Piero Sraffa’s physical price system and reproduction without production

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Abstract
This article examines the physical price system of Piero Sraffa. Sraffa’s system is presented as a physical model of production, which provides an internally consistent and so logically superior alternative to Marx’s allegedly inconsistent labour theory of value. This article does not contest the internal consistency of Sraffa’s logic but demonstrates that its logic contradicts every actual process of production. Production, defined as the human labour process that transforms one set of physical inputs into a different set of physical outputs, is a process of physical change by definition, which in capitalist production means that outputs are physically incommensurate to, or different from, inputs. This article focuses on several key issues: commensurability, the standard commodity or physical numeraire; the relation between Sraffian and Leontief’s production matrix, including Quesnay’s Tableau Économique; and the production of surplus. It finds that Sraffa’s physical price system is not a model of production at all, it is a mathematically correct model of nothing. It is reproduction without production.

Keywords
labour theory of value, Marx, price, Sraffa, value

Introduction
After the Second War, Piero Sraffa co-edited with Maurice Dobb, the Cambridge Stalinist, Ricardo’s collected works (Ricardo 1951–1973). In 1960, Dobb oversaw the publication of Sraffa’s (1960) seminal alternative to labour value theory, the Production
of Commodities by Means of Commodities, subtitled a Prelude to the Critique of Political Economy. Sraffa’s work provides a mathematically consistent or ‘mathematically true’ method for deriving prices and surplus from a set of physical constants (Cogliano et al. 2018: 1). Ian Steedman’s Marx after Sraffa (1977) showed that if prices could be so derived, there was no point to a labour theory of value, which was ‘logically incoherent’ (Steedman 1977: 35). Steve Rankin, a neo-Marxian writing, in 1987, concluded that ‘Marxian economics is in a state of hopeless disarray’ and he went on to say that ‘there can be no meaningful sense in which these prices can be said to derive from, or be based upon, Marxian labour values’ (Rankin 1987: 105). Heinz Kurz (2018) in a critical appraisal of Marx’s value theory on the 200th anniversary of Marx’s birthday claimed that ‘Piero Sraffa has shown, prices of production and the general rate of profits are fully determined in terms of the same set of data from which Marx started his analysis’ (p. 40), so the ‘purely mystical conception’ (p. 69) of labour value is unsustainable. Labour value is a pointless, unnecessary and indeed a mathematically imperfect detour, so ‘why bother?’ (Hahnel 2018).

This article explains why bother. It takes a different direction from theorists such as Andrew Kliman (2007) and Michel Husson (2018) of the temporalist school, who answer the claim of inconsistency through pointing to the sequential nature of production within time, and instead addresses a hitherto neglected element of this debate, the consistency of Sraffa’s model with the world, that is, actual human production. Engels (1946 [1886]) observed that ‘the great basic thought’ of dialectical philosophy was ‘that the world is not to be comprehended as a complex of readymade things, but as a complex of processes’ (Engels’ emphasis). Paradoxically, Sraffa’s model only works as a model of readymade things, and not as a complex of processes. Sraffa’s data are not Marx’s, as even with a given set of common inputs, Marx’s data will have been qualitatively changed by production, into a different and so physically incommensurate set of outputs. Indeed, the very purpose of production is physical change. Sraffa’s premise that inputs are physically identical with outputs, that relative prices do not change and the surplus appears without equivalent, from nowhere, is not just unrealistic, after all models necessarily abstract from details of the thing or process that they aim to model, but more fundamentally unreal – it contradicts the essence of the production process itself.

Sraffa’s lectures on the history of the modern theory of value

In 1929 and 1930, Sraffa authored a set of lectures on The History of the Theory of Value (Sraffa 1916–1983; D2.4.3) which traced the development of value theory from Steuart and the Physiocrats, through Smith and Ricardo, to Marshall, Edgeworth and the marginalists. These lectures provide many clues to Sraffa’s subsequent theory. At the outset, he noted that ‘the general theory of value’ accounted for ‘the common characteristics of the most diverse conditions under which values of different commodities are determined, it is necessary very abstract in character’. At first sight, it appeared that ‘the theory is a purely logical construction’ that must be tested for the ‘extent its assumptions correspond to the facts’ (Sraffa 1916–1983; D2 4 3 f1), but he continued later, ‘a further
disturbing element is that in the background of every theory of value there is a theory of distribution'. This meant that

the real problem to be solved by a theory of value, that is: ‘Why is a commodity exchanged with another in a given ratio?’ was constantly transformed into the entirely different one: ‘How is the price received for the product distributed between the factors of production?’ (Sraffa 1916–1983, D2 3 f4)

Sraffa’s (1916–1983) history traced,

the transformation of the notion of cost from the original one of a stock of material goods, to the conception of an amount of human sacrifice – that is to say the gradual transition from an objective to a subjective point of view. (D2 4 3 f36)

Sraffa considered that classical political economy measured output physically and so objectively, in contrast to marginal economics which measured output (if it measured it all) individually and so subjectively. Following Steuart and the Physiocrats, Sraffa considered that physical quantities of definite things formed the basis for the objective theory of value. For Sraffa, both labour time and utility, which vary from individual to individual, were subjective whereas quantities of physical things formed costs which were ‘something concrete tangible, and visible, that can be measured in tons or gallons’. Classical political economy stood ‘at the opposite extreme of Marshall’s cost, which is private to each individual, and can only be measured (if at all) by means of the monetary inducement required to call forth the exertion’ (Sraffa 1916–1983, D2.4.3 F.23). He continued, ‘I think that the classical notion of costs, as quantities of things used up in production, is the most important from the point of view of the theory of value’ (Sraffa 1916–1983, D2 4 3 f62) or labour values. He concluded,

we are left with two kinds of materials (utility, and ‘costs’ physical) each of which can be used as the only basis of a theory of value: we can therefore have two independent theories, but not one which takes both into account. (Sraffa’s emphasis; Sraffa 1916–1983, D2 4 3 f69)

Profit was a cost

all these profits, together with those of the spinner himself, were again advanced by the weaver, in the price of his material – linen yarn … profits, therefore, as well as wages, enter into the cost of production which determines the value of the produce. (Sraffa 1916–1983, D2 4 3 f43)

But how can profit exist if it is a cost, when profit is the difference between price and cost? Sraffa does not resolve this conundrum.

There was a further problem however, the incommensurability of physical outputs from inputs during the manufacturing process. In a discussion of the Physiocrats, Sraffa (1916–1983) noted that
in agriculture, owing to the identity in the quality of the product and of the materials used up in production, the comparison for the calculation of the surplus is possible to some extent without introducing the disturbing element of price for measuring the quantities: whereas in industry, the quality of the two things is so fundamentally different that only their values can be compared. (D2.4.3 F.26)

Manufacturing changes the physical form of inputs such that they cannot be compared with outputs except as values, but what was value in a physical price system designed to preclude the need for the very values it posited as unnecessary?

**Production as creative destruction**

Marx and Engels, following Vico, explained that production is the process through which people create their world. This process transforms inputs from one, less useful form, into a different, more useful form, that is a form in which they can be consumed. All production physically changed inputs into physically different outputs. As capitalist production only produced commodities for sale that physical difference meant that in capitalist production outputs were incommensurate with inputs. A + B inputs produced were destroyed in the process of making, new output C or A + B = C. C cannot be physically larger than the quantities of A + B as A + B were destroyed in the production of C. C is the product of A + B. While A, B and C are changed into not A, not B and not C, as technological development progressed.

The essential nature of production as a process of creative destruction posed Sraffa with a fundamental dilemma. His argument was premised on the assumption that the cost of production was the physical inputs consumed during production. Sraffa argued that in the process of value transfer to the product and the physical “destruction” of the input are one and the same thing (Kurz & Salvadori 2005: 71). Sraffa considered that the ‘real cost’ of a commodity consists in the commodities actually destroyed in the course of its production. This quantity was the price of production, inputs had been destroyed in their original physical form, and were now embodied in an output with a different physical form. As the original inputs no longer existed, it could not be assumed that these physical quantities were the price of the new physical quantity, as the relative physical quantities of inputs necessary for the production of any given output could change. This change could not be measured through examining the output alone. As outputs were different to inputs, so they were incommensurate with them.

Marx (1976 [1867]) noted that ‘exchange value appears first of all as the quantitative relation, the proportion, in which use values of one kind exchange for use values of another kind. This relation changes constantly in time and place’ (Capital I, p. 126). Insofar as exchange values (the physical proportions in which commodities exchange) reflect these changes, and insofar as prices reflect exchange values, then prices do too. Price signifies that a certain quantity of one commodity equals a certain quantity of another, but as these quantities are not fixed, as these prices constantly change, so relative prices change, so no physical commodity can act as an absolute standard of value or ‘third thing’ against which the exchange value of these physical commodities can be measured (Marx, Capital I, 1976 [1867]: 127). Hence, Marx’s the ‘third thing’ was not
physical but abstract labour, the social standard of value, outside of physical production. Gold or other physical commodities that functioned as money did so precisely insofar as their value represented this third social standard, which was not physical, and so varied with respect to physical commodities. Nonetheless, according to Sraffa, it was a ‘purely mystical conception that attributes to human labour a special gift of determining value’ (Kurz & Salvadori 2005: 75).

Samuel Bailey, the Bentham of Hallamshire, observed in his critique of Ricardo, if exchange value was represented by an infinite variety of different material products then it was purely relative, there could be no common intrinsic measure to any or all value (Furmer 2004). As every physical exchange is different, every exchange value is different, every exchange is incommensurate with every other one and there can be no objective exchange value or price or indeed any exchange or price at all. Bailey (1967 [1825]) considered that value denotes ‘nothing positive or intrinsic, but merely the relation in which two objects stand to each other as exchangeable commodities’. Marx (1971 [1861–1863]) observed that if this were true, then if A exchanged for B and B exchanged for C, then the two exchanges ‘have nothing in common with each other’ and so it would be ‘nonsense to say that they are equivalent expressions’. In explaining why Ricardo’s system failed as a physical measure, Bailey inadvertently justified why only an external social standard, not an internal physical one, could act as a standard of value or price. The process of capitalist production reduces all labour to a common standard through the mechanisation of production. Individuality is crushed under the exigencies of the accumulation process. Marx’s assertion that social or abstract labour was the ‘third thing’ or standard of value was predicated on conditions that continue to define the essence of capitalist production. Production is an economic labour process in which individual labour is transformed into collective labour through the accumulation process. This labour reproduces human society by physically transforming inputs into different outputs that meet human needs, while capitalist competition creates technological advance which means that relative prices, and inputs and outputs constantly change. Brewer and Cutler considered that Marx’s argument for the pre-eminence of social labour were nonetheless ‘unbelievably weak’ (Brewer 1995: 117; Cutler et al. 1977: 6). Were they?

According to Sraffa, price was determined by the quantity of inputs physically destroyed in producing outputs. The physical difference between inputs and outputs made them incommensurate and so immeasurable, so for physical prices to be measurable it was necessary for inputs to be identical with outputs, in other words, for inputs to be ‘unchanged’ to not to be destroyed in producing outputs. The strength of Sraffa’s argument it transpires was that it assumed away production itself. Kurz (2018) considers that ‘any product that is needed directly or indirectly in the production of all commodities produced in the economy could serve as a common third’ (p. 55). Indeed, but what appears to be a statement of the obvious is essentially misconceived. Economics or political economy is an examination of human society, even wheat was an invention of people. The assertion that human social activity is the only common feature of all human production is a necessary consequence of the definition of the problem. In a capitalist mode of production, the only commodity common to all human production is the human commodity itself.
Ricardo, Schmidt, Hilferding and Rubin

David Ricardo (1815) had proposed the original corn/wheat model in his 1815 Essay, where he assumed that one input wheat could produce more of itself as an output. Wheat could serve as the seed for itself and food for the labourer that sowed it. As inputs and outputs were directly commensurate, their quantitative difference could then be measured. As wheat formed a key cost of all other commodities, it could then provide a standard of value against which all other output could be weighed. His correspondent and friend, Thomas Malthus pointed out, however, that even in the production of wheat different inputs were required to produce the single output wheat (fertiliser, water, apples, clothes, spades etc.) aside from wheat itself. Consequently, even in the best case, inputs and outputs were incommensurate. Ricardo’s new political economy, explained in his 1817 Principles of Political Economy, was similarly incommensurate with his old political economy, the physical wheat/corn model. The Principles examined how the produce of the earth was divided between classes of people and sought through the discovery of the laws that regulate this distribution to explain the production process. Developing Smith’s distinction between value in use and value in exchange, Ricardo (1990 [1817]) determined that labour is ‘really the foundation of the exchangeable value of all things’ (p. 2). Manufacturing production is above all a labour process where commodities are not scarce but produced in unlimited varieties and amounts so that the labour time necessary to produce a commodity, to make it useful, was the only factor common to production. Luigi Pasinetti (1977) claimed that ‘the simplest way to grasp the principal ideas of Ricardo is to consider a highly simplified economic system in which only one commodity – corn – is produced (a one-sector model)’ (p. 8). This is practically the opposite of the truth. As Bellino (2012) noted,

The assumption that capital is constituted just by one commodity has two consequences: on the one hand, it establishes a direct connection of the model with the logical structure described by Ricardo in his Essay, rather than in its Principles; on the other hand, it renders unnecessary the introduction of the labour theory of value as an assumption, as we have no necessity to measure aggregates of commodities with different compositions in order to calculate the rate of profits. This rate emerges as a ratio of physical quantities of corn for the entire economic system. This result is the point of departure of Sraffa’s generalization to any number of basic commodities.

A single commodity economy (a contradiction in terms – economic production changes commodities) would be directly commensurate and so exclude the necessity for labour value as a standard of price. Nonetheless, Gary Mongiovi(2002), paradoxically a Sraffian, attributed Sraffa’s reliance on physical quantities to Ricardo and then criticised Ricardo for Sraffa’s mistake:

Marx also grasped the limitations of Ricardo’s corn-ratio argument: its central premise – physical homogeneity of product, means of production, and workers’ means of subsistence – does not carry over to manufacturing sectors, so that the general analysis of the process by which a surplus is created and appropriated requires a theory of value. (p. 413)
The homogeneity of the physical product, the identity of inputs and outputs, excludes manufacturing production, and by extension all production. That is why manufacturing production, and by extension all production, requires a theory of value. There are two alternatives, given that production is a process in which humans produce commodities for their own consumption, either the use value or utility of a commodity or exchange value, the human labour time necessary for its production.

Conrad Schmidt, in a brilliant 1892 critique of subjective marginal utility economics *The Psychological Tendency in Recent Political Economy*, applied Marx’s method to examine the contradictions of the Austrian new marginal economics. He explained that the riddle of value and price can ‘only be solved if we understand the universal, objective and comprehensive law that rules the exchange of commodities for money’ (Schmidt in Day & Guido 2018: 412). Money equated different goods with each other. Despite their differences, ‘a common factor must exist that makes this equalisation, this commensurability of the seemingly incommensurable units, possible’ (Schmidt in Day & Guido 2018: 412), this common factor, and so the unit measured by money, can be nothing other than ‘labour per se’ (Schmidt in Day & Guido 2018: 412). Labour is the only factor common to all human production.

The value of labour on the input side is the same as the value of labour on the output side, necessarily as output is the product of input, but not all of the labour on the input side is paid for. The cost of labour on the input side is lower than labour on the output side. The difference between the worth of labour expended and the amount paid for it is surplus value, the social origin of profit, interest and rent. This value is measured in a variable social numeraire, money. Variable as the value of the numeraire changes with changes in productivity too. Competition ensures that this aggregate of surplus value is redistributed between property owners (according to those same property laws) so that the price of individual commodities will diverge from values, but nonetheless, the origin of surplus value is social, a consequence of property relations. As only humans can own property (cats, radios and water features cannot own humans), so only humans can be underpaid for the value of their product, so only human labour can add value.

The laws of political economy are social not physical ones. There can be no other explanation of social surplus than a social one. These laws are not natural laws akin to physics; they are not necessarily mathematical laws. Mathematical truth, that is, a mathematical result consistent with the laws of mathematics, a subjective truth, may or may not be an objective truth, that is, consistent with the world. The laws of political economy are social laws, objectively true when they describe the real laws of society. This is not a lower standard than a mathematical one, and insofar, as the laws of mathematics contradict reality, it is a higher one. Rudolph Hilferding in Day and Guido (2018) perhaps the most noted political economist of the Second International explained that economic laws are the historically determined ‘laws of social relations between people’ (p. 349). Isaac Rubin in Day and Guido (2018), the leading value theorist of the early Soviet period, considered that Marx’s theory of political economy investigated ‘the change of production relations among people in accordance with development of productive resources’ (p. 543). Political economy is a study of ‘people to people’ (Rubin in Day & Guido 2018: 544). While the ‘categories of political economy express different social
functions of things that correspond to the different production relations of people’ (Rubin in Day & Guido 2018: 553), Marxist political economy studies the social relations between people where physical commodities are the bearers of those social relations.

**Commensurability and basic goods**

Commensurability means that inputs and outputs must share some quality in order that the quantitative change of that quality can be measured. Physical commensurability is impossible in a market economy as inputs are finite but outputs are infinitely varied. Sraffa rejected an external social numeraire (‘mystical’ abstract labour or ‘meaningless’ use value) and so assumed away the actual world and replaced it with direct commensurability, the physical identity of inputs and outputs, either at the level of the firm or the economy as a whole, in his imaginary world. This truly was mystical labour.

At the level of the firm, the identity of inputs and outputs (inputs are ‘unchanged’ during production (Shaikh 2016)) conflicts with the function of a firm, to change inputs into different outputs. Direct physical commensurability requires that with the given data set, a certain set of definite inputs must be identical with the outputs they produce. There can be any finite number of inputs, but once defined, they determine the same exact set of outputs. Nothing new, that is, something not already produced or within the given set of inputs, can be added. One undefined (incommensurate and so immeasurable) commodity is enough to make all commodities undefined. To add plausibility to Sraffa’s system of stasis, Sraffa creates a set of unchanging or ‘basic’ goods. They are the ‘external third’, the standard against which all other goods can be measured. Luigi Pasinetti (1977), a noted Sraffian, explained in his book *Lectures on the Theory of Production*,

> We have seen that a basic commodity is a commodity which is technically necessary for the production of all the other commodities. It follows that a zero production of even just one basic commodity necessarily implies zero production of all commodities (basic and non-basic). (p. 108)

Basic goods do not change and, so logically, can never have been produced, if production is a process that creates new goods. Basic goods have existed forever and every one of them is always used in the production of every other good. ‘New’ goods are but combinations of old ‘basic’ goods. The purpose of production is to increase the scale of physical production in general and of physical surplus in particular. Sraffa’s economy is limited to those sectors that ‘permit the production of at least some commodity in addition to those needed for the replacement of the means of production used upon the production process’ (Pasinetti’s emphasis, Pasinetti 1977: 63). By means of production, Pasinetti means the basic goods from which everything else is produced. Sectors which reduce the scale of physical production, that is, which consume basic goods or raw materials (i.e. which recombine them in a form which does not increase their physical scale) are deemed to be unviable and excluded from the model. This Perron–Frobenius viability condition means the output of every sector must be non-negative in terms of basic goods (i.e. the goods
that all output is a combination of) so no sector may ‘use as means of production greater quantities of commodities than it would be able to produce’ (Pasinetti 1977: 97). This excludes what Pasinetti (1977) deems to be ‘economically meaningless cases’ that use up inputs in aggregate (p. 63). Which economically meaningless sectors are these? All non-raw materials producing sectors, that is, manufacturing and services. Manufacturing is, insofar, as surplus is measured in quantities of physical raw materials, to use the Physiocrats’ expression, ‘sterile’. The output of all services and manufactured goods contain a physically smaller amount of raw materials than was required to produce them. Sraffa’s model is limited to the only ‘economically meaningful’ sector, the only sector which increases the amount of basic goods in aggregate, that is, farming, mining and fishing, the raw materials sector, hence his reference to a stock of inputs and harvest of outputs.

Sraffa’s model of simple production took a quantity of wheat (A), a quantity of pigs (B) and a quantity of iron (C) as inputs to produce the same quantity of wheat, pigs and iron as outputs $A + B + C = A + B + C$. It is assumed that the ‘labour force, technical knowledge, and consumption decisions are all constant through time, and the economic system neither expands nor contracts’ (Pasinetti 1977: 36). Pasinetti (1977) noted that ‘exactly the same physical flows of goods and services are reproduced year after year without any change’ (p. 36) and according to a fixed set of ‘exchange ratios or “prices”’ (p. 38). It answers the question ‘how even simple reproduction can occur without production’ (Mongiovi 2016: 8). Literally, nothing is produced in it. Sraffa’s model of simple production is a model of nothing. It cannot be otherwise $A + B + C = 0$.

Although Sraffa and Pasinetti claim that the inputs ‘reproduce’ outputs, this cannot be true. The physical price system requires that inputs and outputs have to be physically (qualitatively) identical to be commensurate, so they cannot change qualitatively. As they are quantitatively identical in simple reproduction (quantities of inputs and outputs are the same), so they cannot change quantitatively. As they are qualitatively and quantitatively identical, so they are identical. They are identical so they have not changed, as they have not changed, so nothing has happened. As production is a process which physically changes inputs into different physical outputs, the identity of inputs and outputs, this ‘reproduction’, paradoxically precludes production.

Almost by the way it should further be noted, it is impossible to produce pigs, iron and wheat from pigs, iron and wheat. Actually, Sraffa (1916–1983, D3/12/6/10 p2) was aware that outputs must be different from inputs but provided inputs and outputs were given (i.e. identical in aggregate), then price could be directly derived from the structure of production, and surplus was the physical difference between stock and crop:

Since ‘surplus’ is the difference of physical quantity between initial stock and crop, this comparison cannot be made within any one industry, which has an initial stock composed of quantities of various heterogeneous things, and crop composed of one thing only. The comparison is possible only from the point of view of the community as a whole: in this case we can say ‘the stock of A (distributed among the different industries) was 100, and at the end of year the crop of the industry producing A is 100A’.

But if output was incommensurate with input, then Sraffa’s model could not be true of a single individual act of exchange, for as Bailey explained, considered physically, all
prices are relative, so there can be no objective standard of value and so no surplus considered physically. If individual exchange is impossible, how can there be an aggregate of exchanges? If outputs are physically different from inputs, then there must be some standard of value against which their value or price is measured. Hence the need for a standard commodity or physical numeraire, but is this possible either?

**The standard commodity or physical numeraire and Leon Walras’ imaginary economy**

Leon Walras who ‘vehemently denounced’ realism ‘in all its manifestations: in art and in literature as well as in philosophy, science, and economics’ (Jaffé 1980: 532) developed the physical numeraire essentially inherited by Sraffa. This physical numeraire rejected the notion of absolute value, a unit of value against which output can be measured and instead assumed that relative values are fixed, such that relative value is elided with absolute value.

Walras’ problem was that utility is not measurable. It was subjective by definition. As a result, if utility was the standard of value, there was no standard of value at all. Commodities had no intrinsic value to be measured and money had not intrinsic value to measure it ‘the word franc [as a unit of value] is the name of a thing that does not exist’ (Moscati’s interpolation, Moscati 2013: 188). This made the whole application of mathematics problematic. How to count something that cannot be counted? Henri Poincaré, the theoretical mathematician and physicist, pointed out to Walras that the inclusion of arbitrary functions in his premises, while strictly speaking allowable, meant that if these functions were not excluded from the conclusions, then the conclusions ‘are not false, but they are totally without interest’ (Jaffé 1977: 304). Jaffé (1977) sagely noted, ‘in point of fact, Walras did not succeed in eliminating his arbitrary functions from his conclusions’ (p. 304). He simply assumed he did ‘I shall … assume the existence of a standard of measure of intensive utility’ (Moscati 2013:400). How simple life was?

How then to count this thing that could not be counted? Walras considered that in the derivation of demand curves of the two elements given, initial endowments and utility, ‘one is perfectly measurable, namely the quantity of each commodity initially owned by each trader’ (Jaffé 1977: 304). To measure exchange value, the quantity of the initial endowments was assumed to be equal to marginal utility. This assumption had no basis in reality or the theory itself, but it provided a method for avoiding the problem. It is the equivalent of asserting that in order to measure the beauty of the Mona Lisa, all that was required was to measure the distance between its picture frame and the door to the gallery, on the assumption that the relatively fixed distance between frame and door corresponded to the loveliness of the portrait. This produces an answer which is not (mathematically) false but is totally without interest; nonetheless, following this procedure, the unit becomes the initial endowments, the arbitrary set of relative prices themselves. If relative prices are fixed, then any commodity can be substituted for any other commodity so the unit is ‘a certain quantity of a given commodity [what Walras called a numéraire], and not the value of this quantity of the given commodity’ (Moscati’s interpolation, Moscati 2013: 402). The relationship between the Mona Lisa’s frame and the door becomes a universal measure of beauty. The exchange value of a commodity is then measured by the number of units of the numéraire exchanged for one unit of the
commodity in question (Moscati 2013: 402). A multi-commodity economy is then a single commodity economy in disguise. Relative exchange values can only be fixed if different commodities are, in fact, the same single commodity (Marx 1970 [1859]). This single commodity was Walras’ imaginary commodity (E), which he ‘explicitly defined it as a figment of the theorist’s imagination’ (Jaffé 1980: 541). The commodity (E) was ‘a pure abstraction’ serving to reduce the ‘whole complex of heterogenous capital assets to a homogeneous net-income yielding entity’ (Jaffé 1980: 542).

Sraffa replaced Walras’ homogeneous net-income yielding figment of an imaginary commodity with the standard commodity. The standard commodity assumed a certain type and number of inputs and multiplied them by a certain scalar to become a larger physical quantity of the same homogeneous outputs, such that the inputs did not change and their relative proportions or exchange values or prices remained constant. This was in effect a vector, scalar and Eigen vector. The key point was that inputs and outputs are physically identical and that relative prices were fixed. Provided the relative prices or proportions or exchange values were maintained, then any commodity or combination of commodities could act as a standard of value or price for any other. All commodities were produced. There was no explicit social framework or social element, no labour or use value and no social law. The standard of value was a natural emanation of the given system of prices. Although it was ‘merely fanciful to think such a commodity could be found in the real world’, it could ‘always be constructed from the commodities actually produced, by taking them in the proportions defined by the standard system’ (Pasinetti 1977: 120). Profit is an assumed quantity and assuming it is constant then ‘with respect to prices there is no difference between a stationary system and a system with population growth where the same numeraire is kept through time’ (Pasinetti 1977: 198). If growth is assumed, all physical commodities increase by exactly the same proportion ‘so that all proportions remain constant through time’ (Pasinetti 1977: 195) and ‘since all proportions remain constant the physical quantities too may thereby be expressed independently of time’ (Pasinetti 1977: 199). The fanciful assumption of physically identical unchanging and immutable inputs-outputs, with fancifully fixed relative prices, meant fancifully heterogeneous physical quantities could be fancifully multiplied by a price and so valued, fancifully out of time (Pasinetti 1977: 49). The use of Walras’ physical numeraire required the economist, Sraffa, to ‘vehemently denounce’ reality.

**Wassily Leontief and Sraffian Matrices**

Wassily Leontief, who originated input–output matrices in the West, was a Soviet economist who fled to the United States in the late 1920s. A participant in the Balance debates of 1923/1924, his input–output matrix divided production into a number of sectors based on socio-technical use-value categories like construction, transport, entertainment, agriculture, mining, infrastructure and services (Jefferies 2015b). The Balance was in its turn inspired by Marx’s schemas of reproduction from Capital II (Marx 1981 [1894]). Leontief abstracted from the changing nature of physical outputs to establish broad use value categories that enabled the input–output matrix to show the physical inter-relationship between inputs and outputs measured in value terms. Together these categories constituted the economic system. Each category recorded the quantity and
variety of the broadly conceived physical inputs and physical outputs consumed by and produced within it. In so doing, they revealed the inter-relationships between the sectors of the economy. The categories were constant and for the sake of convenience, it was assumed that each industry produces one, more or less narrowly defined, homogeneous good. The assumption of homogeneity did not actually render the commodities homogeneous. Actual output in a market economy is infinitely varied. Some definite quantity of inputs \( n \) produces an unknown and infinitely varied \( \infty \) quantity of outputs. As the Sraffian–Marxians, Cogliano et al. (2018) explain,

In theory we can conceive of an economy producing \( n \) goods and refer to the quantities of all these goods. Yet there are thousands of goods on the market. In the real world even the most detailed statistics cannot record each and every good, and even at the most elementary level several goods are combined into aggregate goods. Thus in applied work the expression ‘quantity of good \( i \)’ must not be taken literally. Rather it means something like: One unit of good \( i \) is a bundle of (physically different) goods that are assigned a category \( i \). (p. 43)

The use value categories of Leontief’s model defines the extent of allowable difference. They do not make the different aggregate commodities actually identical, actually real or actually commensurate. Paradoxically, while the Cogliano and colleagues warn against taking these aggregates literally, this is precisely what they do. None of their mathematical calculations include the qualification that these aggregate homogeneous commodities do not actually exist. To do so would mean that the Sraffian physical alternative to value measures does not exist either. The pre-condition for the mathematically rigorous alternative to the labour theory of value is a set of conditions that excluded change, the very purpose of production. Leontief’s (1951) aggregate physical quantities were multiplied by an external value measure, money as a numéraire, a ‘value’ unit. Leontief (1951) noted that

… in order to obtain the corresponding physical amounts of all commodities and services, we simply define the unit of physical measurement of every particular type of product so as to make it equal to the amount of the commodity that can be purchased by one dollar at prevailing prices. (p. 72)

(Human) labour is the only scarce commodity, that is, the only input not produced by the production matrix (Cameron 1952). An application of the labour theory of value means that the Perron–Frobenius viability condition which requires that the output of every sector must be non-negative is easily met. Every sector adds value (quantities of socially necessary labour time) and creates surplus value (including transfers) irrespective of the quantity of physical inputs incorporated into their outputs. Manufacturing and services destroy physical inputs but add value by incorporating more abstract labour into the commodity as output than is embodied in it as input. That value is not limited to the labour directly paid for wages or expended on the production of raw materials, but includes transfers from other sectors and crucially unpaid labour. This is a cost borne by the labourer not the capitalist. Its value only appears on the output side as it is alienated upon sale. As the origin of surplus is social, it is real labour, a real cost of production but to the workers who expend it, not to the capitalists, who do not pay for it. It is unpaid
labour. The value added or price of matrix A (for intermediate inputs) is less than the total of value added or price $p_A < p$.

Provided prices are already given in money, a universal equivalent, a unit of intrinsic value, then it is possible to ascertain the amount of commodities by the price or the price by the amount of commodities and transform these into amounts into socially necessary labour as ‘the general equilibrium system of Professor W. Leontief … is premised upon a labour theory of value’ (Cameron, 1952: 191). This is necessarily so as it was based on the Soviet Balance, which was in its turn based on Marx’s *Capital II*.

Leontief’s matrix allows new output to be measured in terms of money prices in categories allocated to physical use values of more of less the same type (Leontief 1986). Rows are commensurate as units of different physical inputs and outputs, but columns are not and so must be valued by the external value money numeraire. But, what is the unit of value that money brings into equivalence? What is the common property that it counts? Inputs are finite, and varieties of physical outputs are conceptually infinite. Relative prices constantly change. Every individual exchange must be commensurate with every other, individually and in aggregate. The unit cannot, therefore, be a physical unit. It must then be a social unit. The unit cannot be use value, as use value is subjective, immeasurable. There is no unit of utility. The only possible unit that money measures is abstract (human) labour time. The social average time that people require to produce something is modified by the movement of surplus that tends to equalise profits per unit of capital advanced. The test is whether the operation is reversible. If we know the amount of socially necessary labour time incorporated into a commodity (including transfers of value to equalise profit rates), then establishing a correspondence between the physical output and price is straightforward. But the reverse operation is impossible. It is not possible to know the value of something from its physical characteristics alone. The price of a physical quantity of goods can only be known from that physical quantity if prices are given in advance.

Sraffa substituted Leontief’s matrix, where the inputs and outputs of different sectors of the economy are measured in money, with one of identical and unchanging basic goods produced in strictly fixed proportions. The Sraffian matrix did away with productive categories entirely, instead commodities produce themselves. There were no scarce commodities. It was the production of commodities by means of commodities. Commodities were self-expanding, constant, finite and really homogeneous. They were no actual commodity or anything that existed, they were nothing real. Relative prices were fixed so that a physical numeraire or standard commodity, or any commodity indeed, provided a measure of ‘value’, such that relative prices were absolute prices. There was no conceptual difference between human labour and any other type of physical commodity. Sraffa’s model did not require humans at all. The ‘wages’ were just costs. They could be the ‘wages’ of pigs (swill), machines (oil) or humans (TV dinners). Pig labour (the quality of eating to fatten oneself up for slaughter) was qualitatively identical to human labour. Sraffa considers that the ‘wages of the worker belong to the same class of necessities as the fuel of machines and the hay of horses’ (Sraffa 1916–1983, D2 4 3 f23). If human labour was identical to pig labour, then if pig labour can be produced, labour can be produced and so human labour can be produced. Can the humans be fed for slaughter? But humans cannot be owned by pigs, only pigs by humans, and humans cannot be owned by humans, unless this was a slave economy.
As there were no social laws within it, then maybe it was a slave economy. In a market economy, the human labour of the producers is free in the double sense, free from the ownership of the means of production and free to be sold. If it is assumed, it was not a slave economy but rather a market economy, then human labour cannot be produced. Humans cannot sell themselves, but only their labour power, their potential to labour for distinct periods. Human labour is not then produced by the production matrix. If human labour cannot be produced, then it is qualitatively different from pig labour.

The proviso that the model is one of generalised commodity production, and so not one of slavery, means that human labour is not included in it, not produced by it. If human labour is not produced by it, then the Perron–Frobenius viability condition, that the production of each sector must be non-negative cannot be met, unless production is restricted to raw materials or the basic commodity producing sectors. Consequently, the model cannot include the manufacturing and services sectors that only add human labour but use basic commodities or raw materials; should the economy not be a slave economy and should manufacturing and services exist, then the physical price system cannot be the standard of value in it.

Manufacturing and services increase the usefulness of physical inputs by transforming raw materials into a form in which they can be consumed. They constitute a labour process as the only physical commodity they add to production is (human) labour. As the labour process adds labour, it destroys as it transforms and so reduces the physical scale of all other basic commodities or raw materials:

Thus in the course of the labour process use-values undergo a genuine transformation, whether of a mechanical, chemical or physical nature. In the commodity use-value is a given thing with definite characteristics. Now, however, in the labour process, we find the transformation of things, use-values, functioning as raw materials or means of labour into a new use value – the product. (Marx’s emphasis, Marx, Capital I: 980)

Production is a process of creative destruction, a labour process, in which humans physically transform inputs from a less useful form, a form that cannot be consumed, into a physically different, more useful form that can be consumed and in a market economy sold, the pre-condition for and purpose of the accumulation process. Marx (1976 [1867]) continued,

Looking at the process of production from its real side, i.e. as a process which creates new use-values by performing useful labour with existing use-values, we find a real labour process. (Capital I: 981)

It is a process of human labour, not a process of physical commodities distinct from and independent to that labour. Physical inputs are the stuff from which human make commodities. As something cannot come from nothing, as output is the product of the inputs that made it, so output can only be different to, not larger than, that quantity of inputs that made it. Surplus value arises not because the amount of labour on the output side is larger than the input side but because the cost of that labour on the output side is lower than the input side. A quantity of labour on the input side is not paid for, it is
unpaid labour. Uniquely, humans can be paid less for their labour than its value, hence Marx’s distinction between labour and labour power. As human labour is the only commodity that cannot be produced in the production matrix, as human labour is the only commodity common to all production processes, as only humans have free will, so only humans can own other commodities, and as human labour is the only commodity that can be paid for its own use and the use of another commodities, so all commodities are valued in terms of human labour.

**Quesnay and the Physiocrats**

François Quesnay’s, 1753, Tableau Économique was the original circuit of production. It showed how surplus appeared first in agriculture, represented by a physical increase in the mass of products. The difference between subsistence and surplus ‘appears most palpably, most incontrovertibly, of all branches of production, in agriculture, the primary branch of production’ (Marx’s emphasis, Marx, 1971 [1861–1863]). Quesnay’s schema considered that agricultural sector alone produced surplus as

the sum total of the means of subsistence which the labourer consumes from one year to another, or the mass of material substance which he consumes, is smaller than the sum total of the means of subsistence which he produces. (Marx, 1971 [1861–1863])

As Sraffa (1916–1983) observed, The whole system of the Physiocrats turns upon the conception of cost which I have outlined. The essential question for them is the ‘produit net’ or net produce, that is the difference between the total aggregate of goods advanced and consumed in production and the aggregate of goods produced. (D2.4.3 E.25)

While manufacturing was ‘sterile’, it used up physical inputs and so reduced the quantity of physical output in general and physical surplus in particular. Sraffa (1916–1983) noted that

The Physiocrats’, whose fundamental doctrine was that only agriculture produced a surplus, and therefore added something to wealth, whereas industry simply reproduced in its products the values of the advances (food and raw materials) consumed in the course of production (marginal note ‘They thought that producing meant to increase material, weight’). (D2.4.3 F.26)

Assuming that value is merely a register of quantities of agricultural production, then the quantity or price of matrix A (for intermediate inputs), that is, after agricultural produce have been harvested, but before produce is used up by the manufacturing (the ‘sterile’ sector), is greater than the total of value added or physical production (the same thing) at the end of the circuit so that \( pA > p \). Sraffa (1916–1983) confirms this point:

We can see how the Physiocrats came to hold this view. Measuring both the product and the cost in physical amount it is obvious that in agriculture, say in a corn farm, the amount of corn produced is greater than the amount used for seed and for subsistence of the workers. But in
industry such a calculation leads to opposite results: e.g. in a spinning mill the amount, in weight, of the yarn produced is necessarily smaller than the weight of the raw cotton consumed. (D2 4 3 f26)

Marx (1861–1863 [1971]) explained that the Physiocrats considered,

In manufacture the workman is not generally seen directly producing either his means of subsistence or the surplus in excess of his means of subsistence ... The workman in industry does not increase the material substance; he only alters its form. The material – the mass of material substance – is given to him by agriculture.

Quesnay was justified in this use of aggregates. In 1753, pre-revolutionary French agriculture was still dominated by the corvée system of labour services. It was not a market economy. Surplus labour in the form of aggregates of physical product was directly extorted from serfs at harvest. The chevalier, lord or church did not pay for the production of their serfs, they directly appropriated it. Individual commensurability was irrelevant, as this was not yet an economy based on generalised commodity production. In agriculture, the aggregate quantity of outputs, the sum total, was larger than the quantity of inputs. In manufacturing, the worker did not increase the material substance but consumed it through altering its form, adding their wages to its value, but in so doing increasing ‘material’ costs and reducing ‘material’ surplus. But if manufacturing consumed surplus why did it occur? Adam Smith visited France in 1763 and stayed for 20 months. He was familiar with the work of the Physiocrats and may well have held personal discussions with Turgot and Quesnay (Ross 1984: 184). Smith praised Quesnay ‘the very ingenious and very profound author of this system’ but Smith argued Quesnay’s error lay in defining the ‘class of artificers, manufacturers and merchants, as altogether barren and unproductive’ (Ross 1984: 186). Manufacturing added labour in the process of consuming, destroying and transforming physical inputs, this did not reduce value as the Physiocrats argued, but increased it, as value was not physical but social. Smith realised it was precisely the addition of labour that constituted the essence of value (Smith 1776). This was Smith’s great breakthrough most notably developed by Ricardo and Marx.

Sraffa’s system is then vulnerable to the same criticism that Sraffa made of the Physiocrats. Sraffa’s notion of basic goods excludes manufacturing as outputs are different from inputs and so incommensurate with them and that the physical amount of manufactured goods (as a quantity of basic goods) must be smaller than the physical amount of raw materials or inputs (as a quantity of basic goods) used in their manufacture.

Leontief mentioned Quesnay in his original input–output schema (1951), but only to throw his secret service interrogators off track. Leontief’s input–output system was based on the Soviet Balance of the Soviet central statistical office (TsSU), which included the monetary balances of the mid-1920s Soviet New Economic Policy (NEP) period. Sraffa, on the other hand, followed Gosplan’s Material Product System (MPS), which included the material balances of the post-1928 plan period. The distinction is fundamental. The NEP was a system of managed commodity production, with real markets and market prices. The commensurability of production was ensured through a
convertible currency based on the gold Chernovets. The plan period, on the other hand, had no sales within it. Material inputs and outputs were determined by the planning ministry and prices were allocated post-factum according to political criteria. There was no commodity production and so no exchange value or price. The centrally planned economy had no need for individual commensurability. Maurice Dobb, who oversaw the publication of Sraffa’s *Production of Commodities by Means of Commodities*, was a Cambridge professor and Britain’s leading Stalinist economist and a loyal follower of the party line. He was but a small cog in the wheel, the Stalinists’ (Jefferies 2014) attempt to attribute value categories to the plan was part of a wider attempt to reconcile the existence of really existing socialism with the market (Jefferies 2015b). It did not end well.

The production of surplus

If physical quantities of outputs are produced by given quantities of inputs, what is the origin of surplus? Sraffa (1916–1983) noted that ‘the “cost” of producing an ounce of gold (or a bushel of wheat) will always be … an ounce of gold (or a bushel of wheat) however, much the methods of production are improved’ (Sraffa's punctuation, D2 4 3 f46), but if a thing cannot produce more of itself from itself, from whence did surplus appear? If a change in technology alters the physical relationship of inputs and outputs (something forbidden by the assumption of constant relative prices and no new basic goods), then the process becomes more efficient and less inputs are required to produce the same outputs, but this is not the creation of surplus. The same inputs now produce more outputs, but if prices directly reflect the technical structure of production, then prices simply change in proportion to the technical change. As Sraffa (1960) noted,

> If an invention were to reduce by half the quantity of each of the means of production which are required to produce a unit of a ‘luxury’ commodity of this type, the commodity would be halved in price. (p. 8)

A luxury commodity is a non-basic commodity (and so a contradiction in terms given the viability condition). Elsewhere, Sraffa observed that

If one attempts to take an entirely objective point of view, the very conception of a surplus melts away … in our case, there can be no product for which there has not been an equivalent cost, and all costs (expenses) must be necessary to produce it … This is the great difficulty: the surplus is the object of the inquiry, but as soon as it is explained, a cause is found for it, and [it] ceases to be a surplus. This sounds as if the object of the inquiry had been defined as ‘the unknown’, but if the inquiry is successful it becomes known, and the object of the inquiry ceases to exist! (Kurz & Salvadori 2005: 88)

How to overcome this apparently insurmountable problem? Assume it away, ‘the mistake has been in the initial equations not to make it explicit that product is greater than total factors’ (Sraffa 1916–1983; notes on PCBMC; D3/12/6 1/6 6). Surplus comes from nowhere. It is the product of magic (Jefferies 2015a). The output side is simply multiplied by a number (any number take your pick) so that it is larger than the input
side. This was the ‘solution’ posited by the original physical price economist Dmitriev (1974 [1902]):

Whenever a known quantity of some product $\alpha$ has been used up in the production of $\alpha$ and we can obtain a larger quantity of the same product within some finite period of time as a result of the production process, the profit rate in the given branch of industry will be a fully-determined quantity greater than zero, irrespective of the price of the product $\alpha$. (p. 62)

There was one input $A$ which became a larger $A + a$. The profit was then the difference between $A + a - A = a$, while the rate of profit was $a/A$. Inputs were identical with outputs, while outputs had grown relative to inputs without any separate value measurement. Dmitriev’s most famous example elaborated this idea with self-producing robots. The production of robots is by means of robots. The robots were the only factor of production. There were a given number of robots say 5. Those robots produced say, a new sixth robot identical to the previous five robots. The additional sixth robot was worth the same as each of the original five robots, so price (or value) had been produced by robots (i.e. without humans), and a surplus robot (i.e. without humans) such that rate of surplus was the new robot divided by the original robots or one-fifth or 20%.

The original five robots were unchanged at the end of the production process. Assuming the conservation of energy, the old robots cannot have been what the new robot was produced from, even if they were involved in its production (which given they are unchanged they cannot have been). As the original robots were unchanged through the production process, and as there were no other inputs, the new robot was produced from nothing. Whatever the price or value of the original robots, as they were unchanged during production, and as they have used no other inputs (they cannot or output would be incommensurate with input) and so they have transferred no value to the product. As something cannot come from nothing, so the new robot was the product of nothing, so the new robot was nothing. As the new robot was identical with the old robots, so the old robots were nothing, so Dmitriev’s surplus was nothing, so Dmitriev’s model was a model of nothing. It was the limitless expansion of nothing.

If physical output of the raw materials sector increases in aggregate, this can only be measured if inputs are directly commensurate, identical to outputs, necessarily they are deemed to be ‘unchanged’ during production (Mongiovi 2016). They are then, inputs which are not inputs, not the stuff from which outputs are produced. Nonetheless, these unchanging inputs somehow produce more of themselves from themselves when they are multiplied by some scalar so that outputs are quantitatively larger than inputs. The surplus has been produced without equivalent, indeed without production. It has come from nowhere. It is the product of nothing. As the surplus is qualitatively identical with the inputs that produced it, so the inputs are nothing, so Sraffa’s model of expanded production is a model of nothing. If one takes an entirely objective physical point of view, the very conception of Sraffa’s surplus melts away.

**Conclusion**

Sraffa’s physical price system required the inputs and outputs were identical at the level of the firm. This requirement contradicted the essential purpose of production to
physically change inputs into different and so necessarily incommensurate outputs. But it enabled the commensurability and so measurement of production and so permitted exchange, sale and price. Alternatively, Sraffa’s physical price system required that inputs and outputs were only identical at the level of the aggregate. This requirement enabled the existence of production but contradicted the essential purpose of a market economy, exchange, as purchase and sale require knowledge of price.

The identity of inputs and outputs was ensured through the creation of ‘basic goods’. All output consisted of combinations of these basic goods. These basic goods had existed always and all basic goods were always used in the production of every good forever. If there is one exception, like asbestos, a raw material which was used but is no longer, then the model failed. Not that there was forever, as there was no time in the model either. Technology was fixed, for if technology was inconstant, then relative prices changed, while the discovery of just one new commodity, or basic good, meant all other goods instantly became immeasurable. Yet, at the same time, as it was unchanged, technology changed, to improve the efficiency of production and so create ‘surplus’.

Given a common set of inputs, Sraffian data must retain its physical form through the production process, whether individually or in aggregate. This contradicts the essential purpose of production which is to change the physical form of inputs, individually and in aggregate. It means that Sraffians can never have the ‘same data’ as any theory of real production (Smith, Ricardo, Marx, etc.) as such theories are predicated on the change of data, that is, the physical form of input changes into a different and so incommensurate output during production itself.

For Sraffa, there was no intrinsic or absolute measure of value and following Walras – the irrationalist – it was assumed that all relative prices were fixed. This ‘fanciful’ proviso enabled the construction of a non-monetary, standard commodity or physical numeraire such that relative price became absolute price. In an economy based on generalised commodity exchange where aggregates are composed of millions of individual exchanges, every exchange must be commensurate with every other. Without a measure of intrinsic value, that is, money, no individual producer can know the value of their production and so sales are impossible and production is impossible too. Sraffa’s model included no intrinsic measure of value and so excluded money.

The production of commodities was by means of commodities. There was no distinction between human labour and animal labour or inanimate objects. There were no scarce commodities in the input–output production matrix. This allowed the production of animals and so consistency demands this included human animals, it was a slave economy. If it was not a slave economy, if humans were not produced by it, then it excluded all manufacturing and services, that was, all sectors which only added human labour and consumed physical inputs. All such activity was value destroying in the physical matrix.

At the level of simple reproduction, it was a system of reproduction without production. Inputs and outputs were qualitatively and quantitatively identical. Nothing occurred in it at all. At the level of expanded production, inputs were unchanged and so not the thing from which output was produced. As the surplus was produced from nothing, so it was nothing, as inputs were identical with outputs so the original inputs were also nothing. It was the production of nothing by means of nothing. Reproduction without production.
References


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William Jefferies wrote his doctoral thesis on the History of National Income Measures in the Centrally Planned Economies, which was subsequently published by Routledge in 2015. He currently teaches business and economics at Anglia Ruskin University and SOAS.