of production and consumption.

(I) *The Production Function*

When $(x_1, x_2, \ldots, x_m)$ stand for the amounts produced of the commodities in the first group, and $(y_1, y_2, \ldots, y_n)$ for the amounts produced from the second group, we have the following transformation function, with given supply of factors (including of capital and of work-effort):

$$P(x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n) = 0 \qquad \text{(I)}$$

(II) *The Private Income Function*

The income of the private sector is derived solely from selling the $(m+n)$ commodities. When I refers to total income, and $(p_1^x, p_2^x, \ldots, p_m^x)$ and $(p_1^y, p_2^y,$

..., p_n^y) to the prices (exclusive of indirect taxes), we have the income equation as:

$$I = \sum_1^m p_i^x \cdot x_i + \sum_1^n p_j^y \cdot y_j \tag{II}$$

(III) *The Budget Restriction*

The private sector's purchases are restricted to the total income *minus* direct taxation, D. The prices relevant are, however, those inclusive of indirect taxes, $t_1, t_2, \ldots, t_m$, on commodities $x_1, x_2, \ldots, x_m$, respectively.

$$I - D = \sum_1^m (p_i^x + t_i) \cdot x_i \tag{III}$$

(IV) *Consumers' Equilibrium*

The consumers try to maximize their satisfaction given by a utility function, $U(x_1, x_2, \ldots, x_m)$, within the budget constraint. We take the Lagrangean-multiplier expression:

$$H = H(x_1, x_2, \ldots, x_m) = U(x_1, x_2, \ldots, x_m)$$
$$+ \lambda[(I-D) - \sum_1^m (p_i^x + t_i) \cdot x_i]$$

where λ is an undetermined constant. The first necessary maximizing conditions are now given by:

$$\frac{\partial U}{\partial x_i} - \lambda \cdot (p_i^x + t_i) = 0, \qquad (i = 1, 2, \ldots, m) \tag{IV}$$

These provide us with m differential equations subject to the sufficiency conditions for maximization which can be similarly derived. We also have the constraint, which is the same as the budget-restriction equation III.

(V) *Producers' Equilibrium*

The producers try to maximize their income I (as long as the marginal rate of direct taxation is not greater than unity) within the restriction of the production function. Consider the Lagrangean-multiplier expression:

$$L = L(x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n)$$

$$= \sum_1^m p_i^x \cdot x_i + \sum_1^n p_i^y \cdot y_j + \lambda' \cdot P(x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n),$$

where λ' is an undetermined constant. The first necessary maximizing conditions are now given by:

$$p_i^x + \lambda' \cdot \frac{\partial P}{\partial x_i} = 0, \qquad (i = 1, 2, ..., m) \qquad \text{(V—A)}$$

$$p_j^y + \lambda' \cdot \frac{\partial P}{\partial y_j} = 0, \qquad (j = 1, 2, ..., n) \qquad \text{(V—B)}$$

These provide us with $(m+n)$ differential equations, subject again to the sufficien cy conditions for maximization, which can be derived from above. We also have the constraint, which is the same as the production function, equation I.

(VI) *Government Demand*

The amounts $(y_1, y_2, ..., y_n)$ of the commodities demanded by the government depends upon the amounts $(x_1, x_2, ..., x_m)$ of the commodities consumed by the people. In some cases it is an increasing function, e.g., more roads needed when there are more motor cars, or more hospitals when more tobacco is consumed; sometimes it is a decreasing function, e.g., less need for the National-Health-Service vitamin tablets when the public has a more balanced diet. The equations of both sorts are represented by:

$$y_j = Y_j(x_1, x_2, ..., x_m), \qquad (j = 1, 2, ..., n) \qquad \text{(VI)}$$

Needless to add that y_j need not *necessarily* depend on each of the amounts consumed, $x_1, x_2, ..., x_m$. We shall discuss this question further in the last section.

(VII) *Paretian Optimization*

Since commodities $(Y_1, Y_2, ..., Y_n)$ are only "permissive," the total social satisfaction depends on the utility function $U(x_1, x_2, ..., x_m)$. Ignoring external effects of production and of consumption, optimization requires the maximization of U within the constraint of the production function. Take the Lagrangean-multiplier expression:

$$\begin{aligned} M = M(x_1, x_2, ..., x_m) = U(x_1, x_2, ..., x_m) + \lambda'' P\{&x_1, x_2, \\ &..., x_m, Y_1(x_1, x_2, ..., x_m), \\ &Y_2(x_1, x_2, ..., x_m), ..., \\ &Y_n(x_1, x_2, ..., x_m)\}, \end{aligned}$$

where λ'' stands for an undetermined constant, and the values of $(y_1, y_2, ..., y_n)$ have been translated into functions of $(x_1, x_2, ..., x_m)$ in terms of equations VI.

The first necessary conditions for maximization are now given by:

$$\frac{\partial U}{\partial x_i} + \lambda'' \cdot \left[\frac{\partial P}{\partial x_i} + \sum_{1}^{j-n} \cdot \frac{\partial P}{\partial y_j} \cdot \frac{\partial y_j}{\partial x_i}\right] = 0,$$

$$(i = 1, 2, ..., m) \qquad \text{(VII)}$$

Subject to the sufficiency conditions for maximization, which can be derived from above, these give us m differential equations; we also have the constraint, i.e., the production function, equation I.

(VIII) *The Absolute Price Level*

Finally, we fix the price of any one of the commodities and make it the unit of account. This gives us a set of absolute prices from the set of relative ones.

$$p_1 = 1 \tag{VIII}$$

III

We now have the complete system of consumption, production, and exchange, along with optimization conditions. If all the equations are satisfied, the consumers and producers will be in equilibrium and their positions will also be optimized. In this system we have the following set of "unknowns": m each of x_i, p_i^x, and t_i; n each of y_j and p_j^y; and one each of I, D, λ, λ', and λ''. This makes a total of $(3m+2n+5)$. The number of equations in the system are: 1 equation of type I, 1 of type II, 1 of type III, m of type IV, $(m+n)$ of type V, n of type VI, m of type VII, and 1 of type VIII, which make a total of $(3m+2n+4)$. Thus the system is underdetermined with one degree of freedom[6]. This means that we can drop D, i.e., direct taxation, and still achieve optimization. Direct taxation, in this system, appears to be a totally redundant instrument of public policy. Indirect taxes $(t_1, t_2, \ldots, t_m)$ are, however, not redundant[7]. If we drop them we lose m unknowns without losing equations, and the system will be very much overdetermined. We can, of course, afford to lose one specific indirect tax, i.e., that on any one commodity, and adjust the rest to get the relative exchanges properly arranged, with direct taxation looking after the size of total collection to balance the budget restriction. But we cannot really drop indirect taxation as a weapon of public policy as we can drop direct taxation.

We should pause here for a minute. Have we not been overstating the case for indirect taxation, treating direct taxation as totally redundant? What about the need for balancing the budget of the government? That should, it might appear, give us another equation, and sure enough it will. But that equation will not be independent. The government budget equality can be exprcssed as:

$$\sum_1^m t_i \cdot x_i + D = \sum_1^n p_j \cdot y_j$$

[6] All this with the requirement independent and consistent equations. Further, we assume that the economic functions are "well-behaved" so that not only can we differentiate them, but they are also convex, and they give rise to a unique solution when the system is determined.

[7] Some of these might be negative, i.e., subsidies rather than taxes.

This can, however, be obtained by combining the equations II and III. This will remain so even when $D = 0$, which implies that even in the absence of direct taxes there will be no difficulty in this score.

All this might appear to be, at first glance, a trifle mystifying. Why should the government's budget be automatically balanced? One way of looking at it is to recognize that in an exchange equilibrium involving z participants, if the income and expenditure of $(z-1)$ participants are equal, those of the last must also be equal. Another way of looking at the same thing, which the Keynesians might find easier to appreciate, is that investment and government expenditure must equal the total savings, and that there is no investment in this system and the only savings are the results of taxation. Thus, government expenditure *must* equal taxation, whether it tries to balance its budget or not[8].

IV

We may now discuss a special case that explains why the superiority of direct taxation has often been asserted. If we assume that the values of $(y_1, y_2, \ldots, y_n)$ are fixed independently of $(x_1, x_2, \ldots, x_m)$, i.e., if the government demand pattern is completely independent of what the public consumes, we have to introduce the following changes. First of all, the n equations of type (VI) must be changed into,

$$y_j = \bar{y}_j, \quad (j = 1, 2, \ldots, n),$$

where $\bar{y}_j$ are given constants. (VI′)

Secondly, the optimization equations (VII) will be affected, and will become:

$$\frac{\partial U}{\partial x_i} + \lambda'' \cdot \frac{\partial P}{\partial x_i} = 0,$$

$$(i = 1, 2, \ldots, m) \qquad \text{(VII′)}$$

These will conflict with the combined equations (IV) and (V–A) for producers and consumers equilibrium. We have from (IV) and (V–A)

$$\frac{\partial U}{\partial x_i} - \lambda\left(-\lambda' \cdot \frac{\partial P}{\partial x_i} + t_i\right) = 0,$$

i.e.,

$$\frac{\partial U}{\partial x_i} + \lambda \cdot \lambda' \left(\frac{\partial P}{\partial x_i} - \frac{t_i}{\lambda'}\right) = 0$$

[8] It should be added that when private saving is introduced into the picture, we must also introduce private investment, if the government budget is to be balanced. The budget surplus (or deficit) must equal the excess (or short-fall) of private investment over private saving. To what extent the government's budget equation can be treated as another restriction, even when it *is* an independent equation, is not quite clear, for the government is capable of running on unbalanced budget.

If, however, we make t_i's equal to zero or proportional to p_i^x, we have $t_i = -\lambda' \cdot q \cdot \frac{\partial P}{\partial x_i}$, where q is a constant, so that the equations become:

$$\frac{\partial U}{\partial x_i} + \lambda \cdot \lambda'(1+q) \cdot \frac{\partial P}{\partial x_i} = 0 \tag{VII''}$$

By putting $\lambda'' = \lambda \cdot \lambda' (1+q)$, we translatc them into equations VII′. Thus, with the special assumption made in this section we arrive at the classical result that optimization requires either that there should be no inderect taxes, or that they should all be a fixed proportion of the corresponding prices. The latter represents a general sales tax and is not really very indirect. So, in this special case, an argument against selective excise is obtainable.

All these raise a question of considerable economic significance. What is the economically correct assumption for government spending? It seems that certain types of government spending, e.g., defense, will be virtually independent of private consumption, but many other types of government spending will be very heavily dependent. We have referred earlier to highways and cancer hospitals depending on the private consumption of motor cars and tobacco smoking. In general this dependence on private consumption will be present to a greater or less extent in the case of public works (e.g., bridges, water resource utilization, housing) as well as of health service (e.g., medicine, hospitalization). The same would apply to the administration of justice, e.g., gangster's buying too many revolvers might require the government to expand its police force. Since many of these government services are provided free, or at a nominal price, the question involved is very important.

We should say a few words on the nature of this interdependence to avoid a possible misunderstanding. The form of the equations in general, and our last example, in particular, might give the impression that what we are discussing is a case of external economy. The consumption relationship need not, however, be at all "external" in the usual sense of the term, i.e., it need not concern more than one person's consumption. Person A's consumption of vitamin tablets of the National Health Service will depend on his usual diet; his use of government hospitals will be affected by his day to day consumption; his use of public roads and bridges will depend on his having a motor car; and so on. Thus the relationship might be between different items in the consumption of the same person and need not necessarily involve any external relationship.

To come back to the question of the actual importance of the relationship, we have just seen that certain types of government spending will involve this while others will not. It can be further added that the more a country moves towards a welfare state, in the widest sense of the term, and the less the country faces a cold (or a hot) war, the more will be the importance of the relationship. But something more than these rather vague statements can be made in view of the form of the equations of optimization, i.e., equations VII. For having m independ equations of optimization not reducible to the equations IV and V (of consumers' and producers' equilibrium), we do *not* have to assume that each item of government spending depends on private consumption. Actually, what is necessary is that each item of private consumption should have *some* effect or other on public spending. In so far as certain item (x_r) does not, we can exclude the corresponding equation from the set VII, and will be spared the necessity of a corresponding indirect tax. But the fact that a number of items of government spending (e.g., defense) is unaffected by private consumption does not necessarily mean that we will not need to have an indirect tax on each and every item of private consumption. It will be necessary to have an indirect tax on each commodity purchased by the public whenever each item of (x_i) has some effect, no matter how many items of (y_j) are affected.

The essence of the argument developed in this paper is this. For a number of commodities that are bought by private consumers, an expansion of private purchase involves an expansion of the supply of complimentary goods provided by the government. Since most of these government goods are provided free, optimization requires that their costs should be reflected in the purchase price of private goods. This *can* be done, and can *only* be done, by indirect taxation. Thus, indirect taxes can lead to the optimum whereas direct taxation cannot. It is also of interest to find that indirect taxation can lead to the optimum without the help of direct taxation. This means that as an instrument of allocational policy direct taxation is quite redundant, and the argument for its use must be based on considerations excluded from this paper, which, interestingly enough, are the same as the considerations excluded from the usual proofs[9] of the superiority of *direct* taxation.

---

[9] See Musgrave, pp. 142–147.

Jati K. Sengupta[1] and Gerhard Tintner
U.S.A.

AN APPROACH TO A STOCHASTIC THEORY OF ECONOMIC DEVELOPMENT WITH APPLICATIONS

INTRODUCTION

Modern theoretical analysis of economic growth[2] by means of specific aggregative models, although very helpful in characterizing the processes of development in its different aspects, has been very restrictive from a basic standpoint. This is because the recent growth models neglect the influence of stochastic elements, with the exception of Haavelmo's theory[3]. It is not difficult, however, to show that in any realistic situation the economic variables, like population, demand and capacity variations, etc., which by their interactions specify the course of growth of an economy or some of its sectors, are probabilistic in nature. This is because the decisions underlying demand and capacity variations are made not in a world of complete certainty, but in one of imperfect knowledge and uncertainty. Insofar as this holds good, the analysis of the probabilistic aspects of economic growth acquires a crucial and important role. From purely formal and analytic standpoints this probabilistic approach to the analysis of economic growth serves to generalize the purely deterministic results derived from conventional growth models which neglect stochastic influences altogether. For instance, in the theory of stochastic processes it is known that for linear models restricted to very short periods, the solution of a deterministic model can, under fairly general conditions, be shown to be identical with the mean solution of the corresponding stochastic model[4]. However, such results do not hold for even very simple types of nonlinearities. This generalization has far reaching implications from an operational

[1] Part of the research work of this author has been supported by the United States National Science Foundation, Project No. 401-04-70 at the Department of Economics and Sociology, Iowa State University, Ames, Iowa.

[2] R. F. Harrod, *Towards a Dynamic Economics*, London 1948. E. D. Domar, *Essays in the of Economic Growth*, New York 1957. M. Kalecki, *Theory of Economic Theory Dynamics*, London 1954.

[3] T. Haavelmo, *A Study in the Theory of Economic Evolution*, Amsterdam 1954.

[4] A. T. Bharucha-Reid, *Elements of the Theory of Markoff Processes and Their Applications*, New York 1960.

standpoint, because when a growth model is applied for predictive or policy purposes, one can formulate alternative decision rules for specifying the optimal course of policy under various conditions of uncertainty.

The stochastic considerations may be introduced at various levels into the structural equations of a growth model, e.g., considering the observed set of coefficients as the expected values of corresponding stochastic coefficients or introducing equational errors as shocks to the otherwise deterministic model. But perhaps the most generalized specification would be to consider the basic variable (or variables) of a growth model to be subject to stochastic processes of different types. Let T denote a set of points on a time axis such that each point t in T an observation is made of a random variable $X(t,w)$ occupying a point w in the entire state space Ω, then a stochastic process is completely specified by the family of random variables $\{X(t,w),\ t \varepsilon T,\ w \varepsilon \Omega\}$. Apart from the question of inferring the probability law of the stochastic process from a set of observations (or realizations of the process $X(t,w)$ for fixed w) we may consider the set of alternative solutions of a growth model when the assumptions behind the underlying stochastic process are varied in different ways. This would be useful, not only from the standpoint of predictive power, but also from that of policy applications and optimality interpretations. We shall consider the process of economic growth in terms of stochastic birth and death processes under simplifying assumptions, regarding the type of stochastic processes and the number of independent variables determining the process of over-all growth. Apart from empirical applications, this will be followed by a discussion of a simple interdependent growth model, when probabilistic considerations are introduced at a simpler level through capacity and/or demand variations.

Stochastic Birth and Death Processes

In order to show the relevance and implications of stochastic processes in the aggregative theory of economic development, let us consider the simplest assumption that economic development is measured by a single variable $X(t)$ which may denote real national income over-all or *per capita*. This is here assumed to be a discontinuous variable. Let $p_x(t)$ denote the probability that $X(t)$ will have a given value x.

$$p_x(t) = \text{Prob}\ \{X(t) = x\} \text{ for } x = 0,\ 1,\ 2,\ \ldots \tag{1}$$

We now make the following assumptions about the stochastic process $\{X(t), t \varepsilon T\}$, i.e., about the possible changes in the value of $X(t)$ during a small time interval between t and $(t+\Delta t)$

(i) assumption about stationary independent increments which postulates the following condition:

(a) If at time t the system $\{X(t), t \varepsilon T\}$ takes the value $x = 1, 2, 3, \ldots$ the probability of transition from x to $(x+1)$ in the small interval $(t,$

$t+\Delta t$) is given by $\lambda \Delta t + 0(\Delta t)$, where the symbol $0(\Delta t)$ denotes a value of smaller order of magnitude than Δt.

(b) If a time t, the system $\{X(t),\ t \varepsilon T\}$ takes the value x, $x = 1, 2, 3, \ldots$ the probability of transition from $x \to (x-1)$ in the small interval $(t,\ t+\Delta t)$ is $\mu_x \Delta t + 0(\Delta t)$.

(c) The probability of transition between any two specific values (or states) of the system, x and $(x+s)$ is independent of the initial position.

(ii) The probability of no transition to a neighboring value (or state) is given by $1-(\lambda_x+\mu_x)\Delta t + 0(\Delta t)$.

(iii) The probability of a transition to a value other than a neighboring value is $0(\Delta t)$ which becomes negligible in the limit when Δt tends to zero.

(iv) The transition from $x = 0$ to $x = 1$ is not possible, i.e., in the terminology of Markov chains the value (or state) $x = 0$ is an absorbing state from which no exit is possible.

In view of these assumptions we can derive a recurrence relation for the transition probabilities as follows:

$$p_x(t+\Delta t) = \lambda_{x-1} p_{x-1}(t)\Delta t + [1-(\lambda_x+\mu_x)\Delta t] p_x(t) + \mu_{x+1} p_{x+1}(t) + 0(\Delta t). \tag{2}$$

Taking the limit $\Delta t \to 0$ this relation leads to the following difference-differential equation for the probability $p_x(t)$:

$$\frac{dp_x(t)}{dt} = \lambda_{x-1} p_{x-1}(t) - (\lambda_x+\mu_x) p_x(t) + \mu_{x+1} p_{x+1}(t). \tag{3}$$

The initial conditions governing equation (3) are specified by

$$p_x(0) = \delta_{xx_0} = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{othervise} \end{cases} \tag{4}$$

when the system $\{X(t),\ t \varepsilon T\}$ takes the value $x = x_0$, $0 < x_0 < \infty$ at time zero. Let us further extend the notation for transition probabilities as

$$p_{j,k}(t) = \text{Prob}\,[X(t+s) = k \,|\, X(s) = j] \tag{5}$$

so that $p_{j,k}(t)$ denotes the conditional probability of $\{X(t+s)\}$ taking a specific value k, given that $\{X(s)\}$ has been observed at time point s to take the value j. Then it can be shown[5] that the functions λ_x and μ_x given in equation (3) admit the following interpretation:

$$\begin{aligned} &\lim_{\Delta t \leftarrow 0} \frac{1}{\Delta t}[p_{x,x+1}(t+\Delta t)] = \lambda_x(t) \text{ for } x \geqslant 0 \\ &\lim_{\Delta t \to 0} \frac{1}{\Delta t}[p_{x,x-1}(t+\Delta t)] = \mu_x(t) \text{ for } x \geqslant 1 \\ &\lim_{\Delta t \to 0} \frac{1}{\Delta t}[1-p_{x,x}(t+\Delta t)] = \lambda_x(t)+\mu_x(t) \text{ for } x \geqslant 0 \end{aligned} \tag{6}$$

where we define for all $t \geqslant 0$ the plausible condition

$$\mu_0(t) = 0.$$

[5] E. Parzen, *Stochastic Processes*, San Francisco 1962.

Now it must be apparent from the system of equations (6) that $\lambda_x(t)$ and $\mu_x(t)$ specify the probabilities of transition from a specific value of national income x to $x+1$ and $x-1$, respectively. By making alternative assumptions about these arbitrary functions of time $\lambda_x(t)$ and $\mu_x(t)$, we may generate alternative probabilistic models of economic growth as follows:

(i) linear birth process type growth model: here we make the assumptions that $\mu_x(t) = 0$ and $\lambda_x(t) = \lambda_x$ for all t and x. In this case the solution of equation (3) is known to be

$$p_x(t) = \binom{x-1}{x-j} \exp(-j\lambda t)\,(1-\exp(-\lambda t))^{x-j} \text{ for } x > j \geqslant 1 \tag{3.1}$$

where j denotes the value of x at time zero. Since by appropriate choice of units one can take $j = 1$, one may write (3.1) more simply as

$$p_x(t) = \exp(-\lambda t)\,(1-\exp(-\lambda t))^{x-1} \text{ for } x = 1, 2, 3, \ldots, \infty. \tag{3.2}$$

where exp (m) is a notation for e^m. Denoting the expected value and variance of $\{X(t)\}$ by $M(t)$ and $V(t)$, respectively, we can compute their values from (3.1) as follows:

$$\begin{aligned} M(t) &= \sum_{x=0}^{\infty} x p_x(t) = j e^{\lambda t} \\ V(t) &= \sum_{x=0}^{\infty} \{x - M(t)\}^2 p_x(t) = j e^{\lambda t}(2 j e^{\lambda t} - 1). \end{aligned} \tag{3.3}$$

To consider the economic meaning of the proportional growth rate (or birth rate) λ in terms of economic models, we interpret λ as the product of the two structural coefficients of the Harrod–Domar type growth model, i.e., the marginal output-capital ratio (σ) and the saving coefficient (α). By writing $\lambda = \alpha\sigma$ we can specify the deterministic growth model of the Harrod–Domar type as

$$\begin{aligned} &\frac{dx}{dt} = \lambda x \\ &\text{Solution: } x(t) = j e^{\lambda t} \text{ where } j = x(t) \text{ at } t = 0. \end{aligned} \tag{3.4}$$

By comparing (3.3) and (3.4) two interesting results become readily apparent. In the first place, the mean value function of the stochastic model (3.1) is exactly identical with the solution of the deterministic growth model (3.4) of the Harrod–Domar type. Secondly, it is apparent that the linearity assumption for $\lambda_x = \lambda_x$ makes the two models applicable to very restrictive situations, i.e., short periods. For applications to long-run framework, however, we may still retain the linearity assumption by postulating that the entire long-run time scale T is divided into several subperiods or regimes, for each of which a linearized sequence of birth rates (or growth rates) may be a good approximation. Thus, the new assumption would be

$$\begin{gathered} \lambda_x = \lambda^{(k)} x(t) \text{ for } c_k \leqslant x(t) \leqslant c_{k+1} \\ (k = 1, 2, \ldots, K) \end{gathered} \tag{3.5}$$

where c_k and c_{k+1} are arbitrary constants such that the growth rate parameter $\lambda^{(k)}$ is a good approximation for the k–th regime when the total number of regimes is K. As soon as $x(t)$ moves outside the region defined in (3.5) we must make a new approximation, i.e., we must replace the constants $\lambda^{(k)}$ by new constants, for example $\lambda^{(k+1)}$ or $\lambda^{(k-1)}$, etc. However, it is not difficult to show that the idea of regime changes defined by (3.5) is implicit in the growth model formalized by Domar[6], since he makes a distinction between the actual (or observed) output-capital ratio (σ) and the potential maximum of output-capital ratios ($s = \sigma_{max}$), the gap between the two being caused by misdirection of investment or inoptimal decision-making under various conditions of uncertainty and lack of perfect knowledge.

(ii) linear birth and death process type growth model: here we make the assumption that $\mu_x(t) = \mu x(t)$ and $\lambda_x(t) = \lambda x(t)$, i.e., the proportional rates of birth (λ) and death (μ) are constant parameters characterizing the stochastic distribution of national income over time. From an economic viewpoint the birth rate λ reflects the average increase in real national product (or income) resulting from an additional dose of real capital formation. By the same interpretation the death rate μ represents an average decrease in real national product (or income) resulting from any decrease in investment caused by the increased size of the total capital stock. Any misdirection of investment, nonoptimal depreciation policy or suboptimal capacity variations which result in a reduced proportional rate of growth of real national income may thus be subsumed under the rate μ. The formulas (3.1) through (3.4) now hold as before, except that the net birth rate ($\bar{\lambda}$) has to be defined as $\bar{\lambda} = \lambda - \mu$. Hence, we have to replace λ by $\bar{\lambda}$ in the above formulas and write the variance function as:

$$V(t) = (\lambda+\mu)(\lambda-\mu)^{-1} \exp\left(t(\lambda-\mu)\right)\{\exp\left(t(\lambda-\mu)\right)-1\}. \tag{3.3a}$$

Similarly one may define a set of changes of regime, both by conditions like (3.5) on the birth rate and similar conditions on the death rate parameter μ. The deterministic version of this model would now be equivalent to the Duesenberry type[7] of modification into the conventional Domar-type growth model. For instance, denoting the k-th regime values of the two parameters as $\lambda^{(k)}$ and $\mu^{(k)}$, $k = 1, 2, 3, \ldots, K$, it is easy to see that the mean value function $M^{(k)}(t)$ for the k-th regime may show a negatively proportional rate of growth or even no growth at all, if $\mu^{(k)} > \lambda^{(k)}$, or $\mu^{(k)} = \lambda^{(k)}$. Now if the concept of regime is identified with a succession of short periods into a long-run framework, the cyclical fluctuations due to the negative and positive discrepancy between $\{\lambda^{(k)} - \mu^{(k)}\}$ for different $k = 1, 2, \ldots, K$ may persist under different conditions characterizing the long-run trend of national

---

[6] E. D. Domar, *Capital Expansion, Rate of Growth and Employment*, "Econometrica", Vol. 14, pp. 137–147, 1946.

[7] J. Duesenberry, *Business Cycles and Economic Growth*, New York 1959.

income. For instance, if λ_0 and μ_0 be defined to be the average values of $\lambda^{(k)}$ and $\mu^{(k)}$ for all $k \varepsilon K$ (when the set K is finite), then the long-run trend of the mean national income $M(t)$ would follow a strictly increasing (decreasing) exponential time-path accordingly, as $\lambda_0 \gg \mu_0$, $(\mu_0 \gg \lambda_0)$ where the symbol $\gg$ is an abbreviation for 'sufficiently greater than'. (In other words this is used in the same sense as the dominant root of a characteristic equation of a matrix). If we make the further assumption that the subperiods classified by the sequence of regimes are equispaced and the set K is enumerably infinite, then even a mild restriction $\lambda_0 > \mu_0$ (or $\mu_0 > \lambda_0$) would ensure a positive (or negative) proportional rate of growth of the mean national income.

At this stage a relevant question of economic policy may be raised. Suppose on the basis of the above probabilistic growth model we found that the mean growth path is an increasing exponential function of time, then what sort of policy variables (i.e., instrument variables[8]) should be chosen so as to ensure that the actual growth path converges to the mean growth path. Such convergence may imply a type of stabilizing policy pursued by the national policy-maker (or the planning board). This question may be answered on two different levels. In the first place, we may consider the problem of estimating the parameter $\lambda = \alpha\sigma$ as a part of decision theory, which requires as a datum of the problem the specification of the loss for a given difference between the estimate and the true value of the unknown parameter. The optimality criterion in such a case is naturally the minimization of expected loss. Alternative linear decision rules can be formulated by using the minimum expected squared error and other criteria discussed elsewhere, in order to specify the optimal choice of the estimates of α and σ. Secondly, the parameter λ occurring in the mean value function $M(t)$ in (3.3) may be interpreted in such a way that the mean size of national income $je^{\lambda t}$ may vary from one subperiod (or regime) to another due to the variations of the output-capital coefficient σ. The mean value function $M(t)$ can now be written as (3.6) if λ is regarded as the observed value of a random variable Λ with distribution function $F(\lambda)$,

$$E[X(t)|\Lambda = \lambda] = M(t) = je^{\lambda t}. \tag{3.6}$$

This, of course, would lead to compound distributions[9] because in effect we are now assuming a mixture of two or more homogenous statistical populations over subperiods, each with the conditional probability

$$\text{Prob } [X(t) = x|\lambda = \Lambda] = p_x(t) \text{ in (3.2).}$$

Although this step is a slight generalization of our original assumptions, this permits an easier policy application, when the variations in λ can be associated with variations in the output coefficient σ over a given period. For instance, let us consider now the aggregative coefficient σ to be a weighted average $(q_1\sigma_1 + q_2\sigma_2)$

---

[8] J. Tinbergen, *Economic Policy, Principles and Design*, Amsterdam 1956.

[9] L. Takacs, *Stochastic Processes*, London 1960.

of the sectoral output-capital coefficients where the weights q_1, q_2 may denote for example, the proportion of total national investment allocated to two mutually exclusive sectors such that $q_1+q_2 = 1$. Now we may specify a quadratic loss function (L) similar to that used by Theil[10] and others

$$L = [X(t)-E\{X(t)|\lambda = \Lambda\}]^2 \tag{3.7}$$

and select the optimal set of instrument variables $q_1 = 1-q_2$ by minimizing the expected value of L in (3.7).

(iii) homogeneous linear growth model: in a strictly homogeneous model we make the assumption that $\lambda_x(t)$ and $\mu_x(t)$ do not depend on t for all x. A quasi-homogeneous model is defined by the conditions that at least one of the two parameters $\lambda_x(t)$, $\mu_x(t)$ does not depend on t for all values of x. For instance, with reference to economic models it may be plausible to assume

$$\lambda_x = ac \text{ and } \mu_x = cx \tag{4.0}$$

where a, c are positive constants independent of x, such that $\sum_{x=0}^{\infty} p_x(t) = 1$ i.e., the constants a, c are so normalized that $p_x(t)$ (or the probability that $\{x(t)\}$ will have a given value x at time t) represents an honest probability distribution. The assumptions (4.0) imply that the impact of productivity changes and technology for example, on the time rate of change of national product is a constant, independent of the size of the present national product, while the growth-retarding effects (e.g., misdirection or lack of realization of investment[11] or inoptimal capacity variations etc.) are proportional to the size of the present national product (through the retarding effects of the present volume of capital stock on current realized investment and hence, on national product). The implications of these assumptions may be further clarified by means of $x(t)$ which is now nonstochastic by definition:

$$\frac{dx}{dt} = ac-cx$$

$$\text{Solution: } x(t) = a-(a-j)e^{-et} \tag{4.1}$$

where $j = x(t)$ at $t = 0$ such that $j < a > 0$.

This solution indicates that national income (x) grows in an exponential fashion from the lower asymptotic initial level $x(0) = j$ to the upper asymptotic level given by $a = x(t)$ *at* $t = \infty$. The proportional rate of growth of national income $dx/(xdt)$ is, however, no longer a constant as in the model (3.4), but varies from $c(a-j)/j$ at $t = 0$ to zero at $t = \infty$.

---

[10] H. Theil, *Economic Forecasts and Policy*, Amsterdam 1958.

[11] R. Frisch, *A Reconsideration of Domar's Theory of Economic Growth*, "Econometrica", Vol. 29, pp. 406–413, 1961.

It may be easily shown that the stochastic analogue of the deterministic model (4.1) is given by the equations (3) and (4.0) combined. In this case the solution of the difference-differential equation (3) subject to (4.0) is given by the Poisson distribution

$$p_x(t) = e^{-h(t)}[h(t)]^x/x! \tag{4.1}$$

The mean value function $M(t)$ for this distribution is given by $h(t) = M(t)$ as follows

$$M(t) = E\{X(t),\ t \geqslant 0\} = \sum_{x=0}^{\infty} x p_x(t) = [a - be^{-ct}] = V(t) \tag{4.2}$$

where b is a positive constant of integration such that $b = a - j$, where $j = h(t)$ at $t - 0$ and $V(t)$ is the variance function for the stochastic process $\{X(t),\ t \geqslant 0\}$.

It is easily seen that the mean value function $M(t)$ given in (4.2) for the stochastic model is exactly identical with the solution of the deterministic model (4.1). But the stochastic model is much more general in at least three different respects. In the first place, if the stochastic process holds good, it enables us to estimate the confidence limit for the realization of a given value of national income which may be set as a target by a national policy-maker. Secondly, an estimating procedure-like maximum likelihood method, when applied to the stochastic model (3) and (4.0) would give consistent estimates for the parameters a, b and c, which are likely to contain more information than any direct estimates from the deterministic model (4.1) without the use of the corresponding stochastic model. Thirdly, it is easy to show that the stochastic model possesses a "long-run statistical distribution" since from (4.1)

$$\lim_{t \to \infty} p_x(t) = e^{-a}[a]^x/x! \tag{4.3}$$

hence, no matter what the initial unconditional probability distribution $p_x(0)$, the unconditional probability $p_x(t)$ defined in (4.1) always tends to a limiting probability (4.3) which is independent of t. This last condition implies that the probability distribution function is "infinitely divisible" .Hence, the stochastic model may be said to have a statistical equilibrium which corresponds to the economic equilibrium of the deterministic model. The idea of change of regime and consequent switch-over of the parameters a, b and c may again be applied here.

An immediate theoretical generalization of this linear stochastic process model would be to define λ_x and μ_x as:

$$\lambda_x = \lambda_0 + \lambda_1 x;\ \mu_x = \mu_0 + \mu_1 x \tag{4.4}$$

where λ_0, λ_1, μ_0, μ_1 are positive constants satisfying the regularity conditions[12] that $p_x(t)$ must fulfill the conditions of an honest probability distribution function.

---

[12] W. Feller, *An Introduction to Probability Theory and its Applications*, Vol. 1, New York 1950.

Combining equations (3) and (4.4) we get a difference-differential equation, whose solution is given as follows:

$$p_x(t) = \left(\frac{\lambda_0+\lambda_1 h(t)}{\lambda_0}\right)^{-\lambda_0/\lambda_1} \frac{\lambda_0(\lambda_0+\lambda_1)\ldots(\lambda_0+(x-1)\lambda_1)}{x!}\left[\frac{h(t)}{\lambda_0+\lambda_1 h(t)}\right]^x \tag{4.5}$$

where $h(t) = M(t) = E\{X(t)\} = a-be^{-ct}$.

From this equation (4.5) it is easy to show that by making different assumptions about the limiting value of the ratio $\lambda_3 = \lambda_0/\lambda_1$ we may generate different probability schemes. For instance, when $\lambda_3 \to \infty$ we get the Poisson type distribution given in (4.1), whereas if $\lambda_3 \to -1$ we get a geometric distribution with the probability distribution function as:

$$p_x(t) = \left(1+h(t)\right)^{-1}[h(t)/\left(1+h(t)\right)]^x;\ h(t) > 0 \tag{4.6}$$

where $h(t) = a = be^{-ct}$ as before.

Similarly other discrete distributions like the binomial, negative binomial, Pascal, etc., could be derived as special cases.

(iv) nonlinear stochastic growth model: a generalized form of a stochastic growth model, which has a very close relation with the deterministic form of a generalized logistic-type model of economic growth formalized by Haavelmo,[3] is obtained from the system (3), when we introduce nonlinearities in the birth (λ) and death rates (μ) as follows:

$$\lambda_x = \lambda x \text{ and } \lambda = \alpha(k_2-x)$$
$$\mu_x = \mu x \text{ and } \mu = \beta(x-k_1);\ k_1 < k_2 \tag{4.7}$$

where α, β, k_1 and k_2 are absolute constants such that $x(t)$ at $t = 0$ lies in the closed interval (k_1, k_2). A combination of (4.7) and (3) leads to a system of difference-differential equations, which has not yet been explicitly solved so that the explicit form of the probability function $p_x(t)$ is not known. However, it has been shown by Feller,[13] Kendall[14] and Bartlett[15] that the mean value function $M(t)$ in this case satisfies the following differential equation:

$$dM(t)/dt = (\alpha k_2+\beta k_1)M(t)-(\alpha+\beta)m_2(t) \tag{4.8}$$

where $m_2(t)$ is an unknown function representing the second moment about the origin for the process $\{X(t)\}$. Denoting the variance of the process $\{X(t)\}$ by $V(t)$ we can write (4.8) as

$$dM(t)/dt = (\alpha+\beta)\left[\frac{\alpha k_2+\beta k_1}{\alpha+\beta}M(t)-M^2(t)\right]-(\alpha+\beta)V(t). \tag{4.9}$$

---

[13] *On the Integro-differential Equations of Purely Discontinuous Markoff Processes*, Trans. Amer. Math. Society, Vol. 48, pp. 488 ff, 1940.

[14] D. G. Kendall, *Stochastic Processes and Population Growth*, "Journal of Royal Stat. Society", Series B, Vol. 11, pp. 230–264, 1949.

[15] M. S. Bartlett, *Some Evolutionary Stochastic Processes*, "Journ. of Royal Stat. Society", Series B, Vol. 11, pp. 211 ff, 1949.

Now the deterministic analogue of this model (4.8) can be written in terms of the nonstochastic variable $x(t)$ representing national income total or *per capita* as follows:

$$dx(t)/dt = (\alpha+\beta)\left[\frac{\alpha k_2+\beta k_1}{\alpha+\beta}x(t)-x^2(t)\right]. \tag{4.10}$$

By comparing (4.9) and (4.10) it is readily apparent that for a non-zero value of the variance function $V(t)$, the mean solution of the logistic stochastic model given in (4.9) would be less than the deterministic solution given in (4.10). This rather surprising result (originally discovered by Feller[16]) has two important consequences for the operational application ot such models for policy purposes. In the first place, when we linearize a nonlinear model (4.7) at k stages (or facets) by assuming

$$\lambda_x = \lambda^{(k)}x,\ \mu_x = \mu^{(k)}x;\ k = 1,\ 2,\ \ldots,\ K \tag{4.11}$$

as in the equation (3.5), where $\lambda^{(k)}$, $\mu^{(k)}$ are constants for the k-th stage. We may introduce a specification bias (or error) of a large order and hence, the linearly optimal decision rules for each linearized stage may not necessarily be the optimal for the complete nonlinear model. In particular, let us define for K linearized stages of (4.11) thc mean value function $M^{(k)}(t)$ for each k belonging to the finite subset K, which is necessarily equal to the solution $x^{(k)}(t)$ of the corresponding linearized deterministic model. Further, let $\overline{M}(t)$ and $\overline{x}(t)$ be the average values of $M^{(k)}(t)$ and $x^{(k)}(t)$ so defined above for $k\varepsilon K$ and let $\hat{M}(t)$ and $\hat{x}(t)$ denote the solutions of the complete nonlinearized stochastic model (4.9) and the deterministic model (4.10) respectively, then under certain broad regularity conditions one can state the following interesting result:

$$\overline{M}(t)<\hat{M}(t)<\hat{x}(t)>\overline{x}(t). \tag{14.2}$$

We are still investigating under what general class of nonlinearities one can state (4.12) as a general theorem.

$$\overline{M}(t)\leqslant\hat{M}(t)\leqslant\hat{x}(t)\geqslant\overline{x}(t). \tag{4.13}$$

The close relation of this result with the set of inequalities we derived for stochastic linear programming problems elsewhere[17] is very interesting and suggestive.

The second interesting result of this nonlinear stochastic process model is that it leads to an *inverse* problem of the following kind: given a deterministic model (4.10), what are the conditions under which a *similar* stochastic model may be formulated such that the mean value function for the latter is exactly identical with the solution of the deterministic model[18].

---

[16] W. Feller, *An Introduction to Probability Theory and its Applications*, Vol. 1, New York 1950.

[17] J. K. Sengupta, G. Tintner, and B. Morrison, *Stochastic Linear Programming with Applications to Economic Models*, (To be published in "Economica").

[18] Here similarity to the first order is meant. There may be several stochastic models with the same mean value function as the deterministic solution, but with different variances or different higher order moments.

For instance, it has been shown[19] that if the birth (λ) and death (μ) rates are defined to be linearly dependent on the inverse of the present size of national income x, i.e., then this nonlinear model has a mean value function

$$\lambda_x = \lambda x \text{ and } \lambda = \alpha\left[\frac{k_2}{x} - 1\right]$$
$$\mu_x = \mu x \text{ and } \mu = \beta\left[1 - \frac{k_1}{x}\right] \tag{4.14}$$
$$k_1 < k_2,\ 0 < k_1 \leqslant x \leqslant k_2$$

$M(t)$ which is identical with the solution of the following deterministic model defined in terms of the nonstochastic variable $x(t)$:

$$dx(t)/dt = (\alpha k_2 + \beta k_1) - (\alpha + \beta)x(t). \tag{4.15}$$

Since in economic growth models we have different kinds of nonlinearities, it would be a significantly new line of work to investigate the generality and applicability of the inversion hypothesis satisfied for example, by (4.14), which is sometimes referred to as the Prendiville process.

Extension of Theoretical Results

The theoretical results of the preceeding section may be extended in several ways of which some may be mentioned here. In the first place we may ask whether we can construct a continuous probability scheme $p_x(t)$ which has some reasonable justification from an economic viewpoint such that a stochastic birth and death process type growth model can be formulated along with a corresponding deterministic growth model. In order to indicate such a formulation we use a slightly altered notation and denote by $p(x, t)$ the probability that at time t the continuous stochastic process $\{X(t), t \geqslant 0\}$ takes the value x. We assume that $p(x, t)$ follows an exponential distribution for fixed t as:

$$p(x, t) = g(t)e^{-xg(t)} \quad 0 \leqslant x < \infty \tag{5.0}$$

where $g(t)$ is a certain function of time given by

$$g(t) = a + be^{-ct};\ a,\ b,\ c \text{ are all positive.} \tag{5.1}$$

The main justification for the assumption (5.0) is that the arithmetic mean of a large number of extreme values tends to have under fairly general conditions a limiting distribution which is exponential and the different sectoral components of real national income may be interpreted broadly as the optimal values resulting from optimizing decisions by producers and consumers. Since we have from (5.0) the following relations: $\partial p(x, t)/\partial x = -g^2(t)e^{-xg(t)}$

and

$$\int_x^\infty p(y, t)dy = e^{-xg(t)}$$

[19] M. Takashima, *A Note on Evolutionary Processes*, "Bull. of Math. Stat.", Vol. 7, pp. 18–24, 1956.

therefore it is easy to verify that the time derivative of $p(x, t)$ satisfies the following differential equation like the stochastic birth and death process.

$$dp(x, t)/dt = -cx(dp(x, t)/dx) - (c+acx)p(x, t) + ac\int_x^\infty p(y, t)dy \tag{5.2}$$

with the initial condition

$$p(x, 0) = (a+b)e^{-(a+b)x_0}.$$

This equation may be interpreted in the following way: cx is the probability of $X(t)$ decreasing from a value x to xdx in the small time interval t to $t+dt$; $(1-c-acx)$ is the probability of $X(t)$ not changing from its value x in the same time interval, whereas ac is the probability of a change from a value of y to $y+dy$ in the same time interval if $y > x$. It is easy to show that the characteristic function $\varphi_x(u)$ of our probability distribution is given by

$$\varphi_x(u) = \left[1 - \frac{iu}{g(t)}\right]^{-1}. \tag{5.3}$$

Hence the mean $M(t)$ and variance $V(t)$ are given by

$$\begin{aligned} M(t) &= E\{X(t),\ t \geqslant 0\} = (1/a)[1+(b/a)e^{-ct}]^{-1} \\ V(t) &= E\{X(t)-M(t)\}^2 = (1/a^2)[(b^2/a^2)e^{-2ct}+(2b/a)e^{-ct}]^{-1}. \end{aligned} \tag{5.4}$$

The deterministic form of the corresponding model whose solution is exactly identical with the mean value function $M(t)$ in (5.4) must observe a logistic growth-path over time. Since the various economic assumptions which may generate such a deterministic growth-path had been investigated by these authors elsewhere,[20] two points may be mentioned here, viz, that it represents a long-run trend so that for subperiods when $M(t) < (1/a)$ we have a fast rate of growth $\frac{d}{dt}\left(\frac{dM(t)}{dt}\right) > 0$ like the exponential trend and that the trend function $M(t)$ may have a discrete shift of its upper asymptote and other parameters a, b or c in (5.4) due to changes in technology and other exogeneous forces.

It is easy to generalize the stochastic process model (5.2) for multiple stochastic processes which are independent[21]. Let $X_1(t) = x_1, X_2(t) = x_2, \ldots, X_q(t) = x_q$ be such processes. The differential-integral equation (5.2) becomes

$$\frac{dp}{dt} = \sum_{k=1}^{q} -c_k x_k(\partial p/\partial x_k) - (c_k + a_k c_k x_k)p + ac\int_{x_k}^{\infty} pdy_k \tag{5.5}$$

---

[20] J. K. Sengupta and G. Tintner, *On Some Aspects of Trend in the Aggregative Models of Economic Growth*. (To be published in "Kyklos", 1963).

[21] J. K. Sengupta and G. Tintner, *A Stochastic Programming Interpretation of the Domar-type Growth Model*. (To be published in "Arthaniti", 1962).

with the solution

$$p = g_1(t)g_2(t)\ldots g_k(t)e - \sum_{k=1}^{q} x_k g_k(t)$$

$$\text{where } g_k(t) = a_k + b_k e^{-c_k t}$$

$p =$ joint probability distribution of $x_1, x_2, \ldots, x_q$.

If the economic variables $x_1, x_2, \ldots, x_q$ are not independent it is recommended to compute the principal components (or a sort of index numbers, based on $x_1, \ldots, x_q$) of the set and derive the cquation (5.5).

Lct us now consider a linear interdependent economic model of a very simple type where $x_1(t)$, $x_2(t)$ denote the net outputs of two mutually exclusive sectors such that the second sector produces additional capacity (i.e., capital goods) for expansion of output in the two sectors. The deterministic model is the following:

$$\begin{aligned} dx_1(t)/dt &= \alpha x_1(t) + f(x_1(t), x_2(t)) \\ dx_2(t)/dt &= \beta x_2(t) + g(x_1(t), x_2(t)) \\ f(x_1(t), x_2(t)) &= u x_2(t) \\ g(x_1(t), x_2(t) &= v\, x_2(t) \end{aligned} \tag{5.6}$$

where α, β, u, v are constants and the interaction effects on sectoral growth represented by $f(x_1(t), x_2(t))$ and $g(x_1(t), x_2(t))$ are selected such that the capacity producing sector is 'dominant' (i.e., in the terminology of stochastic process model corresponding to (5.6), the birth rates of the two sectors' outputs depend on the size of the second sector's output). The solution of the deterministic model (5.6) are easily found to be:

$$\begin{aligned} x_1(t) &= \frac{uC_0}{v+\beta-\alpha}[e(v+\beta)t] + D_0 e^{\alpha t} \\ x_2(t) &= C_0 e^{(v+\beta)t} \end{aligned} \tag{5.7}$$

$$H(t) = x_1(t)/x_2(t) = \left[\left\{1 + \frac{D_0(v+\beta-\alpha)}{uC_0} e^{-(v+\beta-\alpha)t}\right\}\Big/\left(\frac{v+\beta-\alpha}{u}\right)\right]$$

where the constants C_0 and D_0 are determined by the composition of national output at the initial time point $t = 0$ and $H(t)$ denotes the ratio of the output-mix. Now consider the stochastic model corresponding to (5.7). It is easy to show that the second sector output follows the rules of a linear birth and death process, the mean size of second sector output denoted by $M_2(t)$ and the variance denoted by $V_2(t)$ given by

$$M_2(t) = e^{(v+\beta)t}\ ;\ V_2(t) = \frac{v-\beta}{v+\beta} e^{(v+\beta)t}\{e^{(v+\beta)t} - 1\} \tag{5.8}$$

where it is assumed that $x_2(t) = 1$ for $t = 0$.

Similarly the mean $M_1(t)$ and variance $V_1(t)$ of the first sector output and the ratio $M_1(t)/M_2(t) = \bar{H}(t)$ are given by

$$M_1(t) = M_1(t) = \frac{u}{v+\beta-\alpha}\, e^{(v+\beta)t} - e^{\alpha_t}$$

$$V_1(t) = \frac{v(-\beta)u^2}{(v+\beta-\alpha)^2}\left\{\frac{e^2(v+\beta)t}{v+\beta} - \frac{e^{2\alpha_t}}{v+\beta-2\alpha} - \frac{2e(v+\beta+\alpha)t}{\alpha}\right\}$$

$$+ue^{(v-\beta)t}\left\{\frac{1}{v+\beta-\alpha} - \frac{2u(v-\beta)}{\alpha(v+\beta)(v+\beta-2\alpha)}\right\}\frac{ue^{\alpha_t}}{v+\beta-\alpha} \tag{5.9}$$

$$H(t) = \left[\left\{1 - \frac{v+\beta-\alpha}{u} - e^{(v+\beta-\alpha)t}\right\}\Big/\left(\frac{v+\beta-\alpha}{u}\right)\right]$$

where $x_1(0) = 1 = x_2(0)$ so that the constants C_0 and D_0 of the deterministic model (5.7) are appropriately satisfied. Interesting policy applications of the equations (5.9) corresponding to the stochastic model may be easily visualized. For instance, the parameters α, β, u and v may be shown to be certain functions of the decision variables q_1, q_2, say where q_1, q_2 may represent, for example, various proportions of new investment allocated to the two sectors. Then the optimal set of the decision variables may be determined by the condition that it maximizes a preference functional F defined as:

$$F = F\big(M_1(t), M_2(t), V_1(t), V_2(t)\big) \tag{5.10}$$

Secondly, the various cases of complementarity in growth of the two sectors may be shown by assuming different signs for the parameters α, β, u and v. For instance, if v is negative, then the rate of change of output $x_1(t)$ per unit time period is adversely affected by the level of $x_2(t)$, but in the long-run this lack of complementarity in growth of the two sectors would have very little effect on the growth trend in output for each sector so long as $\beta > v$ and $\alpha > 0$, $u > 0$. Other different cases may be easily analyzed.

This model can obviously be generalized to more than two sectors and to include the possibility of discrete shifts in the parameters over time, due to exogenous forces like technology, innovation, etc.

Empirical results

The empirical specification of some of the stochastic growth models with reference to economic data may be of some illustrative use. We shall consider here three different types of empirical specification. In the first place we consider that the coefficients of a linear growth model may be subject to a switch-over when the regime changes. For instance, consider the following simple stochastic difference equation

$$x_t = a^{(k)}x_{t-1} + u_{t-1} \tag{6.0}$$

where u_t is random and the parameter $a^{(k)}$ holds only for the regime defined by

$$c_k \leqslant x_{t-1} \leqslant c_{k+1}\,. \tag{6.1}$$

If x_{t-1} falls outside the interval (or regime) defined in (6.1) the parameter $a^{(k)}$ has to be replaced by another constants $a^{(k-1)}$ or $a^{(k+1)}$. The method for estimating the switch-over points c_k is an adaptation of the method of Quandt.[22] All possible values of c_k are studied and the one chosen which gives the maximum of the maximum likelihood estimates. The constants $a^{(k)}$ are computed by the method of maximum likelihood. In order to verify the theory we have used the data of W. Hoffmann[23] for the index of total industrial production excluding building (base year 1913) for the United Kingdom (1700–1939) as one variable x_t. We give, in the following table, the empirical results based on the assumption of a given number of regimes.

TABLE 1. *Estimates of the Switch-over Points for U. K. data* (1700–1939)

Number of regimes	Estimate of $a^{(k)}$	95 percent confidence limits		Correspon-ding year	Estimate of $c_{(k)}$
		Lower	Upper		
2	1.0302	1.0203	1.0402		
				1834	14.20
	1.0108	0.9930	1.0285		
3	1.0239	1.0065	1.0413		
				1791	4.13
	1.0299	1.0191	1.0407		
				1869	40.60
	1.0096	0.9872	1.0319		
4	1.0107	0.9897	1.0317		
				1777	2.47
	1.0267	1.0089	1.0446		
				1820	8.13
	1.0301	1.0170	1.0432		
				1869	40.60
	1.0096	0.9872	1.0319		

It should be noted that the confidence limits given in this table are only approximate, since only the asymptotic theory is available. Tests also show that the difference between the values of $a^{(k)}$ belonging to neighboring regimes are not significant, showing that the transition is a gradual one. On the other hand the variances of the random component u_t connected with different regimes are statistically different. Thus, one may perhaps conclude on the basis of the above approximate test procedure that the exponential trend resulting from the first order stochastic difference equation (6.0) seem to give a reasonably good short-term explanation of the trend of economic development.

Now we consider an application of a linear homogeneous stochastic growth model for which the probability distribution function has been given in (4.1). We

[22] R. Quandt, *The Estimation of Parameters of a Linear Regression System Obeying Two Separate Regimes,* "Journ. Am. Stat. Assoc.", Vol. 53, pp. 873 ff, 1958.
[23] W. G. Hoffmann, *British Industry (1700–1950)*, Oxford 1955.

have used the long-run annual index of total output for the United Kingdom (1700–1940) constructed by Hoffmann, where the latter index is defined as the sum total of the indices of output of consumer goods and producer goods industries with the base year 1913. Denoting the index of total output by x_t and taking yearly values for the whole period (1700–1940), a first order autoregressive equation fitted on the basis of least squares and its approximate solution turn out to be as follows:

$$x_t = \underset{(0.4157)}{2.096751} + \underset{(0.0112)}{0.997297}\, x_{t-1}$$

Solution:

$$x_t = 775.7125 - 772.8825\, e^{-0.0027t} \qquad \text{(approximately)}$$

where the standard errors are specified in respective parentheses. The over-all mean of x_t for the whole period is about 99.3731. Using this as a normalizing factor, such that $h(t)$ given in (4.6) is redefined as $h(t) = x_t/99.3731$ we obtain

$$h(t) = \text{Mean value } M(t) = 7.8061 - 7.7776\, e^{-0.0027t}$$

Hence the difference-differential equation characterizing the process of British economic development for this period becomes

$$dp_x(t)/dt = 0.0211\, p_{x-1}(t) - (0.0211 + 0.0027)\, p_x(t) + 0.0027(x+1)\, p_{x+1}(t). \tag{6.2}$$

This equation has the following interpretation: The probability of x changing from a given value x to $x-1$ in the interval t to $t+\Delta t$ is approximately 0.0211. The probability of x not changing in the same time interval is approximately $0.9789-0.0027x$ and the probability of x changing from x to $x+1$ in the same interval is about $0.0027x$.

By following the same method we may consider some subperiods of the overall period and the normalized index of total output now turns out to be as follows:

1846–1880	$M(t) = h(t) = 11.5898 - 11.0154\, e^{-0.0024t}$
1881–1908	$M(t) = 3.0976 - 2.3211\, e^{-0.0082t}$
1909–1940	$M(t) = 1.0233 - 0.0711\, e^{-0.2653t}$

The corresponding differential-difference equations for the probability $p_x(t)$ can be easily derived. Statistical tests are available[24] for the hypothesis that the underlying stochastic process is of the Poisson-type against the simple alternative that it is not.

From an analytic standpoint, however, it may be more interesting to characterize the process of economic development in terms of more than one variable e.g., capital stock, population and total national output. However, the time series on these three variables are highly serially and temporally interdependent and hence, the assumption of mutual independence underlying the solution of the multiple difference-differential equation (5.5) cannot be empirically justified. We may,

[24] M. S. Bartlett, *An Introduction to Stochastic Processes*, Cambridge 1961.

however, apply the method of principal components. As an illustration we consider the time-series on population (x_{1t}), real value of fixed capital at 1913 prices (x_{2t}) and the index of total output at 1913 prices for the United Kingdom over the period 1870–1940. Since the figures of capital stock constructed by Cairncross[25] are not available prior to 1870, we restrict ourselves to this period. The series x_{1t}, x_{2t}, x_{3t} along with the standardized series are presented in the Appendix.

On the basis of the biennial values for the three variables for the period 1870–1940 we compute the correlation

$$\begin{array}{cc} & \begin{array}{ccc} x_{1t} & x_{2t} & x_{3t} \end{array} \\ \begin{array}{c} x_{1t} \\ x_{2t} \\ x_{3t} \end{array} & \begin{bmatrix} 1.0000 & 0.9778 & 0.7312 \\ 0.9778 & 1.0000 & 0.9324 \\ 0.7312 & 0.9324 & 1.0000 \end{bmatrix} \end{array}$$

The system of linear equations which gives the coefficients of the first and the largest principal component is given by

$$\begin{aligned} 1.0000\,k_{11}+0.9778\,k_{21}+0.7312\,k_{31} &= \lambda\,k_{11} \\ 0.9778\,k_{11}+1.0000\,k_{21}+0.9324\,k_{31} &= \lambda\,k_{21} \\ 0.7312\,k_{11}+0.9324\,k_{21}+1.0000\,k_{31} &= \lambda\,k_{31} \end{aligned} \tag{6.3}$$

for which the three characteristic roots of λ are $\lambda_1 = 2.5657$, $\lambda_2 = 0.3525$ and $\lambda_3 = 0.0818$. Since the total variance of the three standardized variables Z_{1t}, Z_{2t}, Z_{3t} is evidently 3, the first principal component explains about 85.5 percent, the second about 11.8 percent and the third about 2.7 percent of the total variance. Taking the largest principal component[26] (λ_1) we compute from (6.3) its unit normal eigen vector as

$$\begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \end{bmatrix} = \begin{bmatrix} 0.614647 \\ 0.649990 \\ 0.446902 \end{bmatrix}$$

Imposing the condition $\sum_{i=1}^{3} k_{i1}^2 = \lambda_1$ we get the standardized set of coefficients as $k_{11}^* = 0.984531$, $k_{21}^* = 1.041143$, $k_{31}^* = 0.715840$. Similarly the standardized eigen vectors corresponding to second (λ_2) and third (λ_3) principal components are obtained. Then we express the standardized variates $Z_{it} = (x_{it}-\bar{x}_i)/\sigma_i$, ($i = 1, 2, 3$) in terms of the principal components u_{it}, u_{2t}, u_3 as

$$\begin{aligned} Z_{1t} &= 0.984531\,u_{1t}-0.249248\,u_{2t}-0.228515\,u_{3t} \\ Z_{2t} &= 1.041143\,u_{1t}+0.063501\,u_{2t}+0.134201\,u_{3t} \\ Z_{3t} &= 0.715840\,u_{1t}+0.217304\,u_{2t}+0.107493\,u_{3s}. \end{aligned} \tag{6.4}$$

[25] A. K. Cairncross, *Home and Foreign Investment*, Cambridge 1953.

[26] G. Tintner, *Econometrics*, New York 1952, pp. 102–114. H. Hotelling, *Analysis of a Complex of Statistical Variables into Principal Components*, "Journal of Educ. Psyhology", Vol. 24, pp. 417 ff, 1933.

We obtain the principal components by inversion of the nonsingular coefficient matrix in (6.4) as

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} = \begin{bmatrix} 0.376325 & 0.385237 & 0.319074 \\ 0.267029 & -4.539113 & 6.234596 \\ -3.045979 & 6.610685 & -5.425522 \end{bmatrix} \begin{bmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \end{bmatrix}.$$

Now we fit a first order autoregressive equation to each of the principal components u_{it}, $(i = 1, 2, 3)$, the solutions of which turn out to be as follows:

$$u_{1t} = 8.950987 - 14.2860\, e^{-0.032971t}$$
$$u_{2t} = -0.455001 - 1.249905\, e^{-0.249901t}$$
$$u_{3t} = -2.168802 + 15.701004\, e^{-0.181828t}$$

The differential-difference equation (6.2) corresponding to the first principal component (u_1) now reduces to

$$dp_{u_1}(t)dt = 0.295123\, p_{u_1-1}(t) - (0.295123 + 0.032971\, u_1)\, p_{u_1}(t) + 0.032971\, (u_1+1)\, p_{u_1+1}(t).$$

The other principal components, which are mutually orthogonal by construction, may be similarly used to compute the multiple differential-difference equation of a form similar to (5.5).

An alternative approach[27] in this case would be to define an interdependent deterministic model between the three variables as in (5.7) and construct a corresponding stochastic model. Then on the basis of the mean value, variance function and other characteristics of the stochastic process for these three variables, we can derive various types of inference having useful policy implications.

Further Generalizations

At the estimation level we are investigating the possibility of obtaining improved and efficient estimates of the parameter in case of (5.0) with (5.1) and (4.1) with $h(t) = a - be^{-ct}$. Since the maximum likelihood equations are highly nonlinear in our case, we have used the iteration method of scoring[28], using the least square estimates of a, b, c as the first trial estimates. However, since this method requires the inversion of the information matrix, which in our case turns out to be very nearly singular, we are investigating the possibility of a maximum likelihood-like method of estimation, maybe by using some *a priori* estimates for some coefficients.

A second line of generalization would be to investigate various other types of stochastic models corresponding to different kinds of deterministic growth mod-

[27] G. Tintner, *A Stochastic Theory of Economic Development*. In H. Hegeland (ed.) *Money, Growth and Methodology*, Lund 1961, Sweden.

[28] C. R. Rao, *The Utilization of Multiple Measurements in Problems of Biological Classification* "Journal of Royal Stat. Society", Vol. 10, pp. 159–193, 1948.

els. In particular the theory of branching stochastic processes and compound nonlinear processes maybe further investigated, insofar as the generalization of the deterministic economic models is concerned. The different levels of policy implications of the generalized stochastic models of economic growth may also be investigated further.

APPENDIX

TABLE 2. *Population* (*in mill.*) *capital stock* (in £ 100 m. at 1913 prices) *and the index of total output* (base year 1913), *United Kingdom*

Year	Population		Capital Stock		Index of output	
	x_{1t}	z_{1t}	x_{2t}	z_{2t}	x_{3t}	z_{3t}
1870	22.45	—8.165	69.36	—3.785	88.3	—2.522
1872	23.04	—7.716	69.98	—3.655	96.6	—2.203
1874	23.69	—7.222	70.66	—3.513	101.9	—1.999
1876	24.34	—6.728	71.35	—3.368	101.3	—2.022
1878	24.99	—6.234	72.08	—3.215	98.7	—2.122
1880	25.64	—5.740	72.78	—3.068	110.2	—1.680
1882	26.27	—5.261	73.61	—2.894	114.4	—1.519
1884	26.57	—5.033	74.53	—2.701	121.8	—1.234
1886	27.17	—4.577	75.40	—2.519	114.4	—1.519
1888	27.77	—4.120	76.24	—2.343	120.7	—1.276
1890	28.37	—3.664	77.24	—2.133	134.1	—0.761
1892	29.35	—2.919	78.25	—1.922	130.7	—0.892
1894	30.05	—2.387	79.19	—1.725	127.4	—1.019
1896	30.75	—1.855	80.20	—1.513	138.6	—0.588
1898	31.45	—1.323	81.32	1.278	148.6	—0.203
1900	32.15	—0.791	82.51	—1.029	155.1	0.046
1902	32.88	—0.236	83.72	—0.775	153.5	—0.015
1904	33.58	0.296	84.97	—0.513	154.2	0.011
1906	34.28	0.829	86.27	—0.241	166.5	0.484
1908	34.98	1.361	87.68	0.054	173.4	0.750
1910	35.68	1.893	89.00	0.331	169.2	0.588
1912	36.30	2.364	90.44	0.633	185.6	1.219
1914	36.75	2.706	92.00	1.169	196.7	1.646
1916	37.20	3.048	93.47	1.268	178.9	0.961
1918	37.65	3.391	94.92	1.572	164.0	0.388
1920	38.10	3.733	96.34	1.869	176.8	0.880
1922	38.57	4.090	97.38	2.087	136.5	—0.669
1924	38.99	4.409	98.90	2.406	175.6	0.834
1926	39.41	4.729	100.16	2.670	180.5	1.023
1928	39.83	5.048	101.85	3.024	202.5	1.869
1930	40.25	5.367	103.61	3.393	208.2	2.088
1932	40.62	5.648	105.00	3.684	185.9	1.230
1934	40.94	5.892	106.68	4.036	202.1	1.853
1936	41.26	6.135	108.61	4.441	201.2	1.819
1938	41.58	6.378	110.51	4.839	209.8	2.149
1940	41.90	6.622	111.01	4.944	216.3	2.399

В. В. ШВЫРКОВ
СССР

КОЭФФИЦИЕНТЫ ЭЛАСТИЧНОСТИ СПРОСА И ПОТРЕБЛЕНИЯ

1. ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТА ЭЛАСТИЧНОСТИ СПРОСА И ПОТРЕБЛЕНИЯ. МЕТОДОЛОГИЯ ЕГО РАСЧЕТА

Под эластичностью спроса и потребления понимается относительное их изменение под действием определенных факторов. Такими факторами могут быть величина дохода, уровень цен, состав и размер семьи и т.д.

К. Маркс[1] предлагает потребности различать по степени их эластичности, т.е. изменчивости под влиянием вышеперечисленных факторов, и, главным образом, в зависимости от величины дохода. Одни обладают высокой степенью эластичности, изменения расходов на удовлетворение этих потребностей в зависимости от размера дохода значительны. Другие — мало эластичные, при увеличении дохода потребление этих продуктов изменяется незначительно.

К мало эластичным потребностям относятся наиболее настоятельные потребности человека, удовлетворение их происходит в первую очередь. Более эластичные потребности относятся к менее настоятельным потребностям и удовлетворение их происходит во вторую очередь. На такую закономерность в удовлетворении потребностей указывает С.Г. Струмилин в работе *Планирование в СССР*[2]. С.Г. Струмилин пишет, что в первую очередь происходит удовлетворение наиболее настоятельных потребностей, а уже потом — менее настоятельных. Следовательно, по мере уменьшения размера дохода потребление падает, в первую очередь за счет тех продуктов, потребность в которых менее остра, т.е. за счет товаров обладающих большей эластичностью.

Интенсивность изменения потребления под влиянием какого-либо фактора обычно измеряется коэффициентом эластичности. Коэффициент эластичности показывает, на сколько процентов изменится потребление, если причинный фактор увеличить на 1 процент. Эту словесную формулировку можно

[1] К. Маркс, *Капитал*, том III, 1955 г., стр. 196.
[2] С.Г. Струмилин, *Планирование в СССР*, 1957 г.

записать в виде математической формулы[3]. Для этого введем следующие обозначения:

x — доход семьи

Δx — приращение дохода семьи

y — расход семьи на удовлетворение какой-либо потребности

Δy — приращение расхода семьи на удовлетворение какой-либо потребности

Приведем пример расчета коэффициента эластичности по данным о фактических доходах и потреблении масла животного в 8 однородных семьях. Каждая семья состоит из двух взрослых и двух детей. Доходы и потребление рассчитаны на 1 потребительскую единицу.

Исходные данные (условные)[4] запишем в табличной форме (семьи расположены в порядке возрастания их доходности):

Порядковый номер семьи	Месячный доход (без налогов и прочих отчислений) в пересчете на 1 потр. един. (в руб.) x	Суточное потребление масла животного в пересчете на 1 потреб. единицу (в гр.) y
1	29,0	15,2
2	38,0	17,0
3	46,0	25,0
4	54,0	26,3
5	62,0	32,0
6	70,0	34,1
7	79,0	38,0
8	97,3	42,0
Итого	475,3	229,6
В среднем	59,41	28,7

Требуется определить, на сколько изменится потребление животного масла при увеличении дохода на 1%, или выражаясь статистически, — рассчитать эластичность потребления в зависимости от дохода.

Для этого из 8 семей выделим по две крайних — с низким и высоким уровнем дохода, — рассчитаем их средний доход и потребление и определим приращение дохода и потребления между этими крайними группами.

Рассчитаем процентное приращение дохода a, приняв доход семей низкой группы за 100.

$$\begin{aligned} x &- 100 \\ \Delta x &- a \end{aligned} \qquad a = \frac{\Delta x \cdot 100}{y} \qquad a = \frac{54 \cdot 7 \cdot 100}{33{,}5} = 163\%$$

[3] Более подробно см. в статье К. Отто, *Статистические исследования зависимости потребления продуктов питания и потребления товаров от изменения зарплаты и цен* в журн. „Экономическое обозрение", т. 12, № 4, отк. 1961 г. Токио (на японском языке).

[4] Данные заимствованы из работы *Статистика потребления* (учебное пособие). Изд. И-та Нар. Х-ва им. Г.В. Плеханова, Москва, 1962 г.

Затем рассчитаем процентное приращение потребления b по отношению к потреблению семей низкой группы.

$$\begin{matrix} y-100 \\ \Delta y-b \end{matrix} \qquad b=\frac{\Delta y\cdot 100}{y} \qquad b=\frac{23{,}9\cdot 100}{16{,}1}=148\%$$

Группы семей по доходу	Число семей f	Потребление масла животного	Доход	Группы дохода	Число семей f	Потребление масла животного y	Доход в руб. x
Семьи с низким доходом		y	x	Низкая	2	16,1	33,5
Семьи с высоким доходом		y_1	x_1	Высокая	2	40,0	88,2
Приращение		Δy	Δx	Приращение		23,9	54,7

Коэффициент эластичности (обозначим его символом Э) потребления от дохода вычислим как отношение между процентными изменениями потребления и дохода. Разумеется, связь между x и y в нашем примере предполагается линейной. Данное отношение выводится из следующей пропорции:

$$\left.\begin{matrix} a-b \\ 1-Э \end{matrix}\right| \quad \text{Следовательно} \quad Э=\frac{b}{a}=\frac{\Delta y\cdot 100}{y}:\frac{\Delta x\cdot 100}{x}=\frac{\Delta y\cdot x}{\Delta x\cdot y}$$

$$Э=\frac{\Delta yx}{\Delta xy} \qquad Э=\frac{23{,}9}{54{,}7}\times\frac{33{,}5}{16{,}1}=0{,}9$$

Коэффициент эластичности в нашем примере показывает, что при увеличении дохода на 1% потребление масла животного возрастает на 0,9%.

При незначительных приращениях дохода коэффициент эластичности потребления может быть рассчитан по следующей формуле: $Э=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}\cdot\frac{x}{y}$.

А так как $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ есть $\frac{dy}{dx}$

(первая производная), то общая формула для определения коэффициента эластичности примет следующий вид:

$$Э=y'_x\frac{x}{y_x}$$

Нетрудно догадаться, что частные формулы коэффициентов эластичности зависят от вида уравнения.

Рассчитаем коэффициенты эластичности потребления масла животного в зависимости от применений дохода для каждой из 8 семей. Связь между доходом и потреблением выразим уравнением прямой (данные условные).

Расчет коэффициентов эластичности потребления масла животного в зависимости от дохода

(доходы и потребление рассчитаны на 1 потребительскую единицу)

Семьи	Месячный доход (без налогов и прочих отчислений) в руб. x	Потребление масла животного в сутки в гр. (Эмпиричные данные) y	Потребление масла животного рассчитанное по уравнению $y_x = 3{,}87 + 0{,}418x$ (теоретические значения)	$\frac{x}{y_x}$	$Э = y'_x \frac{x}{y_x}$
1	29,0	15,2	15,99	1,81	0,76
2	38,0	17,0	19,75	1,91	0,80
3	46,0	25,0	23,10	1,99	0,83
4	54,0	26,3	26,44	2,04	0,85
5	62,0	32,0	29,79	2,08	0,87
6	70,0	34,1	33,13	2,11	0,88
7	79,0	38,0	36,89	2,14	0,89
8	97,3	42,0	44,54	2,19	0,91
Итого:	475,3	229,6	229,63		
В среднем	59,41	28,7	28,7	2,07	0,87

$y_x = 3{,}87 + 0{,}418x \qquad y'_x = 0{,}418$

Вычисленные коэффициенты эластичности показывают интенсивное увеличение масла животного с ростом дохода, нигде, однако, не превышая единицы. Последнее обстоятельство объясняется более быстрым ростом дохода по сравнению с потреблением. Для среднего дохода коэффициент эластичности равен

$$Э = y'_x \frac{\bar{x}}{\bar{y}_x} \qquad Э = 0{,}418 \quad \frac{49{,}41}{28{,}7} = 0{,}87$$

Исследуем изменение коэффициента эластичности потребления, рассчитанного для уравнения прямой. Известно, что уравнение прямой в системе координат принимает пять существенно отличающихся друг от друга положений, в зависимости от значений параметров. Вычислим для каждого такого случая коэффициенты эластичности:

а) при $a < 0 \qquad Э = y'_x \frac{x}{y_x} \qquad \frac{bx}{a+bx} > 1$

б) при $a = 0$ $\qquad Э = \frac{bx}{bx} = 1$

в) при $a > 0$ $\qquad Э = \frac{bx}{a+bx} < 1$

г) при $b = 0$ $\qquad Э = \frac{bx}{a+bx} = 0$

д) при $b < 0$ $\qquad Э = -b\frac{x}{y_x} < 0$

Известно, что любую криволинейную функцию можно заменить кусочно-линейной. Следовательно, коэффициенты эластичности для любой точки криволинейных уравнений можно определить, применяя уравнение прямой. Так, например, коэффициенты эластичности для параболы второго порядка ($y_x = a+bx+cx^2$) в зависимости от величины x могут принимать значения от $Э \gtreqless 0$.

Выше была рассмотрена методология вычисления коэффициента эластичности потребления в зависимости от дохода. Аналогично вычисляют коэффициенты эластичности для уравнений выражающих связь между любыми явлениями. Так, исчисляются коэффициенты эластичности качества потребления в зависимости от дохода, коэффициенты эластичности сезонных колебаний от уровня потребления и др. При расчете всех этих коэффициентов следует помнить, что они вычисляются по совокупностям с двумя меняющимися признаками. Такое условие (принцип гомогенности) необходимо для определения влияния на потребление одного какого-либо фактора при элиминировании всех остальных. В связи с этим, расчету коэффициента эластичности должен предшествовать отбор однородных семей и метод аналитической группировки бюджетных данных. Принцип отбора семей определяется целью исследования. Так, например, для анализа влияния дохода на потребление отбирают семьи одинакового размера и состава, но разные по уровню дохода. Для изучения влияния размера семьи на потребление надо отобрать семьи однородные по составу и с одинаковым уровнем материальной обеспеченности, но различающиеся по размеру. Для исследования влияния состава семьи на потребление отбираются семьи одинаковые по размеру и уровню материальной обеспеченности, но различного состава.

После того как подобраны однородные совокупности составляются аналитические группировки. Зависимость, выявленная аналитической группировкой, может быть уточнена методом простой регрессии. Для этого подбирают уравнение, достаточно хорошо выражающее связь между двумя признаками аналитической группировки. Только после этого вычисляется коэффициент эластичности для исследования реакции потребителя на изменение интересующего нас фактора.

2. ЭЛАСТИЧНОСТЬ ПОТРЕБЛЕНИЯ ОТ ДОХОДА

Настоящий раздел озаглавлен „эластичность потребления от дохода". В действительности же коэффициенты эластичности будут рассчитываться как в зависимости от дохода, так и от общей суммы расходов. Связь между этими двумя коэффициентами эластичности несложная: коэффициент эластичности потребления от дохода равен коэффициенту эластичности потребления от общей суммы расходов, помноженный на коэффициент эластичности общей суммы расходов от дохода:

$$\frac{\Delta y}{\Delta x}\cdot\frac{x}{y}=\frac{\Delta y}{\Delta z}\cdot\frac{z}{y}\cdot\frac{\Delta z}{\Delta x}\cdot\frac{x}{z}$$

где: y — потребление;
x — доход;
z — общий расход;
Δy, Δx, Δz — приращения потребления, дохода и общего расхода.

Для измерения влияния дохода на потребление вычисляется коэффициент эластичности потребления от дохода. Он показывает относительное изменение потребления или расхода в результате увеличения дохода на 1%.

Область практического применения коэффициента эластичности потребления широка и разнообразна. Так, например, при помощи коэффициента эластичности можно измерить степень удовлетворения потребностей, определить степень материальной обеспеченности семей разного состава и размера, статистически рассчитать нормы потребления продовольственных и промышленных товаров[5].

Рассмотрим рассчитанные на фактическом материале коэффициенты эластичности потребления от дохода.

Приведем коэффициенты эластичности потребления от дохода по некоторым предметам потребления и расходным статьям, рассчитанные по сгруппированным бюджетам рабочих-одиночек Москвы за 1960 г. (160 бюджетов).

Коэффициенты эластичности потребления и расходов от дохода

Виды расходов и потребления	
Хлеб пшеничный (кг)	0,47
картофель (кг)	0,58

[5] Применение коэффициента эластичности при изучении уровня жизни см. в работе В. Швыркова *Коэффициент эластичности потребления и его применение при изучении уровня жизни трудящихся* (в сбор. *Математическая статистика*, М. 1962). В данной работе для специальных экономических исследований предлагается коэффициент эластичности в абсолютных величинах. Он выводится из следующей пропорции: $\begin{matrix} y-100 \\ E-Э \end{matrix}$ | $E=\frac{y\cdot 3}{100}$

Этот коэффициент показывает приращение в абсолютных величинах при увеличении x на 1%.

Масло животное (кг)	0,65
Молоко, свежее, квашеное (кг)	0,54
Яйца (штук)	0,79
Мясо, птица, дичь (кг)	0,58
Говядина, телятина (кг)	1,05
Фрукты, ягоды свежие (кг)	0,44
Кондитерские изделия (кг)	0,52
Сахар (кг)	0,58
Расходы на продукты питания (руб.)	0,46
Покупка одежды, тканей, обуви (руб.)	1,22
Покупка промышленных товаров (руб.)	1,29
Расходы на жилищно-коммунальные услуги (руб.)	0,24

По размеру коэффициентов эластичности потребления от дохода можно судить о степени удовлетворения потребностей в тех товарах и услугах, рынок которыми насыщен достаточно полно. (Чем больше коэффициент эластичности, тем в меньшей степени удовлетворены потребности в данном виде товара). Так, например, величина коэффициента эластичности расходов на продукты питания ($Э = 0{,}46$) свидетельствует о том, что потребности рабочих в продуктах питания удовлетворены гораздо полнее чем в промышленных товарах ($Э = 1{,}29$).

Выше было сказано о необходимости самого строгого соблюдения принципа гомогенности при исследовании влияния дохода на потребление. Однако, иногда при расчетах коэффициента эластичности потребления от дохода этот принцип нарушается — расчеты ведутся по семьям разного состава и размера. В целях сохранения принципа гомогенности доходы и расходы семей рассчитываются на I потребительскую единицу (или на душу) и по этим данным вычисляются коэффициенты эластичности. Однако, однородность изучаемой совокупности с помощью таких расчетов полностью восстановить не удается[6]; влияние состава и размера семьи на потребление элиминируется только частично. Поэтому на практике в целях фундирования этого принципа при расчетах коэффициентов эластичности применяют метод отбора семей приблизительно одинакового размера. Семьи объединяются в более или менее однородные группы, например, от 1 потребительной единицы до 1,5 п.ед., от 1,5 п.ед. до 2,0 п.ед., от 2,0 п.ед. до 2,5 п.ед. и т.д.

Однако наиболее правильные расчеты коэффициентов эластичности могут быть получены лишь при подборе однородных типов семей. Общий же коэффициент эластичности потребления для всех семей рассчитывается как средневзвешенный показатель по отдельным типам семей.

---

[6] См. статью П.П. Маслова, *Некоторые пути использования коэффициента эластичности потребления* в сборнике статей *Применение математики в экономических исследованиях*, т. 2, М. 1961 г.

Труднее всего сохранить принцип гомогенности в расчетах по рядам динамики. Он нарушается даже в случае расчета коэффициентов эластичности потребления от дохода по семьям одинакового размера и состава. Это объясняется тем, что потребление семей в динамике изменяется не только под влиянием изменений дохода (который мы учитываем), но изменения цен, насыщенности рынка товарами, изменения потребительских привычек и т.д., влияние которых в расчетах мы не учитываем.

Приведем некоторые коэффициенты эластичности потребления от дохода, рассчитанные по рядам динамики за 1953–1960 г.г. Ряды динамики построены по 160 бюджетам рабочих-одиночек, г. Москвы. Выравнивание рядов динамики произведено по прямой.

Коэффициенты эластичности потребления от дохода

Виды расходов и потребления	Коэффициенты эластичности
Хлеб пшеничный (кг)	—1,17
Картофель (кг)	—0,48
Яйца (штук)	5,55
Говядина, телятина (кг)	2,80
Фрукты, ягоды свежие (кг)	4,58
Расходы на покупку продуктов питания (руб.)	1,94
Потребление продуктов питания (кг)	1,24
Расходы на покупку промышленных товаров (руб.)	2,82

Из таблицы видно, что с 1953 г. по 1960 г. под влиянием роста дохода сильно возрасло потребление мяса, яиц, молока и фруктов, потребление же хлеба и картофеля сократилось.

Хотя приведенные коэффициенты эластичности потребления рассчитаны только от дохода, однако они характеризуют изменение потребления под влиянием не только дохода, но и ряда других причин. Эти коэффициенты эластичности, сложившиеся под влиянием всей совокупности факторов, в отличие от коэффициентов эластичности, отражающих влияние одного какого-либо фактора (частичные) назовем условно полными коэффициентами эластичности.

3. ЭЛАСТИЧНОСТЬ КАЧЕСТВА ПОТРЕБЛЕНИЯ ОТ ДОХОДА

Известно, что хорошо обеспеченные семьи покупают продукты питания лучшего качества, по цене гораздо более высокой чем семьи малообеспеченные. Поэтому средняя цена покупки может служить косвенным показателем уровня благосостояния семьи. Влияние дохода на качество потребления продуктов питания измеряется коэффициентом эластичности, который показывает на сколько процентов увеличивается средняя цена покупки при увеличении дохода на один процент. Ниже приведены некоторые из рассчитанных нами коэффициенты эластичности качества потребления продуктов питания от дохода. $\left(Э = y'_x \frac{\bar{x}}{y_x}\right.$ где y — цена покупки, x — доход$\left.\right)$. Выравнивание сгруппированных 9800 бюджетов рабочих семей и одиночек (РСФСР в 1961 г.) произведено по уравнению прямой.

Наименование продуктов питания	Коэффициенты эластичности качества потребления от дохода
	куплено в госторговле
Мука пшеничная	0,12
Мясо, птица, дичь	0,53
Молоко, свежее, квашеное	0,48
Масло животное	0,13
Яйца	0,11
Рыба (кроме сельдей)	2,31
Варенье, джем, мед и т.п.	1,80

Коэффициенты эластичности качества потребления продуктов питания показывают, что с увеличением дохода в наибольшей степени возрастает средняя цена покупки тех продуктов питания, которые обладают значительными качественными различиями. Рост дохода вызывает обязательное улучшение качества потребляемых продуктов питания. В меньшей степени меняется средняя цена покупки питания первой необходимости (хлеба, муки и др.)

Заканчивая рассмотрение коэффициента эластичности качества потребления продуктов питания от дохода, нельзя не остановиться на связи этого показателя с коэффициентами эластичности расходов, доходов и коэффициентов эластичности количества потребления от дохода.

Зависимость между ними такова: коэффициент расходов = коэффициенту эластичности качества + коэффициент эластичности качества.

Алгебраически это записывается в виде следующей формулы:

$$\frac{\Delta y}{\Delta x} \cdot \frac{x}{y} = \frac{\Delta p}{\Delta x} \cdot \frac{x}{p} + \frac{\Delta q}{\Delta x} \cdot \frac{x}{q} \qquad \text{где}$$

p — цена
q — количество
y — расход
x — доход

Поскольку коэффициент эластичности качества от дохода, как правило, имеет положительное значение, то коэффициент эластичности расходов, как правило выше коэффициента эластичности количества[7].

4. ЭЛАСТИЧНОСТЬ СЕЗОННЫХ КОЛЕБАНИЙ ОТ УРОВНЯ ПОТРЕБЛЕНИЯ

Сезонные волны потребления формируются под влиянием таких факторов как климатический, физиологический, бытовой и экономический. Первые три фактора носят устойчивый характер, в короткие промежутки времени они не изменяются. Последний, — экономический фактор (насыщенность рынка товарами, величина дохода и потребления и т.д.) претерпевает значительные изменения даже в пределах одного года. Исследования показали, что происходит сглаживание сезонных колебаний таких продуктов питания как масло, яйца, овощи, фрукты и другие с повышением уровня их потребления[8]. Этот процесс сглаживания может быть выражен коэффициентами эластичности сезонных колебаний от уровня потребления. Для этого внутригодичные колебания за ряд лет, вычисленные в процентах к уровню потребле-

Продукты питания	Кварталы[9]	Коэффициенты эластичности сезонных колебаний
Молоко (свежее и квашеное)	I	0,43
	II	−0,10
	III	−0,12
	IV	−0,39
Яйца	I	0,73
	II	−0,18
	III	0,18
	IV	0,64
Овощи и бахчевые	I	0,70
	II	1,10
	III	−0,29
	IV	0,18
Фрукты свежие	I	−0,31
	II	−0,12
	III	−0,44
	IV	0,03

[7] Вопросы вычисления коэффициентов эластичности качества потребления от дохода детально исследованы в монографии: S. J. Prais, H. S. Houthakker, *The Analysis of Family Budgets*, Cambridge, 1955.

[8] Вопросы народного хозяйства СССР, изд. АН СССР, М. 1962 г., стр. 368–387.

[9] Следует заметить, что квартальные изменения являются в высшей степени грубыми показателями сезонности. Однако, статистические данные не позволили нам произвести расчет по месяцам.

ния, выравниваются по уравнению простой регрессии в зависимости от уровня потребления. Далее, рассчитываются коэффициенты эластичности, которые показывают процентное изменение сезонных колебаний с повышением уровня потребления на 1%.

Приведем коэффициенты эластичности сезонных колебаний, вычисленные по 10 т. бюджетов рабочих семей РСФСР с 1952 г. по 1959 г. Выравнивание произведено по уравнению прямой.

Вычисленные коэффициенты эластичности показывают сглаживание сезонных колебаний с увеличением уровня потребления. Сглаживание происходит в результате увеличения потребления в несезонное время года и уменьшения — в сезон (увеличение и уменьшение относительно к уровню потребления).

5. ЭЛАСТИЧНОСТЬ СПРОСА ОТ ТОВАРООБОРОТА

Коэффициенты эластичности могут быть рассчитаны и по данным торговой статистики. В этом случае они называются коэффициентами эластичности спроса. П.П. Маслов предлагает рассчитывать коэффициенты эластичности спроса на отдельные группы товаров в зависимости от общего размера товарооборота. „... планируя общий размер товарооборота, — пишет профессор П.П. Маслов, — можно рассчитывать товарооборот по группам товаров на основе коэффициента эластичности, рассчитанного по функциональной модели, где независимой переменной будет товарооборот отдельных областей на душу населения"[10]. Функциональную модель автор предлагает строить в виде уравнения параболы второго порядка. Покупки различных товаров выравниваются от товарооборота по отдельным областям. Предложенный показатель эластичности спроса имеет свои положительные и отрицательные стороны. Положительным моментом является то, что товарооборот в отличие от дохода семьи является синтетическим показателем: он впитывает в себя элементы дохода и производства. Следовательно, рассчитанные коэффициенты спроса от товарооборота складываются под воздействием множества факторов.

Однако привлекательность этого показателя заключающаяся в его простоте, таит в себе и отрицательные элементы, не позволяющие использовать его для перспективных расчетов. Коэффициенты эластичности спроса являются полными, они характеризуют изменение спроса под влиянием товарооборота, объема производства, доходов, состава и размера семей, средних цен и т.д. При расчетах его становится невозможным сохранение принципа гомогенности, который единственно только может обеспечить необходимую

---

[10] П. Маслов, *Применимость коэффициентов эластичности в статистике и планировании товарооборота*, „Вестник статистики" № 10, 1961 г.

их точность. Поэтому нам представляется заманчивым использование коэффициента эластичности спроса только в текущем планировании, но для перспективных расчетов, очевидно, он не может быть применен.

6. ФАКТОРНЫЙ АНАЛИЗ ЗАКОНОМЕРНОСТЕЙ ПОТРЕБЛЕНИЯ ПРИ ПОМОЩИ КОЭФФИЦИЕНТОВ ЭЛАСТИЧНОСТИ

Частные коэффициенты эластичности. Для исследования влияния одного признака на другой (например, дохода на расход) необходимо, как выше было сказано, отобрать однородную совокупность бюджетов, в которой бы обследуемые семьи отличались друг от друга только двумя признаками. Однако, в исследованиях это не всегда удается. Так, например, при изучении небольшого числа бюджетов отбор однородных семей приводит к тому, что вся исследуемая совокупность разбивается на очень мелкие группы. Такие материалы не позволяют произвести необходимые исследования, рассчитать сколько-нибудь достоверные коэффициенты эластичности. В этих случаях прибегают к расчету частных коэффициентов эластичности по уравнению множественной регрессии.

Частные коэффициенты эластичности иногда называются чистыми. Они количественно (в процентах) характеризуют реакцию результативного явления на изменение одного из причинных факторов, влияние же других факторов закрепляется на постоянном уровне, т.е. элиминируется[11].

Приведем пример вычисления частных коэффициентов эластичности расходов в зависимости от дохода и размера семьи.

Мы располагали данными по 10 семьям, однородным по составу, но неодинаковым по доходу и размеру. Разбить данную совокупность на однородные группы для определения степени влияния дохода и размера семьи (раздельно) на потребление невозможно в силу малого числа выборки. Поэтому прибегаем к расчету частных коэффициентов эластичности, вычисленных по уравнению множественной регрессии.

Предположим, что расходы на одежду находятся в линейной зависимости от дохода и размера семьи:

$$y = a_0+a_1x_1+a_2x_2$$

где: y — расход семьи на одежду

x_1 — доход семьи

x_2 — размер семьи

Параметры данного уравнения были определены по способу наименьших квадратов:

$$y = -0{,}996+0{,}2633x_1-3{,}2870x_2$$

[11] См. нашу статью *К вопросу о нахождении некоторых исходных данных для составления оптимального плана производства по динамическим моделям потребления*. В сборнике трудов научной конференции МГУ, М. 1963 г.

Расчетная таблица для вычисления частных коэффициентов эластичности

Семьи	Месячный расход семьи на одежду (в руб.) y	Месячный доход семьи (в руб.) x_1	Размер семьи в потребительских единицах x_2	Теоретические значения расхода на одежду $y = -0{,}996 + 0{,}2633\,x_1 - 3{,}2870\,x_2$	Частная регрессия по „x" ${}_2y_1 = -0{,}996 + 0{,}2633\,x_1 - 3{,}2870 \cdot 2{,}29$	Частная регрессия по ${}_1y_2 = -0{,}996 + 0{,}2633 \cdot 130{,}671 - 3{,}2870\,x_2$
1	18,63	80,87	2,0	13,72	12,77	26,83
2	19,47	90,73	2,1	15,99	15,35	26,50
3	12,89	109,17	2,2	20,51	20,22	26,17
4	21,17	113,07	2,2	21,54	21,24	26,17
5	25,22	120,22	2,3	23,09	23,13	25,85
6	17,05	135,66	2,3	27,16	27,19	25,85
7	33,60	140,99	2,4	28,23	28,59	25,52
8	27,03	151,07	2,4	30,89	31,25	25,52
9	40,62	175,96	2,5	37,11	37,80	25,19
10	43,10	188,97	2,5	40,54	41,23	25,19
Итого:	258,78	1306,71	22,9	258,78	258,77	258,79
В среднем	25,878	130,671	2,29	25,877	25,878	25,879

Для определения влияния дохода семьи на расходы на одежду вычислим частный коэффициент эластичности расходов от доходов. Он рассчитывается по формуле: $Э = \frac{\partial y_{1 \cdot 2}}{\partial x_1} \cdot \frac{x_1}{{}_2y_1}$ где ${}_2y_1$ — частная регрессия по x_1. Символ „2" перед y говорит о том, что влияние данного фактора закреплено, т.е. не учитывается при расчете коэффициента эластичности. При расчете частного коэффициента эластичности расхода от размера семьи элиминируем влияние фактора (дохода). Этот показатель рассчитывается по формуле: $Э = \frac{\partial y_{1 \cdot 2}}{\partial x_2} \frac{x_2}{{}_1y_2}$ где ${}_1y_2$ частная регрессия по x_2 при элиминировании x_1.

Частные коэффициенты эластичности расходов на одежду для величин дохода и размера семьи могут быть рассчитаны по формулам:

$$Э = \frac{\partial y_{1 \cdot 2}}{\partial x_1} \cdot \frac{\overline{x}_1}{{}_2\overline{y}_1} \text{ и } Э = \frac{\partial y_{1 \cdot 2}}{\partial x_2} \cdot \frac{\overline{x}_2}{{}_1y_2}$$

Для нашего примера они будут равны (см. табл. на стр. 24).

$$Э \text{ от дохода} = a_1 \frac{\overline{x}_1}{{}_2y_1} \qquad Э = 0{,}2633 \cdot \frac{130{,}671}{25{,}877} = 1{,}33 \text{ и}$$

$$Э \text{ от размера семьи } a_3 \frac{\overline{x}_2}{{}_1y_2} = -3{,}2870 \cdot \frac{2{,}29}{25{,}879} = -0{,}291$$

Следовательно, при увеличении дохода на 1% расходы на одежду возрастают на 1,33%, при условии, что размер семьи постоянный и равен 2,29 потребительских единиц. А при увеличении размера семьи на 1% расходы на одежду уменьшаются на 0,291% при условии, что месячный доход семьи постоянный и равен 130,67 руб.

Полные коэффициенты эластичности. Для анализа закономерностей потребления, наряду с расчетами частных коэффициентов вычисляются полные коэффициенты эластичности как корректированные, так и некорректированные.

Эти коэффициенты по методике расчета несколько напоминают полные коэффициенты корреляции.

При помощи некорректированных полных коэффициентов эластичности устанавливается связь между функцией и одной из независимых переменных. Влияние других независимых переменных во внимание не принимается. Так, например, можно рассчитать полный некорректированный коэффициент эластичности между месячным расходом семьи на одежду y и месячным доходом семьи x_1, а также между месячным расходом семьи на одежду и размером семьи x_2. Вычисление полного коэффициента эластичности для средних значений y и x производится по формуле:

$$Э = y'_x \frac{\overline{x}}{y_x}$$

где:

$$y_x = a_1 + в_1 x_1$$

Расчетная таблица для вычисления (некорректированных) коэффициентов эластичности

1. От дохода

y	x_1	x_1^2	x_1y	$y_x=a+b_1x_1$	$Э=b_1\frac{x_1}{y_{x_1}}$
18,63	80,87	65,40	1507	13,53	1,48
19,47	90,73	8232	1767	15,97	1,41
12,89	109,17	11918	1407	20,54	1,32
21,17	113,07	12785	2394	21,51	1,30
25,22	120,22	14453	3032	23,28	1,28
17,05	135,66	18404	2313	27,11	1,24
33,60	140,99	19878	4737	28,44	1,23
27,03	151,07	22822	4083	30,94	1,21
40,62	175,96	30962	7147	37,11	1,18
43,10	188,97	35710	8145	40,33	1,16
258,78	1306,71	18170,4	36532	258,77	

$$a_1 = \frac{47021362}{1817040} = \frac{47736730}{1707491} = \frac{715369}{109549} = -6,5301$$

$$b_1 = \frac{365320-33810}{109549} = \frac{27170}{109549} = 0,2480$$

$$y_x = -6,5301+0,2480x_1$$

$$Э = b_1\frac{\overline{x}_1}{\overline{y}_{x_1}} \quad Э = 1,25$$

2. От размера семьи

y	x_2	x_2^2	x_2y	$y_x=a_2+b_2x_2$	$Э=-b_2\frac{x_2}{y_{x_2}}$
18,63	2,0	4,00	37,26	11,30	8,89
19,47	2,1	4,41	40,89	16,33	6,46
12,89	2,2	4,84	28,36	21,35	5,18
21,17	2,2	4,84	46,57	21,35	5,18
25,22	2,3	5,29	58,01	26,38	4,38
17,05	2,3	5,29	39,22	26,38	4,38
33,60	2,4	5,76	80,64	31,41	3,84
27,03	2,4	5,76	64,87	31,41	3,84
40,62	2,5	6,25	101,55	36,43	3,45
43,10	2,5	6,25	107,75	36,43	3,45
258,78	22,9	52,69	605,12	258,78	

$$a_2 = \frac{13635,1182-13857,248}{526,9-524,41} = -89,209$$

$$b_2 = \frac{6051,2-5926,062}{526,9-524,4} \quad \frac{125,138}{2,5} = 50,256$$

$$y_x = -89,209+50,256$$

$$Э = b_2\frac{\overline{x}}{\overline{y}x_2} \quad Э = 4,44$$

Следует заметить, что познавательное значение этого коэффициента не велико, так как потребление изменяется не только под влиянием учтенного фактора — дохода, но и неучтенного — размера семьи.

Полный коэффициент эластичности между y и x_2 вычисляется аналогично:

$$Э = y'_x \frac{\overline{x}_2}{\overline{y}_{x_2}}$$

$$\text{где: } y_x = a_2 + b_2 x_2$$

(см. табл. на стр. 25), пример условный[12]. Полный коэффициент эластичности расходов на одежду от дохода (1,25) меньше частного коэффициента эластичности (1,33). Это объясняется тем, что рост расходов на одежду с увеличением дохода замедляется с увеличением размера семьи.

Полный коэффициент эластичности расходов на одежду от размера семьи (4,45) значительно отличается от частного коэффициента эластичности (—0,291). Величина этого показателя преувеличивает влияние размера семьи на рост расходов на одежду в результате того, что рост размера семьи сопровождается одновременным увеличением ее дохода. Этим и объясняется столь большое различие сравниваемых показателей.

Гораздо большую ценность представляют полные корректированные коэффициенты эластичности. Для нашего примера такой коэффициент может быть рассчитан только в зависимости от дохода. Чтобы элиминировать влияние неучтенного фактора (размера семьи), данные о доходах и расходах делятся на размер семьи и все вычисления ведутся в расчете на душу. Затем, по полученным рядам вычисляется, как обычно, коэффициент эластичности Этот корректированный коэффициент эластичности также является полным так как влияние размера семьи на потребление полностью не элиминировано

Произведем расчет полного корректированного коэффициента эластичности расходов на одежду в зависимости от дохода, для этого составляем расчетную таблицу.

Полный корректированный коэффициент эластичности расходов от дохода (1,92) выше частного коэффициента эластичности (1,33), так как влияние фактора ,,дохода'' усиливается с уменьшением размера семьи.

Частные коэффициенты эластичности, вычисленные по рядам динамики. Частные коэффициенты эластичности незаменимы в факторном анализе динамических рядов потребления.

Влияние таких факторов как производство и цены на потребление нельзя изучить в чистом виде при помощи аналитических группировок. В этом случае необходимо применять множественную регрессию. Влияние же различных факторов на потребление можно рассчитать с помощью частных коэффициентов эластичности по уравнению множественной регрессии. Кроме того, частные коэффициенты эластичности применяются и при отборе неза-

[12] См. нашу статью в журнале ,,Вопросы экономики'' 1962 г., № 5.

Расчетная таблица для вычисления коррктированных коэффициентов эластичности расходов на одежду от дохода

Семьи	Месячный расход на одежду в пересчете на душу (в руб.) y	Месячный доход в пересчете на душу (в руб.) x	x^2	xy	$y_x = a+bx$	$Э = b\frac{x}{y_x}$
1	9,32	40,44	1635	377	5,09	3,01
2	9,27	43,20	1866	400	6,14	2,66
3	5,86	49,62	24,62	291	8,57	2,19
4	9,62	51,40	2642	494	9,24	2,11
5	10,97	52,27	2732	573	9,57	2,07
6	7,41	58,98	3479	437	12,11	1,94
7	14,00	58,75	3452	823	12,03	1,85
8	11,26	62,95	3963	709	13,62	1,75
9	16,25	70,38	4953	1290	16,43	1,62
10	17,24	75,29	5714	1303	18,40	1,56
Итого:	111,20	563,58	32898	6697	111,20	20,66

$$a = \frac{365258-3774295}{328980-317622} = \frac{-116037}{11358} = -10,2163 \qquad b = \frac{66970-62670}{428980-317622} = \frac{4300}{11358} = 0,3786$$

$$y_x = -10,2163+0,3786x$$

$$Э = b\frac{\overline{x}}{\overline{y}x} = 1,92$$

висимых переменных для включения их в уравнение множественной регрессии. Так, например, по средним бюджетным данным 1500 малообеспеченных рабочих семей г. Москвы с 1950 г. по 1960 г. было построено уравнение множественной регрессии и вычислены расходы семей рабочих на питание в зависимости от дохода, товарооборота, цен на продовольственные и промышленные товары, уравнение имело следующий вид:

$$\log y = -0,21157+0,73604 \log x_1+0,00411 \log x_2+0,51066 \log x_3-0,23196 \log x_4$$

Выпишем частные коэффициенты эластичности: 1) коэффициент эластичности от дохода $+0,74$; 2) коэффициент эластичности от товарооборота $+0,004$; 3) коэффициент эластичности от цен на продовольственные товары $+0,5$; 4) коэффициент эластичности от цен на промышленные товары $-0,23$.

Совершенно очевидно, что при составлении такого уравнения товарооборотом, как фактором, влияющим на потребление, можно в данном случае пренебречь (в силу его незначительной роли в формировании уровня потребления) и заменить другим фактором.

Таковы основные моменты применения коэффициентов эластичности спроса и потребления.

W. W. Shvirkow

COEFFICIENTS OF THE ELASTICITY OF DEMAND AND OF CONSUMPTION

Summary

The paper contains a brief outline of some selected problems concerning the techniques of measuring the demand and consumption elasticity.

At the beginning, the author deals with the problem of the nature of elasticity and gives the definition of the coefficient of consumption elasticity. The technique of computing this elasticity with respect to the income level is presented by a theoretical numerical example referring to incomes and to the consumption of animal fats.

Next, the author draws a distinction between the coefficient of the consumption elasticity with a respect to income level, and, with respect to the total level of expenditure. The author quotes coefficints of the consumption elasticity derived from 160 budgets of single workers in Moscow in 1960, and on this basis proceeds to a short analysis of the role these coefficients can play as measures of saturation of population needs. The next problem dealt with is that of analysing coefficients of elasticity computed from time series data.

The author proceeds to an analysis of the coefficient of income elasticity of the quality of consumption. The influence of income increase on the quality of food-stuffs consumption is measured by the percentage change of the purchase price in relation to the percentage change of income. The author draws our attention to the relation between the coefficient of the consumption elasticity, of coefficient of the elasticity of total expenditure and the coefficient of the elasticity of the quantity of consumption. Empirical data are presented and an analysis of the coefficients of the elasticity of quality of consumption is made, based on data derived from 9800 budgets of workers, families in the Soviet Union.

A further section of the paper is devoted to the problem of the coefficient of seasonal consumption variations and to the problem of the demand elasticity coefficient with respect to the total retail trade turnover. The latter coefficient is considered as the demand elasticity coefficient for a given group of commodities computed with respect to the total retail trade turnover.

The last problem dealt with by the author is that of the estimation of the consumption elasticity coefficients by multiple regression techniques. The paper is concluded with a detailed numerical analysis of the coefficients of elasticity with respect to income level and to size of the family.

V. B. SINGH
INDIA

INSTITUTIONAL ASPECTS OF ECONOMIC DEVELOPMENT

WARNING against the tendency to overestimate the role of the New Polish Economic Model, Professor Kalecki wrote: "It is easy to prove that the attainment of equilibrium between demand and supply is to a great extent dependent on factors 'outside the model' "[1]. A careful reader will notice that he refers *only* to economic factors outside the Model. Our contention is that Professor Kalecki's observation should be given a wider meaning to show that in the contemporary discussions on problems of economic development institutional factors are not being adequately discussed and it is sometimes on the *rock of institutions* that the ship of economic planning (or programming, whatever name may be given) fonders. We propose to discuss, in brief, this theme in the context of History of Economic Thought; and in relation to the contemporary Indian experience.

I. INSTITUTIONAL ASPECTS IN ECONOMIC THOUGHT[2]

When the classicists spoke of Political Economy, they clearly underlined the fact that 'economic' activities were to be discussed in the context of the social setting of man. This is why philosophy provided a background for economic discussions.

Classical economics itself has grown out, as is aptly pointed out by Professor Lange, "of the search of the industrial bourgeoisie for a way to achieve freedom of initiative, aiming at economic development, from the crippling fetters of feudal institutions and guild, as well as from mercantilist policies[3]". It was in opposition to some of the existing institutions, while within the framework of the others, that the Classical Political Economy advanced its analysis and prescription. The Mercantilist and feudal institutions were ruthlessly opposed and critisized; while the capitalist institutions of private property and *laissez faire* were advocated in the framing of the theories regarding the growth of the Wealth of Nations. The clas-

[1] *Polish Facts on File*, No. 1, April 1958, New Delhi.

[2] In the first part of this essay the writer has leaned on his *Theories of Economic Development*. which is being published by Asia Publishing House (P) Ltd., Bombay, London, New York (etc.).

[3] O. Lange, *Marxism and Bourgoisie Economics*, "Enquiry", No. 1, New Delhi, pp. 1–2.

sical illustration of the pin industry in the context of Smith's (1723–90) analysis of the division of labour, which for Smith, was the *prime mobile* of economic development is rooted in his appreciation of the existing institutions.

Again in the context of wages, the advantages of the employer in his attempt to lower wages due to the legal institutions prevailing at that time, which went against combining to raise wages; and favoured lowering the price of work: while the whole magistracy was on the side of the master[4]. Further, the entire analysis of the relation of wages and profit[5], and rent[6] was worked out by Smith on the basis of existing property relations as well as social institutions. Coming to the analysis of 'Productive and Unproductive Labour, or Accumulation of Capital', Smith's sense of institutional touch is again explicit in the description of habits of different classes of society in regard to the maintenance of unproductive labour —prodigality.

These instances reveal that Adam Smith's economic world is not a self-propelling mechanism isolated from the society in which it functions. Therefore, its conduct is subject to the nature of the socio-economic institutions which Smith interpreted in the light of the natural philosophy that he took from his teacher Francis Hutcheson. His faith in the material philosophy was *a priori*. He adopted empiricism to deepen it as well as verify the validity of his philosophy.

Although Ricardo (1772–1823) is better known for his rigorous methods of deduction and abstraction, it can be said with equal force that his generalizations were more or less the reflections of the existing institutional set up of the British society. Institutions like Poor Law, Corn Law and protective tariffs concerned Ricardo throughout his life. His theory of rent was at the same time an attack on the remnants of feudal institutions and the existing Corn Laws. His analysis of the distributive shares, which for him was the chief theme of Political Economy, was very well suited to the growing institution of private property in the means of production. He looked at the problem of economic development from the viewpoint of the existing institutions in the society. Some of them (like landlordism) were regarded as obstacles to progress while others (like use of machinery) were to be taken account of, while prescribing economic policy. Landed aristocracy was an anathema to him, protective tarriffs and Corn Law were checking the pace of development, the Law of Diminishing Returns was operating due to the absentee landlordism which prevented the use of higher technology. High price of corn, wchich was an obstacle in economic development, was a result of this tendency as well as of the restrictions on the import of corn. All these institutions were coming in the way of development and Ricardo revolted against them with the help of his theory. Like Smith or Marx, Ricardo did not have a systematic philosophy or sociology,

[4] A. Smith, *Wealth of Nations*, p. 74.
[5] *Ibid*, Chapter X, Part II, pp. 132 ff.
[6] *Ibid*, pp. 197–274.

but his political actions were informed, throughout by the spirit of Bentham and the older Mill; who had taken into account the existing institutional framework.

John Stuart Mill (1806–73), who thought of Principles of Political Economy as having their "Applications to social Philosophy", ventured to write his well-known treatise, as he considered the resurvey of Political Economy necessary as a result of the new development arising out of a political institution, namely Colonization[7]. This clearly shows that he did not consider such principles complete in themselves, which fail to keep pace with the changing institutions by reflecting them in their body. For him the way to economics lies through psychology, ethnology and sociology[8]. Mill's *Political Economy* glowed with a temperature optimism concerning the future, because he saw that economic institutions were malleable. He was pre-occupied with the role of institutions in economic change. He praises customs and institutions, prevailing in Indian agriculture, because they "make provision for joint action in the cases in which it is seen to be necessary"[9]. The malady he finds in the wretched nature of implements and processes of agriculture. The excellent discussion of property and communism, land tenure and the sphere of government action, are good examples of what we may call 'social economics'. It is indeed, the institutional framework of the capitalist system, that made him realize that socialism is a preferable way for the advaneement of society.

The polemics between the classicists and the historicists (who emphasized the relevance of the variation in time and place to economic analysis), provided the historical background for Karl Marx (1818–1883) to create a synthesis between the two schools: and he used deduction as well as historicism. His analysis is based on successive approximations and in his theoretical system the economic order is an integral part of the society. There is intermingling of politics, economics and history. This is a refined extension of the theoretical foundations laid by Smith. In his preface to *A Contribution to the Critique of Political Economy*, 1857, (the year which witnessed the publication of Darwin's *The Origin of Species*) he reaches the conclusion: "that legal relations as well as forms of state could neither be understood by themselves nor explained by the so-called general progress of the human mind, but that they are rooted deep in the material conditions of life... 'civic society'; the anatomy of that civic society is to be sought in political economy... The mode of production in material life determines the general character of the social, political and spiritual processes of life". The mode of production, for Marx, is the totality of economic organization and technology. From the sociological viewpoint, as indicated above, the mode of production is the foundation and the political (e.g. government) legal (e.g. property relations) and social (e.g. family) institutions are the super-structures: but this is not to deny the mutual impact

---

[7] See *Principles*, Preface.

[8] O. F. Bouke, *Development of Economics*, New York 1921, p. 134.

[9] J.S. Mill, *Principles*, p. 92.

of the two on each other. The importance of institutional factors in the Marxian system is clearly borne out by one single illustration—the necessary political changes must precede for changing a given economy. In his analysis of Wages (*Capital* Vol. I) and Rent (*Capital* Vol. III), Marx brings in factors like customs, habits and traditions. This shows how his economic analysis is interrelated to the socio-economic functions of institutions. The confluence of Economics and Sociology has been the source of Marxism as a social science.

Marx was followed by a troop of institutionalists of varying shades—like Thornstein Veblen, Werner Sombart and Sidney Webb—but because of political implications, the dominant fashion had been to isolate the economic analysis from its social setting. Reflecting on this aspect of the growth of economic analysis during the period 1870–1914, Schumpeter writes: "Nations remained amorphous agglomerations of individuals, social classes were not living and fighting entities, but were labels affixed to economic functions (or functional categories). Nor were the individuals themselves living and fighting beings; they continued to be mere clothlines on which to hang propositions of economic logic"[10].

This unfortunate tendency was temporarily checked by Alfred Marshall, (1842–1924) who considered Economics "a study of man in the ordinary business of life". This understanding led him to study stock and produce exchanges, insurance, corporate organization, land tenure, banking, labour-management relations etc. His analysis of savings is related to habits and customs, which are determined by social institutions: and he emphasizes the economic aspects of the caste system[11]. He tried to co-ordinate marginalism and institutionalism, and his approach to the influence of environments and group attitudes, is akin to that of J. S. Mill[12].

In general, there is a misconception that John Maynard Keynes' (1883–1946) theory is devoid of institutional background and is like carpenter's tool. Controverting this viewpoint Streeten (P.P.) invites attention to the following classical traditions in Keynes: (i) the classical liberal utilitarianism, (ii) the *harmony*; (iii) the advocay of the regulation of aggregates is qualified by the liberal tradition, namely, the removal of the specific factors on the free pursuits of the common good; and (iv) the problems of class conflicts and the distribution of economic power[13]. If one takes up broad problems of macro-economics, like the functioning of capitalism, he cannot but have to go deeper into social and political institutions. Keynes assigns a positive role to the state in the economic sphere, namely, to organize and socialize savings and investment (those opposed to this viewpoint are characterized by him as "a nineteenth century publicist" or "a contemporary American financier"[14].

---

[10] J. A. Schumpeter, *History of Economic Analysis*, pp. 886–87.

[11] A. Marshall, *Principles of Economics*, p. 203.

[12] J. A. Schumpeter, *op. cit.* p. 889.

[13] K. K. Kurihara (Ed.), *Post-Keynesian Economics*, London 1955, pp. 345–64.

[14] J. M. Keynes, *The General Theory*, p. 380.

This function of the state is in confirmity with the traditions of regulated free enterprise; and the state apparatus is supposed to be run by an "intellectual aristocracy"[15]. Therefore, without saying in so many words, Keynes propounds a theory of state capitalism as opposed to State Socialism—the political goal of the British Labour Party. Once the principle of state functioning in the economic sphere is accepted, it will have its own logical course. Its limits will be determined not by individuals, but by social movement. In any case, the state cannot enter and exit from the economic sphere according to the rythm of economic fluctuations (a short-period phenomenon), but it is bound to be a cumulative and continous governmental programme (a long-period phenomenon). True, Keynes wants the state to function within the general requirements of the capitalist system, but it is not his theory that, in practice, determines the function of the state.

Even in his "pure" economic analysis of interest and profit, Keynes assigns an *undue* role to psychological factors. By introducing the elements of "expectations" and "liquidity preference"; as the determinants of profit and interest, Keynes abstracts them from the realm of production and attributes them to psychic behaviour. "Explanation acquires explanatory value", says Schumpeter "only if we are made to understand *why* people except *what* they expect. Otherwise... expectation conceals problems instead of solving them"[16]. The same may be said about the 'liquidity preference'. Thus we see that Keynesianism implies, or assumes, an institutional framework.

J. A. Schumpeter (1883–1950) had a much more comprehensive theoretical framework than his contemporaries. He, like Marx, considers capitalism to be a specific stage in economic evolution—which has its growth and decay. Sweezy aptly suggests that Schumpeter's essay on *Social Classes* embodies his view on the theory of the origins of capitalism; the *Theory of Economic Development* and the *Business Cycles* are devoted to the analysis of the capitalist process; while *Capitalism, Socialism and Democracy* enjoys the same position *vis-a-vis* his views on the theory of the decay of capitalism. With Sweezy we may say that scope of the Schumpeterian system is akin to Marxian social science, but not Marxism as a whole[17].

In his *History of Economic Analysis* (chap. 2) Schumpeter clearly shows the integral relationship between Economics, Statistics, History and Economic Sociology. It is the institutional approach that prompts Schumpeter to remark that because of the dependence of the economic history on universal history: "it is not possible to explain *economic* change by previous *economic* conditions alone. For the economic state of a people does not emerge simply from the preceding economic conditions, but only from the preceding total situation"[18].

---

[15] R. F. Harrod, *The Life of J. M. Keynes*, p. 331.

[16] Quoted by J. M. Gillman, in on *Evaluation of John Maynord Keynes* "Science & Society", Vol. XIX No. 1, 1955, p. 121.

[17] P. M. Sweezy, in *Schumpeter: Social Scientist*, Edited by Seymour, E. Harriss, p. 121

[18] J. A. Schumpeter, *The History of Economic Development*, p. 58.

Exactly it was this approach that was adopted by the Indian Classical Economists[19] (Dadabhai Nairoji (1825–1917) Romesh Chandra Dutt (1848–1899) whose rise is synchronized with the growth of national movement against British colonialism in India; and the rise of a socialist movement abroad. The Indian classicists explained Indian poverty in terms of the alien rule. The political economy as an enquiry into the nature and causes of the Wealth of Nations was used to enquire into the nature and causes of Indian poverty. Ranade took the position that the process of development involved mutual interaction between psychological and institutional factors; and ascribed numerous functions to the state in solving the problems of Indian squalor and ignorance. For a variety of socio-economic reasons the tradition (of linking of economic problems with the socio-political movements), was given up, by and large, with the turn of the century. But under the impact of the national movement the consciousness roused by the sufferings of the war and the post-Independence efforts of the country to march along the path of economic development and the climate for the study of the growth problems generated by international competition between the two conflicting social systems, we have once again started looking at economic problems against their social setting[20]. This corresponds to the current international economic thinking on growth problems. Arthur Lewis, for example, emphasizes that economics should go behind the 'proximate causes' (i) the effort to economise, (ii) the increase of knowledge and (iii) its application and increase in the amount of capital and other resources per head of economic growth", to ask: why it is that they are found strongly operating in some societies, but not in others, or at same stage of history but less so in others... First, we must enquire which kinds of institutions are favourable to growth, and which are inimical to effort to innovation or to investment. Then we must move into the realms of beliefs and... valuations..."[21]. This statement fully goes in the traditions of economists' war against economic and noneconomic factors that have hampered economic development. Here we are inevitably reminded of Smith's onslaught on Mercantilism, Ricardo's opposition to landed aristocracy and Marx's advocacy of the abolition of the private ownership of the means of production. Even Lord Keynes, who stood to reform capitalism, did not hesitate to critisize the capitalist class for its "euthanasia of the cumulative oppressive power... to exploit scarcity value of capital"[22].

---

[19] See P. K. Gopal Krishna, *Development of Economic Ideas in India 1880–1950*, Delhi 1959. Dr. Radhakamal Mukerji's *The Foundation of Indian Economics* (1916) and the *Principles of Comperative Economics* (1921) occupy a foremost place in Indian institutionalism.

[20] D. R. Gadgil, *Pre-conditions of Economic Development*, "Indian Economic Review", Vol I, No. 1, Feb. 1952. See also present writer's *On Economic History* in Singh, V. B. (Ed.), *Economic History of India: 1857–1956*, Bombay 1962, pp. 13–19.

[21] W. A. Lewis, *The Theory of Economic Growth*, London 1955, p. 11.

[22] J. M. Keynes, *op. cit.* pp. 375–76.

II. THE INDIAN EXPERIENCE

In 1951, the Indian planning was started with the understanding that: "An underdeveloped economy is characterized by the co-existence, in greater or less degree of unutilized or underutilized manpower on the one hand of unexploited natural resources on the other. This state of affairs may be due to stagnancy of techniques or to the certain inhibiting socio-economic factors which prevent the more dynamic forces in the economy from asserting themselves. Corresponding to each stage of development, there tends to grow a certain economic and social stratification which is conducive to the conservation of the gains from the use of known techniques. Such stratification has a part to play in social progress. But, beyond a point, it hampers innovation and change, and its very strength becomes a source of weakness. For development to proceed further, a re-adaptation of social institutions and social relationship thus becomes necessary. In planning for a better economic order, the close interrelation between the technical and social aspects of the process of development has to be continually kept in view"[23].

As the process of Indian Planning is unfolding itself, a number of problems are coming to the fore, and their solutions have to be provided on theoretical, programmatical and practical levels. To our mind the most crucial problem today, after ten years of planning, seems to be the institutional[24] barriers to economic development; which may be discussed under Economic, Social and Political heads.

An underdeveloped economy, like that of India, may be divided into two main sectors. Sector A, the *capitalist* sector, which is *surplus* yielding, consists of organized factories, commercial and trading enterpries, plantations and organized farming. Here two qualifications are necessary. Firstly, it is to be remembered that some of the aforesaid capitalist enterprises are tiny and scattered. Secondly, there is a pocket, a crucial one, owned by foreign monopolies. Sector B, the *subsistence* sector, consists of the bulk of the peasantry, cottagers, retailers, traders and money lenders. This sector embraces bulk of the Indian population. Accumulation (the crux of economic development), is being hampered by certain institutional barriers in the aforesaid two sectors.

For a rapid economic development it is essential to reorganize sector *A* so as to make it yield the optimum rate of capital accumulation. Here one has to remember that sector *A* has developed under a colonial framework, and the organized industries still bear stamp of semi-feudal and colonial structure. For example, the industrial management has a pre-capitalist attitude towards workers. The industrial bureaucracy resembles to a semi-feudal hierarchy. The working class has yet not fully stabilized itself as permanent city-dwellers; and its link with the village is not

---

[23] Government of India Planning Commission, *The First Five-Year Plan.* p. 7.

[24] By institution is meant: "*established forms or conditions of procedure* characteristic of group activity. See R. M. Mac Iver, and C. H. Page, *Society*, London 1952, p. 15.

only a cause of high absenteeism (which fluctuates with agricultural operations) but is also responsible for accentuating low labour productivity. The working class dwellings are caste oriented. Radical reforms in all these sections are urgently needed.

The pocket, owned by foreign (mainly British) capital consists of exportable commodities (e.g. jute and plantations); or having an impact on foreign exchange (e.g. oil). One of the lessons of planned economies, is that through the instrumentality of nationalization, profits are ploughed back in the economy to achieve an accelerated rate of growth. Recently a section of the organized working class has started demanding the nationalization of banking, oil industry and export and import. This type of democratic movement should be related to the larger question: how and in how many years, do the Government propose to bring Schedule *B* (privately-owned) industries, within the meaning of the Industrial Policy Resolution, 1956, into Schedule *A* industries (state-owned)? The working of the nationalized sector in the country may not have been properly evaluated, but this is not to deny that with its nationalization, the life insurance business has not only increased but become a direct aid to economic development. Even this limited experience underlines the necessity of furthering of nationalization in *selected sectors*, with a view to breaking monopolistic advantages and leaving at the disposal of the state an increasing investible funds, to be used in a planned manner.

But mere nationalization is not enough. Its democratic content and economic efficiency are to be created through workers' participation in management, factory committees and increasing association of the trade unions with production, consumption and distribution. There is no reason why the trade unions should not be associated with the administration of social security schemes.

With regard to Sector *B*, the crucial problem is to transform it into a surplus yielding sector, without undergoing the painful experiences of capitalist development. Within the framework of Indian planning, co-operative institutions can, and must, replace the existing peasant economy, cottage industry, retail trade and rural credit. This alone appears to be the way to liberate the peasantry from the age old burden of exploitation in various ways; to enable the cottagers to have the benefits of power and modern technology; to enable the consumer to purchase more with given income, and insure the producers to increase income by eliminating the profitering practised by the middlemen; and finally liberate the peasantry, along with other sections of the rural poor, from the clutches of the userer. The institution of co-operativization and state trading of foodgrains will not only guarantee freedom to the self-employed in the primary and tertiary sectors, but will also increase their income and employment and thereby ensure a higher rate of accumulation.

The reorganization of Sector *A* and *B* involve transformation of archaic and exploitative economic institutions into those of dynamic and progressive ones. Without these fundamental institutional changes, neither can the prevailing economic *impasse* be ended, nor can the distance between developed and underdeveloped economies be reduced.

In the pre-Independence days it was quite fashionable for a section of British economists[25] (who dominated the scene) to ascribe Indian poverty, squalor, ignorance and disease to religious beliefs, caste and increasing population—dividing itself into joint family. This type of analysis—no doubt, helped the colonial administration, but it had nothing to do with scientific investigation. Social institutions like caste and the joint family, by reducing individual initiative and precluding unrestricted occupational mobility, have retarded economic development[26]. The persistence of the dowry system, in spite of legal ban, functions as an anti-accumulation institution. But these could not be the fundamental cause of a stagnant economy. One has to go deeper into the socio-economic content of these institutions as barriers (or aid) to economic development. A glance at the occupational-cum-caste statistics of India brings to light the fact that business leaders and owners of capital and land as well as high-ups in services and even trained captains of commerce and trade or overwhelming members of a given community, belonging to certain caste, and coming from specific regions. Against this occupational background if economic, social and political conflicts arise, it is a job of Indian planners to change the occupational pattern and remove the causes of conflict. Let us not forget that economic relationships are expressed through property rights and political organizations as well.

The Third Five-Year Plan rightly states that: "The socialist pattern of society provides a major line of advance in a developing economy, which is becoming increasingly complex, and in which there is a constant interplay of a variety of social-economic and other elements... It is true that the economic foundations must be well laid; if the social objectives are to be attained"[27]. The economic foundations of any progressive society, today, is to be laid on the public ownership and the control of the means of production. It is the planned efforts of a people to reconstruct the economy for an increasingly better standard of living that enables it not only to conquer hunger, disease and obscurantism; but this very victorious march "radically recreates man's intellectual and psychic structure".

This intellectual approach to Indian problem is generally blurred by the presence of an educational system, which is the creation of an alien rule with a view to recruiting administrative staff. This is not to deny either the limited good that the system has done; nor our efforts to give a democratic and dynamic content to the prevalent system. But the fact that matters is that our educational system, as a whole, is quite unsuited to planned productive efforts of the nation. The Indian Commission on University Education (presided over by Dr. Radhakrishnan—the

---

[25] See Anstey, Vera, *The Economic Development of India*, London 1949, pp. VIII, 52–54, 58.

[26] See T. N. Madan in *Social Organization* V. B. Singh, (Ed.), *op. cit.* pp. 59–84.

[27] *Report*, p. 18.

present President of the Indian Republic) favoured the view that education be combined with productive labour[28].

Looking at the problem functionally education has to provide a variety of skills ranging from unskilled labour to highly trained scientists. In a planned economy education has to fulfill a threefold task: (a) to supply skilled manpower—human capital (without which physical capital will be a waste), (b) to generate a climate for growth; and (c) to teach the cultivators and artisans simple and elementary skills which will yield a small surplus over subsistence—a tangible contribution to physical accumulation[29]. But education is not playing these roles in India because, the productive role of education and investment in human capital are not being fully appreciated by the planning and the administrative authorities. A complete overhauling of the educational system, with a view to gearing it to the needs of a developing economy, is urgently needed.

Planning, social or economic, (and more social than economic) is implemented through the instrumentality of a political institution, that is, government. Thus political independence, that gives rise to a free democratic government, is only a mean to economic, cultural and spiritual regeneration, as the Indian Pledge of Independence pointed out, it is a special characteristic of Indian National Movement, during the 'thirties and the forties', that it became a confluence of political and economic aspirations of resurgent India. Against this background it was natural for the Indian parties, fighting for complete Independence, with a clear vision of the shape of things to come, to unite together, to fight the *alien* rule and its Indian allies. But with the dawn of Independence the parties of the left seceded from Congress and came out with their independent policies and programmes. One of the impacts of the British Rule on Indian Constitution is the acceptance of parliamentary democracy, which, by definition, requires an opposition to the party in power. This theory has been accepted in India without any critical examination. In Britain the Labour and the Conservative parties fight the election with divergent socio-economic programes; and often the battle is on the question of nationalization. In India, however, the situation is quite different. All the parties of opposition (with the exceptions of the Jana Sangh and the Swatantra Party which are post-Independence parties) have declared their faith in socialism. In this context opposition to the present ruling party should only mean a left to the Congress opposition. This would imply that the parties of Indian socialism, other than the ruling party, should pose before the electorate the issue of Indian socialism and their solutions as opposed to the official solutions. Such issues may be the role of nationalization with its democratic content, the growth of the public sector, level of employment,

[28] In this connection the Commission quotes Marx to say: "Education of the future will in the case of every child over a certain age, combine productive labour with education ... and athletics ..., and merely as one of the methods of raising social production but as the only method of producing fully developed human being". 1. *Report*. pp. 41–42.

[29] See: J. Vaizy, *Economics of Education*, London 1962.

agrarian reforms and more particularly land redistribution and co-operative farming, working conditions for the industrial workers and the middle classes, cultural regeneration and the forms in which democratic institutions can be promoted along with economic development. Once such issues are raised and discussed, it will be found that there is a large area of agreement between the parties of Indian socialism. In fact, there will be only two broad Fronts—a Planning Front and an Anti-Planning Front. This vital need of the country is not being reflected through the traditional parliamentary form of government. There is opposition for the sake of opposition. The success of Indian Planning depends on the development of the Planning Front, may be, within the framework of parliamentary democracy.

A government is run by a hierarchy of officials, that is, bureaucracy. With the growth of socio-economic activities it is only natural that a bureaucratic set-up enlarges itself. Therefore, often a rapid programme of planning and expansion of bureaucracy simultaneously take place. In such a situation it is the task of national leadership to curb influences of bureaucracy in the formulation and implementation of planning; and by setting up *effective* indogeneous democratic institutions at various levels of planning—like village Panchayats, Workers Committees, Staff Councils, Co-operatives etc.

The present Indian leadership is a product of national liberation movement, which has deep democratic roots, and has formulated a positive programme of post-Independence national reconstruction. Such a leadership is an asset to any democratic movement that aims at a socio-economic transformation of the society.

Thus, economic development is just not the function of economic calculus: it is a complex socio-economic process involving a transformation of economic, political, social and administrative institutions in a desired direction, which becomes a socio-political decision backed by democratic movement. The success of such a transformation, as of any other social reconstruction, depends on a definite correlation of social forces expressed in class alliances, internally as well externally. In its theoretical framework, the Indian economic development seeks the alliance of the peasantry (along with other sections of rural poor), working class, middle classes, traders and manufacturers and the enlightend intelligentsia. There is a growing consciousness that without such a social dynamics the Indian planning[30] cannot achieve its goal of building a socialist pattern of society.

---

[30] For the evolution, working and achievements of Indian Planning see V. B. Singh, *Gospodarka Indii wczoraj i dziś* Warsaw 1961. *Indian Economy: Yesterday and Today*, New Delhi 1962.

JOSEF STEINDL
AUSTRIA

ON MATURITY IN CAPITALIST ECONOMIES

What has become of maturity?

In this paper I want to deal briefly with a few questions which readers of my "Maturity and Stagnation in American Capitalism" (Oxford 1952) have been inclined to ask. They refer to

(1) the contrast in the U.S. and West European employment situation between the period before and since the Second World War

(2) the weakening of the incentive to invest in advanced stages of capitalism, and

(3) the role of innovations in the secular trend movement.

From Keynes to Haavelmo

I shall argue, in relation to the first of the above points, that increased public spending has played a major role, both in U.S. and in Western Europe. There has not been more deficit spending since the war, it is true, even rather less than before. However, the increase in public spending financed out of taxation tended to raise output and employment. Following Haavelmo[1], we use a simple model to demonstrate this. Let consumption be a linear function of income, and assume that the taxation does not alter the distribution of incomes. In the absence of taxation national income will be

$$Y_0 = aY_0+B+I$$

$$Y_0 = \frac{B+I}{1-a}$$

where I is investment, a is the marginal propensity to consume and aY_0+B is consumption. Assume now that a proportion λ of income is taxed away and spent by the government. In addition the government incurs a deficit of the amount d. We have then for the new income Y

$$Y = a\,Y(1-\lambda)+B+I+\lambda Y+d$$

[1] *Multiplier Effects of a Balanced Budget*, "Econometrica" 1945.

where $\lambda Y+d$ is the government's spending on goods and services. From this we find

$$Y=\frac{Y_0}{1-\lambda}+\frac{d}{(1-\lambda)(1-a)}=\frac{Y_0}{1-\lambda}\left(1+\frac{\delta}{1-a}\right) \qquad \left(\delta=\frac{d}{Y_0}\right).$$

If the deficit spending is zero the original income Y_0 is raised by a multiplier $\frac{1}{1-\lambda}$ determined by the rate of taxation (we might call it the "tax multiplier"). This result is based on the assumption that the government's spending does not directly affect the consumers' real income, or rather, what they think their real income is (unlike the case of free medical service, for example, which directly affect consumers' real income). In U.S. the additional government spending was on arms; in Western Europe, mainly on public investment and arms. The above assumption therefore broadly holds. Another important assumption is that the taxation is "neutral" with regard to income distribution. The additional post-war taxation in U.S. and Europe was hardly regressive; in so far as it was progressive the effect on employment was greater than appears from the above simple model.

We shall now use the following figures to make it plausible that public spending was in fact a major factor in the achievement of high levels of employment after the war.

U.S. (in percent of national income)

	Government purchases of goods and services	Government deficit spending
average 1929 to 1937	16.4	2.6
average 1951 to 1961	24.3	0.4

U.K. (in percent of gross national product at factor cost)

	Public authorities current expenditure plus public investment	Deficit in public accounts
1938	18.8	4.2
average 1951–1958	27.2	0.8

Using the U.S. data to evaluate the algebraic formula given above we find, assuming, rather arbitrairly, a multiplier of 1.5

$$Y \text{ (pre-war)}=\frac{Y_0}{1-0.138}\left(1+\frac{0.031}{1-a}\right)=1.21\,Y_0$$

$$Y \text{ (post-war)}=\frac{Y_0}{1-0.239}\left(1+\frac{0.005}{1-a}\right)=1.32\,Y_0.$$

The net effect of public spending and taxation appears to have increased the national income by a factor of 1.32/1.21, i.e. by about 9 percent above the level it would have reached without the additional taxation.

If employment is taken to be proportionate to the spending, then, with a full time equivalent labour force of about 62 millions (including self-employed) in the period concerned, the 9 percent additional spending corresponds to $5\frac{1}{2}$ million employees. A *prima facie* case seems to be made out in favour of the assumption that public spending has played a major role in the reduction of unemployment.

We have, however, to consider also the secondary effect of the public spending. It increases the demand for the goods and services produced by the private sector, but, unlike private investment, it does not increase the productive capacity. Now this is flagrantly untrue in Western Europe where much of the public spending goes into investment in certain key industries (especially the production of energy); however, it remains true that the capacity of *private* industry is not increased. Therefore, the utilization of capacity in the private sector is increased as a consequence of an increased public spending-*cum*-taxation. (In fact, the investment in public utilities, by obviating bottlenecks, in many cases only ensures that the stimulating effect of the demand on the private sector leads to a smooth growth.) The effect of the increased utilization is a rise in private investment to a level permanently higher than it would have been without the public spending.

As the figures given earlier on show, the increase in public spending is important also in the case of Great Britain. The same could be shown for Sweden and Holland, and probably for France, if the pre-war data were not so inadequate in the case of this country.

Thus the pre-war technique of deficit-spending has been replaced since the war by the technique of increased public spending financed by taxation, but this has been practised on a much greater scale, and with correspondingly greater effect on employment. A full analysis would have to take account, of course, of the possible redistributive effects of the post-war taxation, and of the effects of the tax relief accorded to business in various forms as an incentive to invest.

The Role of Consumers' Credit

We start from a consideration of the special character of durable consumers' goods. The studies of consumers' behaviour have often treated spending on new durables in the same way as spending on non-durables, and related both to the current income. It would seem more logical to relate the depreciation of durables, or the stock of them, to the current income of the consumer. The standard of life, which is thought to depend on the income, is shown in equilibrium by the current spending on non-durables and by the stock of durables owned. When the income changes from a lower level to a higher one, however, the stock of durables will change only after a certain time, because the consumer has to save up until he can buy all the durables appropriate to his new higher standard of life (we exclude consumers' credit for the time being). We can imagine that, with a jump in income, the "depreciation" of a fictitious durable good is started, and at the end of the depreciation period this durable is actually bought (being afterwards depreciated in the same way, if the owner's standard of life remains unchanged). Thus, there will be a lag between income and consumption of durables corresponding to the life time of the durable good. (In a less abstract model the lag will be shorter, because people often reduce current consumption of non-durables in order to reduce the time of waiting for the purchase of a new durable good.)

It seems very plausible that this lag is the reason for a phenomenon described by Modigliani and Duesenberry, namely the discrepancy between short term and long term propensity to consume. If we take 10 year moving averages of income and consumption, then these figures will reflect the average spending on durables over a ten year period, corresponding more or less to the "depreciation" notion used above: the long term data will be a "true" propensity to consume in the sense that they take account of the spending on durables. In a regression of *annual* data of income and consumption, however, a different pattern will appear: the rise in incomes over a few years' boom will not be fully reflected in consumption in the same period, because much of the spending on durables can only materialize at a later date owing to the lag explained above. In a deep slump, on the other hand, the spending will never be fully adapted to the current income, because some-spending on durables will take place out of savings made in more prosperous years. Thus the short term propensity to consume will be lower than the "true" propensity to consume; part of what appears as saving in the short run is in fact only saving for durable goods.

It is evident that the introduction and wide acceptance of consumers' credit will very much reduce the lag between income and consumption of durables. (The lag will probably not disappear entirely, even if nobody is excluded from or refuses the credit facilities, because the permanence of an increase in income may be doubted by the consumer until the increase has persisted for some time.) If the preceding explanations are correct, the reduced lag must lead to a closing of the gap between long term and short term propensity to consume: the short term propensity to consume will increase as a consequence of the habit of buying durables on credit and it will tend to approach the long term propensity to consume. A cursory examination of the time series of consumption and disposable income in the U.S. (in real terms per head of population) does seem to confirm this expectation: we get a marginal propensity to consume of 0.78 for 1929–40, and of 0.91 for 1950–59[2].

[2] The regression equations are:
consumption per head (pre-war) $= 0.78y + 190$
consumption per head (post-war) $= 0.91y + 26$
(where y is real income in $ of 1954 per head).
The "basic consumption" (consumption at zero income) has declined to a small amount, so that the pattern more nearly approximates to the proportionality of consumption and income which is supposed to hold in the long run.

The introduction of consumers' credit on a very large scale has, in fact, been a characteristic feature of the U.S. post-war economy. What are the consequences of such a reduction in the spending lag on the dynamics of the system?

Intuitively it is obvious that a considerable lag in spending on durables must act somewhat as a stabilizer. The additional money earned in the boom is in part withheld and is spent later, presumably largely in the course of the following slump, and this may help to bring about the turning point of the slump[3]. If, in consequence of the prevalent use of consumers' credit, the lag in spending on durables is reduced to a small interval, so that any increase in income has its full consequence on the demand for durables almost immediately, then the effect on the cycle is *un-damping*. The sequence income—consumption—income ... (representing a *positive* feed-back) will proceed more quickly to higher and higher (or lower and lower) levels. The countereffect (*negative* feed-back) which appears once the new equipment has become ready to produce, is correspondingly greater (in view of the greater investment during the construction period). Intuition tells us that the effect of the shortened lag in spending will be to un-damp the cycle and to shorten it. The amplitude of the cycle will probably in practice not be much effected; it depends to a large extent on the magnitude of the random shocks which are essential for keeping the cycle going[4].

Another effect is not so obvious: the undamping will affect the secular growth of the economy by increasing the long term rate of growth. This can best be explained by reference to Prof. Kalecki's model of the trend and cycle[5]. The movement of *net* investment i round the trend value is governed by the equation

$$\dot{i}_{t+\theta} = n\,\dot{i}_t + \mu \frac{d\,\dot{i}_{t-\omega}}{dt} \qquad (1)$$

while the movement of the trend values themselves is governed by

$$y_{t+\theta} = ny_t + m\dot{y}_{t-\omega} + [(1-n)\beta + \gamma]K_t \qquad (2)$$

this equation referring to the trend value of *gross* investment. K_t is the stock of capital.

It will be seen that the parameters n and μ which occur in the equation of the cycle, are met also in the equation of the trend (m is, in fact, a sum of μ and some other term). The close connection between the two phenomena of trend and cycle could perhaps be shown as follows: the term K_t, the capital stock, which occurs in the trend equation, depends obviously on the investment; we can determine it, if a uniform lifetime τ is given, as the integral of gross investment over a past period τ. This integral $\int_{t-\tau}^{t} y_{\tau'}\, d_{\tau'}$ can be replaced, making use of the intermediate

[3] The spending is not necessary on durables. The money saved for this purpose may be spent on necessities in the slump.

[4] M. Kalecki, *Theory of Economic Dynamics*, London 1954, p. 129.

[5] *Theory of Economic Dynamics*, London 1954, Part 6. p. 146.

value theorem, by the approximation $\tau y_{t-\sigma}$ where σ is a value intermediate between 0 and τ. If we introduce this expression $\tau y_{t-\sigma}$ instead of K_t into (2), we have a homogenous equation in gross investment; now we may add to this the equation (1) for the oscillations round the trend, and the equation so constituted will govern at the same time the cycle and the trend. It will yield an exponential solution, due to the term in $y_{t-\sigma}$, which represents the impact of innovations, and an oscillation due to the action of the other two terms: The term in $\dot{y}_{t-\omega}$ is a destabilizer, and the greater its coefficient m, the smaller the damping of the oscillation. The term in y_t embodies the action of the negative feed-back, which is due to the depressing effect of the growing capital stock on the rate of profit and to the incomplete reinvestment of internal savings.

Both trend and oscillation are determined by the same set of data. It can be shown that the parameter m, which acts as a destabilizer in the oscillation (the greater m, the smaller the damping) will at the same time promote the long term growth. This is apparent from Prof. Kalecki's analysis of the exponential solution[6]: it follows from it that if an exponential trend is obtained at all (i.e. as long as m is not too large) the rate of growth will be the greater, the smaller $\theta - m$, (i.e. the greater m) given the other parameters. Thus, the parameter which un-damps the cycle at the sometime stimulates secular growth.

Now the decrease of the lag in spending on durables, if it were introduced explicity into this system, would play the same role as the increase of the coefficient m. The essence is in both cases the strengthenging of a positive feed-back, which leads at the same time to anti-damping in the cycle and to a larger growth rate in the long run.

It is possible that changes in lags have played an important role apart from consumption. The tax-relief accorded to businessmen on condition of investment of their profits has, in certain European countries at least, speeded up the process of investment of internal savings: the lag between earning and investment of profits which in the above equation is denoted by θ, has become shorter. Since this is the lag with which the "negative feed-back" operates, it is plausible that its reduction has tended to shorten the cycle. At the same time, the reduction of θ must have stimulated the secular rate of growth, as the above quoted analysis of Prof. Kalecki shows: the smaller $\theta - m$, the larger the real exponential solution of the equation.

In countries with an endemic price increase there is, of course, still another reason for a shortening of θ. On the other hand, difficulties on the supply side, for example in construction, or delays in deliveries of capital goods, may work in the opposite direction. It is, nevertheless, a fair guess that the factors making for a reduction of θ in the post-war era have dominated.

---

[6] Op. cit., p. 152–155.

Safety versus investment

My original explanation of maturity rests on the idea that the economy can move upwards only if some capital is knocked out of existence, and that this happens by competition; once only a few oligopolists remain in an industry, however, competition to the point of the knife involves too much loss. Since "knocking out" of capitals does not take place, there is a tendency to low utilization of capacity and a fear of excess capacity dominates the investment decisions of business.

In elaborating the theory I have made use of some arguments which can not stand up to the stress of time. I have argued that the oligopolists have some difficulty in moving into other sectors of the economy which are not yet oligopolised, and where they could consequently expand, knocking out some of the existing capital without too much competitive effort. The movement into a different branch requires know-how and this takes time, there is therefore a delay in investment which is tantamount to a disincentive to investment. But the big oligopolists in the U.S. nowadays are so organized as to spread their tentacles into a host of the most diverse lines of business. Since they have their finger in every pie, it is quite possible to argue that the impediments against moving into another branch do not exist for them any more. This situation which has arisen only in the post-war era, involves no doubt a much easier flow of capital from the point of inception (earning) to the investment.

Whatever the possibilities of movement of funds between industries, the fact remains, however, that the competitive sector of the economy, which offers better prospects for investment than the oligopolistic sector, has been *greatly reduced* in proportion to the whole economy. This alone is sufficient to account for a weakening of the incentive to invest.

There exists, however, another reason for the weakening of the incentive to invest in advanced capitalism which is supplementary to my original explanation. The reason is that large concerns prefer to barter the chances of great profit for greater safety, and the policies designed to meet this aim involve in most cases less investment than would otherwise have been decided. The larger, therefore, the concerns become, and the greater the relative weight of large concerns in the economy, the smaller is the incentive to invest. The oligopolistic market situation is partly an incidental consequence of bigness, partly a symptom of the safety-mindedness of big business.

The striving for safety is, of course, common to all business. It is patent, however, that most of the roads to safety are blocked to small business. The small man has to indebt himself heavily, or he will in many cases not be able to run his business at all. He cannot afford to keep reserves and he has to put his one egg willy-nilly into one and the same basket. The bigger the business becomes, the more opportunities open up for a leisurely decision whether to choose a high mathematical expectation of profit associated with great risk, or a lower one with greater safety.

The safety preference of large concerns is an assumption which we do not make just for the purpose of explaining a weakening of the incentive to invest in mature capitalism, but which is forced on us by data which hardly admit of another interpretation. It is undeniable, on the one hand, that big firms have the advantages of large scale economies, that they can, therefore, earn a higher profit rate than small firms. It should be expected, therefore, that firms grow more quickly the greater they are. Such data as we have on the correlation of growth rate and size of firms suggest that it is rather uncertain, sometimes positive, sometimes negative, but always very small[7]. There is no strong evidence that large firms grow more quickly and earn higher profit rates than small firms. On the other hand, there is pretty good evidence that the mortality rate of firms decreases with the size; big firms manage to be safer. Linking the two strands of evidence together we should say that the larger firm uses the greater opportunities open to it due to larger scale economies to increase its safety rather than its profits. This involves reducing the relative indebtedness (or even holding bonds), and therefore investing less than could be done at a fixed proportion of debt to own capital. A smaller proportion of debt involves a reduction of the mathematical expectation of profit rate, and a reduction of the variance of the profit rate[8]. It is, therefore, an ideal way of bartering away profit for security.

Not all the methods of buying safety involve a smaller investment, but it is clear that the whole investment policy is affected by the relative value put on safety. The security preference may lead to the elimination of risky investment projects, and it may also lead to a greater lag in spending on investment, if the aim of collecting more experience about a new process etc. is coming to weigh more strongly than the aim of getting in first and reaping the profits.

Technically speaking, what happens in maturing capitalism (U.S. in the period from 1890 to 1939) is therefore this: the share of large concerns in the market and in the total internal saving grows. For the large concerns the effect of a given internal saving on investment is smaller than for the medium and small business, and possibly also the time lag of spending on investment is bigger. With the growing share of the big concerns the investment effect of a given internal saving in the total economy will therefore decline.

The Role of Innovations and the Generation of the Trend

In denying the active role of innovations in the investment process I have formerly taken up an extreme position which I have no wish to uphold. I was reacting against the view that maturity had arisen from a drying up of the flow of innovations—why it should have done so, nobody had explained, and the fact of drying up itself seemed not very well documented (except to the extent that the concepts of innovation and investment were quietly merged, which made the result

[7] I give some of these data and elaborate on the above argument in my forthcoming book *Growth and Survival of Business Firms*. London, Charles Griffin & Co.

[8] Cf. my paper *On Risk*, "Oxford Economic Papers" 1943.

obvious). I have been wrong, however, to disregard the economic function of innovations in capitalism which, as Prof. Kalecki's work has made clear, is to make a more than temporary enlargement of the capital stock profitable. The capitalist system as represented by Kalecki's model of the pure business cycle is subject to a badly working servo-mechanism (or rather, a controller) which in the absence of innovations (and certain other influences, like outside savings) keeps the capital stock constant. It does so, because it happens to work in this way, not because anybody designed it. Whenever the capital stock grows, and a boom develops, a negative feed-back operates so as to push the investment back. This is due to the depressing effect which a growth of the capital stock has on the rate of profit. This feed-back, which operates with a not inconsiderable lag (up to one year) produces an oscillation round a stable position where net saving is zero and the capital stock constant. Innovations make it possible to break through this closed circle and set an upward path for the capital stock; the controller henceforth pins the system down to oscillations round this path. The function of innovations is to offer the prospect of additional profits which make it possible to enlarge the capital. In Kalecki's formulation: an innovation is *analogous* to an increase in profit, and a steady stream of innovations is *comparable* in its effects on investment to a steady rate of increase in profits[9].

One might wish to go a little further and ask: is it not actually the belief of the innovator-investor that he will get the additional profit, and if so, must we not exclude that his belief is consistently disappointed, or else the innovations would loose their fascination for the investor? There is room for comment here and I offer the following interpretation of the effect of innovations:

A steady stream of innovations means that a given proportion of the existing capital stock is knocked out every year owing to technical obsolescence (it does not pay to work it any more, because the current cost are not covered, owing to the competition of new processes or products). An equal amount of investment in proportion to the capital stock becomes then possible every year without the negative feed-back operatin gagainst it. This investment (which will embody the innovations) causes an additional stream of profits which makes it possible to actually enlarge the capital stock by some further investment without depressing the rate of profit.

The way in which Kalecki introduces the innovations into the last version of his model (1954) makes it actually an *endogenous* theory of the trend. (Seep. above). The effect of innovations (represented by the term γK_t in (2) depends on the size of the capital stock, which evidently depends on gross investment over a certain long period in the past. The innovations are, therefore, an influence of gross investment with a certain relatively long lag. I suggest this could also be interpreted as follows: innovations arise in a stochastic process of learning[10], the knowledge

[9] op. cit. p. 158 (my italics).

[10] T. Haavelmo, *A Study in the Theory of Economic Evolution*, Amsterdam 1954: K. Arrow, *Economic Implications of Learning by Doing*, "Review of Economic Studies", June 1962.

being embodied in the capital stock, and new innovations are thrown up in proportion to the capital stock with a given intensity γ. The innovations would then represent random positive shocks proportionate to the capital stock.

Kalecki in his model of 1954 has in fact brought about the synthesis of cycle and trend in form of an endogenous theory which I desired but failed to achieve in my book. I think that this synthesis is essential for the analysis of economic development. It permits us to analyse the effects which certain parameters of the trade cycle, as for example saving propensities or lags, have on the secular development. Naturally a closer understanding of the dynamic process of learning which throws up the innovations remains desirable.

I should like briefly to mention a modification or variant which could be made of Kalecki's model. The negative feed-back which operates in this model acts *via* the rate of profit. One could imagine it to act, in an analogous way, *via* the degree of utilisation. This requires only that the investment is made to depend not on the rate of profit on existing capital, but on the degree of utilisation of capacity. The boom, in this version, will break because of the accumulation of new capacity which will depress the degree of utilisation. In this version the business cycle represents a controller which tends to keep capacity constant by a feed-back operating *via* utilisation. This view is, in fact, implicit in the theory contained in *Maturity and Stagnation*. It might seem that the problem of the trend—the question how the economy breaks out of the closed circle and sets on an upward path—takes a different form in this version: required is a steady stream of *additional demand* of a type which will not set the negative feed-back operating. Innovations will again fullfill the requirement, in so far as they destroy existing capacity. But will not also a steadily increasing stream of export surplusses, government deficits, or public expenditure financed by taxation (*vide* p. 1–3 above) do the trick?

Historically the stimulants to growth have been very strong at times and it is not entirely perverse to look for a systematic brake. It may be found in the assumption that the share of profits in the national income is elastic in the long run and influenced by utilization, so that it tends to adapt itself to some extent to the rate of growth. This theory has been extensily discussed and motivated in my book. This kind of damping effect acting in the secular development might explain how the secular evolution has most of the time avoided running headlong into inflation.

An alternative explanation, which I did not mention, suits the post-war conditions, at any rate, much better: If they cannot get labour, entrepreneurs are driven to use their investment funds for automation rather than for creating additional employment capacity. This adaptiveness of investment explains why full employment in Western Europe has been maintained for many years now without hyperinflation.

Richard Stone
Great Britain

THE CHANGING PATTERN OF CONSUMPTION

1. INTRODUCTION

The object of this paper is to describe the system of demand functions which we are using in our model of British economic growth [2]. The purpose of these functions is to enable us to divide a given total of consumption into its constituent commodities and so to take us a step nearer our goal, a balanced statement of the economy in a future year.

Our method has been to start with a simple system of relationships which possesses a number of generally accepted theoretical properties and then to elaborate this model to take account of what we know about consumption patterns, in the past. The development of the model is thus an example of the iterative process of induction and deduction commonly found in scientific work.

In the following section I shall explain the economics of the model and in section 3 I shall show how we have solved or how we propose to solve the various statistical problems to which the model gives rise. In section 4 I shall set out some of the results and in section 5 I shall give a brief summary of conclusions. Section 6 contains a list of works cited.

2. THE ECONOMIC OF THE SYSTEM

The system of demand functions I am discussing relates to the average consumer. In its simplest form, it can be described as follows.

The average consumer has a concept of the standard of living he expects to be able to achieve. This concept is expressed in terms of a set of quantities whose elements are the amount of each commodity which must be consumed if the standard of living is to be realised. The average consumer buys these quantities nationally at their current market prices and then compares the total cost of what he has bought with the amount of money he allows himself for spending on consumption. If he finds he has some money over he allocates this to the different commodities in certain fixed proportions; if he finds that he has overspent the money available he reduces his expenditure on the different commodities by applying the same proportions to the amount of overspending.

Let us stop at this point and express this set of relationships in algebra. Let p denote the vector of commodity prices and e the vector of quantities bought by the average consumer; then $\hat{p}e$ denotes the vector of expenditures, a circumflex accent on a vector being used to denote a diagonal matrix formed from it. Let μ denote total expenditure, the elements of a vector c denote the constituents of the basic standard of living and the elements of b denote the proportions in which uncomitted expenditure is devoted to the different commodities. Then

$$\begin{aligned} \hat{p}e &= \hat{p}c+b(\mu-p'c) \\ &= b\mu+(I-bi')\hat{c}p \end{aligned} \tag{1}$$

where i and I denote respectively the unit vector and the unit matrix and a prime denotes transposition. Since $i'b \equiv 1$, premultiplication of (1) by i' yields the identity $p'e \equiv \mu$. Premultiplication of (1) by $\hat{p}^{-1}$, the inverse of $\hat{p}$, shows that the elements of e are homogeneous linear functions of degree zero in μ and p, so that e is unchanged if μ and p are changed to $\lambda\mu$ and λp where λ is any positive constant. The equation for an element, β say, of e can be written in the form

$$e_\beta = (1-b_\beta)c_\beta+b_\beta(\mu-\sum_{\gamma\neq\beta} p_\gamma c_\gamma)/p_\beta \tag{2}$$

from which it follows that

$$\frac{\partial e_\beta}{\partial \mu}\cdot\frac{\mu}{e_\beta} = \frac{b_\beta \mu}{p_\beta e_\beta} \tag{3}$$

$$\frac{\partial e_\beta}{\partial p_\beta}\cdot\frac{p_\beta}{e_\beta} = -\frac{b_\beta(\mu-\sum\limits_{\gamma\neq\beta} p_\gamma c_\gamma)}{p_\beta e_\beta} \tag{4}$$

and

$$\frac{\partial e_\beta}{\partial p_\gamma}\cdot\frac{p_\gamma}{e_\beta} = -\frac{b_\beta p_\gamma c_\gamma}{p_\beta e_\beta} \tag{5}$$

so that

$$\frac{\partial e_\beta}{\partial \mu}\cdot\frac{\mu}{e_\beta} = -\sum_{\gamma=1}^{\nu}\frac{\partial e_\beta}{\partial p_\gamma}\cdot\frac{p_\gamma}{e_\beta}. \tag{6}$$

The price elasticities in (4) and (5) are elasticities along uncompensated demand curves and can be divided into an income effect and a substitution effect. Thus if we denote by w_γ the proportion of total expenditure devoted to commodity γ, so that

$$w_\gamma \equiv \frac{p_\gamma e_\gamma}{\mu} \tag{7}$$

and by $s_{\beta\gamma}$ the elasticity of substitution between commodities β and γ, then

$$s_{\beta\gamma} = -\frac{(\delta_{\beta\gamma}-b_\beta)b_\gamma(\mu-p'c)}{w_\beta w_\gamma \mu} \tag{8}$$

where $\delta_{\beta\gamma} = 1$ if $\beta = \gamma$ and is other wise equal to zero. Thus we can write

$$\frac{\partial e_\beta}{\partial p_\gamma} \cdot \frac{p_\gamma}{e_\beta} = w_\gamma \left[s_{\beta\gamma} - \left(\frac{\partial e_\beta}{\partial \mu} \cdot \frac{\mu}{e_\beta} \right) \right]. \tag{9}$$

By appropriate substitutions (9) can be reduced to (4) if $\beta = \gamma$ and to (5) if $\beta \neq \gamma$. The first term, $w_\gamma s_{\beta\gamma}$, on the right-hand side of (9) measures the substitution effect of a change in the price of γ on the demand for β and the second term measures the income effect. This is Slutsky's equation, and, as can be seen from (8), Slutsky's condition that $s_{\beta\gamma} = s_{\gamma\beta}$ is satisfied by this system of equations.

In theory the own-elasticities of substitution, $s_{\beta\beta}$, must be negative. If, as seems reasonable, we assume that uncommitted expenditure, $\mu - p'c$, is positive, this condition requires that $0 < b_\beta < 1$. Thus inferior goods are ruled out. But with this restriction on the elements of b it follows that $s_{\beta\gamma} > 0$ for all $\beta \neq \gamma$. Thus complementary goods are ruled out too, and the system can only represent a set of commodities, or commodity groups, that are substitutes for one another. With a careful choice of commodity groups, this may not be a serious limitation in practice.

There are, however, other limitations to this simple formulation which are serious. The most obvious, which I shall discuss in some detail in this paper, is that so far the elements of b and c have been assumed constant. As time progresses, the average consumer's conception of his standard of living is likely to change; and with it his allocation of uncommitted expenditure is likely to change too. What can we do about this?

Fortunately, it is not very difficult to allow for systematic changes in b and c. The formal properties of the model are not affected if the parameters are made functions of predetermined variables. The simplest possibility is to make the linear functions of time. Thus, at time Θ, we should have

$$b_\Theta = b^* + \Theta b^{**} \tag{10}$$

and

$$c_\Theta = c^* + \Theta c^{**} \tag{11}$$

say.

The introduction of these linear trends removes the main rigidity of the original formulation in (1). Nevertheless, cases arise in which linear trends are too crude an approximation. For example, one of the categories we have used in our empirical work is transport which includes expenditure on cars and their running expenses as well as on public transport. At the beginning of the century, cars were of negligible importance; by the nineteen thirties they were generally accepted but were still too expensive for many people; nowadays they are coming to be bought by the whole community. As a consequence the trends in the parameters for transport have tended to move along an accelerating curve. Such a tendency can be represented in the model by adding quadratic terms to (10) and (11).

We shall see in section 4 below that we can improve by this means the ability of the model to describe past observations. But in doing this we risk the possibility that our projections will be less reliable than they would have been with linear trends. For example, the accelerating trends for transport will eventually pass through a point of inflection and begin to slow down. In other cases we may find that a quadratic trend passes through its maximum or minimum near the end of the period of observation and so will change its direction in the period of projection. Though possible in exceptional cases, for example the gradual disappearance of the carriage after the advent of the motor car, such changes of direction are, in general, not very plausible. They can be avoided by giving up time as the variable on which the parameters depend and by making them functions of the past history of the branch of demand to which they relate. For example if e^*_Θ denotes a vector of five-year moving averages of the components of consumption ending in year $\Theta-1$, then we could replace (10) and (11) by

$$b_\Theta = b^* + \hat{e}^*_\Theta b^{**} \tag{12}$$

and

$$c_\Theta = c^* + \hat{e}^*_\Theta c^{**} \tag{13}$$

We shall see in the next section that this formulation presents no more statistical difficulties than the earlier ones.

Having thus freed the basic model from its failure to allow for changing tastes and habits, we must now consider another limitation. Even if the parameters change systematically through time, the model still implies that consumers are capable of rapid adaptation so that in each year they are in equilibrium. This is probably a reasonable assumption in the case of perishable goods but it is certainly not reasonable in the case of major durable goods which involve a large initial expenditure. As explained in [2] the method described in [6, 8] can be adapted to the present model. A computing sequence for this extension of the model is given in [7]. I shall not discuss this problem further here as we have not so far made use in our calculations of this extension of the model.

3. THE STATISTICS OF THE SYSTEM

In applying the system of equations just described we have used annual observations over the period 1900 to 1960. The first results, using linear trends in the parameters, were set out in [7]. In this paper only broad groups were analysed and no attempt was made to base the estimates of the parameters on cross-section data as well as time series. Accordingly, in this section I shall consider three problems: (i) a computing sequence for the model consisting of (1), (12) and (13); (ii) a decomposition of the model to enable the subgroups of main groups to be analysed; and (iii) the combination of cross-section data and time series in estimating the parameters.

(i) *The computing sequence.* In order to estimate the parameters b^*, b^{**}, c^* and c^{**} we have used an iterative two-stage least squares procedure. If we consider the model consisting of (1), (12) and (13), the computing sequence is as follows.

We begin by guessing values of b^* and b^{**} which I shall denote by b_0^* and b_0^{**}. The values of the elements of b_0^* are the average expenditure proportions; those of b_0^{**} are zero.

We then form a vector of type $v x$ 1, y_Θ say, as follows

$$y_\Theta \equiv \hat{p}_\Theta e_\Theta - (b_0^* + \hat{e}_\Theta^* b_0^{**})\mu_\Theta \tag{14}$$

and a matrix of order v, Y_Θ say, as follows

$$Y_\Theta = [I - (b_0^* + e_\Theta^* b_\Theta^{**})i']\hat{p}_\Theta \tag{15}$$

Apart from a random element, y_Θ and Y_Θ are connected by the relationship

$$y_\Theta = [Y_\Theta : Y_\Theta \hat{e}_\Theta^*]\begin{bmatrix} c^* \\ \cdots \\ c^{**} \end{bmatrix}. \tag{16}$$

If we now define

$$y \equiv \{y_1, y_2, \ldots, y_\tau\} \tag{17}$$

$$Y \equiv \{Y_1, Y_2, \ldots, Y_\tau\} \tag{18}$$

and

$$Y \equiv \{Y_1\hat{e}_1^*, Y_2\hat{e}_2{}^*, \ldots, Y_\tau\hat{e}_\tau^*\} \tag{19}$$

we can write, apart from a random element,

$$y = Xg \tag{20}$$

where $X \equiv [Y : Y^*]$ and $g \equiv \{c^* : c^{**}\}$

The least squares estimator, g_1, of g is

$$g_1 = (X'X)^{-1}X'y \tag{21}$$

Given g_1 we can form a vector of type $v x$ 1, w_Θ say, as follows

$$w_\Theta \equiv \hat{p}_\Theta[e_\Theta - (c_1^* + \hat{e}_\Theta^* c_1{}^{**})] \tag{22}$$

and a matrix of order v, W_Θ say, as follows

$$W_\Theta \equiv [\mu_\Theta - p_\Theta'(c_1^* + \hat{e}_\Theta^* c_1^{**})]I \tag{23}$$

It will be noticed that W_Θ is a scalar matrix. Apart from a random element w_Θ and W_Θ are connected by the relationship

$$w_\Theta = [W_\Theta : W_\Theta \hat{e}_\Theta^*]\begin{bmatrix} b^* \\ \cdots \\ b^{**} \end{bmatrix} \tag{24}$$

so that if we define

$$w \equiv \{w_1, w_2, \ldots, w_\tau\} \tag{25}$$

and

$$W \equiv \{W_1, W_2, ..., W_\tau\} \tag{26}$$

and

$$W^* \equiv \{W_1\hat{e}_1^*, W_2\hat{e}_2^*, ..., W_\tau\hat{e}_\tau^*\}. \tag{27}$$

We can write, apart from a random element,

$$w = Zh \tag{28}$$

where $Z = [W : W^*]$ and $h \equiv \{b^* : b^{**}\}$. The least squares estimator, h_1, of h is

$$h_1 = (Z'Z)^{-1}Z'w. \tag{29}$$

Given h_1 we can return to (16), replace b_0^* and $\{b_0^{**}$ by b_1^* and b_1^{**} and calculate the next approximation $g_2 \equiv \{c_2^* : c_2^{**}\}$ of g. We continue in this way until the process converges.

In estimating h, the system breaks down into v separate equations since W_Θ and W_Θ^* are diagonal matrices. At the same time the adding-up, theorem ensures that $i'b^* = 1$ and $i'b^{**} = 0$ for the estimated values of b^* and b^{**}. In estimating g on the other hand, since Y_Θ and Y_Θ^* are not diagonal matrices, the v equations all contribute to a single, average estimator of g.

(ii) *The analysis of subgroups.* A feature of the system I am describing is that it is decomposable: once we have analysed total consumption divided into a certain number of main groups, we can then carry out exactly similar analyses on the components of each of the main groups. If necessary we can continue this process in a hierarchy of subanalyses.

The method is as follows. The equations for group j in a complete system can be written as

$$\hat{p}_j e_j = \hat{p}_j c_j + b_j(\mu - p'c) \tag{29}$$

where the suffix j denotes that the vector to which it is attached contains elements relating only to group j. If we premultiply (29) by i' we obtain

$$\begin{aligned} p_j' e_j &= \mu_j \\ &= p_j' c_j + i' b_j(\mu - p'c) \end{aligned} \tag{30}$$

so that

$$\mu - p'c = (i'b_j)^{-1}(\mu_j - p_j' c_j) \tag{31}$$

If we substitute for $\mu - p'c$ from (31) into (29) we obtain

$$\hat{p}_j e_j = \hat{p}_j c_j + b_j(i'b_j)^{-1}(\mu_j - p_j' c_j) \tag{32}$$

From an analysis of the main groups, we can obtain estimates of $i'b_j$ and $i'c_j \equiv {}_jc$, say. From the analysis of the components of group j we can obtain estimates of c_j and $b_j(i'b_j)^{-1}$. Thus the whole system will fit together consistently provided that $i'c_j$ in the subanalysis is equal to ${}_jc$ in the main analysis. To ensure this equality we must carry out stage 1 of the subanalysis subject to this constraint.

If, for example, we consider the version of the system in which the parameters are linear functions of time, then we must ensure that ${}_jc^* = i'c^*j$ and that ${}_jc^{**} = i'c_j^{**}$. If λ_1 and λ_2 denote two undertermined multipliers, then the normal equations of stage 1 of the computing sequence for the subanalysis take the form

$$\left[\begin{array}{cc:cc} Y_j'Y_j & \Theta Y_j'Y_j & i & 0 \\ \Theta Y_j'Y_j & \Theta^2 Y_j'Y & 0 & i \\ \hdashline i' & 0 & 0 & 0 \\ 0 & i' & 0 & 0 \end{array}\right] \left[\begin{array}{c} c_j^* \\ c_j^{**} \\ \hdashline \lambda_1 \\ \lambda_2 \end{array}\right] = \left[\begin{array}{c} Y_j'y_j \\ \Theta Y_j'y_j \\ \hdashline {}_jc^* \\ {}_jc^{**} \end{array}\right]. \tag{33}$$

The equations in the second stage of the computing sequence are unaffected.

(iii) *Cross-section data and time series.* Although our empirical work on the model I am describing has been based so far on time series, it would obviously be desirable to check the conclusions derived from it with the estimates of the relationships between individual expenditures and total expenditure derivables from budget studies. Since there are obvious difficulties in comparing derivatives or elasticities obtained from time series with those obtained from budgets, the first step is to make independent calculations on the two bases and find out if they differ significantly. If they do, we must conclude that apparently comparable measures are in fact not really comparable. A possible reason for this which seems to be borne out by a limited amount of analysis [9] is that for some types of good and, in particular, for durable goods, long-term total expenditure elasticities are typically different from short-term elasticities. Analyses of time-series which do not concern themselves with the time needed for adaptation, may reasonably be supposed to yield estimates of short-term elasticities. Elasticities from budgets on the other hand may better approximate to longterm elasticities. We could test this approximation by using the dynamic version on the model based on time series. If we find that the two sets of estimates are, on the whole, not very different we should combine them to give better estimates of the parameters. Following Durbin [3], there are two ways of doing this.

(a) We estimate a time-series of b from the budget studies, supposing these to be sufficiently numerous, and use these as extraneous estimators in (1). With this information we can rewrite (1) as

$$u = Vc \tag{34}$$

where $u = \hat{p}e - b\mu$ and $V = (I - bi')\hat{p}$. Thus we could estimate c from the equation

$$c = (V'V)^{-1}V'u \tag{35}$$

In all likelihood the budget studies of the past will not be sufficiently numerous for this purpose. If this is so we ought to carry out the fitting simultaneously, subject to an appropriate constraint on the time-form of b.

(b) Statistically speaking, the method just suggested does not make an efficient use of the available data. A method which made the fullest use of the data from both sources would have to follow the lines suggested in section 3 of [3].

By these means it should be possible to build up a fairly detailed picture of the structure of demand consistent with all the information about the past that we possess. In the course of doing this we may be led away from the particular, simple approximations that I have described. Our problems, then, are: first to find a suitable specific form of the model; and, second, to generalise it so as to take account of important features of the real world that it leaves out. I have already mentioned the question of adaptation and adjustment rates. But we should not abandon the idea of a coherent and manageable approximation to reality or we shall fail to obtain even an approximate picture of the structure of demand.

4. SOME RESULT OF THE MODEL

I wish that I could now give a complete set of results obtained from the model I have described. Unfortunately I cannot because the series of analyses I have outlined are not yet completed. So far we have only analysed eight main groups; we have made no subgroup analyses and we have made no systematic comparison with budgets.

The data we have used relate to expenditure per head on eight commodity groups and the corresponding price index-numbers measured over the years 1900 to 1960. In estimating b and c we left out the years 1914 through 1919 and 1940 through 1947 because of the abnormal conditions of war periods. This means that we have 376 observations from which to determine 30 independent parameters in the case of the model with linear trends and 45 in the case of the model with quadratic trends. The method of fitting is the one described in the preceding section with ΘI in place of $\hat{e}_\Theta{}^*$ in the case of the linear version and with additional terms of the form $\Theta^2 b^{***}$ and $\Theta^2 c^{***}$ in the case of the quadratic version.

The estimates of the parameters are shown in the following two tables and the goodness of fit of the two models is indicated in the double page diagram at the end of the paper.

The first table compares the components of b. The letters L and Q denote respectively the linear and the quadratic model.

In these tables $\Theta = 0$ in 1960 and so b^* and c^* show the estimates of b and c in the year. All these estimates showed considerable trends. For example, in the linear model the proportion of uncommitted expenditure spent on the food fell by 0.39 percentage points in each year until in 1960 it was as low as 8.05 per cent. At the same time committed purchases per head of food measured in 1938 prices rose each year by £0.332 until in 1960 they reached £33.73. In 1960 uncommitted expenditure according to the linear model was £38.32 per head and committed expenditure on food at 1960 prices was £93.1 per head. Thus, according to the linear model, food expenditure per head in 1960 should have been £[93.1+

(38.32×0.0805)] = 96.2. The corresponding figure from the quadratic model is £95.3. The observed value is £95.8. In each of these cases the error is about one half of one per cent.

TABLE 1

	b^*		b^{**}		b^{***}	
	L	Q	L	Q	L	Q
Food	0.0805	0.0948	—0.00390	0.00039	—	0.000086
Clothing	0.1569	0.1813	0.00210	—0.00008	—	—0.000035
Household	0.2263	0.1724	0.00210	—0.00961	—	—0.000137
Communications	0.0023	—0.0018	—0.00040	0.00010	—	0.000002
Transport	0.2342	0.2588	0.00160	0.01080	—	0.000117
Drink and tabacco	0.0956	0.0618	—0.00190	—0.00161	—	—0.000038
Entertainment	—0.0143	—0.0018	—0.00080	0.00236	—	0.000035
Other	0.2186	0.2346	0.00120	—0.00236	—	—0.000029
Total	1.0001	1.0001	0.00000	—0.00001	—	0.000001

The second table compares the compenents of c.

TABLE 2

	c^*		c^{**}		c^{***}	
	L	Q	L	Q	L	Q
Food	33.73	35.06	0.3320	0.2775	—	—0.000518
Clothing	9.26	12.25	0.0430	0.0557	—	—0.000544
Household	24.59	28.63	0.0540	—0.0383	—	—0.004702
Communications	1.13	1.15	0.0260	0.0358	—	0.000366
Transport	8.97	14.62	0.1650	0.3979	—	0.003135
Drink and tabacco	10.20	11.56	0.0580	0.1572	—	0.003829
Entertainment	5.05	4.86	0.0750	0.1035	—	0.000778
Other	9.99	14.15	0.0640	—0.0292	—	—0.002401
Total	102.92	122.28	0.9170	0.9464	—	—0.000057

The diagram shows that both models perform well. In many cases, for example throughout the period for food and in the post war years for all the series, they show very similar results. In other cases, where they differ, the quadratic model is usually the better. For example, it shows to advantage in reproducing the interwar series for clothing and household almost exactly and gives a much better reproducing of the earlier part of the series for transport and communications. The rather flat series for transport obtained for the interwar period from the quadratic model is probably due to the substantial element of net investment in the purchases of cars in that period. Since neither version of the general model given here takes any account of consumers' problems of adaptation, each is likely to reproduce the underlying movement of consumption, that is purchases minus net investment. The other case in which this kind of effect might be important is the durable component of the category household. But here the net investment component of purchases is smaller than in the case of cars and the category itself, which includes expenditure on rent and fuel, is very large.

Although the two war periods were left out in calculating b and c, the actual and estimated series for these periods are shown in the diagram. On the whole they show what one might expect. For example, during the second world war the average consumer systematically devoted less of his expenditure to food and clothing and more of it to drink and tobacco than he would have done in normal circumstances.

It will be noticed that in the post war years the fit of both models is uniformly good. This is not surprising because it is the sum of squares of all the absolute discrepancies that is being minimized by the statistical procedure, and in the post war period all series were relatively high because prices were high compared with most earlier periods. What is perhaps more interesting is that the models also fit reasonably well in the early part of the century when prices were very much lower and also in the case of the very small groups, communications and entertainment.

As I have said, we have not yet made a systematic comparison between the total expenditure elasticities derived from the model and those derived from budgets. So far we have only looked into two cases, food and clothing, in both of which the alternative estimates are in fairly close agreement. For food, the linear model gives estimates of this elasticity of 1.0 for 1900, 0.6 for 1938 and 0.3 for 1960; the quadratic model gives corresponding estimates of 1.1, 0.4 and 0.3; budgets are not available in sufficient detail for the earlier part of the period but give estimates of 0.6 for 1938 [1, 10] and 0.3 for 1960 [11], in complete agreement with the linear model. For clothing, the linear model gives estimates of 0.3 for 1900, 1.1 for 1938 and 1.4 for 1953; the quadratic model gives corresponding estimates of 0.7, 1.6 and 1.7; budgets give estimates of 1.1 for 1938 [5] and 1.4 for 1953 [4], again in complete agreement with the linear model.

It is hard to say why the linear model should have appeared the better one in the comparisons. It may be that the budget estimates are not the best that could be made; it may be that the curvilinearity introduced in the quadratic model is not of the appropriate kind. Reasons for this belief have already been given though no calculations have yet been made with the third time-series model or with the model in which budgets and time series are combined for estimation purposes. In the present case it is the parameter b that is important and it is, perhaps, significant that, in the quadratic model, min b for food occurs in 1958 and that max b for clothing occurs in 1959. Thus in both cases there is a considerable degree of curvature over the observation period which has already changed direction. Certainly we might be doubtful of such results in making projections.

Another curious feature of the quadratic model is that in almost all years uncommitted expenditure is negative. This is in contrast with the result obtained from the linear model in which uncommitted expenditure is positive except during wars and the years immediately following them. However, even with this model uncommitted expenditure is not very big; in the interwar and postwar periods it fluctuated between 10 and 15 per cent of total expenditure.

This result of the linear model seems to me to be much what one might expect. The average consumer has a great many commitments, whether they are legal commitments or not, and can only make marginal adjustments to his expenditure pattern. But over time he tends to get richer and as he does his conception of his commitments rises. As a consequence he can still only make marginal adjustments. In war and immediate postwar year the average consumer cannot maintain his conception of his standard of living and, in one way or another, is forced to cut back his expenditure. As conditions improve a margin of uncommitted expenditure reasserts itself.

The quadratic model goes further than this and suggests that the average consumers' conception of his standard of living always outruns the money he has to spend and so he is always preoccupied not with what to buy but with what not to buy. In a society in which emulation plays an important part and which is not divided by sumptuary laws into non-competing groups, even this picture may not seem unduly far-fetched. But, if we accept it, the whole theoretical interpretation of the model needs to be reconsidered since, as we can see from (8), a negative value of uncommitted expenditure would cause the own-elasticities of substitution, $s_{\beta\beta}$, to be positive. The position would be still worse if the commitments were so high that the term in round brackets on the right-handed side of (4) became negative, since in this case the Marshallian demand curves would get into the wrong quadrant.

The Slutsky condition (8), is an equilibrium condition, and we might argue that the average consumer cannot be in equilibrium if his uncommited expenditure is negative. With the quadratic model this makes the theory somewhat remote for reality, unless, as is possible, we have so far been able to make only a poor approximation to the true values of the parameters in this model.

5. CONCLUSIONS

My conclusions from the work discussed in this paper can be summarised as follows.

(1) The kind of demand model I have described is capable of a sensible interpretation and gives a reasonably good representation of the past.

(2) This model is capable of considerable generalisation and of making very full use of past experience.

(3) In it, the influence of income and prices are introduced in a simple and approximate way. The important innovation, on which I have concentrated in this paper, is the allowance for changing responses. This is done by allowing the parameters, which have a clear economic interpretation, to change over time.

(4) An outline of our practical experience with the model shows that it is promising but that there is still room for experiment and improvement. Two methods of improvement were suggested: the introduction of more sophisticated trends in the parameters and the combination of time series and budget studies.

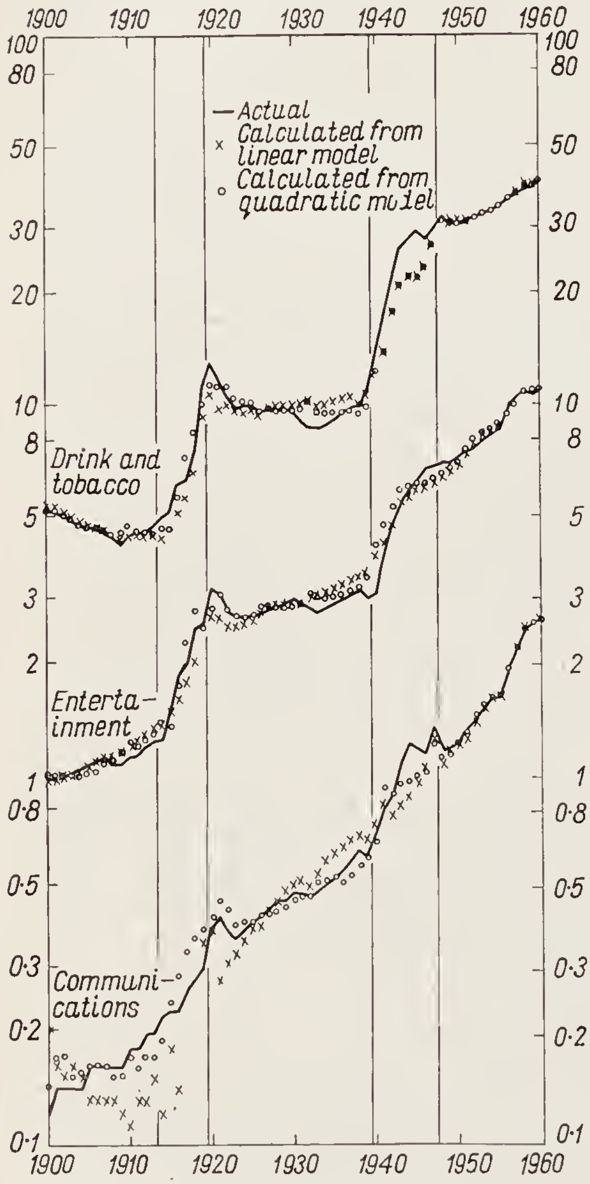


Fig. 1

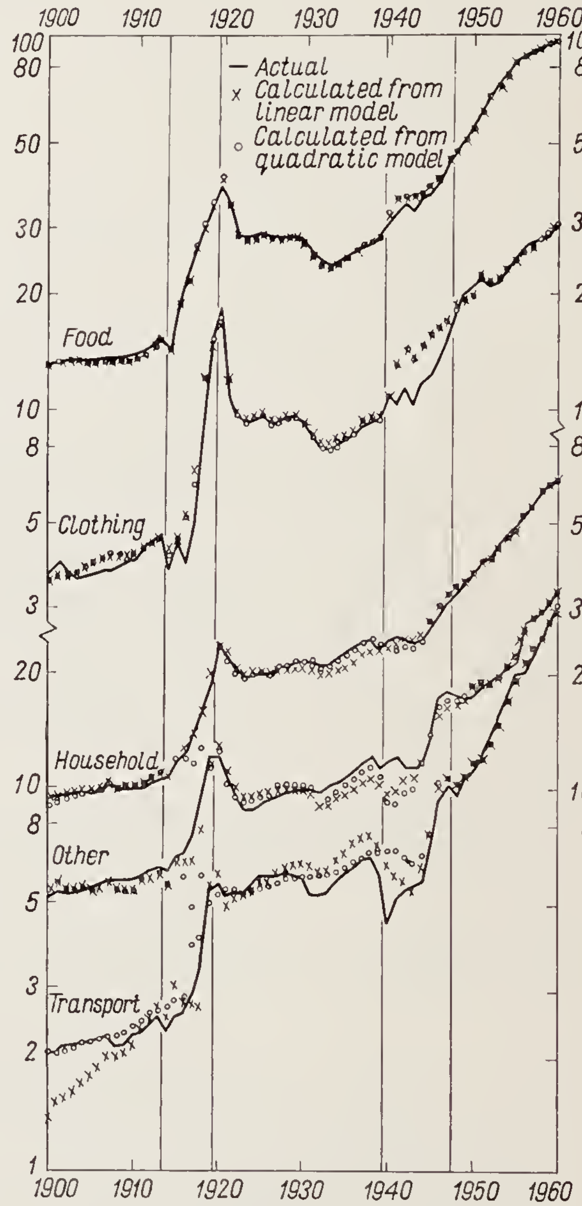


Fig. 2

(5) The model is hierarchical: it deals first with main groups, then with subgroups of main groups and then with subgroups of subgroups. The thought here is that main groups are relatively little affected by changes either from the side of supply or of demand which cannot be represented by simple systematic changes in the parameters. As the subgroups become smaller the effect of these changes become more important until at some point in the subdivision of commodities the

usefulness of the model is likely to be exhausted. This is as it should be; it is easier to formulate relationships to describe broad categories than to describe narrow ones. It is helpful to be able to see how far we can go with any proposed formulation.

(6) In all our work on economic growth we are concerned to produce the best picture we can of the future based on the changing relationships of the past. By this means we expect to be able to provide a worthwhile basis of discussion with those engaged in the different branches of economic activity. From such discussions we hope that a plan will finally emerge which is realistic as well as consistent, and which reaches a reasonable compromise between individual and social aims.

6. A LIST OF WORKS CITED

1. Aithison, J. and J. A. C. Brown, *A Synthesis of Engel Curve Theory*, "The Review of Economic Studies", vol. XXII, 1954–55, pp. 35–46.
2. Cambridge, Department of Applied Economics. *A Computable Model of Economic Growth.* No. 1 in A Programme for Growth, Chapman and Hall, London 1962.
3. Durbin, J. *A Note on Regression when there is Extraneous Information about One of the Coefficients*, "Journal of the American Statistical Association", vol. 48, no. 264, 1953, pp. 799–808.
4. Forsyth, F. G. *The Relationship between Family Size and Family Expenditure*, "Journal of the Royal Statistical Society, Series A (General), vol. 123, pt. 4, 1960, pp. 367–97.
5. Prais, S. J. and H. S. Houthakker. *The Analysis of Family Budgets.* Cambridge 1955.
6. Stone, Richard, *Dynamiczny model kształtowania się popytu*, "Przegląd Statystyczny", vol. VII, No. 3, 1960, pp. 255–70.
7. Stone, Richard, Alan Brown and D. A. Rowe, *Demand Analysis and Projections for Britain*: 1900–1970. *A Study in Method.* To be published for the Association Scientifique Européenne pour la Prévision Economique à Moyen et à Long Terme, Amsterdam 1963.
8. Stone, Richard and D. A. Rowe, *The Market Demand for Durable Goods*, "Econometrica", vol. 25, No. 3, 1957, pp. 423–43.
9. Stone, Richard and D. A. Rowe, *Dynamic Demand Functions: Some Econometric Results*, "The Economic Journal", vol. LXVIII, no. 270, 1958, pp. 256–70.
10. Stone, Richard and Others, *The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom*, 1920–1938, Vol. I, Cambridge 1954.
11. U. K. Ministry of Agriculture, Fisheries and Food. *Domestic Food Consumption and Expenditure*: 1960. H.M.S.O., London 1962.

ADAM SZEWORSKI
POLAND

GOVERNMENT INTERVENTION AND THE LONG-TERM GROWTH IN DEVELOPED CAPITALIST ECONOMIES

THE PURPOSE of this contribution is to sketch the ways in which the Government can influence upon the rate of long-term growth, and to appreciate briefly the effectiveness in this respect of various policy measures used in the developed capitalist economies.

The problem will be considered on a theoretical background as provided by M. Kalecki's theory of dynamics of capitalist economy[1]. So far as our subject is concerned, the main point of this theory is that long-run development is not inherent in that economy, and specific "development factors" of semi-exogenous nature, are required to sustain a long-run upward movement.

These main factors are considered to be innovations in the broadest sense, i.e. technological developments which make certain investment projects more attractive than they would be otherwise, introduction of new products and opening up of new sources of raw materials which make necessary new investment in production, transport facilities, etc. Their influence does manifest itself in raising the long-run level of investment above the depreciation level at which it would otherwise maintain, thus adding to the stock of capital equipment. Since that investment effect can be generally assumed to be the higher the larger is the stock of capital, it follows that, with a given stream of innovations, a continuous upward movement in the long-run level of investment, thus in the development of the economy, is obtained.

On the other hand, it is argued that there is a factor, i.e. savings being accumulated outside firms, or the so called rentiers' savings, which tend to generate a negative trend in the long-run level of investment in a similar way that innovations do a positive one. Thus, it is the net effect of innovations and rentiers' savings which determines the long-run development of the economy.

The rate of this development is dependent on the intensity of the two factors in relation to the stock of capital equipment; if the intensity of innovations is increasing or that of rentiers' savings declining, a higher rate of growth of the economy is the result; and if there is a decline in the intensity of innovations or an increase in that of rentiers' savings, this leads to a retardation in growth.

[1] M. Kalecki, *Theory of Economic Dynamics*, London, George Allen and Unwin, 1954, pp. 178.

It results from the above that, if a change occurs in the long-run level of investment, owing to an intervention of some other factor than the two mentioned above, the effect of this will lead to a similar change in the rate of growth of the economy. This is because given intensity of innovations and rentiers' savings will be related to a stock of capital which has been changed as result of that intervention, so that a corresponding change in their investment effect will follow. This may be particularly the case of a rise in the degree of monopoly which, as it is known, results in a fall in the long-run level of investment below that at which it would be otherwise. This may also be the case of a Government intervention which may obtain a rise in that level.

It is largely in terms of those four factors that a rise in the long-term rate of growth can be explained as it was observed in the economies of developed capitalist countries after the World War II.

There was, indeed, a considerable rise in the degree of monopoly in most of the countries considered, the evidence of which is a rise in relative shares of corporate profits in their national incomes in the postwar years as compared with the prewar period. This had, of course, a negative long-run effect on the level of investment. On the other hand, there was also an increase in the intensity of innovations, the effect of which combined with a decline in real value of rentiers' savings in relation to real value of capital stock, was superior to that of monopoly, so that the net effect was a rise in the long-run level of investment, thus in the rate of growth of the economies considered. And in addition, there was also an intervention of the Government in current development of these economies which contributed, in many cases, to a further rise in the long-run level of investment.

It is true that a powerful expansion in foreign trade has been also an important factor in speeding up the rate of growth of those countries. But this can also be explained in terms of the influence of the factors already enumerated which brought about the changes outside the area considered leading to that expansion. So, for instance, large-scale development programmes which have been launched by Governments in the primary producing countries, gave rise to introduction of new techniques and products not known so far in those countries, thus contributing to an increased intensity of innovations, whereas rentiers' savings were brought to a relative decline. Moreover, the extension of the socialist system over large areas, while cutting off important sources of raw materials exploited so far by capitalist countries, has caused the latter to search for new sources of supply, both domestic and foreign. This, in turn, strenghhened the purchasing power of primary producing countries to buy investment goods for their development programmes in developed countries, and so forth.

From the above it may be concluded that the Government action contributed to the rise in the rate of growth of the countries in question both in a direct and indirect way. In fact, the changes in the intensity of the factors determining that rate were, even in the developed countries, dependent on the influence of their Governments.

This is especially the case of the rise in the intensity of innovations. It has been essentially based, on a big progress in science and technology which to a considerable degree, was financed by an ample Government expenditure on research for was purposes during the World War II and the postwar armament race. This, in fact, opened the way to many important technological developments and to introduction of a lot of new products which were accompanied by exploitation of new raw materials, etc. altogether enlarging the scope for profitable investment. And as inventive activities are becoming more and more institutionalised, this enables, especially big corporations, to engage the Government into research for their own purposes.

So far as the relative decline in rentiers' savings is concerned, it was mainly the result of a general rise in the price level owing to a heavy war and postwar demand but the Government action has also contributed to thus fact, through rent controls monetary policies, etc. both during the war and early postwar years.

On the other hand, it was also due, to a large extent, to the influence of the Government that there has been a rise in the degree of monopoly. Fostering investment in heavy industries for both the war production and postwar reconstruction purposes by means of policies which favoured accumulation of profits within the big corporations, has considerably contributed to promote the concentration of production and capital, which is the main source for a rise in the degree of monopoly. And with an increased influence of monopolistic corporations on the Government, its policies have been more and more subjected to their interests.

As it may be seen from the above, the indirect influence of the Government on the rate of growth of the economies considered has been both a positive and a negative one. So far as the direct influence is concerned, it consists mainly in a current intervention in the short-run developments of the economy. A relative attenuation in cyclical fluctuations and an improvement in the rate of employment of the labour force available as observed in those countries in the postwar period, have been mainly the results of such intervention. And, as already mentioned, this has been not without significance for their long-run developments.

It is to that intervention that our further considerations will be devoted. Given its main objectives, which are to prevent the unemployment from growing excessively and thus leading to heavy social conflicts, and at the same time to protect the economy against excessive inflationary pressures, we will try to show what implications the policy measures may have on the long-run level of investment, and thus on the rate of growth of the economy. Two main groups of those measures which are as usually applied in the countries considered will be reviewed, i.e. monetary policy and fiscal policy measures, and some words will also be said on direct controls.

The most important of the measures belonging to the first group is a policy of changing the rate of interest for short-term credits in order to stimulate sluggish activity in recession, or to prevent inflationary pressures in the up-swing of the cycle. The effectiveness of this policy for the level of investment is obviously very limited, because the latter (except for dwelling construction) is dependent—so far as credit conditions are concerned—upon the long-term rate of interest and, therefore, can be influenced by those measures in an indirect way only and with a certain time-lag. This may be the case, if the long-term rate of interest which is based on the short-term rate expected in the next few years, does response to a given policy when applied for a longer period.

Thus, for instance, an "easy money" policy which consists in keeping the short-term rate at an average relatively low level, may bring about a decline in the long-term rate which, in turn, may cause a rise in investment above the level at which it would be, if the interest rate were determined by market conditions. Moreover, it may also cause the rentiers' savings to fall in relation to both the total savings and capital stock, which will be growing at a faster rate. In these ways such policy may lead to a positive, although not very significant, change in a given rate of growth of the economy.

An opposite effect may result, if the short-term rate of interest is kept at an average relatively high level. The long-run level of investment may then be depressed and rentiers' savings may rise in relation to the capital stock, thus slowing down the rate of economic growth.

It appears then that a Government intervention which relies on this kind of policy measures may contribute to a higher rate of growth, if it can disregard the interests of rentiers' groups. If the latter prevail in the policy considerations of the Government, this cannot but lead to a retardation in economic growth.

This problem may be exemplified by the experience of Great Britain, a country where the Government intervention has heavily relied on the measures considered above. A shift from an "easy money" policy as practicised in the early post war years towards a high rate of interest policy in recent years, which has taken place to comply with interests of the rentiers', has been certainly one of the factors which contributed to a relative stagnation of the British economy in the latter period.

Another main measure of the monetary policy which consists in changing the supply of money, does not seem to be relevant for the subject of our considerations. For, neither a rise in that supply in order to stimulate investment, nor a reduction in order to prevent it from growing excessively, can lead to desired results; its raising does not assure *per se* a rise in profit expectations which is needed for a new capital to be invested, and its reducting can be easily offset by a rise in the velocity of money circulation. And so far as their possible influence on the short-term rate of interest is concerned, it is rather improbable that it could be of a kind to produce a change in the long-term rate, thus to affect the level of investment[2].

Of much more importance from the point of view considered, are fiscal policy measures, i.e. adjustments in the level of Government expenditure or/and the rate of taxation. For, under a tendency inherent in capitalist economy for the demand to fail in growing pari passu with the growth in investment, these are exactly the measures which, due to their direct influence on the level of overall demand, are the most suitable to counteract the recurring falls in investment being a result of that tendency. They may be used in order to create an additional demand to fill in the "demand gap" or to cause investment to rise above the level as determined by market conditions, but also in order to check the growth in investment, if it is considered excessive.

When viewed from the point of the level of demand, the changes in the rate of taxation which affects, directly or indirectly, the incomes of broad masses of population, are the most effective ones. For, in this case, a tax reduction is immediately followed by an almost equivalent rise in demand, and a tax increase by a corresponding decline. Instead, such effects are not certain so far as the taxes on capitalists' incomes are concerned. This is because the latter do not adjust immediately their consumption to their changed incomes, and may also not adjust immediately, and the more so later on, their investment, unless the given measures are consistent with changing profit expectations. And since they are used mainly to counteract the tendencies prevailing on the market, their result can be, at best, only a limited one, even if tax changes are directly tied with investment decisions, as it is the case of depreciation allowances.

Generally, however, there is little scope for using tax reductions as means to raise the level of investment in the long-run, even if the tax rates are high. The expenditure policy is, therefore, of essential importance in this respect.

The crucial points here is not the amount of the expenditure itself, but the ways in which it is financed. For, to the extent to which it is covered by tax revenues, it does not affect the overall level of demand; the Government demand being then equal to the amount by which the private demand has been reduced due to the taxation. Thus, if the Government policy is aimed at to create an additional demand, the rise in expenditure must be in excess of the amount of taxes—and, if they are raised too, in excess of the rise in those affecting the purchasing power of broad masses of population, It follows that the only way to have an additional demand is a deficit spending—and, possibly, also a rise in capitalists' incomes taxation.

The latter may be the case, if capitalists, and especially big monopolist cor-

[2] The same holds for the supply of foreign capital which may be influenced on through changes in the rate of interest. The "hot money" flows which follow such changes, do not affect the level of investment activity, whereas the flows of long-term capital can be induced only by general profit considerations which underlie the investment decisions.

porations, do not throw over their increased tax burden through raised prices on consumers, while an increasing demand due to the increased Government expenditure does raise their profits before tax so as to compensate the negative effect of higher taxation on their profits after tax. Since, however, a rise in capitalists' taxation is inconceivable, under conditions actually prevailing in the countries considered without a simultaneous rise in taxes on population, this way of financing the Government expenditure cannot bring about significant results for the level of overall demand[3].

The problem may be illustrated by the experience of increased armament expenditure in the United States and Great Britain in the early fifties. In United States it was financed, to a large extent, by means of a budget deficit and increased taxation of corporate profits, and the result was a rise in the national product. No such result was obtained in Great Britain, where the rise in expenditure was financed overwhelmingly by increased taxes on population incomes; on the contrary—there was a fall in national product owing to the fact, that armaments came into being at a time when the heavy industry was under a strong pressure of demand for investment and exports, so that either had to be reduced.

The economic doctrine underlying the Government policies in the most of the countries considered, does recognize the necessity of deficit spending, within certain limits, in the recession, but considers it to be inadmissible in the upward stages of the cycle. This approach is based on considerations of a possible depressing effect of such spending on private investment or its possible inflationary effect on the economy as a whole.

Despite what is argued, a deficit spending while reducing the supply of capital for private borrowing has no adverse effect on private investment because, as already said, changes in money supply do not affect the level of investment. On the contrary, it raises the profits above the level at which they would otherwise maintain, by an amount nearly equivalent to the budget deficit, thus creating powerful incentives for capitalists to expand their investment.

Neither it is necessarily bound to produce inflationary pressures as long as production capacities of the economy are not utilized to the extent. And in so far as this may actually happen despite existing idle capacities, it may be due to the kind of expenditure rather than to the deficit spending itself. This may be the case of an increased Government expenditure—may it be financed by means of taxes or budget deficit—which is directed toward industries already working at full capacity.

The Government policy following the above doctrine tends to shrink the budget deficit as soon as the recession has given way to an expansion, and even to shift to a budget surplus in an advanced phase of the up-swing. This leads to

[3] The problems of financing of economic development are a field to which the contribution of M. Kalecki is as outstanding as it has been to the theory of economic dynamics. The present considerations of the subject follow the lines of his argumentation.

a relative stabilization of fluctuations in investment over the cycle at an average level which is rather close to that determined by market forces. This means that such a policy does not affect the long-run level of investment, except, possibly, for some psychological effects of a relative stability in the future on the profit expectations of capitalists and their investment decisions. In consequence, such a policy cannot but little contribute to a higher rate of growth of the economy.

This kind of policy is exemplified by the United States and Great Britain, where the balanced budget considerations in the long-run have been the main guideline of the Government policies. The result was a relatively low rate at which their national products have been growing in the postwar period.

In fact, the objective of a high rate of growth can be obtained only if there is a purposeful growth-oriented policy which does not take into account the doctrinal considerations of a "sound finance", and which is striving to keep investment at a level which is necessary to assure a rate of growth. However, the possibilities for the Government to obtain such a level of investment are limited in a capitalist economy by the nature of the system itself.

The scope for expanding the Government expenditure which may create an additional required demand, is relatively large for unproductive purposes, like investment in dwelling, social and administrative construction—or armaments. This, however, may contribute to solving the problem of unemployment, but it does not ensure a high rate of growth. It is only by means of productive investment that such a goal can be obtained. And in this respect the Government direct activities are limited, as a rule, to the fields like economic infrastructure, extraction of raw materials or power production, which are rather of secondary importance for the long-term growth. Investment in these fields cannot, in fact, but create favourable conditions for expanding industries like manufacturing, which in that respect are of decisive importance, and which are governed by profit motivations of private capitalists.

Thus, it is only to the extent to which the Government can directly influence upon investment decisions of private capitalists that significant results in rasing the rate of growth may be obtained. Direct controls, however, are usually resorted to, in general, in critical situations. And even if applied, they are capable rather of preventing only an undesirable investment from being undertaken than to enforce its implementation in desirable lines—unless this complies with the interests of the given capitalists. In the latter case, direct controls have to be substituted for by direct financing, if the desired results have to be obtained.

The problem may be illustrated, once again, by the experience of Great Britain in in the early postwar period. Despite an expansion in Government direct investment and despite the direct controls largely imposed on private sector, no essential change has been obtained in the industrial structure of the economy required to improve its foreign balance position in the long-run. The consequences were recurring crises in the balance of payments over the whole postwar period which gave rise to restrictive Government policies, thus leading to a low rate of growth in the British economy, the lowcst among all the countries considered.

An opposite example is offered by France, where besides an expansion in Government direct investment, a direct financing of private investment, by means of hig budget deficits, was for several years an essential feature of the Government policy. The main ways of that financing were open subsidics and cheap long-term credits which, under strong inflationary pressures which were the result of that policy, underwent a rapid depreciation. It is mainly in this way that France has achieved a realtively high rate of growth in the postwar period.

To sum up—the only effective way in which the Government can obtain a higher rate of growth in a capitalist economy, is a policy creating an additional demand in order to raise investment in the long-run above the level determined by the given intensity of innovations and rentiers' savings and the degree of monopoly. This is, however, not possible, if the policy is governed by the balanced budget considerations. Mobilization of idle resources of capitalists and their current profits for productive purposes is needed, if a high rate of growth has to be obtained.

J. Tinbergen
Holland

ECONOMIC DEVELOPMENT AND INVESTMENT INDIVISIBILITIES

1. *Formulation of the Problem*

It is well known that part of the productive equipment of an economy shows the feature of "*indivisibility*" or "bulkiness", that is that some capital goods must have a certain minimum size in order to be useful at all. As so many concepts the concept of indivisibility is a simplification. Strictly speaking there may not be this minimum size as a precisely defined quantity, but rather the phenomenon that the bigger sizes of a given type of capital good are more productive than the smaller sizes. This is well illustrated by the rule of thumb that the quantity of capital needed in a number of heavy industries grows with the 0.6th power of the volume of production. The cases usually quoted as examples for strict indivisibilities are roads and railways. In order to be useful a road must connect two centres, that is, extend over the full distance between these centres. But of course the "size" may be reduced by reducing the width or the quality of the road and this may be carried as far as to leave us with a path only. So in principle there is not a real minimum investment; but it is so much more useful to let the road be one for trucks, say, that as a practical approximation we may in fact assume that there is this sort of a minimum. The same applies to the railway; often we will state that the minimum of a railway connection between two cities is one track—forgetting about the theoretical possibility to make the track narrower and narrower until we were left with a toy railway. We assume therefore the existence of such minimum-sized investment projects.

They are particularly interesting when they will not be operated at full capacity. For roads, railways, harbours, information centres and so on it is certainly realistic to say that they may already be very useful when they are not at all used to the full. In such cases we will speak of *technical overcapacity.* They are interesting because the marginal costs of an additional unit of production will be lower, sometimes much lower, than average costs. In these cases the famous dilemma of welfare economics becomes important, what prices should be charged for these products: should they cover marginal costs only or should they also cover fixed costs? [1]

The problem which will be discussed in this article is the one of *investment decisions* with regard to indivisible assets. More precisely we are going to discuss how new investment in a growing economy must be distributed over the various sectors of the economy, among which there are some showing indivisibility as a feature. An important reason for discussing this question is that *decentralized decisions* such as those by private firms based on criteria such as the profit rate are not compatible with the welfare optimum of the economy. This negative statement may first be discussed, as an introductory exercise.

There are three ways in which decentralized decisions could be conceived of, depending on how prices of the services (or products) of the indivisible assets are being determined. The first way would be to charge *prices equal to marginal costs*. This would lead to permanent losses of the operators of the assets and there would not exist any willingness on their part to make investments in this sort of assets. The second way would be to charge *monopoly prices*; in this way part or all of the fixed costs could be covered and even more than that and there might be a willingness to invest. But there is no guarantee that this policy is the correct one: monopoly prices are not in conformity with the welfare optimum and they may well create an undue desire to invest; alternatively there may be restriction of production and hence of investment below the optimum. Since monopoly prices are not the correct prices to apply the corresponding investment policy may not be correct either. The third possibility for price formation is a *two-part price system*. This may be such as to comply with the condition for the welfare optimum that additional units can be obtained at marginal costs. It may also solve the problem for the enterprise to cover fixed costs; but the fixed amount to be raised and its distribution over the various customers is arbitrary—there do not exist precise welfare economic criteria for it—and again this means that there is no guarantee that an investment policy based on such a policy conducted by a single enterprise is optimal. The reason why the fixed amount is arbitrary may be given a mathematical formulation. The optimum equations of welfare economics—expressing the conditions for maximum welfare—only specify the total fixed amount plus lump-sum redistribution tax to be paid by each individual consumer, but not the component due to any single supplier. [3]

The welfare optimum does define, however, as we will show for a few simple cases, the amounts of capital to be invested in each of the sectors of the economy. We may therefore characterize the situation by saying that *these investments*—at least for the sectors showing indivisibilities—*can only be determined by central planning*. The important practical question which must occupy us most in this context is, however, where we can indicate *measurable criteria* for this investment policy. This we are going to investigate with the aid of a very simple model in order, first of all, to clarify the nature of the solution.

2. *A Simple Demonstration Model*

To this end we assume that a *social welfare function* Ω is known expressing the satisfaction of the population as a function of the quantities x_1, x_2 and x_3

available for consumption and the quantities of labour a_1, a_2 and a_3 needed to produce x_1, x_2 and x_3 respectively. We then assume that $a_h = a_h x_h$ ($h = 1, 2, 3$) and substitute this into Ω; the result being the "net" satisfaction, which is supposed to be a quadratic function of x_h of a simple (*separable*) type:

$$\Omega = \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 - \frac{1}{2}(\omega_{11} x_1^2 + \omega_{22} x_2^2 + \omega_{33} x_3^2) \ldots\ldots \quad (2.1)$$

We assume that labour is abundant.

Furthermore, we assume that the economy has at its disposal a *quantity K of capital, rising over time.* This capital is invested in the three sectors or industries in quantities K_1, K_2 and K_3 where

$$K = K_1 + K_2 + K_3 \quad (2.2)$$

Sectors a and 2 are characterized by indivisibilites meaning that K_1 must be a multiple of a given ("minimum") quantity k_1 and K_2 a multiple of k_2. When used fully these quantities of capital will produce quantities of product

$$x_1 = \beta_1 K_1 \quad (2.3)$$

$$x_2 = \beta_2 K_2 \quad (2.4)$$

where β_1, β_2 are output-capital ratios and are assumed given and constant. Capital K_3 in the third sector can take any value and

$$x_3 = \beta_3 K_3 = \beta_3 (K - K_1 - K_2) \quad (2.5)$$

with β_3 constant and given as well.

Our *problem* will be to *determine* K_1 *and* K_2 *for rising values of* K so as, at any moment, to maximize Ω.

In the situation described there exists for each of the x_h a "*saturation level*" $\bar{x}_h$ for which $\frac{\delta \Omega}{\delta x_h} = \omega_h - \omega_{hh} \bar{x}_h = 0$ When this equation is fulfilled for all values of h we are in the situation sometimes described as "*bliss*" [2]. We assume that the saturation level for x_3 is not reached in the interval of K considered. As long as

$$\beta_h K_h < \bar{x}_h \qquad (h = 1, 2) \quad (2.6)$$

the capitals invested in sectors 1 and 2 will be fully used, because underutilization would mean that Ω can be raised without using more capital, whereas the marginal gross utility derived from an increase in x_h still surpasses the marginal disutility of the corresponding increase in a_h.

Situations satisfying (2.6) will not show technical overcapacity. Whenever for one of the h a situation is reached where

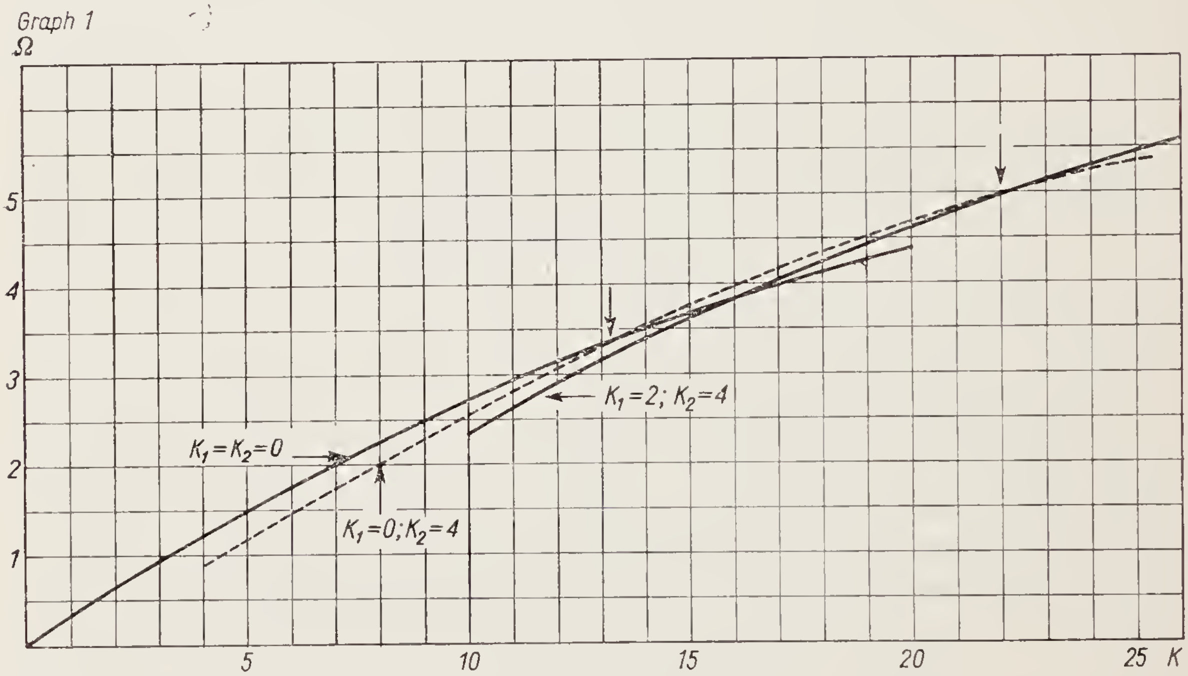
$$\beta_h K_h > \bar{x}_h \quad (2.7)$$

a situation of technical over-capacity for that h will exist. We will treat these two situations one after the other.

3. *Solution of Main Problem in Terms of Utility Coefficients: Situation without Technical Over-Capacity*

During the "development" of the economy considered, that is, with increasing values of K, satisfaction derived from it will rise. For many values of K this increase will take place by an increased investment in sector 3, while the capitals K_1 and K_2 remain unchanged. Only occasionally will it be preferable to let either K_1 or K_2 make a jump by k_1 or k_2, at the expense of the capital K_3 invested in sector 3. It may even happen that K_1 and K_2 will jump simultaneously. The investment policy at any time—that is, at any value of K—evidently will depend on the relative "attractiveness" of increased production in each of the three industries. For constant values of K_1 and K_2, Ω will be a continuous function of K_1 represented in diagram 1, by a rising curve

$$\Omega = \Omega\{\beta_1 K_1, \beta_2 K_2, \beta_3(K-K_1-K_2)\} \tag{3.1}$$



Graph 1

There are a large number of such curves, each corresponding with given values of K_1 and K_2, respectively. These curves will sometimes intersect; that is, for some value of K a curve for a lower value for say K_1 will be "overtaken" by a curve for the next higher value of K_1, that is K_1+k_1. We must find the values of K for which such intersection occurs In any such point we will have

$$\Omega(K,K_1,K_2) = \Omega(K,K_1+k_1,K_2) \tag{3.2}$$

Writing out the expressions at the left-hand and the right-hand side we will have:

$$\omega_1\beta_1K_1-\frac{1}{2}\omega_{11}\beta_1^2K_1^2+\text{terms with } x_2+\omega_3\beta_3(K-K_1-K_2)-\frac{1}{2}\omega_{33}\beta_3$$

$$(K-K_1-K_2)^2=\omega_1\beta_1(K_1+k_1)-\frac{1}{2}\omega)_{11}\beta_1^2(K_1+k_1)^2+\text{terms with}$$

$$x_2+\omega_3\beta_3(K-K_1-k_1-K_2)-\frac{1}{2}\omega_{33}\beta_3(K-K_1-k_1-K_2)^2 \tag{3.3}$$

The terms with $x_2(=\beta_2K_2)$ have not been written out since they are the same on both sides. Leaving out all terms which are the same on both sides we transform (3.3) into:

$$0=\omega_1\beta_1k_1-\frac{1}{2}\omega_{11}\beta_1^2(2K_1k_1+k_1^2)-\omega_3\beta_3k_1-\frac{1}{2}\omega_{33}\beta_3^2\{-2k_1(K-K_1-K_2)+k_1^2\} \tag{3.4}$$

which, by division through $\omega_{33}\beta_3^2k_1$ leaves us with:

$$K=\gamma_3-\gamma_1+\frac{1}{2}(1+\gamma_{11})k_1+(1+\gamma_{11})K_1+K_2 \tag{3.5}$$

where

$$\gamma_1=\frac{\omega_1\beta_1}{\omega_{33}\beta_3^2} \tag{3.6}$$

$$\gamma_{11}=\frac{\omega_{11}\beta_1^2}{\omega_{33}\beta_3^2} \tag{3.7}$$

and

$$\gamma_3=\frac{\omega_3\beta_3}{\omega_{33}\beta_3^2}. \tag{3.8}$$

Similarly, a jump from K_2 to K_2+k_2 will be justified, whenever

$$K=\gamma_3-\gamma_2+\frac{1}{2}(1+\gamma_{22})k_2+(1+\gamma_{22})K_2+K_1 \tag{3.9}$$

where

$$\gamma_2=\frac{\omega_2\beta_2}{\omega_{33}\beta_3^2} \tag{3.10}$$

and

$$\gamma_{22}=\frac{\omega_{22}\beta_2^2}{\omega_{33}\beta_3^2} \tag{3.11}$$

The *solution of our main problem*—as long as *no technical overcapacity* exists in sectors 1 and 2—can therefore be formulated as follows. Starting out with values 0 for both K_1 and K_2 we can find the values for which K—which initially will be fully invested in sector 3 only—must either be invested in sector 1 (to an amount of k_1) or in sector 2 (to an amount of k_2); the lowest of these two values of K will be decisive. In our numerical example, to be treated in section 6, we will find that the lowest of these two critical values of K is the one where K_2 has to jump from zero to k_2. This then must be done and from that moment on we must repeat the exercise for $K_2 = k_2$ and $K_1 = 0$. Again we must calculate the values of K for which either K_1 has to jump by k_1 or K_2 by k_2. Again the lower of these two values for K is decisive. In this way (see again section 6) we will be able to determine the optimum path of investment of K. For the time being our formulae assume that we know the coefficients γ; in section 5 we will discuss how these can be estimated from measurable phenomena instead of from utility functions which have not been measured so far and are not available therefore for practical decisions. First, we discuss what change in our formulae is needed whenever a situation of technical over-capacity develops.

4. *Situation with Technical Over-Capacity*

The calculations just discussed must be changed when the point of saturation for commodity 1 or 2 is approached, that is when the next extension of the sector's capacity by either k_1 or k_2 will create a production capacity surpassing the quantity $\bar{x}_1$ or $\bar{x}_2$. In such a case the satisfaction to be derived from the next extension of capacity will be maximum if that capacity is not fully utilized and this maximum addition of satisfaction will now be obtained by sacrificing relatively more of x_3 which instead could have been produced.

Since $\bar{x}_1 = \dfrac{\omega_1}{\omega_{11}}$ we can estimate the increase in utility now by substituting this value into the utility function and the addition of a new quantity k_1 to K_1 will now lead to an increase in satisfaction of

$$\omega_1\left(\frac{\omega_1}{\omega_{11}}-\beta_1 K_1\right)-\frac{1}{2}\,\omega_{11}\left(\frac{\omega_1^2}{\omega_{11}^2}-\beta_1^2 K_1^2\right). \tag{4.1}$$

In order that it be worthwhile to give up the increase in utility to be obtained from investment of the amount of k_1 into sector 3 this increase

$$\omega_3\beta_3 k_1-\frac{1}{2}\,\omega_{33}\beta_3^2\,\{-2k_1(K-K_1-K_2)+k_1^2\} \tag{4.2}$$

must now be just below the value (4.1); the point where it becomes worthwhile increasing K_1 by k_1 now will be found by equating (4.1) and (4.2) and solving for K. This leaves us with an equation:

$$K = K_1+K_2+\frac{1}{2}\,k_1+\gamma_3-\frac{1}{k_1}\left(\frac{\omega_1}{\omega_{11}\beta_1}-K_1\right)+\frac{1}{2}\,\frac{\gamma_{11}}{k_1}\left(\frac{\omega_1^2}{\omega_{11}^2\beta_1^2}-K_1^2\right) \tag{4.3}$$

and similarly for a jump in K_2 under the same circumstances:

$$K = K_1+K_2+\frac{1}{2}k_2+\gamma_3-\frac{\gamma_2}{k_2}\left(\frac{\omega_2}{\omega_{22}\beta_2}-K_2\right)+\frac{1}{2}\,\frac{\gamma_{22}}{k_2}\left(\frac{\omega_2^2}{\omega_{22}^2\beta_2^2}-K_2^2\right) \tag{4.4}$$

5. *Approximation of Utility Coefficients by Measurable Concepts: the "Demand Pattern"*

Equations (3.5), (3.9), (4.3) and (4.4) solve our problem—in the simplified case considered—and a numerical example will be given in section 6. Before discussing the results we will first ask ourselves the very important question whether the coefficients we need to know in order to use the equations just mentioned can be determined from measurable phenomena. We will show that this is the case if we assume that we *know the demand pattern* for the products of our sectors *from the experience of a much "larger" country* than the one considered so far and if we are confident that *that pattern can be applied to our country.* By the demand pattern we mean the influence exerted by a rising capital stock (and hence income level) on the quantities demanded of each of the products considered. Any confidence that we can apply the evidence from another country must be based on some belief in the similarity of demand patterns for different countries after correction for income differences. In other words: we must of course use the demand pattern for incomes (or capital stocks) comparable to the ones which prevail in the country for which the planning must be done.

Our reference to a much "larger" country must be understood as a *methodological device to eliminate the phenomenon of indivisibilities.* For a large country we may assume that the latter play a much lesser role than for small countries, since in comparison to a large country's demand the capacity of a single enterprise is much less important than in comparison to a small country's demand.

If indeed we assume that production in the three sectors can be expanded continuously, it is easy to find out how a growing quantity of capital K will be distributed over the sectors. At any value of K the marginal utilities of the capitals invested in each of the sectors will be equal. Since now $K_h = \frac{x_h}{\beta_h}$ $(h = 1, 2, 3)$ we have:

$$\frac{\delta \Omega}{\delta K_h} = \beta_h \frac{\delta \Omega}{\delta x_h} = \beta_h\,(\omega_h - \omega_{hh} x_h) = \Lambda \tag{5.1}$$

where Λ is the marginal utility of capital and at the same time

$$\sum_1^3 {}_h \frac{x_h}{\beta_h} = K. \tag{5.2}$$

From equations (5.1) and (5.2) we can solve the x_h as functions of K. The result is

$$x_h = \frac{\omega_h}{\omega_{hh}} + \frac{K - S'}{\beta_h\, {}_{hh} S} \tag{5.3}$$

where

$$S = \sum_1^3 {}_k \frac{1}{\beta_k^2 \omega_{kk}} \tag{5.4}$$

$$S' = \sum_{1}^{3} k \frac{\omega_k}{\beta_k \omega_{kk}}. \tag{5.5}$$

Equations (5.3) are what we called the *demand pattern* in a large country; if we write them:

$$x_h = \xi_h^0 + \xi_h^1 K \tag{5.6}$$

it follows that

$$\xi_h^0 = \frac{\omega_h}{\omega_{hh}} - \frac{S'}{\beta_h \omega_{hh} S} \tag{5.7}$$

and

$$\xi_h^1 = \frac{1}{\beta_h \omega_{hh} S}. \tag{5.8}$$

We will assume that the ξ_h^0 and ξ_h^1 can be estimated statistically from the large country's experience:

It can now be shown that all the coefficients appearing in the solutions (3.5), (3.9), (4.3) and (4.4) can be expressed in terms of the measurable coefficients ξ_h^0, ξ_h^1 and β_h.

This is just a question of solving (5.4), (5.5), (5.7) and (5.8) for the unknowns ω_h, ω_{hh}, S and S' and expressing the coefficients of our solution equations in terms of the measurable coefficients. The results are:

$$\gamma_3 - \gamma_1 = \frac{\xi_3^0}{\beta_3} - \frac{\xi_3^1 \xi_1^0}{\beta_3 \xi_1^1} \tag{5.9}$$

$$\gamma_3 - \gamma_2 = \frac{\xi_3^0}{\beta_3} - \frac{\xi_3^1 \xi_2^0}{\beta_3 \xi_2^1} \tag{5.10}$$

$$\gamma_{11} = \frac{\beta_1 \xi_3^1}{\beta_3 \xi_1^1} \tag{5.11}$$

$$\gamma_{22} = \frac{\beta_2 \xi_3^1}{\beta_3 \xi_2^1} \tag{5.12}$$

$$S' = \frac{\sum \frac{\xi_k^0}{\beta_k}}{1 - \sum \frac{\xi_k^1}{\beta_k}} \tag{5.13}$$

$$3 = \frac{\xi_3^0}{\beta_3} + \frac{\xi_3^1}{\beta_3} S' \tag{5.14}$$

$$\frac{\omega_1}{\omega_{11} \beta_1} = \frac{\xi_1^0}{\beta_1} + \frac{\xi_1^1}{\beta_1} S' \tag{5.15}$$

$$\frac{\omega_2}{\omega_{22} \beta_2} = \frac{\xi_2^0}{\beta_2} + \frac{\xi_2^1}{\beta_2} S' \tag{5.16}$$

$$\gamma_1 = \frac{\xi_3^1}{\beta_3}\left(\frac{\xi_1^0}{\xi_1^1} + S'\right) \tag{5.17}$$

$$\gamma_2 = \frac{\xi_3^1}{\beta_3}\left(\frac{\xi_2^0}{\xi_2^1} + S'\right) \tag{5.18}$$

6. *Numerical Example*

We will now illustrate our solution of the problem of investment in indivisible assets by a numerical example. For simplicity's sake we will use the formulae in their original form, expressed in terms of the utility coefficients ω_h and ω_{hh}; from the previous section it can be understood, however, that we could just as well have assumed the ξ_h^0 and ξ_h^1 as given.

Our numerical assumption are:

$k_1 = 2;\ k_2 = 4;\ \beta_1 = 0.5;\ \beta_2 = 0.4;\ \beta_3 = 0.32$

$\omega_1 = 0.35;\ \omega_2 = 0.6;\ \omega_3 = 10.;\ \omega_{11} = \omega_{22} = \omega_{33} = 0.1$

From them we can derive: $\bar{x}_1 = 3.5;\ \bar{x}_2 = 6;\ \bar{x}_3 = 10.$

$\gamma_1 = 17.5;\ \gamma_2 = 24;\ \gamma_3 = 32;\ \gamma_{11} = 2.5;\ \gamma_{22} = 1.6.$

The general form of equations (3.5) and (3.9) becomes:

$$K = 14.5 + 1.75k_1 + 3.5K_1 + K_2 \tag{3.5'}$$

$$K = 8 + 1.3k_2 + K_1 + 2.6K_2. \tag{3.9'}$$

In order to determine the values of K for which investments in sector 1 or 2 will be made, we must now start with the assumption $K_1 = K_2 = 0$, which for small values of K must anyway be true and find out which of the two values for K supplied by (3.5′) and (3.9′) is lower. We find:
from (3.5′) $: K = 18$ and from (3.9′) $: K = 13.2$

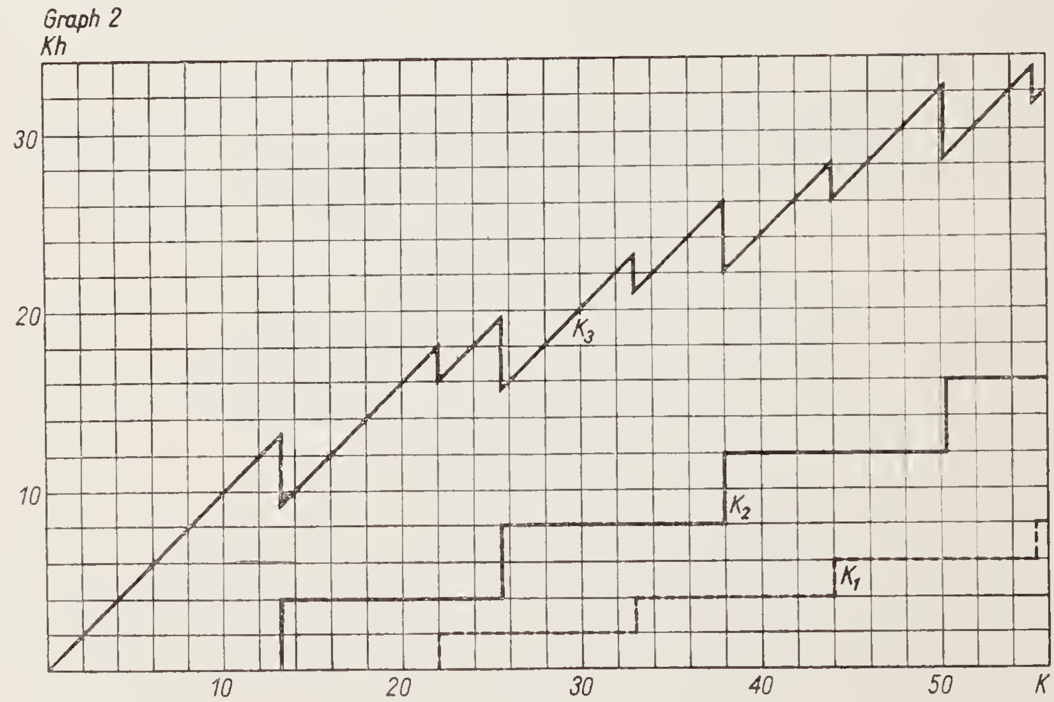
This means that the first jump in investment will be in sector 2, leading, at $K = 13.2$ to $K_2 = 4$, while K_1 remains 0. We now repeat application of the equations (3.5′) and (3.9′) but for $K_1 = 0$ and $K_2 = 4$ and try to find out again which of the two equations yields the lower value for K. We obtain from (3.5′)$: K = 22$ and from (3.9′) $: K = 23.6$. This time there will be, at $K = 22$, a jump of K_1 from 0 to 2. Continuing this way we will find that jumps take place at the following values of K:

$K = 13.2$	$K_1 = 0$	$K_2 = 4$
$K = 22$	$K_1 = 2$	$K_2 = 4$
$K = 25.6$	$K_1 = 2$	$K_2 = 8$
$K = 33$	$K_1 = 4$	$K_2 = 8$
$K = 38$	$K_1 = 4$	$K_2 = 12$
$K = 44$	$K_1 = 6$	$K_2 = 12$

Since the saturation values of x_1 and x_2 are, respectively, $\bar{x}_1 = 3.5$ and $\bar{x}_2 = 6$ and the corresponding capitals would be (without indivisibilities) $K_1 = 7$ and $K_2 = 15$, we have now reached the point (for both at the same time, which is a coincidence)) where the next extension of production capacity would not be fully used. Hence we must now apply formulae (4.3) and (4.4) instead of (3.5) and (3.9). We find from (4.3) : $K = 50.4$ and from (4.4) : $K = 50.2$, meaning that the jump will be in K_2 (becoming 16) and in a similar way we obtain as the last value of K for which a jump will take place : $K = 55.4$, where K_1 will be brought at 8. This completes our list as follows:

$K = 50.4$	$K_1 = 6$	$K_2 = 16$
$K = 55.4$	$K_1 = 8$	$K_2 = 16$

Since the saturation level of K_3 is 32, the economy considered will be completely saturated for $K = 56$.



Graph 2

Graph 2 shows the development, as a function of K, of capitals invested in the three sectors.

In this example the phenomenon of *unused capacity* because of indivisibilities does not play a considerable role; it only occurs when K has reached the level of 50.4. If we had chosen the value for k_1 or k_2 or both larger, we might have met the occurrence of technical overcapacity at relatively lower levels of K, that is, longer before saturation for industry 3 occurs.

This may be shown for a few still simpler cases, in which industry 2 does not occur at all, meaning that $\omega_2 = \omega_{22} = K_2 = 0$. For this case, and assuming at the same time that now $\beta_1 k_1 > \bar{x}_1$, we know that only equation (4.3) has to be used to find the only jump in investment which will take place. It takes the form:

$$K = \frac{1}{2} k_1 + \gamma_3 - \frac{\gamma_1}{k_1} \frac{\omega_1}{\omega_{11}\beta_1} + \frac{1}{2} \frac{\gamma_{11}}{k_1} \frac{\omega_1^2}{\omega_{11}^2 \beta_1^2} \tag{4.3'}$$

which, with the numerical values we have assumed, becomes:

$$K = \frac{1}{2} k_1 - \frac{61.3}{k_1} + 32 \tag{4.3''}$$

For $k_1 = 10$ this yields $K = 30.9$, whereas the saturation value for K_3 remains $\frac{\bar{x}_3}{\beta_3} = 32$ and hence the saturation level in sector 3 will only be reached when[1]

$$K = k_1 + 32 = 42$$

7. *What Would Happen under a Private Regime?*

Finally we may illustrate, with the figures of our last example, the *difference between the centrally planned development and the development under a private regime.* As we already saw, the centrally planned development is characterized by two phases; up till values of $K = 30.9$ all capital will be invested in sector 3, showing no indivisibilities. At that value an investment $K_1 = 10$ in sector 1 will occur and production of the corresponding good will be at the saturation level $\bar{x}_1 = 3.5$.

Development under private decisions, we assume, will be one of two possibilities. The *first* possibility is that a single price p_1 will be charged for the product of sector 1 and a price p_3 for sector 3. Demand will distribute itself over the two products according to the rule that marginal utilities show the same ratio as the two prices:

$$\frac{\omega_3 - \omega_{33} x_3}{\omega_1 - \omega_{11} x_1} = \frac{p_3}{p_1} \tag{7.1}$$

[1] For a much higher value of k_1 we meet still another phenomenon, not to be discussed here now. It may then be best first to invest all capital into sector 3, until saturation is reached, i.e. until $x_3 = \bar{x}_3$. After that, for some time there will be no possibility to invest at all and idle capital will accumulate until it becomes worth while to disinvest out of sector 3 and to invest both the capital released from that sector and the accumulated idle capital into sector 1. This intermediary phase of "hoarding" will occur as soon as the minimum capital k_1 needed for sector 1 surpasses the value $\frac{\bar{x}_3 - \tilde{x}_3}{\beta_3}$, where $\tilde{x}_3$ represents the level of x_3, to which we are prepared to fall back if instead we obtain the utility derived from investing k_1 into sector 1. This latter utility being $\frac{1}{2}\frac{\omega_1^2}{\omega_{11}}$, $\tilde{x}_3$ satisfies:

$$\frac{1}{2}\frac{\omega_3^2}{\omega_{33}} - \omega_3 \tilde{x}_3 + \frac{1}{2}\omega_{33}\tilde{x}_3^2 = \frac{1}{2}\frac{\omega_1^2}{\omega_{11}}$$

Producers will distribute their capital over the two sectors so as to equalize returns which in our simple case will take the form

$$\frac{p_1 x_1}{k_1} = \beta_3 p_3 \,. \tag{7.2}$$

Finally, total capital equals the sum of the capitals invested in the two sectors:

$$K = k_1 + \frac{x_3}{\beta_3} \text{ (if } k_1 \neq 0). \tag{7.3}$$

Equations (7.1), (7.2) and (7.3) can be used to express x_1, x_3 and $\frac{p_3}{p_1}$ in terms of K. Filling out the numerical values and eliminating p_3/p_1 we find:

$$\frac{1-0.1x_3}{0.35-0.1x_1} = \frac{x_1}{10\times 0.32} \tag{7.4}$$

$$K = 10 + \frac{x_3}{0.32} \tag{7.5}$$

For x_1 we then find:

$$x_1(0.35-0.1x_1) = 3.2\{1-0.1(0.32K-3.2)\}$$

or

$$x_1^2 - 3.5x_1 + 42 - K = 0.$$

Solution for x_1 yields:

$$x_1 = 1.75 \pm \sqrt{1.75^2 - 42 + K} \qquad (7.6)^2$$

which can be interpreted as follows:
No real solution for x_1 exists, that is, no x_1 will be produced, as long as $K - 42 + 1.75^2 < 0$

or

$$K < 38.94.$$

At that value of K production of x_1 will start, at a volume of 1.75. This volume will rise with rising K and it will only reach the value 3.5 for $K = 42$ which is the value for which saturation in x_1 and x_3 is reached. In plain words: production of x_1 will start too late and remain too low up till the situation of "bliss".

The *second* possibility is that a two-part price is applied by private producers in sector 1 and that a fixed amount of $f_1 p_1$ is charged in addition to the price p_1 per unit sold. Receipts then become $(f_1 + x_1)p_1$ and equation (7.4.) now runs)

$$\frac{1-0.1x_3}{0.35-0.1x_1} = \frac{f_1 + x_1}{10\times 0.32}. \tag{7.7}$$

Using the same method as before we now find instead of (7.6):

$$x_1 = 1.75 - \frac{1}{2} f_1 + \sqrt{\left(1.75 + \frac{1}{2} f_1\right)^2 + K - 42}. \tag{7.8}$$

[2] Clearly only the + sign is economically relevant.

From this formula we may conclude that, for positive values of f_1 the production of x_1 will now start at a lower value of K, that it will always be higher than for the same K in the absence of a fixed amount f_1, but that it will only reach the level of 3.5 again for $K = 42$. By an appropriate choice of f_1 we are able to let x_1 approach the optimum value, but we can never reach it entirely. The value to be given to f_1 appears to be

$$f_1 = -3.5 + \frac{42 - K}{\varepsilon} \tag{7.9}$$

where ε is small; the smaller ε_1 the more will x_1 approach its optimal value.

The general conclusion to be drawn from our two alternatives is that for our model *private decisions will lead to a sub-optimal value of* x_1.

REFERENCES

1. C. J. Oort, *Decreasing Costs as a Problems of Welfare Economics*, Diss. Leiden, Amsterdam 1958.
2. F. P. Ramsey, *A Mathematical Theory of Saving*, "Economic Journal" XXXVIII (1928), p. 543.
3. This can be shown with the aid of the method used in J. Tinbergen, *The Theory of the Optimum Regime*, Selected Papers, Amsterdam 1959.

Józef Zagórski
Poland

THE CONCEPT OF TECHNICAL PROGRES IN ECONOMIC ANALYSIS

The object of this paper is to derive the definition of technical progress against the background of the basic Marxian concept of production and to establish its relation with the, what may be called, economic progress.

We shall begin with the discussion of the concept of factors of production.

Factors of Production and their "Productivity". If we look at the process of production from a purely physical point of view then we can define it as the process of combining different forms of energy (human and that derived directly from nature) and matter (formed or unformed) the result of which is a product, or a new material object (or a new form of energy) with different physical and use characteristics. All the elements participating (combined) in this process are defined as factors of production. Input and output tables reflect statistically the process of combining different elements into a new product. And depending on how detailed the criteria of classification are we can obtain a larger or a smaller number of factors of production (in the same way as the number of rows and columns in input–output tables). However, for the purposes of a more general analysis a very detailed classification is superfluous and, therefore, since the time of the classical school very broad criteria have been used for classification (they are physical and economic categories at the same time) and thus the number of factors of production is reduced to the three basic ones: labour (direct), capital (accumulated labour) and land (natural factors).

All these factors are indispensable in the process of production and none of them can appear in isolation. Therefore, it is impossible to determine on the basis of physical criteria to what extent each of these factors contributes to the creation of the product and which one contributes more.

There was a time in the history of economics, when the introduction of the concept of marginal productivity of production factors seemed to have done it. This, however, proved to be an illusion. The isolation of particular factors, as achieved in the calculation of marginal productivities, is a purely artificial device, very useful for some types of analysis, but unable to serve as a gauge of the real contribution of particular factors. To this end it is necessary to analyse the real nature of the factors and their function in the process of production. This kind of analysis, as is well known, was carried out by Karl Marx.

The Essential Features of the three Classical Factors of Production. Marx does not formally refer to this classification (except casually). Nevertheless, this classification is implied in his analysis.

The definition of labour given by Marx is of key importance here: "Primarily, labour is a process going on between man and nature, a process in which man, through his own activity, initiates, regulates, and controls the material reactions between himself and nature. He confronts nature as one of her own forces, setting in motion arms and legs, head and hands, in order to appropriate nature's productions in a form suitable to his own wants. By thus acting on the external world and changing it, he at the same time changes his own nature. He develops the potentialities that slumber within him, and subjects these inner forces to his own control"[1]. A little farther Marx amplifies this definition: "The labour process resolved into its simple rudimentary factors, is as we have seen, purposive activity carried on for the production of use-values, for the fitting of natural substances to human wants; it is the general condition requisite for effecting an exchange of matter between man and nature; it is the condition perennially imposed by nature upon human life, and is therefore independent of the forms of social life—or, rather, is common to all social forms"[2].

Capital begins with the theory of value and the whole analysis is carried in value terms, although, incidentally, the physical categories are considered as well. However, the problem of technical progress cannot be properly handled in terms of the theory of value. The process of production is, primarily, a physical phenomenon and should be handled in physical terms.

Our first object is to vindicate the Marxian thesis that only labour creates new use-values (which we treat as physical category because it can be reduced to physical characteristics), without resorting to the theory of value.

Human labour is first of all "a purposive activity". Marx describes its specific character in the following way: "We have to consider labour in a form peculiar to the human species. A spider carries on operations resembling those of the weaver; and many a human architect is put to shame by the skill with which a bee constructs its cell. But what from the very first distinguishes the most incompetent architect from the best of bees, is that the architect has built a cell in his head before he constructs it in wax. The labour process ends in the creation of something which, when the process began, already existed in the worker's imagination; already existed in an ideal form. What happens is, not merely that the worker brings about a change of form in natural objects; at the same time, in the nature that exists apart from himself, he realizes his own purpose, the purpose which gives the law to his activities, the purpose to which he has to subordinate his own will"[3].

[1] Karl Marx, *Capital*, Vol. I, translated by Eden and Cedar Paul, Everyman's Library, London 1930, p. 169.

[2] *Ibid.*, p. 177.

[3] *Ibid.*, pp. 169–170.

It follows from the above that only the human labour can endow natural objects with the capacity of serving needs of man, in other words, only human labour can create new use-values; and in this sense only human labour is a productive factor. Of course, in result of the self-existent processes going on in nature, or rather as an incidental by-product of these processes, some natural objects may happen to possess already the form fit for satisfying directly human needs, i.e. possess use-values. This was the basic condition of man's survival in the early stages of his existence when his labour was of an instinctive character. However, as soon as labour has become a purposive activity—and this is the most essential feature of human labour—man began to subordinate the forces of nature to his own ends and all "the free gifts of nature" ceased to play any important role.

What is, then, the part played by other factors? Let us begin with the capital. According to Marx, there are two original factors, "foundations of all wealth—land and the workers"[4]. Capital is a derivative factor consisting of matter ("land") and past labour. Matter or nature in general given once for all and being the background of all human activity, a datum, can be left out of account (see below). Therefore, there remains only the past labour embodied in the form which endows matter with the use-value. However, the use-value of the capital goods is only potential. "The machine—writes Marx—which does not serve the purposes of labour is useless"[5]. This value is activated only in the hands of man and is transferred to the product "not in virtue of the addition of labour considered in the abstract, but in virtue of labour that has a specifically useful character, in virtue of the specific form of this supplementary labour. As such purposive productive activity (spinning, weaving, or forging), the labour is able, by its mere contact with the means of production, to raise them from the dead, to quicken them so that they become living factors of the labour process, to enter into combination with them in order to form products"[6]. Man "makes use of the mechanical, physical and chemical properties of things as means of exerting power over other things, and in order to make these other things subservient to his aims. (...) Thus nature becomes an instrument of his activities, an instrument with which he supplements his own bodily organs, adding a cubit and more to his stature, scripture notwithstanding"[7].

It is true that capital not only substitutes and magnifies human labour but also enables us to produce things that man by himself would never be able to make. In this practical sense we can consider capital as "productive". But essentially it is an apparent productiveness, reflecting the productiveness of human knowledge, skill and toil. For stressing the practical importance of capital we should rather talk of "the productivity of the use of capital".

---

[4] *Ibid.*, p. 548.
[5] *Ibid.*, p. 176.
[6] *Ibid.*, p. 195.
[7] *Ibid.*, p. 171.

There is, of course, a very essential economic difference between living labour and labour embodied in means of production. The latter denotes the "freezing" of living labour for the construction period of means of production and of their gradual "unfreezing" as they are being used up, which temporarily excludes this labour from the process of the reproduction of means of consumption causing a temporary—absolute or relative—decrease in the flow of consumer goods. But—as we mentioned before—the choice between current consumption and future consumption which can be expanded at the cost of current consumption by increasing means of production is an economic problem, and not a technical one. If the construction period of the means of production and their useful life were relatively short the distinction between living labour and embodied labour would be of no practical consequence.

The matter is also relatively simple as far as the productivity of "land" is concerned, "land" in the sense of all natural factors. All that happens on our planet, including organic life in spite of its specific characteristics, and all human activity is, from a physical point of view, a transformation of different forms of matter and energy. The general prime mover of all these transformations is solar energy[8]; the earth provides matter with definite properties. Thus, matter and energy combine to form the notion of nature. From the point of view of primitive man whose life depended on the grace, or rather the whims of nature, it was the source of all well-being, the only life-giving and "productive" factor. This view was quite logical in the epoch of the beginnings of human civilization, but in the minds of agricultural societies which saw the whole power of nature primarily in the "earth" it persisted until the era of industrialization[9].

In the classical doctrine land appears only as one of the three factors of production—beside labour and capital. This approach, however, is, in some sense, illogical from the point of view of any epoch. Nature acts not only through the "land", but also through capital and human labour which is also only one of the links in the transformation of matter and energy, the same as the power of draft animals (capital) or a waterfall (nature). The only rational solution of this problem has been given by Marx. He did not place man beside nature, but opposed him to it. He did it because human labour is not just another link in the chain of transformations of matter and energy. It is more than that, it is something that transcends the concept of nature—it is a "purposive activity". Man learns to know the laws regulating the transformation of matter and energy and uses them for creating use-values which satisfy his needs and which—before he actually creates them—he must first create "in his own mind". Nature, as the environment in which a man

---

[8] Including stored-up solar energy in the form of fuels and excluding other cosmic influences, the nucleus of the earth and the "native", earthly source of energy—nuclear energy.

[9] In the history of economic thought the last serious exponents of this view were the Physiocrats. Although Quesnay himself sensed the weakness of this view, he finally accepted it, even though formally. Cf. J. Zagórski, *Ekonomia F. Quesnay'a* (The Economics of F. Quesnay), Warsaw 1963.

lives and acts and of which he is a part, is completely indifferent with respect to his needs; some natural phenomena are favourable to man and some detrimental. The balance of advantageous over disadvantageous phenomena provided man only with limited possibilities of subsistence of the animal type. Man embarked upon a struggle with nature to expand these possibilities; he began consciously—as far as his knowledge of the laws of nature permitted—to direct the processes of transformation of matter and energy in a way most advantageous to himself, he began to produce. Being the pre-condition of production, nature is a basic factor of production, but she is a completely passive factor—an object of human activity—whose amount can be neither increased nor decreased, though it can be exploited to a lesser or greater advantage. The Marxian definition of the process of labour given at the beginning of this paragraph puts the problem of production in its proper historical and philosophical perspective. As we have tried to show, it follows logically from this definition that the only creative and productive factor in this process is man and his labour.

It can be argued, of course, that even conventional, purely arbitrary definitions may lead to correct practical solutions. This is undoubtedly true. But the definition that emphasizes the key elements of a given problem facilitates the whole analysis and provides the shortest and surest way to the final objective.

The above considerations were, perhaps, of a somewhat elementary nature. However, they are an indispensable link in this study.

The ability to acquire knowledge and to put it to practical use is the essential source of the productive power of man. What we shall say below will be in the nature of dotting the *i*, emphasizing certain logical inferences which proceed from our arguments.

Man, as a power plant producing energy and transforming it into mechanical work (force times distance moved) is, of course, subject to the laws of thermodynamics—the amount of effective energy produced is smaller than the amount of energy absorbed (in different forms). Also, with regard to the efficiency of his energo-mechanism man comes second to animals[10]. In this respect, of course, he was completely at the mercy of nature.

The factors that enabled man to change his role from being a passive toy of the forces of nature to being a tamer imposing his will and his objectives were two elements of his physical structure: his highly developed brain and the dexterity of his hands. His brain enabled him to acquire and to accumulate knowledge about nature (and about himself) and his hands, these "first instruments of human labour", enabled him to put this knowledge to practical use through designing new and ever more complicated instruments multiplying his native power and his ability to subjugate the forces of nature to his own practical ends. The notion of knowledge in this context is understood in a very broad sense of comprising not only

[10] e. g. draft animals or beasts of burden.

a systematized theoretical lore which could have been achieved only at a fairly advanced stage of the development of human civilisation, but also all kinds of knowledge about the properties of nature; including the kind manifest in simple human labour that endows this labour with the characteristics of purposive activity[11].

Thus we can distinquish three essential elements of human labour: knowledge, the practical ability to use it in the process of production, and expenditure of energy. The last element, as we know, is a deficit item in the input–output balance-sheet of energy, and were man not endowed with other weapons in his struggle for living, he would be compltetely at the mercy of nature. The capacity for executing mechanical work is in man a very static element, as nature itself, although it is still an indispensable condition of his survival. Knowledge and the ability to use it for practical purposes are dynamic elements and they alone enable man to undertake production and to develop it by the ever widening process of substituting man's effort by energy derived directly from the resources of nature.

The Technique of Production. By the term technique of production we understand a strictly defined quantitative combination of factors of production in a given process of production whose result and aim is the product. In other words, the technique of production is a kind of a technical prescription for a given process of production and we can express it as a vector of technical parameters similar to, although not identical with, Leontief's technical coefficients. The latter express the extent to which a given factor of production is used-up per unit of product and our parameters express the necessary amount of a given factor of production, per unit of product, that is engaged in the process of production. Thus the proposed parameters do not reflect the consumption of fixed capital and materials, but the amount of fixed and working capital engaged in the process, per unit of product, per unit of time.

Sticking strictly to physical terms we have to consider as many various factors of production, as there are elements—different in respect of physical characteristics participating in a given production process. Thus, we have to divide living labour into as many different factors as many various specializations with different skill are required in a given process. The same applies to the subject matter of labour, both produced by man or natural, like, for instance, the area of land occupied by a given process (in agriculture with consideration given to the kind of soil), as well as to the instruments of labour (each type of machinery is a different factor).

The significance of this definition as a tool of analysis is, of course, very limited. We have introduced it here as a basic concept defining precisely the whole problem and constituting a point of departure for all generalizations.

In every day use (and in other uses) the notion of technique is usually directly associated with equipment of labour: technique is equipment. This is justified, as

---

[11] It is worth remembering that in ancient Greece the Goddess of Wisdom, Athene, was also the Goddess of Crafts.

far as common use is concerned, because to day the type of equipment determines, in a broad sense, (although not exclusively) the whole process of production. The simplification, therefore, is vindicated. But the transposition of this simplification to the theoretical analysis causes complications.

It is a well known fact that the same production equipment gives different production results (different technical parameters) in a well developed country and in an underdeveloped country. Also, the results obtained with the same equipment are different at the time when its exploitation is beginning and different after a certain period of time. This fact is explained by not very precisely defined organization factors or in some similar way. But, it follows from our definition that always when technical parameters are distinctly different we are dealing with a different technique. And even though from the point of view of the common use of the term "technique" this approach may appear to be queer (the equipment is the same), it ensures greater precision in theoretical analysis. In both examples given above main differences in the technique of production stemmed from the fact that labour force used with the same equipment in two different places or moments of time was characterized by different skill.

Identifying technique only with the equipment diminishes the role of living labour: technique is not only the machine but also (disregarding the subject matter of labour) man who knows how the machine works, can put it in motion and operate it; in other words, technique also means the technical skill of human labour.

Marx defines simple labour force as: "such as, on the average, the ordinary man, without any special development of faculty, is equipped with in his bodily organism". On the other hand "Skilled labour counts only as intensified, or rather multiplied simple labour, so that a smaller quantity of skilled labour is equal to a larger quantity of simple labour"[12]. The skilled labour, therefore, can be expressed in terms of simple labour and divided into two parts: that of simple labour that need be constantly reproduced, and accumulated simple labour embodied in the skill of man. This distinction may be of practical importance for the problems of economic growth of developing countries. "The accumulated simple labour" possesses of the character of capital; indeed, it is "a living capital" in contrast to the inanimate capital.

Our definition of technique enables us also to eliminate certain concepts which sometimes appear in economic considerations concerning technical progress. What we have in mind here is the notion of "technology" as something different from "technique", and the notion of the "organization of work".

In our terminology "technique" includes the notion of technology (the method of processing matter). There is no change in technology which would not be reflected in our technical parameters (and even if this could conceivably happen it would not have any economic consequences).

[12] Op. cit., p. 13.

Somewhat greater difficulties are presented by the problem of "the organization of work" because this concept is not very precisely defined. However, if by the "organization of work" we understand the organization of the process of production itself, i.e. the activities that fall within the competence of the technical and engineering personnel, then this concept is included in our definition of technique and is expressed in technical parameters pertaining to living labour with appropriate skill. But excluded from the concept of technique are all other organization factors which in this paper we shall define as "management" on all levels of the social organization of production.

As we have already mentioned, our definition of production technique, though possessing of some abstract virtues can serve only very limited purposes. If we wanted, for instance, to deduce on this basis the definition of technical progress it would be: technical progress is every lasting lowering of any one of the technical parameters not involving an increase in the others or the introduction of new parameters. As we can see, this definition would limit very radically the notion of technical progress by excluding from it progress involving substitution of factors. We have therefore to seek for some other less rigid a definition.

Synthetic Characteristic of Production Technique. First of all we have to reduce a practically infinite number of factors to basic economic categories: labour and capital. There arises the problem of reducing different kinds of labour to one, and different means of production to units of capital.

How should labour be measured? To reduce skilled labour to simple labour is undoubtedly a very difficult problem from the practical point of view. In empirical and theoretical studies, differences in qualifications are, as a rule, disregarded, and labour is counted in physical units which is, in fact, tantamount to counting only simple labour (the number of units is the same) and eliminating what we have defined as accumulated in the living form simple labour. There would be some economic sense in it were we to include this accumulated labour in capital. From the traditional point of view, however, we should convert skillled labour into units of simple labour.

The different physical capital goods may be reckoned in units of time of labour required to produce them. This, however, leaves out the differences in the most essential feature of capital goods—their time patterns. Mrs. Joan Robinson proposes to account for these differences in the time pattern by means of interest[13]. Much more ambitious, and very ingenuous indeed, is the solution elaborated by Prof. Michał Kalecki and Mieczysław Rakowski[14] with a view to find the formula

[13] Joan Robinson, *The Accumulation of Capital*, Macmillan, London 1956, pp. 121–122 and 422–425. See also M. Dobb, *An Essay on Economic Growth and Planning*, Routledge and Kegan Paul, London 1960, section III.

[14] *Uogólnienie wzoru efektywności inwestycji* (The Generalization of the Formula of Efficiency of Investment), "Gospodarka Planowa", No. 11, 1959. See also M. Rakowski (edit.), *Efektywność inwestycji* (The Efficiency of Investment), Warsaw 1963.

of the efficiency of investment, which could guide the choice of techniques in a planned economy[15]. The whole problem is considered within the fremework of a model of growth with constant parameters; the time pattern of the gestation period of investments, of their service life and of output and input streams, connected with particular investments, are being dealt with separately and accounted for on the basis of the social opportunity costs. Thus this solution goes too far to be applied directly to our problem.

Without going into further detail of this involved problem we can assume here, that the problem is solved, so that the time pattern of a capital good can be treated on the same footing as its cost (measured in units of time of labour) thus adding up to "the social opportunity cost of capital".

If we include into the notion and measure of capital the factor of time, capital ceases to represent the embodied labour only and thus cannot be added at current labour, though both can be measured in the same units.

So far we have reduced the indefinite number of factors of production to the two basic homogenous factors: labour and capital. There are still two different factors, which cannot be added together. But there is no obstacle for multiplying them one through the other. In this way, instead of two separate factors we get now only one "compounded" factor. Having thus one output and one factor we can now calculate the efficiency of the given process of production, i.e. the efficiency of its technique. The composition of the "compounded" factor of production need not bother us here, the main characteristic of the technique being, according to our present definition, the ratio of the output to the "compounded" factor.

We propose, therefore, the following formula as a measure of the efficiency of technique (denoting by E—the efficiency of technique, by P—output, by N—labour and by K—capital):

$$E = P : NK.$$

The higher is the E, the superior is the production technique, and the faster the E rises, the faster is the rate of technical progress.

The Problem of Aggregation. We have devised our formula primarily with the view to an individual plant producing a single, homogenous good. Can this measure be generalized for the economy as a whole? It would be possible, if all the magnitudes entering into the formula could be aggregated, each taken separately. We have already aggregated N (labour) and K (capital), but it is impossible to aggregate physical outputs—P. In practice, however, outputs are being aggregated by means of valuation at some constant prices, the index problem being abstracted out. This is not a neat procedure, but it is the only one available, and besides useful enough for practical ends. By adopting such a makeshift, method, i.e. by valuing outputs at corresponding (constant) prices, we can calculate the aggregate efficiency of technique for the economy as a whole.

[15] The formula has been oficially approved in this role and is effective in Poland since 1960.

The absolute magnitude of E has no practical meaning whatever. The usefulness of the above formula consists in making possible the measurement of the rate of changes in the efficiency of technique. Yet, there is a big drawback to it: the measurement is correct and makes sense as long as the pattern of output remains constant. The better practical solution would be the aggregate index of individual efficiencies of technique weighted by values added, involving another index problem. There is, however, onc application of our aggregated, item by item, E, which can prove useful.

The P in our formula does not represent the actual output. It is a technical parameter calculated on the assumption of the normal full capacity working of the plant. The same concerns other magnitudes entering into the formula except the fixed equipment, which determines normal full capacity. The calculated E represents, therefore, the potential efficiency of the existing technique.

Let us now define the concerned magnitudes in a different way: let P' denote the actual sum of actual outputs (values added), N'—the employed labour force (the sum total) and K'—the stock of the existing capital. Now, on the basis of our formula we can calculate the E', which in this context measures the actual efficiency of the existing technique.

Both E and E' have one factor in common—the stock of capital ($K = K'$). This makes possible the calculation of the ratio E'/E, which measures the degree of utilization of the potentialities of the existing technique.

The difference between the E' and E is accounted for by the interference of the new factor, as yet not introduced into the picture—the factor of management at the level of an enterprise and at the level of the economy as a whole.

Technical Progress and Economic Progress. Among various circumstances, determining the productivity of labour, Marx mentions the following: "the workers average skill; the development of scientific theory, and the degree to which this theory is applicable in practice; the social organization of production (in the original 'die gesellschaftliche Kombination des Produktionsprozesses'); the supply and the efficiency of the means of production; physical conditions"[16]. The "social organization of production" we define here as the management.

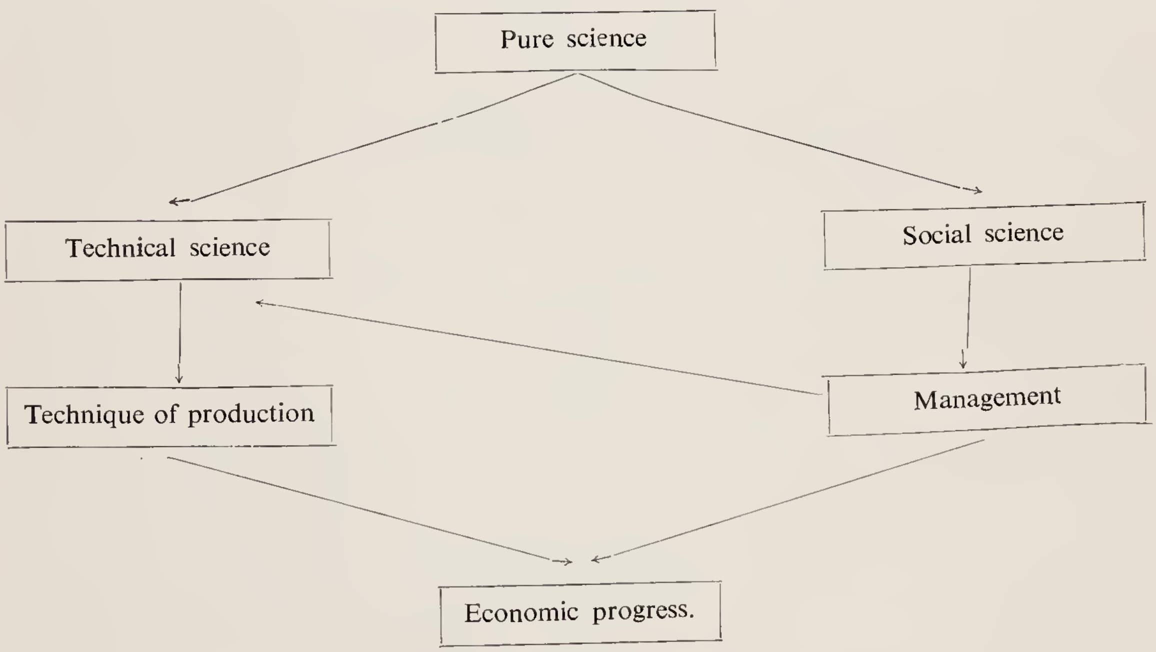
Management is the all important factor in the socialist economy. While in the capitalism an important part of the problems of social organization of production is handled by market forces, in the socialism all the problems of social organization of production fall within the sphere of management. The role of the management does not consist only in the proper utilization of the existing technique, but also in changing the existing technique in the right way. Thus technical progress and progress in management are most closely interwoven, though each can follow

[16] Ibid, p. 8. Also Kalecki in his known formula for national income introduces the term u providing for the effect of "improvements independent of investment outlays": M. Kalecki, *Dynamika inwestycji i dochodu narodowego w gospodarce socjalistycznej* (Dynamics of Investment and National Income in the Socialist Economy), "Ekonomista", No. 5, 1956.

its own way. There may be progress in management without any commensurate technical progress (e.g. the Chinese experiment), and vice versa. Both contribute jointly to what may be properly called "the economic progress". In the colloquial use "the economic progress", usually identified with the technical progress, is understood as the increase in the productivity of labour. So far as the progress in management (in its narrowest sense—the stock of capital being given) is concerned, it is quite correct. And it is also correct as to the technical progress, if this progress is of a neutral character. It ceases, however, to be correct, if the bias in technical progress is labour-saving or capital-saving. The increase in the productivity of labour overrates the, what we consider, real technical progress, when the bias is labour-saving—the increase in the productivity of labour is achieved to the cost of increased accumulation—and underrates it, for similar reasons, when the capital is capital-saving.

In our opinion the economic progress is best reflected by the changes in our E'—the actual level of the efficiency of technique.

To sum up the article, we present here the figure depicting the relationship between science, technique, management and economic progress.



The arrows show the direction the influences operate.

BIBLIOGRAPHY OF WORKS 1927–1963

1927

1. *Zbliżenie gospodarcze amerykańsko-rosyjskie a Polska* (Economic American-Soviet Approach and Poland), "Przegląd Gospodarczy" 1927, No. 23, p. 1029.

1928

2. *Amerykański kartel miedziany* (American Copper Cartel), "Przegląd Gospodarczy" 1928, No. 14, pp. 732–734.
3. (crypt.) M. K., *Dalszy rozwój brytyjsko-sowieckich stosunków gospodarczych* (Further Development of British-Soviet Economic Relations), "Przegląd Gospodarczy" 1928, No. 6, p. 260.
4. (crypt.) M. K., *Europejski trust linoleumowy* (Linoleum Trust in Europe), "Przegląd Gospodarczy" 1928, No. 7, pp. 329–330.
5. (crypt.) M. K., *Kauczuk* (Caoutchouc), "Przegląd Gospodarczy" 1928, No. 22, pp. 1154–1155.
6. Konsekwencje gospodarcze zatargu angielsko-sowieckiego (Economic Consequences of British-Soviet Conflict), "Przegląd Gospodarczy" 1928, No. 1, pp. 19–20.
7. (crypt.) M. K., *Metale* (Metals), "Przegląd Gospodarczy" 1928, No. 17, pp. 886–887; No. 19, pp. 981–982; No. 21, pp. 1100–1102; No. 23, pp. 1202–1204.
8. *Monopolistyczne tendencje niemieckiego przemysłu żelaznego* (Monopolistic Trends of German Iron Industry), "Przegląd Gospodarczy" 1928, No. 11, pp. 542–544.
9. *Szwedzki trust zapałczany* (Swedish Match Trust), "Przegląd Gospodarczy" 1928, No. 3, pp. 101–104.
10. *Światowa wytwórczość aluminium* (World Production of Aluminium), "Przegląd Gospodarczy" 1928, No. 10, pp. 503–504.
11. *Światowa wytwórczość cyny* (World Production of Tin), "Przegląd Gospodarczy 1928, No. 13, pp. 678–679.
12. *Światowa wytwórczość ołowiu* (World Production of Lead), "Przegląd Gospodarczy" 1928, No. 15, pp. 777–778.
13. *Trust zapałczany a Sowiety* (Match Trust and Soviet Union), "Przegląd Gospodarczy" 1928, No. 20, pp. 1027–1028.
14. (crypt.) M. K., *Trust zapałczany w I-ej połowie 1928 r.* (Match Trust in the First Half of 1928), "Przegląd Gospodarczy" 1928, No. 13, pp. 661–663.
15. *Walka o niemiecki rynek węglowy* (Struggle for German Coal Market). "Przegląd Gospodarczy" 1928, No. 6, pp. 274–275.
16. *Wojna naftowa* (Petroleum War), "Przegląd Polityczny" 1928, No. 1–2, pp. 77–85.
17. *Zniesienie angielskich restrykcyj kauczukowych* (Cancellation of English Restriction on Caoutchouc Market), "Przegląd Gospodarczy" 1928, No. 9, pp. 451–453.

1929

18. (crypt.) M. K-cki, *Bawełna* (Cotton), "Przegląd Gospodarczy" 1929, No. 12, pp. 609–611; No. 14, pp. 698–699; No. 16, pp. 788–790; No. 18, pp. 871–872; No. 20, pp. 976–978; No. 22, pp. 1070–1071; No. 24, pp. 1179–1180.
19. (crypt.) M. K-cki, *Bawełna. Światowy rynek bawełny* (Cotton. World Cotton Market), "Przegląd Gospodarczy" 1929, No. 10, pp. 509–511.

20. (crypt.) M. K–cki, *Bawelna. Wytwórczość i spożycie światowe* (Cotton. Production and World Consumption), "Przegląd Gospodarczy" 1929, No. 8, pp. 417–419.
21. *Brytyjsko-sowieckie stosunki gospodarcze* (British–Soviet Economic Relations), "Przegląd Gospodarczy" 1929, No. 5, pp. 237–240.
22. *Europejski kryzys węglowy* (Coal Crisis in Europe), "Przemysł i Handel" 1929, No. 26, pp. 1154–1155.
23. (crypt.) M. Kal., *Francuska polityka naftowa* (French Petroleum Policy), "Przemysł i Handel" 1929, No. 22, pp. 972–974.
24. (crypt.) M. K., *Kauczuk* (Caoutchouc), "Przegląd Gospodarczy" 1929, No. 4, p. 207.
25. (crypt.) M. K., *Kauczuk. Z międzynarodowego rynku kauczuku* (Caoutchouc. On the International Caoutchouc Market), "Przegląd Gospodarczy" 1929, No. 10, pp. 511–512.
26. *Koniunktury przemysłu włókienniczego w okresie 1927–1929* (Business Cycle of Textile Industry in 1927–1929), "Przemysł i Handel" 1929, No. 43, pp. 1823–1826.
27. (crypt.) M. K., *Metale* (Metals), "Przegląd Gospodarczy" 1929, No. 1, pp. 36–37; No. 3, pp. 161–163; No. 5, pp. 259–261; No. 7, pp. 365–367; No. 9, pp. 462–465; No. 11, pp. 561–563; No. 13, pp. 655–657; No. 15, pp. 751–753; No. 17, pp. 827–828; No. 19, pp. 919–922; No. 21, pp. 1017–1020; No. 23, pp. 1122–1124.
28. *Międzynarodowe współzawodnictwo węglowe* (International Coal Competition), "Przegląd Polityczny" 1929, No. 1–2, pp. 49–53.
29. (crypt.) M. Kal., *Opanowanie przemysłu przez kapitał zagraniczny* (Subduement of Industry by Foreign Capital), "Przemysł i Handel" 1929, No. 40, pp. 1710–1711.
30. *W sprawie aktywizacji bilansu handlowego* (On Activating the Balance of Trade), "Przemysł i Handel" 1929, No. 30, pp. 1295–1297.
31. *Wyniki działalności międzynarodowego kartelu stalowego* (Activity Issues of the International Steel Cartel), "Przemysł i Handel" 1929, No. 14, pp. 612–614.
32. *Zakończenie wojny naftowej* (The Close of Petroleum War), "Przegląd Gospodarczy" 1929, No. 6, pp. 301–302.

1930

33. (crypt.) M. K–cki, *Bawelna* (Cotton), "Przegląd Gospodarczy" 1930, No. 2, pp. 95–96; No. 4, pp. 183–184; No. 6, p. 288.
34. *Dynamika spożycia pieczywa pszennego oraz mięsa w Polsce* (Dynamics of Consumption of Wheat Bakery and Meat in Poland), "Koniunktura Gospodarcza" 1930, No. 6, pp. 181–182.
35. *Marża zarobkowa drewna tartego* (Profit Rate in Lumber Wood), "Koniunktura Gospodarcza" 1930, No. 5, pp. 157–158.
36. (crypt.) M. K., *Metale* (Metals), "Przegląd Gospodarczy" 1930, No. 1, pp. 40–42; No. 3, pp. 140–141; No. 5, pp. 237–239; No. 7, pp. 330–332; No. 9, pp. 423–424; No. 11, pp. 502–504; No. 13, pp. 593–594; No. 15, pp. 670–672; No. 17, pp. 749–750; No. 21, pp. 911–912; No. 23, pp. 1001–1002.
37. *Wpływ zniżki cen artykułów przemysłowych dla konsumpcji na przebieg koniunktury* (Influence of Reduction of Industrial Consumption-Goods Prices upon the Run of Business Cycle), "Polska Gospodarcza" 1930, No. 48, pp. 2089–2090.
38. *Wskaźnik cen półfabrykatów inwestycyjnych* (Index of Prices of Semi-finished Investment Goods), "Koniunktura Gospodarcza" 1930, No. 3, pp. 106–107.
39. *Wskaźnik wysokości opodatkowania towarów* (Index of Goods Taxation), "Koniunktura Gospodarcza" 1930, No. 12, p. 330.
40. *Wskaźniki symptomatyczne dochodów mas konsumentów oraz ruchu inwestycyjnego* (Symptomatic Indexes of Consumer Incomes and Building Activity), "Koniunktura Gospodarcza" 1930, No. 12, pp. 327–329.
41. *Zbyt wapna jako wskaźnik ruchu budowlanego* (Sale of Lime as an Index of Building Activity), "Koniunktura Gospodarcza" 1930, No. 8, pp. 227–229.

1931

42. *Ankieta o handlu hurtowym* (Enquiry about Wholesale Market), "Koniunktura Gospodarcza" 1931, No. 7, pp. 202–207.
43. (co-author: Wiśniewski Jan), *Eliminowanie sezonowości z odsetka bezrobocia* (Elimination of Seasonality from the Rate of Unemployment), "Koniunktura Gospodarcza" 1931, No. 3, pp. 82–83.
44. (crypt.) M. K–cki, *Kauczuk. Sytuacja na rynku kauczuku* (Caoutchouc. Situation on the Caoutchouc Market), "Przegląd Gospodarczy" 1931, No. 8, pp. 364–365.
45. *Konsekwencje dumpingu* (Consequences of Dumping), "Koniunktura Gospodarcza" 1931, No. 4, pp. 108–112.
46. (crypt.) M. K–cki, *Metale* (Metals), "Przegląd Gospodarczy" 1931, No. 1, pp. 42–43; No. 3, pp. 132–133; No. 5, pp. 234–235; No. 7, pp. 328–330; No. 9, pp. 408–409; No. 11, pp. 495–496; No. 13, pp. 572– 573; No. 15, pp. 642–643; No. 19, pp. 793–794; No. 21, pp. 864–865; No. 23, p. 949.
47. (co-author: Landau Ludwik), *Szacunek rozmiarów ruchu inwestycyjnego w Polsce* (Estimation of Size of the Investment Activity in Poland), "Koniunktura Gospodarcza" 1931, No. 11, pp. 276–285.
48. (pseud.) Henryk Braun, *Światowy kryzys finansowy* (The World Finance Crisis), "Przegląd Socjalistyczny" 1931, No. 1, pp. 1–3.
49. *Zbyt nici jako symptomat koniunktury włókienniczej* (Sale of Threads as a Symptom of Textiles Market Trend), "Koniunktura Gospodarcza" 1931, No. 2, pp. 56–59.

1932

50. (pseud.) Henryk Braun, *Czy możliwe jest "kapitalistyczne" wyjście z kryzysu*? (Is "Capitalist" Crisis Overcoming Possible?), "Przegląd Socjalistyczny" 1932, No. 10, pp. 1–3.
51. (co-author: Landau Ludwik), *Handel włókienniczy w Polsce. Zarys struktury* (Textile Trade in Poland. Outline of Structure), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1932, No. 3–4, pp. 30–42.
52. (pseud.) Henryk Braun, *Inflacja i wojna* (Inflation and the War), "Przegląd Socjalistyczny" 1932, No. 13, pp. 1–2.
53. (pseud.) H. Br., *Ivar Kreuger* (Ivar Kreuger), "Przegląd Socjalistyczny" 1932, No. 9, pp. 5–6.
54. *Koniunktura a inflacja* (Business Cycle and Inflation), "Polska Gospodarcza" 1932, No. 48, pp. 1411–1415.
55. (crypt.) M. K–cki, *Metale* (Metals), "Przegląd Gospodarczy" 1932, No. 1, pp. 35–36; No. 3, pp. 125–126; No. 5, pp. 203–204; No. 7, pp. 293–294; No. 9, pp. 376–377; No. 11, pp. 468–469; No. 13, pp. 537–538; No. 15, pp. 607–608; No. 19, pp. 742–743; No. 21, pp. 814–815; No. 23, pp. 894–895.
56. *Na marginesie planu Papena* (On the Margin of the Papen Plan), "Polska Gospodarcza" 1932, No. 39, pp. 1157–1158.
57. (pseud.) H. Br., *Na marginesie wydarzeń niemieckich* (On the Margin of the German Events), "Przegląd Socjalistyczny" 1932, No. 15, pp. 4–5.
58. (crypt.) M. Kal., *"Nowe" przemysły a wyjście z kryzysu* ("New" Industries and Issue from Crisis), "Polska Gospodarcza" 1932, No. 31, pp. 907–908.
59. (pseud.) Henryk Braun, *Obniżka płac w czasie kryzysu* (Reduction of Wages during Crisis), "Przegląd Socjalistyczny" 1932, No. 2, pp. 1–2.
60. (pseud.) H. Br., *Podstawy konfliktu mandżurskiego* (Bases of Manchurian Conflict), "Przegląd Socjalistyczny" 1932, No. 2, pp. 6–8.
61. (pseud.) H. Br., *Przewidywania p. Keynes'a* (Mr Keynes' Anticipations), "Przegląd Socjalistyczny" 1932, No. 6, pp. 4.

62. (co-author: Landau Ludwik), *Szacunek rozmiarów ruchu inwestycyjnego w Polsce i wskaźnik kwartalny inwestycji* (Estimation of Size of the Investment Activity in Poland and Quarterly Index of Investment), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1932, No. 1, pp. 7–10.
63. (pseud.) H. Br., *Uwagi o hitleryzmie i "sferach gospodarczych"* (Remarks on Hitlerism and "Economic Circles"), "Przegląd Socjalistyczny" 1932, No. 3, pp. 5–6.
64. (pseud.) Henryk Braun, *Wojna na wschodzie* (War in the East), "Przegląd Socjalistyczny" 1932, No. 5, pp. 1–3.
65. (crypt.) M. K–cki, *Wpływ dewaluacji funta sterlinga na ceny światowe* (Influence of Devaluation of Pound Sterling upon World Prices), "Przegląd Gospodarczy" 1932, No. 2, pp. 67–68.
66. *Wpływ kartelizacji na koniunkturę* (Influence of Cartelization upon Business Cycle), "Polska Gospodarcza" 1932, No. 32, pp. 932–933.
67. *Wskaźniki spożycia i ich analiza* (Indexes of Consumption and Their Analysis), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1932, No. 1, pp. 3–6.
68. (pseud.) H. Br., *Zmierzch Deterdinga* (Deterding's Decline), "Przegląd Socjalistyczny" 1932, No. 12, pp. 4–5.
69. (pseud.) H. Br., *Tandeta w produkcji kapitalistycznej* (Trash in Capitalistic Production). Review: *Jerzy Kulczycki, Upadek jakości w produkcji kapitalistycznej,* Warszawa 1931, "Przegląd Socjalistyczny" 1932, No. 11, pp. 5.
70. (crypt.) M. K–cki, *Metale* (Metals), "Przegląd Gospodarczy" 1933, No. 1, pp. 30–31; No. 4, pp. 144–145; No. 6, pp. 225–226; No. 8, pp. 316–317; No. 10, pp. 387–388; No. 12, pp. 459–460; No. 16 pp. 614–615; No. 18, pp. 681–682; No. 20, p.757; No. 22, pp. 833–834; No. 24, pp. 912–913.
71. *Nakręcanie koniunktury światowej* (Winding up the World Business Cycle), "Polska Gospodarcza" 1933, No. 37, pp. 1111–1115.
72. *O handlu zagranicznym i "eksporcie wewnętrznym"* (On Foreign Trade and "Internal Export"), "Ekonomista" 1933, No. 3, pp. 27–35.
73. *Obecna faza kryzysu światowego* (The Actual Phase of World Crisis), "Polska Gospodarcza", 1933, No. 1, pp. 9–11.
74. *Odpowiedź na "Uwagi krytyczne o jednej z matematycznych teoryj koniunktury" Aleksandra Rajchmana* (A Rejoinder to "Critical Remarks on one of the Mathematical Theorics of Business Cycle" by Aleksander Rajchman), "Kwartalnik Statystyczny" 1933, No. 4. pp. 497–502. Cf. pos. 75.
75. *Próba teorii koniunktury* (Essay on Business Cycle Theory), Warszawa 1933, Instytut Badania Koniunktur Gospodarczych i Cen, pp. 55.
Cf. pos. 74.
76. *Udział karteli w działalności przemysłowej na rynku polskim* (Share of Cartels in Industrial Activity on Polish Market), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1933, No. 3, pp. 3–6.
77. *W sprawie wskaźnika wytwórczości skartelizowanej. Odpowiedź na art. "W sprawie wskaźnika wytwórczości skartelizowanej" p. Wł. C[iechomskiego]* in No. 22 "Przeglądu Gospodarczego" 1933 (On the Index of Cartelized Production. A Rejoinder to Mr. W. Ciechomski's paper: "On the Index of Cartelized Production"), "Przegląd Gospodarczy" 1933, No. 23, pp. 872–873. Cf. pos. 76.

1934

78. (co-author: Landau Ludwik), *Comparabilité internationale des indices de la production industrielle,* Varsovie 1934, 8°, pp. 16.
79. *Koniunktura a dobrobyt* (Business Cycle and Welfare), "Polska Gospodarcza" 1934, No. 50, pp. 1531–1534.

80. *Losy eksperymentów* (Ventures of Experiments), "Polska Gospodarcza" 1934, No. 21, pp. 637–640.
81. (crypt.) M. K–cki, *Metale* (Metals), "Przegląd Gospodarczy" 1934, No. 2, pp. 70–71; No. 4, pp. 153–154; No. 6, pp. 229–230; No. 8, pp. 313–314; No. 10, pp. 389–390; No. 12, p. 459; No. 16, pp. 587–588; No. 18, pp. 656–657; No. 20, pp. 723–724; No. 22, pp. 807–808; No. 4, p. 884.
82. *Nowy wskaźnik inwestycji* (The New Investment Index), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1934, No. 1, pp. 10–15.
83. *Odpowiedź na art. A. M. Neumana, o "Trzech układach"* (A Rejoinder to Paper of A. M. Neuman Article: On "Three Agreements"), "Ekonomista" 1934, No. 4, pp. 97–100.
Cf. pos. 86.
84. (co-author: Landau Ludwik), *Porównalność międzynarodowa wskaźników produkcji przemysłowej i próba jej porównania* (International Comparability of Industrial Production Indexes and an Attempt of Comparison), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1934, No. 2–3, pp. 3–8.
Cf. pos. 78.
85. (co-author: Landau Ludwik), *Szacunek dochodu społecznego w r. 1929* (Estimation of Social Income in 1929), Warszawa 1934, pp. 54.
86. *Trzy układy* (Three Agreements), "Ekonomista" 1934, No. 3, pp. 54–70.
Cf. pos. 83.
87. (co-author: Landau Ludwik), *Wahania produkcji przemysłowej a wahania konsumpcji* (Fluctuations of Industrial Production and Fluctuations of Consumption), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1934, No. 4, pp. 3–6.

1935

88. (co-author: Landau Ludwik), *Dochód społeczny w r.* 1933 *i podstawy badań periodycznych nad zmianami dochodu* (Social Income in 1933 and Bases of Periodical Investigations on Income Fluctuations), Warszawa 1935, Instytut Badania Koniunktur Gospodarczych i Cen, pp. 48.
89. *Eksperyment niemiecki* (The German Experiment), "Polska Gospodarcza" 1935, No. 49, pp. 1574–1576.
90. (co-author: Landau Ludwik), *Encore sur la comparabilité internationale des indices de la production industrielle.* "Travaux de l'Institut de Recherches sur le Mouvement Général des Affaires et sur la Formation des Prix" 1935, No. 3–4.
91. *Essai d'une théorie du mouvement cyclique des affaires,* "Revue d'Economie Politique" 1935, pp. 285–305.
92. *Istota poprawy koniunkturalnej* (The Essence of Improvement of Business Cycle), "Polska Gospodarcza" 1935, No. 43, pp. 1320–1324.
93. *Koniunktura a bilans płatniczy* (Business Cycle and Balance of Payments), "Polska Gospodarcza" 1935, No. 45, pp. 1385–1387.
94. *Koniunktura a zbrojenia* (Business Cycle and Armaments), "Polska Gospodarcza" 1935, No. 22, pp. 701–703.
95. *A Macrodynamic Theory of Business Cycles,* "Econometrica" 1935, No. 3, pp. 327–344.
Cf. pos. 100.
96. *Maszyny krajowe i importowane a charakter inwestycyj przemysłowych* (Home-made and Imported Machines and the Character of Industrial Investment), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1935, No. 2, pp. 68–69.
97. (co-author: Landau Ludwik), *W sprawie porównywalności międzynarodowej wskaźników produkcji przemysłowej* (On the Comparability of International Indexes of Industrial Production), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1935, No. 3–4, pp. 108–110.

98. (co-author: Landau Ludwik), *Wahania cen i kosztów a wahania produkcji przemyslowej w Polsce* (Price Index and Costs Index and Fluctuations of Industrial Production in Poland), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1935, No. 2, pp. 51–61.

99. *Wskaźnik zbytu produktów przemyslowych* (Index of the Sale of Industrial Goods), "Prace Instytutu Badania Koniunktur Gospodarczych i Cen" 1935, No. 3–4, pp. 105–107.

1936

100. *Comments on the Macrodynamic Theory of Business Cycles*, "Econometrica" 1936, No. 4, pp. 356–360.
Cf. pos. 95.

101. *Mechanizm poprawy koniunkturalnej* (The Mechanism of Improvement of Business Cycle), Warszawa 1936, pp. 34.

102. *Parę uwag o teorii Keynesa* (Some Remarks on Keynes' Theory), "Ekonomista" 1936, No 3, pp. 18–26.

1937

103. *The Principle of Increasing Risk*, "Economica" 1937, pp. 440–447.
Cf. pos. 106.

104. *A Theory of the Business Cycle*, "Review of Economic Studies", vol. 4, 1936/1937 No. 2, pp. 77–97.

105. *A Theory of Commodity, Income and Capital Taxation*, "Economic Journal" 1937, No. 3, pp. 444–450.

1938

106. *A Comment on the "Principle of Increasing Risk": A Reply*, "Economica" 1938, pp. 459–460.
Cf. pos. 103.

107. *The Determinants of Distribution of the National Income*, "Econometrica" 1938, No. 2. pp. 97–112.

108. *The Lesson of the Blum Experiments*, "Economic Journal" 1938, No. 1, pp. 26–41.

109. Review: *Havas F., Grundgedanken zu einer Theorie der stoerungsfreien Geldschoepfung* (Jena 1936, G. Fischer, SS. 120), "Economic Journal" 1938, No. 4, pp. 713–714.

110. Review: *Lukas E., Die Aufgaben des Geldes* (Stuttgart–Berlin 1937, W. Kohlhammer, SS. 657), "Economic Journal" 1938, No. 1, pp. 76–77.

111. Review: *Manoilesco M., Die Nationalen Produktivkraefte und der Aussenhandel* (Berlin 1937, Junker u. Dünnhaupt, SS. 400), "Economic Journal" 1938, No. 4, pp. 708–711.

1939

112. *Essays in the Theory of Economic Fluctuations*, London 1939, Allen a. Unwin, pp. 154.

113. *Place nominalne i realne* (Nominal and Real Wages), Warszawa 1939, Instytut Gospodarstwa Społecznego, 8°, pp. 59.

114. *The Supply Curve of an Industry under Imperfect Competition*, "Review of Economic Studies", vol. 7, 1939/1940, pp. 91–110.

115. Review: *Landau L., Gospodarka światowa, produkcja i dochód w liczbach* (World Economy, Production and Income in Figures, Warszawa 1939, Instytut Gospodarstwa Społecznego, pp. VII, 151), "Economic Journal" 1939, pp. 513–515.

1940

116. *The "Mysteries" of Money Market*, "Bulletin of the Oxford University Institute of Statistics" 1940, No. 8, pp. 2–5.

117. (co-author: Tew B.) *A New Method of Trend Elimination*, "Econometrica" 1940, No. 2, pp. 117–129. Cf. pos. 130.

118. *A Scheme of Curtailment of Consumption,* "Bulletin of the Oxford University Institute of Statistics" 1940, No. 3, pp. 7–9.
119. *The Short-term Rate and the Long-term Rate,* "Oxford Economic Papers" 1940, No. 4, pp. 15–22.
120. *A Supplement.* "Bulletin of the Oxford University Institute of Statistics" 1940, No. 6, pp. 10–13.
121. *The Supply Curve of an Industry under Imperfect Competition,* "Review of Economic Studies", vol. 7,1939/1940, No. 2, pp. 91–112.
122. *The Third Quarter,* "Bulletin of the Oxford University Institute of Statistics" 1940, No. 10, pp. 1–5.
123. *Wage Bill and Cash Circulation,* "Bulletin of the Oxford University Institute of Statistics" 1940. No. 5, pp. 2–6.
124. *War Finance in the First Half of 1940,* "Bulletin of the Oxford University Institute of Statistics" 1940, No. 7, pp. 7–18.
125. Review: *Guillaume G. E., Economique Rationelle* (Paris 1937, vol. 1—*Méthode,* pp. 44, vol. 2—*L'économie pure,* pp. 76, vol. 3—*Interférence avec le domaine juridique,* pp. 55, vol. 4—*Interférence avec des mondes étrangers,* pp. 58, vol. 5—*Modèles mathématiques des mondes économiques,* pp. 316), "Economic Journal" 1940, No. 2–3, pp. 276–278.

1941

126. *The Budget and Inflation,* "Bulletin of the Oxford University Institute of Statistics" 1941, pp. 112–113.
127. *Changes in Stocks of Commodities in* 1940, "Bulletin of the Oxford University Institute of Statistics" 1941, pp. 335–338.
128. *The Financial Position on the Eve of the Budget,* "Bulletin of the Oxford University Institute of Statistics" 1941, pp. 61–65.
129. *General Rationing,* "Bulletin of the Oxford University Institute of Statistics" 1941, No. 1, pp. 1–6.
Cf. pos. 132, 138, 143, 149, 169.
130. (co-author: Tew B.), *A New Method of Trend Elimination: A Correction,* "Econometrica" 1941, No. 1, pp. 93–94.
Cf. pos. 117.
131. *Notes of Finance,* "Bulletin of the Oxford University Institute of Statistics" 1941, No. 3, pp. 51–53.
132. *Notes on General Rationing,* "Bulletin of the Oxford University Institute of Statistics" 1941, pp. 103–105.
Cf. pos. 129.
133. *Recent Trends of the Financial Situation,* "Bulletin of the Oxford University Institute of Statistics" 1941, pp. 389–393.
134. *The Share of Wages in the National Income,* "Bulletin of the Oxford University Institute of Statistics" 1941, No. 9, pp. 106–198, Erratum p. 262.
135. *The Short-term Rate of Interest and Velocity of Cash Circulation,* "Review of Economic Statistics" 1941, No. 2, pp. 97–99.
136. *A Theorem on Technical Progress,* "Review of Economic Studies", vol. 8, 1940/1941, No. 3, pp. 179–184.
137. *The Theory of Long-run Distribution of the Product of Industry,* "Oxford Economic Papers" 1941, pp. 31–41.
138. *Towards Comprehensive Rationing,* "Bulletin of the Oxford University Institute of Statistics" 1941, No. 3, pp. 51–53.
Cf. pos. 129.

139. *Wartime Changes in Employment and the Wage Bill,* "Bulletin of the Oxford University Institute of Statistics" 1941, No. 13, pp. 294–298.

140. *What is inflation*? "Bulletin of the Oxford University Institute of Statistics" 1941, pp. 159–164.

1942

141. *The Budget,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 129–132.

142. *The Burden of War. B: The Burden of Wages and Other Incomes,* "Bulletin of the Oxford University Institute of Statistics" 1942, No. 1, pp. 10–11.

143. *Differential Rationing,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 215–217.
Cf. pos. 129.

144. *Employment, Wage Bill, and Cash Circulation,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 67–70.

145. *Excess profits Tax and Government Contracts,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 40–43.

146. *The Fall in "Small" Savings,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 290–293, Erratum 1943, p. 22.

147. *The Financial Situation in the First Half of 1942,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 247–252.

148. *The Problem of Profit Margins,* "Bulletin of The Oxford University Institute of Statistics" 1942, pp. 114–117.

149. *Some Problems on Non-food Rationing,* "Bulletin of the Oxford University Institute of Statistics" 1942, pp. 325–328.
Cf. pos. 129.

150. *A Theory of Profits,* "Economic Journal" 1942, pp. 258–267.

151. *Wages and the National Income in 1940 and 1941,* "Bulletin of the Oxford University Institute of Statistics" 1942, No. 7, pp. 150–153.

152. *Mr. Whitman on the Concept of "Degree of Monopoly". A Comment,* "Economic Journal" 1942, pp. 121–127.

1943

153. *The Budget,* "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 96–97.

154. *The Burden of the National Debt,* "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 76–80.

155. *Economic Implications of the Beveridge Plan,* "Bulletin of the Oxford University Institute of Statistics" 1943, Suppl. No. 4, pp. 2–7.

156. *The Financial Situation in the First Half of* 1943, "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 216–220.

157. (co-author: Schumacher E. F.), *International Clearing and Long-term Lending,* "Bulletin of the Oxford University Institute of Statistics" 1943, Suppl. No. 5, pp. 29–33.

58. *Labour in the War Industries of Britain and USA: A. Sources of Manpower in the British War Sector,* "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 2–11.

159. *Political Aspects of Full Employment,* "Political Quarterly" 1943, No. 4, pp. 322–331.

160. *The Problem of "Small" Savings,* "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 260–265.

161. *Profits, Salaries and Wages,* "Bulletin of the Oxford University Institute of Statistics" 1943, No. 8, pp. 125–129.

162. *Sources of Manpower in the British War Sector in 1941 and 1942,* "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 173–178.

163. *War Finance in 1940, 1941 and 1942,* "Bulletin of the Oxford University Institute of Statistics" 1943, No. 9, pp. 137–142.

164. *The War-time Trend of Deposits*, "Bulletin of the Oxford University Institute of Statistics" 1943, pp. 63–66.

1944

165. *The Budget: A. The Stabilisation Policy*, "Bulletin of the Oxford University Institute of Statistics" 1944, pp. 101–104.

166. *Employment in the United Kingdom during and after the Transition Period*, "Bulletin of the Oxford University Institute of Statistics" 1944, pp. 265–287.

167. *Excess Profits Tax and Post-war Re-equipment*, "Bulletin of the Oxford University Institute of Statistics" 1944, pp. 58–61.

168. *Professor Pigou on "The Classical Stationary State". A comment*, "Economic Journal" 1944, pp. 131–132.

169. *Rationing and Price Control*, "Bulletin of the Oxford University Institute of Statistics" 1944, pp. 29–32.
Cf. pos. 129.

170. *Studies in Economic Dynamics*, New York 1944, Farrar a. Rinehart, 16°, pp. 92.

171. *The White Paper on Employment Policy*, "Bulletin of the Oxford University Institute of Statistics" 1944, pp. 131–135.

172. *The White Paper on the National Income and Expenditure in the Years 1938–1943*, "Bulletin of the Oxford University Institute of Statistics" 1944, No. 9, pp. 137–144.

1945

173. *Full Employment by Stimulating Private Investment*? "Oxford Economic Papers" 1945, No. 7, pp. 83–92.

174. *The Maintenance of Full Employment in the Transition Period*, "International Labour Review" 1945, pp. 449–464.

175. *On the Gibrat Distribution*, "Econometrica" 1945, No. 2, pp. 161–170.

176. *The Work of Erwin Rothbarth*, "Review of Economic Studies", vol. 11–12, 1943–1945, No. 2, pp. 121–122.

1946

177. *A Comment on "Monetary Policy"*, "Review of Economic and Statistics" 1946, No. 3, pp. 81–84.

178. *The Maintenance of Full Employment after the Transition Period. A Comparison of the Problem in the United States and the United Kingdom*, Montreal 1946, pp. 16 (Reprinted from the "International Labour Review" 1945, No. 5).
Cf. pos. 174.

179. *Multilateralism and Full Employment*, "Canadian Journal of Economics and Political Science" 1946, pp. 322–327.

1947

180. *The Maintenance of Full Employment after the Transition Period. A Rejoinder to Mr. Woytinsky's Note*, "American Economic Review" 1947, No. 3, pp. 391–397.
Cf. pos. 174.

1948

181. *Determinations of the Increase in the Cost of Living in the United States*, "Review of Economic and Statistics" 1948, No. 1, pp. 22–24.

182. *Further Comments on the Department of Commerce Series*, "Review of Economics and Statistics" 1948, No. 3, pp. 195–197.

1949

183. *Glos w dyskusji*—Contribution into the Discussion (concerning O. Lange's Report: *The Practice of Economic Planning Another Optimum Allocation of Resource*), "Econometrica" 1949, Suppl., pp. 176–177.
184. *A New Approach to the Problem of Business Cycles*, "Review of Economic Studies", vol. 16, 1948/1949, No. 2, pp. 57–64.

1950

185. *A Note on Long-run Unemployment*, "Review of Economic Studies", vol. 18, 1949/1950, No. 1, pp. 62–64.

1951

186. *Report of Main Current Economic Problems of Israel*, 1951, Governenen Printing Press, Israel, pp. 23.

1954

187. *El problema del financiamento del desarollo económico*, "Trimestre Economico" 1954, No. 4, pp. 381–401.
188. *Theory of Economic Dynamics. An Essay on Cyclical and Long-run Changes in Capitalistic Economy*, London 1954, G. Allen a. Unwin, 8^0, pp. 178. (Ibid.: New York 1954, ed. Rinehart).
Cf. pos. 193–195, 216, 231.

1955

189. *The Problem of Financing of Economic Development*, "Indian Economic Review" 1955, No. 3, pp. 1–22.
Cf. pos. 196.

1956

190. *Dynamika inwestycji i dochodu narodowego w gospodarce socjalistycznej* (The Dynamics of Investment and National Income in Socialistic Economy), "Ekonomista" 1956, No. 5, pp. 61–70.
Cf. pos. 208, 230.
191. *Rady robotnicze a centralne planowanie* (Workers-Councils and Central Planning), "Nowe Drogi" 1956, No. 11–12, pp. 38–45.
Cf. pos. 229.
192. *Sytuacja gospodarcza Stanów Zjednoczonych w zestawieniu z okresem przedwojennym* (The Present Economic Situation of the USA as Compared with that of the Pre-war Period), "Ekonomista" 1956, No. 4, pp. 3–13.
193. *Teoria de la dinámica económica*, Transl. F. Pozos, V. Urquidi, Mexico 1956, Fonde de Cultura Econômica, pp. 184.
194. *Teoria della dinamica economica*. Transl. from English. S. Steve, Torino, 1956, Edizioni Scientifiche Einaudi, pp. VII, 217.
Cf. pos. 188.
195. *Theory of Economic Dynamics. An Essay on Cyclical and Long-run Changes in Capitalistic Economy*, London 1956, G. Allen a. Unwin, 8°, pp. 178.
Cf. pos. 188.
196. *Zagadnienie finansowania rozwoju ekonomicznego* (The Problem of Financing of Economic Development), "Ekonomista". Translations from foreign magazines 1956, No. 1–2, pp. 181–202.
Cf. pos. 189.

197. Translation: Keynes J. M., *Ogólna teoria zatrudnienia, procentu i pieniądza* (The General Theory of Employment, Interest and Money), Warszawa 1956, PWN, 16°, pp. XVIII, 532.

1957

198. (co-author: Polaczek St.), *Dyskusja o cenach surowców podstawowych. Odpowiedź M. Kaleckiego i St. Polaczka na art. B. Pryzińskiego* (Discussion on Prices of Basic Raw Materials. A Rejoinder of M. Kalecki and St. Polaczek to the Article of B. Pryziński), "Gospodarka Planowa" 1957, No. 9, pp. 66–68.
Cf. pos. 205, 209.
199. (co-author: Szeworski A.), *Ekonomiczne problemy automatycznej produkcji w krajach kapitalistycznych* (The Economic Problems of Automation in Capitalistic Countries), "Ekonomista" 1957, No. 3, pp. 105–115.
200. *Jak łagodzić sprzeczność: inwestycje—konsumpcja. O pracach w zakresie badania efektywności inwestycji* (How to Alleviate the Contradiction between Investment and Consumption. On the Works of Economic Efficiency of Investment), "Życie Warszawy" 1957, No. 117, p. 3
201. *Jak wyobrażam sobie polski model gospodarczy* (How I Imagine the Polish Economic Model), "Trybuna Ludu" 1957, No. 33, p. 3.
202. *Koncern pionowy? Propozycje ekonomiczne* (A Vertical Concern? The Economic Proposals), "Polityka" 1957, No. 8, p. 1.
203. *Schemat nowego systemu bodźców i nakazów* (Scheme of New System of Incentives and Orders), "Życie Gospodarcze" 1957, No. 29, p. 1.
204. (co-author: Szeworski A.), *Studia z zakresu koniunktury współczesnego kapitalizmu* (Studies in the Sphere of Business Cycle of Contemporary Capitalism), Warszawa 1957, PWN, vol. 1, pp. 136.
205. (co-author: Polaczek St.), *Uwagi w sprawie metody ustalania cen surowców podstawowych* (Remarks on the Method of Fixing Prices of Basic Raw Materials), "Gospodarka Planowa" 1957, No. 4, pp. 18–22.
Cf. pos. 198.
206. *Wpływ czasu budowy na współzależność inwestycji i dochodu narodowego a "współczynnik zamrożenia"* (The Influence of the Time of Construction on the Interpendence of Investment and National Income and the "Coefficient of Freezing"), "Ekonomista" 1957, No. 1, pp. 3–13.

1958

207. *Czynniki określające tempo wzrostu dochodu narodowego w gospodarce socjalistycznej* (Factors Determining the Rate of Growth of the National Income in Socialistic Economy), "Gospodarka Planowa" 1958, No. 8, pp. 1–15.
208. *Dynamika inwestycji i dochodu narodowego w gospodarce socjalistycznej* (Dynamics of Investment and National Income in Socialistic Economy) in: *Zagadnienia ekonomii politycznej socjalizmu*, Warszawa 1958, Książka i Wiedza, pp. 441–456.
209. (co-author: Polaczek St.): *Dyskusja o cenach surowców podstawowych* (Discussion on the Prices of Basic Raw Materials) in: *Spór o ceny*, Warszawa 1958, Książka i Wiedza, pp. 121–135.
Cf. pos. 198.
210. *Kilka uwag na marginesie organizacji koncernu angielskiego* (A Few Remarks on the Organization of an English Concern), "Gospodarka Planowa" 1958, No. 6, pp. 43–45.
211. *Model mechanistyczny zjawiska losowego* (Mechanistic Model of a Random Phenomenon), "Zastosowania Matematyki" 1958, No. 2, pp. 113–129.
212. *O cenach surowców podstawowych* (On the Prices of Basic Raw Materials), "Ekonomista" 1958, No. 3, pp. 574–578.
Cf. pos. 213.

213. *O cenach surowców podstawowych* (On the Prices of Basic Raw Materials) in: *Spór o ceny*, Warszawa 1958, "Książka i Wiedza", pp. 282–288.
Cf. pos. 212.

214. *O współczynniku zamrożenia* (On the Coefficient of Freezing), "Ekonomista" 1958, No. 6, pp. 1393–1400.

215. *Plan perspektywiczny na lata 1961–1975* (The 1961–1975 Long-run Economic Plan), "Nowe Drogi" 1958, No. 8, pp. 27–45.

216. *Teoria dynamiki gospodarczej. Rozprawa o cyklicznych i długofalowych zmianach gospodarki kapitalistycznej* (Theory of the Economic Dynamics. An Essay on Cyclical and Long-run Changes in Capitalistic Economy), Warszawa 1958, PWN, 16°, pp. 237.

217. (co-author: Polaczek St.), *Uwagi w sprawie metody ustalania cen surowców podstawowych* (Remarks on the Method of Fixing Prices of Basic Raw Materials) in: *Spór o ceny*, Warszawa 1958, "Książka i Wiedza", pp. 62–75.
Cf. pos. 205.

218. (scientific editorship): *Studia z zakresu koniunktury współczesnego kapitalizmu* (Studies in the Sphere of Business Cycle of Contemporary Capitalism), Warszawa 1958, PWN, vol. 2, p. 157.

1959

219. *The 1961–1975 Long-run Economic Plan*, "Polish Perspectives" 1959, No. 3, pp. 3–19.

220. *O wąskich gardłach planu perspektywicznego* (On Bottlenecks in the Long-run Plan). "Nauka Polska" 1959, No. 1, pp. 52–58.

221. *Podstawowe zagadnienia planu pięcioletniego na lata 1961–1965* (The Main Problems of the New Five-Year Plan), "Gospodarka Planowa" 1959, No. 1–2, pp. 1–4.

222. (co-author: Rakowski M.), *Uogólnienie wzoru efektywności inwestycji* (Generalization on the Formula of the Efficiency of Investments), "Gospodarka Planowa" 1959, No. 11, pp. 4–7.

223. (co-author: Rakowski M.), *Uogólnienie wzoru efektywności inwestycji* (Generalization on the Formula of the Efficiency of Investments), "Praca i Materiały Zakładu Badań Ekonomicznych Komisji Planowania" 1959, No. 17, pp. 1–16.

224. (co-author: Rakowski M.), *Uogólnienie wzoru efektywności inwestycji* (Generalization on the Formula of the Efficiency of Investments), "Inwestycje i Budownictwo" 1959, No. 11, pp. 6–9.

225. *Wypowiedź w dyskusji w sprawie metodologii badań efektywności inwestycji* (A Discussional Notice on the Method of Investigation of the Investments Efficiency), "Inwestycje i Budownictwo" 1959, No. 7–8, pp. 9–10.

226. *Z zagadnień teorii dynamiki gospodarki socjalistycznej* (On Problems of Dynamics Theory of Socialistic Economy) in: *Zagadnienia ekonomii politycznej socjalizmu*, 2 ed., Warszawa 1959, "Książka i Wiedza", pp. 145–175.

227. *Rozmawiamy z prof. M. Kaleckim* (Conversations with Professor M. Kalecki), "Życie Gospodarcze" 1959, No. 25, p. 1.

228. (scientific editorship): *Studia z zakresu koniunktury współczesnego kapitalizmu* (Studies in the Sphere of Business Cycle of Contemporary Capitalism), Warszawa 1959, PWN, vol. 3, pp. 96.

1960

229. *Consigli operai e pianificazione centralizzata* in: *Vecchia e nuova pianificazione economica in Polonia*, Milano 1960, Feltrinelli Edit., pp. 127–137.
Cf. pos. 191.

230. *Dinamica degli investimenti e del reddito nell' economia socialista* in: *Vecchia e nuova pianificazione economica in Polonia*, Milano 1960, Feltrinelli Edit., pp. 299–312.
Cf. pos. 190.

231. *Keizai hendô no riron* (Theory of Economic Dynamics), Tokyo 1960, Shin Hyôron pp. 254. Cf. pos. 216.
232. *Financing Problems of Third Plan* (Some Observations), "Economic Weekly" 1960, No. 28, pp. 1119–1123.
233. *Pitanja tempa rasta socijalističke privrede* in: *Poljski economisti o problemima socijalističke privrede*, Beograd 1960, Nolit, pp. 71–108.
Cf. pos. 207.
234. *Rozwój sytuacji gospodarczej w Stanach Zjednoczonych w okresie 1956–1959* (Development of Economic Situation in United States in the 1956–1959) in: *Studia z zakresu koniunktury współczesnego kapitalizmu*, Warszawa 1960, PWN, vol. 4, pp. 5–19.
235. *Unemployment in Underdeveloped Countries*, "Indian Journal of Labour Economics" 1960, No. 2.
236. *Uwagi na temat wydajności pracy* (Remarks on the Labour-Productivity), "Życie Gospodarcze" 1960, No. 31, p. 1,3.
237. *Uwagi o problemach finansowych trzeciego planu pięcioletniego w Indiach* (Remarks on the Financial Problems of the Third Five-Year Plan in India), "Ekonomista" 1960, No. 6, pp. 1278–1284.
Cf. pos. 232.
238. *Z zagadnień teorii dynamiki gospodarki socjalistycznej* (On Problem of Dynamics Theory of Socialistic Economy) in: *Zagadnienia ekonomii politycznej socjalizmu*, 3 ed., Warszawa 1960, "Książka i Wiedza", pp. 145–175.
Cf. pos. 226.
239. (scientific editorship): *Studia z zakresu koniunktury współczesnego kapitalizmu* (Studies in the Sphere of Business Cycle of Contemporary Capitalism), Warszawa 1960, PWN, vol. 4, pp. 194.

1961

240. *Gospodarcze aspekty militaryzacji Niemiec Zachodnich* (Economic Aspects of the Militarization of West Germany), "Ekonomista" 1961, No. 6, pp. 1245–1250.
241. *Polityczne aspekty pełnego zatrudnienia* (Political Aspects of Full Employment), "Ekonomista" 1961, No. 5, pp. 1072–1076.
242. *W odpowiedzi B. Święcickiemu* (A Rejoinder to A. Swięcicki), "Ekonomista" 1961, No. 5, pp. 1173–1177.

1962

243. *Akumulacja a maksymalizacja spożycia* (Accumulation and the Maximization of Consumption), "Ekonomista" 1962, No. 3, pp. 706–708.
244. *Efektywność "programu" inwestycyjnego* (Efficiency of Investment "Program"), "Gospodarka Planowa" 1962, No. 3, pp. 9–12.
245. *Kilka uwag o "Gospodarce Planowej" i planowaniu* (Some Remarks on the Polish Monthly "Planned Economy" and on Planning in General), "Gospodarka Planowa" 1962, No. 1, pp. 7–8.
246. *A Model of Hyperinflation*, "Manchester School" 1962, No. 3, pp. 275–281.
247. *Observations on the Theory of Growth*, "Economic Journal" 1962, pp. 134–153.
248. *Outline of a Method of Constructing a Perspective Plan*: *in Four Papers Prepared for International Conference to be Held in Geneva*, 1963, Warszawa 1962, Szkoła Główna Planowania i Statystyki, pp. 1–15, The Higher Course of National Economic Planning, Teaching Materials, vol. 8.
249. *Prace z teorii koniunktury 1933–1939* (Studies on the Theory of Business Cycle), Warszawa 1962, PWN, 8°, pp. 103.
250. *Próba wyjaśnienia zjawiska przestępczości gospodarczej* (An Attempt of Explanation of the Economic Criminality Phenomenon), "Kultura i Społeczeństwo" 1962, No. 2, pp. 73–77.

251. *Short Papers on Economic Growth,* Warszawa 1962, Szkoła Główna Planowania i Statystyki, pp. 89, The Higher Course in Economic Planning, Teaching Materials, vol. 1.
252. *Szkice o funkcjonowaniu współczesnego kapitalizmu* (Essays on Functioning of Contemporary Capitalism), Warszawa 1962, PWN, pp. 68.
253. *Uwagi o teorii wzrostu* (Observations on the Theory of Growth) in: *Teorie wzrostu ekonomicznego a współczesny kapitalizm,* Warszawa 1962, "Książka i Wiedza", pp. 49–76. Cf. pos. 247.
254. *W sprawie działania bodźców ekonomicznych. Uwagi dyskusyjne* (On the Influence of Economical Incentives. A Discussional Remarks), "Nowe Drogi" 1962, No. 2, pp. 170–171.

1963

255. *Model hiperinflacji* (A Model of Hyperinflation), "Ekonomista" 1963, No. 2, pp. 312–316.
256. *O podstawowych zasadach planowania wieloletniego* (On the Fundamental Principles of the Long-term Planning), "Życie Gospodarcze" 1963, No. 24, p. 1, 2.
257. *An Outline of a Method of Constructing a Perspective Plan* (*Based on Polish Experience*) *in*: *Essays on Planning and Economic Development,* Warszawa 1963, PWN, pp. 9–22, Center of Research on Underdeveloped Economies, vol. 1.
258. *Różnice pomiędzy planowaniem perspektywicznym w socjalistycznych i mieszanych gospodarkach* (Differences between Perspectivic Planning in the Socialist and Mixed Economies), "Prace i Materiały" 1963, No. 2, pp. 13–14. Międzyuczelniany Zakład Problemowy Gospodarki Krajów Słabo Rozwiniętych.
259. *Zagadnienie optymalnej struktury spożycia* (The Problem of Optimal Structure of Consumption), "Gospodarka Planowa" 1963, No. 7, pp. 3–7.
260. *Zagadnienie finansowania rozwoju gospodarczego w malorozwiniętych krajach kapitalistycznych* (The Problem of Financing of Economic Development in Underdeveloped Capitalist Countries), "Ekonomista" 1963, No. 6, pp. 1114–1124.
261. *Zarys metody opracowania planu perspektywicznego* (An Outline of a Method of Constructing of a Perspectivic Plan), "Prace i Materiały" 1963, No. 2, pp. 3–13, Międzyuczelniany Zakład Problemowy Gospodarki Krajów Słabo Rozwiniętych. Cf. pos. 257.
262. *Zarys teorii wzrostu gospodarki socjalistycznej* (Outline of Theory of Growth of a Socialist Economy), Warszawa 1963, PWN, pp. 119.
263. *Przedmowa do*: *Efektywność inwestycji* (Foreword to: Efficiency of Investment), Warszawa 1963, Państwowe Wydawnictwo Ekonomiczne, pp. 5–8.