THE LABOUR THEORY OF VALUE: A MARGINAL ANALYSIS

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Abstract: This article presents the marginal approach to the labour theory of value. The difficulties of the classical and Marxian labour theory of value are overcome when labour value is understood as marginal labour value analogously to marginal cost. Marginal labour value is the reciprocal of the marginal productivity of labour. Under perfect competition relative prices are equal to the ratio of marginal labour values; indeed, Pareto-optimality implies the validity of the labour theory of value but in general, even in a Pareto optimal state, there is exploitation. It is shown that in principle, a capitalist system can never be in a Pareto-optimal state. To assure a maximum productivity of labour and therefore minimum socially necessary labour values, society has to assure the socially necessary accumulation of capital and to organise the formation and control over capital democratically and collectively, a Pareto-optimum without exploitation.

JEL classifications: B51, D24, D46

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“Marginal Labor ... is the only kind that can measure value.”

John Bates Clark (1892)

Introduction

It is a widespread belief that the labour theory of value is inconsistent and in contradiction with modern economic theory. Those economists who still adhere to it are typically regarded as heretics. In the history of economic thought it is referred to the ‘Marginalists’, in particular W. S. Jevons, Léon Walras and Carl Menger, who are said to have overcome the impasses of the labour theory of value. Jevons is quoted accusing Ricardo of having placed political economy on the wrong track. Marginal analysis is presented as being incompatible with the labour theory of value obviously because otherwise questions like how to calculate the minimum socially necessary labour time of producing a commodity would pop up, questions of the highest importance for any real existing socialist system. After all, marginal analysis is a mathematical method to determine optima, e.g. maximum profits, minimum cost, etc., a method extremely important in particular for socialist economists. Those Western Economists, orthodox or Marxist, who are opposed to the marginal analysis of labour values are obviously opposing more than that, they have been and are opposing real existing Socialism all together.

The exchange model of general economic equilibrium without production shows that exchange values exist also in the absence of labour and production processes. However, this argument ignores that without labour there are no quantities of goods to be exchanged[1] and therefore also no people to perform the exchange as they simply could not exist. We still concede that exchange values can exist when there are quantities of goods with use value available to be exchanged. The act of the exchange of goods can increase the use value for the participants, but producing for the market

[1] The free gifts of Nature are ignored as they would not suffice to maintain humanity alive without labour.

implies that the commodities produced have no direct use value for the producer and therefore it cannot be equal use values which explain the exchange of commodities on markets.

It should be noted that the labour values as the ultimate cost are proportional to prices only when labour is used efficiently, that is when the law of value holds. Only then we can speak of the law of value establishing prices according to labour values. It is a bourgeois ideology that this can be achieved only with markets and competition amongst *private* producers. The economic system must assure that marginal costs equal prices. This is well known from the debates on the feasibility of a Socialist economy amongst Oskar Lange, Abba Lerner and others. But they have failed to recognize that behind marginal costs are *marginal labour values*. In a competitive system marginal cost is the monetary form of marginal labour value. This relationship ceases to hold in the case of imperfections, e.g. monopolies etc.

Closer study of the sources, especially the work of Hermann Heinrich Gossen (1854), who is highly praised by Walras and Jevons, reveals that the mainstream's interpretation of the history of economic thought is wrong. Marginal analysis has not replaced the labour theory of value but improved on its analysis.[2] Jevons, who most fervently tries to reject the labour theory of value, even writes in one place that commodities exchange according to their labour values: “thus we have proved that commodities will exchange in any market in the ratio of the quantities produced by the same quantity of labour” (Jevons 1871, 182).[3]

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[2] Alfred Marshall comments on Jevons "[Jevons'] success was aided even by his faults. For under the honest belief that Ricardo and his followers had rendered their account of the causes that determine value hopelessly wrong by omitting to lay stress on the law of satiable wants, he led many to think he was correcting great errors; whereas he was really only adding very important explanations." (Marshall 1890, 84-85).

[3] Marginal analysis has been introduced long before Jevons, Walras and Menger in the 1870ies. Augustin Cournot (1838) used marginal analysis even before Gossen (1854). It is Cournot's work

Of course, discrediting and rejection of the labour theory of value and those who defend it is wanted by the bourgeoisie to avoid the accusation of the exploitation of the labourers. But could orthodox economic theory really be so powerful in guiding the organisation of society without having a labour theoretical foundation? In this paper we present the development of the labour theory of value using marginal analysis as it has been done historically.

Properly understood, the marginal analysis of labour values is the foundation of economic analysis in particular the analysis of supply and demand. It is the appropriate method to resolve the contradictions of the Classical analysis of value. Marginal analysis focuses on the relationship between use value and labour value and offers a synthesis of use and exchange value by introducing marginal utility and marginal pain (Gossen 1854).

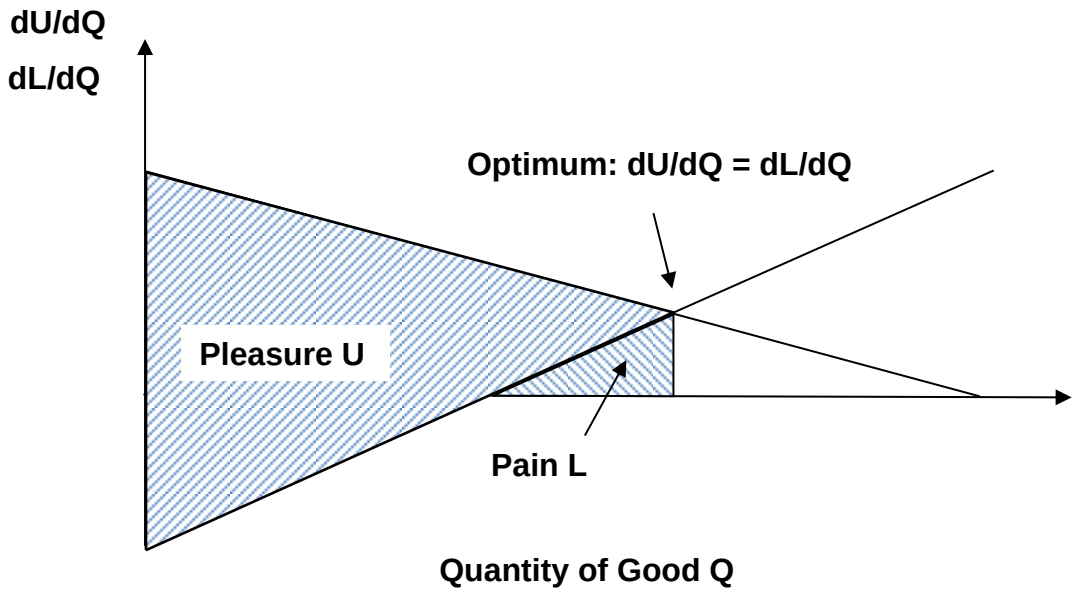

Figure 1: H. H. Gossen's Functions of Marginal Pleasure and Pain

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which has inspired Walras general equilibrium model. Furthermore, also in Germany marginal analysis was well established even if one believes that Gossen's work had been totally ignored.

Through the introduction of the time constraint the optimizing rule becomes one of optimizing net utility per labour time. The subjectivist calculus of utility and pain or disutility is transformed to the objective expression in the materialistic form of marginal labour value derived from the production conditions.

Equally important is the insight gained from marginal analysis, under what conditions surplus value is part of the socially necessary labour time and when on the other hand it is simply exploitation of the labourers for the pleasures of the capitalists. The most powerful insight may be that profits gained as the *returns of capital or land*[4] (natural resources) have to be entirely reinvested in order to assure the optimal use of labour, a requirement in plain contradiction with the institutions of capitalism as the ultimate objective of the capitalist is to make profits in order to consume.

The marginal analysis of labour values allows determining the minima of socially necessary labour values. By giving a precise meaning to socially necessary labour, marginal analysis provides a means to understand the underlying economic processes by which the mode of production of capitalism is transformed into the mode of production of socialism, a process of "crowding out capitalism". The labour theory of value is not only at the core of modern economic theory but it is also fundamental for the *Theory of Historical Materialism,* the general sociology of Marxism. The basic failure of Marxism after Marx was to have abandoned the labour theory of value by defending the concept of the *transformation of values into prices*, known as the *Transformation Problem*. This concept had not been published by Marx but only posthumously in the third volume of *Capital* by Engels. First, we discuss the concept of labour value as it is by no means obvious what has to be understood by labour values.

[4] See Richard Goodwin (1982) and Branko Horvat (1989) for a discussion on this point.

Labour Values and the Theory of Cost

Few are the writings on the labour theory of value which discuss the meaning of labour values at length[5]; indeed here we have the ultimate cause for the misunderstandings concerning the subject. Commonly, also amongst Western Marxists, one considers all the inputs of the production process and according to this bourgeois definition, the labour value of a commodity is understood to be the sum of labour time spent upon producing a good as well as its components. To compute this value, it is most convenient to use the Leontief input-output model (see **Appendix I**), its solution is the vector of the quantities of labour time per unit of the commodities. A more general mathematical method to calculate these values is linear programming[6], but from an economic point of view both methods are essentially the same as can be seen solving a Leontief input-output model via linear programming.

However, it is not at all clear why the values calculated this way should be equal to the *socially necessary labour time* to produce the commodity because there is a difference between the value of the sum of the values of the inputs of production and total cost per unit of output, regardless of expressing this value in terms of money or labour time. Not only that there can be and usually are

[5] An exception is Flaschel (2010), but he provides only an axiomatic definition claimed as being plausible. No discussion of labour values in relation to the theory of cost is found. On the other hand, in Soviet economics, in contrast to Western Marxism, this was a core issue.

[6] This has been introduced by Morishima (1974). We shall see below that this method can easily be adjusted to calculate the proper labour values as total costs by incorporating the "**κ**-rate" which is based on Kantorovich's "norm of effectiveness" (*see* Kantorovich & Vainshtein, 1976) into the program.

externalities but there are costs which occur because of *the use of capital*, i.e. the means of production. We have to focus on the precise meaning and understanding of the causes of these costs in order to provide a useful concept of labour value and to unravel the Gordian knot of the relationship between labour values and prices.

Bourgeois economists offer a wide range of explanations for the causes of the cost of using capital[8] that is interest, *e.g.* waiting, abstinence, time preferences, etc., trying to justify that the owner of capital[9] is given the right to claim some return upon it by lending it to others. We shall not get into the discussion and refusal of all these arguments but simply provide a reason which appears to be self-evident when the dynamics of the process of production is properly taken account of: capital accumulation. It is most important to realize that this part of cost or of labour value is surplus labour, in monetary terms, profits.[10] This is already a glimpse that there is no *real* difference between labour value and cost of production, the later simply being the monetary expression of the former. With other words, there is no *Transformation problem*[11] once labour values are understood as socially necessary cost, expressed in labour time. [12]

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[8] Maurice Allais provides a long list of them (Allais 1947).

[9] It must be noted here that, contrary to orthodox Marxists but in conformity with reality, we consider wages not as capital.

[10] The reader familiar with Morishima's work (1974) will recognize this as a devastating criticism of his approach.

[11] This is the Marxian term referring to *Capital* (1867) on how labour values would be transformed into prices of production (= average cost).

[12] It needs to be emphasised here that at this stage we are not discussing a capitalist economic model but the general economic equilibrium model which does not represent, in its pure form, a capitalist economic system. In fact, in a capitalist economic system prices are usually not proportional to labour values, which is one of the causes of the inefficient and wasteful use of

Cost in Terms of Labour Values

In Marx's *Capital* we find a very touching analysis of the *working day*, a penetrating analysis of the process of exploitation at the work place (Marx 1867 [1906], 255 ff.). We shall attempt here to present the analysis of exploitation in the context of the modern theory of cost but in contrast to the orthodox theory of cost we shall conduct it in terms of labour values. After we have succeeded in presenting the labour theory of value along these lines we shall re-examine and correct the Marxian analysis.

At the outset we have to assume that the economy is using labour efficiently and the producers lack the power to influence the prices on the markets, *i.e.* the firms are price takers on the product- as well as the factor markets. In orthodox theory this is known as an economic system of perfect competition. This condition is crucial for the analysis of labour values as obviously any deviation from perfect competition opens up the chance of extra profits due to some kind of monopoly power and this would create a difference between the cost of production and price and therefore invalidate the labour theory of value and the optimal use of labour. The equality of (marginal) cost – including economic profits – and price is essential; it is the competition within the economic system which enforces the law of value.[13]

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labour. In order to have a general economic equilibrium model of capitalism important modifications of the basic perfect competition model have to be introduced as imperfections, some of them being systemic.

[13] This is a criticism of Rosa Luxemburg's position “In order to find the value of a commodity, we must start by assuming that demand and supply are in a state of equilibrium, that the price of a commodity and its value closely correspond to one another. Thus the scientific problem of value begins at the very point where the effect of demand and supply ceases to operate." (Luxemburg

There is an intrinsic relationship between the process of production and cost; cost theory being the dual to production theory (McFadden and Fuss 1978). Cost theory traces the costs occurring in the production processes, when the process of production changes so usually does cost. We present here only the most rudimentary elements of cost theory in terms of labour values.

The ultimate source of all cost is labour[14] because labour time is the sacrifice of human life time in order to produce goods with a use value.[15] This places the labour theory of value at the centre of Political Economy. Generally speaking, labour is the act of humans necessary for the reproduction of man. The changing of matter in order to gain use values involving the sacrifice of human life time and by this supporting human life, that is the activity of *working* is the foundation of Political Economy and the Theory of Historical Materialism.

Cost can be classified depending on the purpose of analysis. When we consider basic categories of Political Economy we distinguish between *labour cost* (wages), *capital cost* (depreciation and interest), and *rent*.[16] When we speak of *labour cost* we refer to *direct labour*, the labour directly

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1913, [1951], 36). On the contrary, supply and demand are the forces which establish via the establishment of equilibrium prices the law of value. One may concede to Rosa Luxemburg that she wanted to exclude the erratic movements of prices from their equilibrium values, but by eliminating supply and demand she throws the baby out with the bath water. Our cost analysis centres on the establishment of supply and demand functions in terms of labour values. The labour value of a commodity is determined on the market by supply and demand.

[14] This is by no means an exclusively Marxian proposition, e.g. (Fisher 1906, 173 ff.)

[15] Tugan-Baranovsky (1905, 133ff.) explains this point very well but we do not share his distinction between cost and labour value which is due to his (and Ricardo's) error regarding profits; see below the explanation of the **κ**-rate.

[16] In what follows we ignore rent as our analysis is strictly limited to perfect competition, implying the assumption that the rent for marginal land (natural resources) is zero.

used in the production process and to capital cost or *dead* or *indirect labour* as labour contained in the means (also accumulated means) of production, *i.e.* the labour content of the depreciated capital plus surplus labour or profits.[17] The sum of these costs, representing total cost is expressed mathematical in the *cost equation* (see below).

In the analysis of exploitation occurring in the production process, *socially necessary labour cost* can be classified as *paid labour* or *unpaid labour*. Paid labour is that part of the working day which is used to produce the value equivalent to the wage whereas unpaid labour (surplus labour) is the labour time which is used to produce the value equivalent of profits in all its forms. In Marxian analysis the wage is the cost of reproducing *labour power* that is the reproduction of the labourer.

Another criterion is the dependence on the quantity of output. If cost is independent of the quantity of output it is *fixed cost* whereas if it changes with output it is *variable cost*. *Total cost* is the sum of all costs occurring through the act of producing a quantity of output. The *classical cost function* expresses total cost as a function of output.

When we relate *variable* or *fixed cost* to output we speak of *average variable* or *average fixed cost* etc. or if we use derivatives we speak of *marginal cost*. *Marginal cost* expresses the cost of an incremental unit of output whereas the *cost of production*[18] expresses the *average cost* of a unit of

[17] This is loosely speaking as the labour value of the means of production is not the labour time having been spent to produce them but the labour time being socially necessary at the time of evaluation to produce them. So the labour value is not a substance of the commodity but an attribute an economic agent attaches to a commodity.

[18] *Cost of production* is the Marxian and Neo-Ricardian term for average cost. Marxian terminology distinguishes between *constant cost* (capital cost), *variable cost* (paid labour cost), and *surplus value* (value of unpaid labour time). Surplus value is either *absolute* or *relative surplus value* depending on being obtained via the lengthening of the working hours at a constant wage or the intensification of work.

output.

An important distinction of labour time as cost goes back to Adam Smith. If it is seen from the point of an investor spending money one speaks of *labour commanded*, which is calculated simply by dividing an amount of money by the wage rate, the result is the amount of labour time which can be *commanded*. Alternatively, one may look at the commodity from the point of view of production and then the labour time used up in production is considered as *labour embodied*. Under perfect competition, when the labour theory of value holds, both are equal.[19]

When we have *cost-*, *profit-* or *revenue functions* we can use marginal analysis to determine the relative extrema of these functions, e.g. minimum average cost, maximum profit or revenue etc. In order to determine the minimum cost per unit of output we need to find the minimum of the *average cost curve*. This is an important exercise and marginal analysis is most useful to find the solution. It shall give us the *minimum socially necessary labour time* of the production of a commodity and anyone claiming this analysis to be anti-Marxist must be regarded as a fool. But this was the typical Western Marxist attitude at the times of the Cold War, and it still is, even now! In fact, it is common practice amongst Western Marxists to discuss models which imply constant average cost.[20] This is how Marxian economics and Marxists are excluded and exclude themselves from micro-economic analysis and the relation between supply and demand and labour values.

Average fixed costs, i.e. fixed costs divided by the quantity of output, are diminishing with the increase of output. Given a certain size of the production unit, it is most likely that above a certain amount of output additional output becomes more costly, i.e. *marginal cost* rise. Both, diminishing *average fixed cost* and increasing *marginal cost,* lead to an *average cost curve* of a U-shaped type.

[19] Orthodox economists and even Western Marxists usually deny this as they exclude the surplus labour embodied in profits (capital accumulation) from their definitions of labour value, e.g. Morishima, Steedman, H. D. Kurz etc.

[20] See the authors in the previous footnote.

This remains true when we consider cost in terms of labour time. Average labour value, **L/Q**, is a function of output and this function typically has a U-shape.

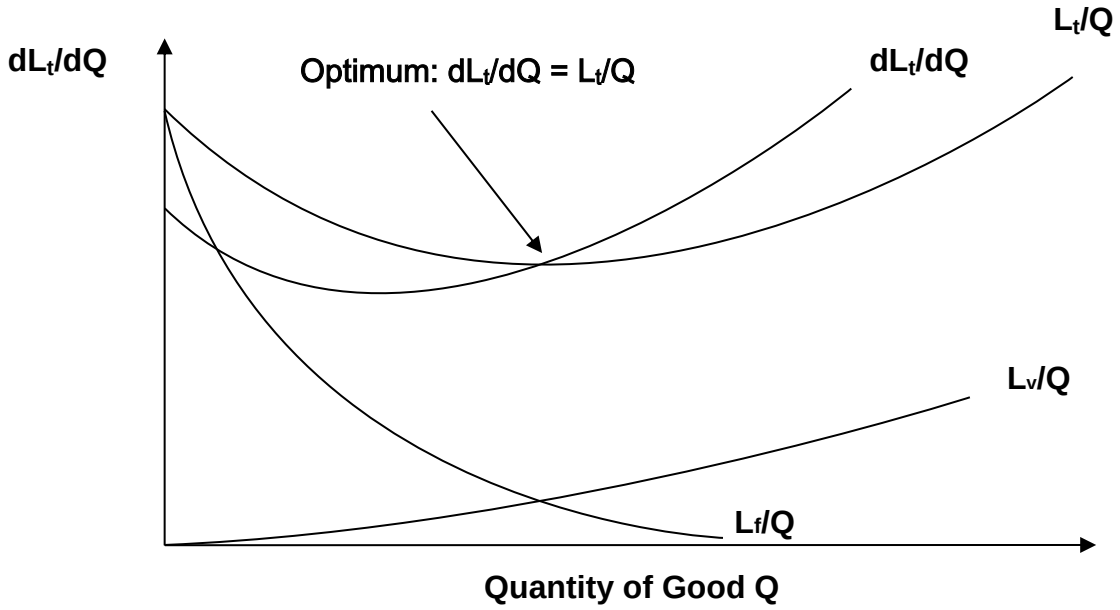

Figure 2: Cost Functions in Terms of Labour Values

Figure 2 presents the cost functions in terms of labour values. In order to interpret the curves properly it is very useful to consider capital as fixed. Therefore the value of capital as well as the cost of using it is included in fixed cost. Here the term cost means labour time. The variables are:

L_f/Q – average fixed cost (cost of means of production **plus** use of them);

L_v/Q – average variable cost (here this is only direct labour costs);

L_t/Q – average total cost of production is the sum of average fixed and variable costs.[21]

dL/dQ – marginal cost.

[21] The meaning of $\boldsymbol{L_t}$ is total labour time including direct, indirect and surplus labour. It is $\boldsymbol{L_t = L_v + L_f}$, equation **(27)** in the text. Surplus labour is included in **L_f**. In the following the symbol $\boldsymbol{L_t}$ is often used. See also footnote 23.

Notice that ***dLt/dQ* = $\mathbf{dL_v/dQ = dL/dQ}$** because fixed cost $\mathbf{L_f}$ vanish in the differential. This is extremely important in practice as one does not know the labour time involved in one's inputs other than labour. One can still calculate total average labour time per unit of output because at minimum cost this total average labour time is equal to marginal labour time. This is the significance of John Bates Clark's remark "Marginal labor is the only kind that can measure value."

The curve of *average total cost*, $\mathbf{L_t/Q}$, is cut at its minimum by the curve of marginal cost, **dL/dQ**. The point of minimum average cost is the point of *minimum socially necessary labour value*.

We shall prove below that one obtains the functions of average and marginal cost in terms of money - as they are usually presented in orthodox economics - by multiplication of the labour value functions with the wage rate.

With the help of the *cost equation* and the *production function* we establish the optimal quantities of production at minimum average cost, i.e. at *minimum socially necessary labour values*.[22]

The cost equation[23] is

$$C = wL + (1 + \kappa)K \qquad \textbf{(1)}$$

C – Total cost, w – wage rate, L – direct paid labour input,
κ – κ-rate (in orthodox economics the rate of interest), K – Value of capital goods

The production function[24] is

[22] The mathematics of the following discussion can be found in (Henderson Quandt 1980), but of course one does not find any analysis of labour values.

[23] In these equations the meaning of **L** is the usual one of direct paid labour input only; in the following ***L*** is also denoted as $\boldsymbol{L_v}$. In equation **(23)** onwards we use the term $\boldsymbol{L_v}$ to distinguish it from *total labour time*, $\boldsymbol{L_t}$, including direct labour, indirect labour and surplus labour.

[24] We conduct the analysis using the term **K** denoting the value of capital goods. This is for convenience only as the analysis can also be conducted by using quantities of heterogeneous

$$Q = f(L, K) \quad \textbf{(2)}$$

Q – Output

We find the quantity of output at minimum cost by differentiating the Lagrangian, **£**, with respect to the factors of production.

The Lagrangian is:

$$£ = wL + (1 + \kappa)K + \lambda[Q - f(L, K)] \quad \textbf{(3)}$$

The first order conditions are:

$$\begin{aligned} \frac{\partial £}{\partial L} &= w - \lambda \frac{\partial Q}{\partial L} = 0 \\ \frac{\partial £}{\partial K} &= (1+\kappa) - \lambda \frac{\partial Q}{\partial K} = 0 \\ \frac{\partial £}{\partial \lambda} &= Q - f(L_v, K) = 0 \end{aligned} \quad \textbf{(4)}$$

λ – Lagrangian multiplier

and resolved gives:

$$\begin{aligned} w &= \lambda \frac{\partial Q}{\partial L} \\ (1+\kappa) &= \lambda \frac{\partial Q}{\partial K} \\ Q &= f(L_v, K) \end{aligned} \quad \textbf{(5)}$$

Because the production function is always invertible we can make use of the inverse of a function rule $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ and write the term for the factor labour as

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capital goods.

$$\lambda = w\frac{\partial L}{\partial Q} \quad \textbf{(6)}$$

On the left side we have the Lagrange multiplier, $\boldsymbol{\lambda}$, and on the right side the product of the wage rate, **w**, and marginal labour value, $\frac{\partial L}{\partial Q}$.

It is important to know that the Lagrange multiplier, **λ**, is marginal cost, **dC/dQ**. This can be shown as follows:

From the cost equation **(1)**, $C = g(L, K)$ we can derive the total differential of cost as

$$dC = \frac{\partial C}{\partial L}dL + \frac{\partial C}{\partial K}\,dK \quad \textbf{(7)}$$

and for the production function, $Q = f(L, K)$, the total differential is

$$dQ = \frac{\partial Q}{\partial L}dL + \frac{\partial Q}{\partial K}\,dK \quad \textbf{(8)}$$

Using the cost equation **(1)** we get the derivatives of cost with respect to labour and capital respectively as

$$\frac{\partial C}{\partial L} = w$$

$$\frac{\partial C}{\partial K} = (1 + \kappa) \quad \textbf{(9)}$$

Substituting these into **(7)** gives

$$dC = w\,dL + (1 + \kappa)dK \quad \textbf{(10)}$$

At minimum cost, from the first order conditions **(5)** we can rewrite **(10)** as

$$dC = \lambda\,\frac{\partial C}{\partial L}dL + \lambda\,\frac{\partial C}{\partial K}\,dK$$

or

$$dC = \lambda\left[\frac{\partial C}{\partial L}dL + \frac{\partial C}{\partial K}\,dK\right] \quad \textbf{(11)}$$

From **(8)** we see that the term in brackets is equal to ***dQ***. Substituted in **(11)** is

$$dC = \lambda dQ \quad \textbf{(12)}$$

Therefore **λ** can be interpreted as marginal cost

$$\lambda = \frac{dC}{dQ} \tag{13}$$

We conclude that in an optimal economic system marginal cost is nothing else but the monetary expression of marginal labour value

$$\lambda = \frac{dC}{dQ} = w\frac{\partial L}{\partial Q} \tag{14}$$

From this follows that if prices are equal to marginal costs, which is the condition for profit maximization under perfect competition (*see below*)

$$p_i = \lambda_i; i = 1, \ldots, n \tag{15}$$

prices are proportional to marginal labour values

$$p_i = w\ \frac{\partial L}{\partial Q_i}; for\ i = 1, \ldots, n. \tag{16}$$

In orthodox economics one finds equation **(16)** expressed in the following form: the wage rate is equal to the value of the marginal product of labour

$$w = p_i\ \frac{\partial Q_i}{\partial L}\ ;\ \text{for}\ i = 1, \ldots, n \tag{17}$$

This is equivalent to **(16)** because $\frac{\partial Q_i}{\partial L} = \frac{1}{\frac{\partial L}{\partial Q_i}}$.

From **(16)** follows that relative prices are equal to marginal labour values

$$\frac{\boldsymbol{p_i}}{\boldsymbol{p_1}} = \frac{\boldsymbol{\partial L/\partial Q_i}}{\boldsymbol{\partial L/\partial Q_1}}\ ;\ \textbf{for}\ \boldsymbol{i = 1, 2, \ldots, n} \tag{18}$$

This shows the validity of the labour theory of value.

Profit maximization, taken prices as constant, e.g. set by planning authorities, requires that marginal cost equals price. This can be seen using the profit function, expressing the difference of revenue over cost.

$$\Pi = pQ - C \tag{19}$$

Π – Profits, p – Price, Q – Quantity, C - Cost

The first order condition for profit maximization requires that marginal profit is zero:

$$\frac{d\,\Pi}{dQ} = p - \frac{dC}{dQ} = 0 \tag{20}$$

From this follows that at the quantity of output yielding maximum profits marginal cost equals price.

$$\frac{dC}{dQ} = p \tag{21}$$

These results are extremely important also for the calculation of labour values as we can conclude from **(16).** One simply has to divide the monetary value of some commodity, its price, by the wage rate to obtain the corresponding labour value. The total labour time[25] embodied as *socially necessary labour time* to produce some quantity of a commodity is obtained by dividing the monetary value of that quantity by the wage rate.

$$p\,Q = w\,\frac{\partial\,L}{\partial\,Q}\,Q \tag{22}$$

$$\frac{p\,Q}{w} = \frac{\partial\,L}{\partial\,Q}\,Q = L_t \tag{23}$$

Equation **(23)** shows also that labour commanded (the left part of the equation) is equal to labour embodied (the right part).

Because of this we can also express demand functions in terms of labour value by dividing the ordinary inverse demand function by the wage rate.

From this follows also that the marginal labour function (or in monetary terms, the marginal cost function) is the supply function of the competitive firm for that part of the function which is

[25] See footnote 23.

positioned on and above the average labour value function (average cost curve).

Now we prove that the marginal labour value function cuts the average labour value function at its minimum as it has been presented in **Figure 2**.

The cost equation **(1)** can be represented in terms of labour values as

$$L_t = \frac{C}{w} = L_v + (1+\kappa)\frac{K}{w} \qquad \textbf{(24)}$$

In the short term the means of production are given and so the capital costs are fixed cost as also the **κ**-rate is given. In terms of labour values we denote these embodied labour values as $\mathbf{L_f}$, recalling that it stands for the labour time used to produce the means of production, **K/w**, as well as the labour time equivalent to the cost of using this capital, **κ K/w**. We know already that the latter part represents surplus value.

According to our model the only labour that varies with the quantity produced - the variable cost - is direct labour, $\mathbf{L_v}$. This is the labour time that also turns up in the production function **(2)** but now we have given it a subscript **v** to identify it as variable labour. We can express $\mathbf{L_v}$ as a function of output because the production function is invertible and capital, **K**, is fixed.

$$L_v = h(Q, \overline{K}) \qquad \textbf{(25)}$$

The right term in equation **(24)** is fixed labour, $\mathbf{L_f}$

$$L_f = (1+\kappa)\frac{\overline{K}}{w} \qquad \textbf{(26)}$$

Substituting the expressions $\mathbf{L_v}$ from **(25)** and $\mathbf{L_f}$ from **(26)** in the equation **(24)** we get the *labour value function*[26]

$$L_t(Q) = L_v(Q) + L_f \qquad \textbf{(27)}$$

[26] Notice that this is leading to the solution of Adam Smith's paradox of labour values and the adding-up-theorem of labour, profit and rent (we ignore rent) as the **L** comprises them all, including also indirect labour, stored up in the means of production (constant capital).

L_t – TOTAL labour (including indirect + surplus labour),
$L_v = h(Q, \overline{K})$ – variable labour,
Lf – fixed labour (including surplus labour)

This function is the *labour value function* and represents total cost in terms of labour time as a function of output.

Differentiating the labour value function with respect to output, the fixed term, $\mathbf{L_f}$, is eliminated and it remains

$$\frac{dL_t}{dQ} = \frac{\partial L_v}{\partial Q} \qquad \textbf{(28)}$$

We see that the derivative of the labour value function, is equal to the reciprocal of the *marginal productivity of labour* as calculated via the production function **(2)**, because

$$\frac{\partial L_v}{\partial Q} = \frac{1}{\dfrac{\partial Q}{\partial L_v}} \qquad \textbf{(29)}$$

$\dfrac{\partial Q}{\partial L_v}$ - partial derivative of the production function with respect to labour

Now we are ready to find the minimum of the average cost function in terms of labour, *i.e.* the minimum of the *average labour value function*. The *average labour value function* is equation **(27)** divided by output, **Q**.

$$\frac{L_t}{Q} = \frac{L_v}{Q} + \frac{L_f}{Q} \qquad \textbf{(30)}$$

The derivative of this function with respect to **Q** is

$$\frac{d(L_t/Q)}{dQ} = \frac{\left[\dfrac{\partial L_v}{\partial Q}Q - L_v - L_f\right]}{Q^2} \qquad \textbf{(31)}$$

And as $L_t = L_v + L_f$

$$\frac{d(L_t / Q)}{dQ} = \frac{\left[\frac{\partial L_v}{\partial Q} Q - L_t\right]}{Q^2} \qquad \textbf{(32)}$$

The first order condition for the *average labour value function* to be at a minimum is that equation **(32)** is equal to zero. As the denominator, $\mathbf{Q^2}$, must be positive, the nominator must be equal to zero:

$$\frac{\partial L_v}{\partial Q} Q - L_t = 0 \qquad \textbf{(33)}$$

From this follows that at the quantity which is produced with a minimum of average labour values these average labour values are equal to marginal labour value.

$$\frac{L_t}{Q*} = \frac{\partial L_v}{\partial Q} \qquad \textbf{(34)}$$

$Q*$ - optimal quantity of output

This is an important result as it means that a firm under perfect competition or in a planned economy produces that amount of output at which marginal labour value equals average labour value.[27] This value multiplied with the wage rate is equal to the price. Equilibrium prices in perfect competition are just monetary expressions of marginal labour values.

$$p = w\frac{L_t}{Q*} = w\frac{\partial L_v}{\partial Q} \qquad \textbf{(35)}$$

L_t - average TOTAL(including indirect + surplus labour),
$Q*$ - optimal quantity of output

[27] The reader must be careful here. Commonly the term *average productivity of labour* refers to the productivity of *direct labour* only that is to **Q/L** or **Q/*Lv*** and not to **Q/L_t**, the indirect labour in the capital goods (constant capital) is ignored and therefore the reciprocal of the so defined productivity is *not average labour value*. *Average labour value*, **L_t/Q**, is the reciprocal of the average productivity of total labour, **Q/L_t**, (direct, indirect and surplus labour).

Notice, $p = w\ \frac{\partial L_v}{\partial Q}$ is equation **(16)**, combining the first order condition of minimizing cost for labour, equation **(6)**, with profit maximization, equation **(21)**.

Multiplication of equation **(35)** with **Q** shows that the monetary value of output is equal to total labour value, $\mathbf{L_t}$, multiplied with the wage rate, ***w***, or equivalently, marginal labour value times output, multiplied with the wage rate, ***w***.

$$pQ^* = wL_t = w\frac{\partial L}{\partial Q}Q \qquad \textbf{(36)}$$

The monetary value of output is equal to the total socially necessary labour time used to produce it multiplied with the wage rate. But this is equal to the total cost of production.

$$pQ = wL_t = w\frac{\partial L}{\partial Q}Q = wL_v + (1+k)K \qquad \textbf{(37)}$$

This is a remarkable result as the value of output is equal to the sum of wages and (gross) profits derived from labour values. This is the solution of the paradox in Adam Smith's *Wealth of Nations* of the labour theory of value and the adding-up-theorem of wages, profits and rent.[28]

The value of total output is equal to wages plus gross profits, expressed in Marxian terms as being equal to the sum of variable capital, constant capital and surplus value.

$$pQ = v + c + s$$

$$v\text{ - variable capital}, c\text{ - constant capital},\ s\text{ - surplus value} \qquad \textbf{(38)}$$

where $pQ = wL_t; v = wL_v; c = K; s = \kappa K$ and therefore

$$pQ = v + c + s = wL_v + (1+\kappa)K \qquad \textbf{(39)}$$

[28] We have omitted rent but there is no problem if the return of marginal land is zero. If there is no marginal land than nature contributes to the creation of value, but marginal labour value remains proportional to price.

But the rate of surplus value

$$\frac{s}{v} = \frac{kK}{wL_v} = k\frac{K/w}{L_v} \qquad \textbf{(40)}$$

is not unique as orthodox Marxists want it, but dependent on the wage rate, **w**, the **κ**-rate, and the capital-labour ratio, $\mathbf{K/L_v}$, which varies amongst industries.

We take the labour used in production as being determined by cost minimization and profit maximization. From the inverse of the production function we can always determine the amount of direct labour to be used when the optimal quantity to be produced is known and this is determined via the first order conditions **(6).** The stock of capital has been treated as being exogenously given in the short run. The wage rate is being determined on the labour market, influenced by industrial relations and collective bargaining and in our model is also treated as a fixed parameter, exogenously given. The wage, $\mathbf{w\ L_v}$, represents the monetary value of labour power. But the cost of using capital is left to be explained. Only when we are able to explain the **κ**-rate and with it surplus labour our theory of labour values is complete.

The Dynamic Characteristic of Labour Value

Capital represents socially necessary labour stored up in the means of production. The problem to explain properly why there is a cost involved using this dead labour, in addition to replacing it, is a very difficult one indeed as neither Smith and Ricardo, Marx, Rosa Luxemburg or Tugan-Baranovsky and his critics have managed to find the proper answer.

In *Capital* the process of accumulation is presented as an extraction of surplus labour and its realisation on the market. The competition amongst the individual capitalists leads to the concentration of capital, a tendency of the rate of profit to fall and a tendency of over-accumulation of capital with respect to the limitations of purchasing power of the working classes. However, from

the point of view of the efficiency of the use of labour there is a *socially necessary accumulation of capital* to guarantee its optimal use. Considering the production process over time, that is, to perform a dynamic analysis, we find it is this *socially necessary* aspect of capital accumulation that makes surplus labour a part of the socially necessary labour time to produce a commodity.

The partial derivative of labour with respect to output presupposes a given amount of capital. The marginal labour value can only be calculated on the basis of this amount of capital. If we assume that this amount is optimal, that is producing at minimum average cost it has to be augmented in the case of a growing labour force and/or liberation of labour through the impact of technical progress. In this sense the accumulation of capital is socially necessary.

In the cost equation **(1)** this is expressed by the κ-rate:

$$C = v + c + s = wL_v + (1 + \kappa)K \qquad \textbf{(41)}$$

This reasoning has been developed only rather late by the Soviet mathematical economists, notably by Kantorovich (Kantorovich & Vainshtein 1976) and Novozhilov (1970) in their attempt to organize the socialist economic system of the Soviet Union.

In *Capital* (1867 [1906], 24: 648 ff.) Marx criticises harshly the spending habits of the new class of the bourgeoisie. He does recognize that the bourgeoisie had already violated its historical mission, the accumulation of capital. Indeed in an optimal economic system the returns of capital have to be entirely used for capital accumulation only to maintain the optimal productivity of labour and the optimal consumption per capita, a condition the capitalist system cannot fulfil. Under capitalism the rate of return on capital is higher than the rate of growth resulting in a higher rate of unemployment, the reserve army. The solution is to introduce the investment wage, a part of the wage devoted to the accumulation of capital, pooled into collective investment funds and controlled by the workers.

Critique of Marx' Conception of Labour Value

With the theoretical explanation of the **κ**-rate we have concluded our analysis of labour values in the context of an idealized general economic equilibrium model where labour values correspond to the socially necessary labour time as ultimate cost to produce a commodity. In this model labour values are proportional to prices and there is no *transformation of values into prices*; prices are simply the monetary expressions of labour values. The unique wage rate is the price of a unit of labour. In this context *all* surplus labour is labour time devoted to the accumulation of capital to assure the optimal productivity of labour in the dynamics of a growing economy. All surplus labour is part of the socially necessary labour time.

We have contrasted this optimal economic system with the real existing capitalist system and elaborated on the fact that capitalists strive for profits (surplus labour) in order to consume them, this being in fundamental contradiction to the requirements of an optimal economic system. But this aspect of the capitalist mode of production is not the only feature which distinguishes the capitalistic system from an optimal economic system. Another central issue, arising from the private strive for profits, is the divergence of prices from actual or optimal (socially necessary) cost. In our model we have assumed that prices are parameters and the firms being unable to exercise any control over them. In practice this is rarely the case and prices are subject to deliberate manipulations. The general rule of profit maximization is marginal revenue equal marginal cost and if prices depend and change with the quantity of the firm's output produced, prices do not equal marginal costs anymore. This leads to a mark-up of price above marginal cost and the break-down of the labour theory of value and with it the optimal use of labour and other economic resources.

Bearing in mind these qualifications of our analysis we can turn to a comparison with Marx' original theory of surplus value. Marx, like Ricardo understood his analysis as being conducted in the framework of an abstract economic system in which certain assumptions are introduced to

reveal the essentials of the capitalist mode of production. His abstract model, unlike ours, was meant to serve directly to explain the capitalist system and his analysis of the working day is basically correct as it remains true that the labourer is working part of the day to produce the value of his labour power and another part of the day to produce surplus value appropriated by the capitalist.

But here he found something difficult to explain: he did not know the concept of a production function and could not perceive marginal labour value and that there is a social cost of using capital which has to be taken account of, although he did mention in his *Critique of the Gotha Programme* that there is a social need for augmenting the means of labour, that is capital accumulation. His view on surplus labour is centred on the aspect that it is exploited from the labourers dependent on the outcome of the class struggle between capital and labour, i.e. the wage negotiations and labour laws.

Not even modern day progressive economists recognize that surplus labour is the difference between output evaluated at marginal labour value, $\frac{\partial L}{\partial Q}$, and the value of labour power.

$$pQ - wL_v = c + s$$

$$c + s = \text{gross surplus value} \qquad \textbf{(42)}$$

This becomes more obvious when we substitute $w\ \frac{\partial L}{\partial Q}$ for $\boldsymbol{p}$, and write:

$$w\frac{\partial L}{\partial Q}Q - wL_v = c + s$$

$$c + s = \text{gross surplus value} \qquad \textbf{(43)}$$

From which we can derive

$$w\left[\frac{\partial L}{\partial Q} - \frac{L_v}{Q}\right] = \frac{c+s}{Q} \qquad (44)$$

To elaborate on the function of capital in the production process it is important to understand the considerations leading to the substitution of direct labour by means of production in order to augment the productivity of the direct labour remaining in use. Obviously, cost-minimizing requires that as long as additional capital goods introduced are less expensive than the direct labour saved this substitution is economical. The substitution is continued up to the point where the value of constant capital is just equal to the value of wages saved by reducing direct labour time.

But if the cost of the use of that capital, that is accumulation, has also to be taken into account, than the substitution of capital for labour goes on only until the value plus interest to be paid for[29] are equal to the saved wage costs. Here the interest as part of profits or surplus value is proportional to constant capital; the greater the value of constant capital the greater is the surplus value. Therefore the ratio of profits (surplus labour) to wages (variable capital), which is the rate of surplus labour in Marxian terminology, is a function of the capital-labour ratio (equation **(40)** above).

Orthodox Marxists, ignoring this argument, insist that the rate of surplus labour must be equal for all labourers because of competition amongst them. However, this modelling is less than convincing as obviously the labourer does not bother at all how the working day is divided into paid and unpaid labour time. He considers only the wage in relation to his whole labour time.

In Vol. III of *Capital*, posthumously edited and published by Engels (Marx Engels 1894 [1909]), one finds this thesis of a unique rate of surplus labour and the so called *transformation problem.* If surplus labour (surplus value) is strictly proportional to direct labour time, the working day is

[29] Notice, that here interest, the price for money has its rational as being the monetary counterpart for the real cost in terms of labour time spent for the accumulation of the means of production, that is capital.

partitioned for all labourers in the same proportions of paid labour time and unpaid labour time, exploited by the capitalists, and then the surplus labour value (profits) cannot generally be proportional to the value of constant capital (capital goods) as is required by an average rate of profit but only in the special case of equal capital-labour ratios. Under this view there is no proportionality between labour values and costs of production (= price); it is unbelievable, but the discussion continues up to the present. Of course the discussants never refer or quote Kantorovich or Novozhilov. One must have the impression that these pseudo-theoretical debates are maintained only in order to isolate and disable critical intelligence.

Considering Marx' first volume of *Capital* and the discussion of the working day and exploitation his reasoning is a very appropriate analysis deeply reflecting the daily antagonistic conflict in capitalist reality. Marx describes in detail the motives and methods of the capitalists to reduce cost by extending the working day, increasing absolute surplus value or intensifying the production processes, by this increasing relative surplus value. The capitalist does not know[30] about the **κ**-rate or the golden rule path of economic development and he cares only about the difference between revenue and cost, selling at the highest price possible and paying the lowest wages for a working day as long and efficient as possible. The essential point is that that part of value which is created in the production process but not appropriated by the labourers - the producers of that value - is exploited labour. But when we realize that that surplus labour is socially necessary for capital accumulation the labourer can claim it only if he uses it for this purpose, and for this only! The solution is the introduction of an investment contribution as part of the wage or salary and the collective democratic control of the accumulation of capital.

[30] At present times this is no longer true and we are facing a monopoly capitalistic system where the well informed ruling classes are planning the exploitation of labour on a global scale.

Conclusion

It is surely a sobering insight that there is a labour theoretical foundation in orthodox economics. Marginal analysis has overcome various difficulties of classical economics, but not, as so many economists of all colours claim, by abandoning the labour theory of value. On the contrary, modern general equilibrium analysis has perfected it and insofar there is some considerable continuity in the development of economic theory and it is the Marxists who have abandoned the labour theory of value with their invention of the transformation of values into prices. But Marx was not a Marxist.

However, we should always be aware that the model of general economic equilibrium of "perfect competition" is not and in principle cannot be a representation of the real capitalist world. To reach a state of dynamic Pareto-optimality without exploitation, Capitalism has to be abandoned. Only where all profits and all rents are productively used and reinvested, society can claim to use its economic resources in an efficient manner. But this condition is identical with the abolishment of exploitation (in production) and the overcoming of capitalism.

Let us finish with the words of Simonde de Sismondi:

> I beg for your attention; it is not against the machines, it is not against the discoveries, it is not against the civilization that are directed my objections, it is against the modern organization of society, an organization, stripping the man who works *of all property but that of his arms*, not giving him any guarantees against competition, against a wild-auction led to his prejudice, of which he must necessarily become a victim. (Sismondi 1827, vol. 2:433, translation and emphasis by the editor).[31]

Progressive bourgeois economists conclude that workers have to *participate* in the wealth of Society and surely nobody is opposed to improving the wealth of the labourers. But we have found,

[31] The French original:

> Je prie qu'on y fasse attention; ce n'est point contre les machines, ce n'est point contre les découvertes, ce n'est point contre la civilisation que portent mes objections, c'est contre l'organisation moderne de la société, organisation qui, en dépouillant l'homme qui travaille *de toute autre propriété que celle de ces bras*, ne lui donne aucune garantie contre une concurrence, contre une folle-enchère dirigée à son préjudice, et dont il doit nécessairement être victime." [emphazis by the editor](Sismondi 1827:433).

that it is not the privately owned wealth of individuals which is the solution but the abolishment of the use of returns on capital and natural resources for individual consumption. It is not simply *participation* which is wanted but the *control* over capital by the workers.

It is precisely the social emancipation of the workers, it is the collective formation of capital via an investment contribution as part of the wage in addition to the social contributions and the social and democratic control over capital which overcomes the antagonistic contradictions of capitalism and breaks the *ultima ratio* of the capitalists, the supply of and control over capital. It is this the method to the prevention of the impoverishment of the individuals by organizing the collective management of the means of production and the rate of capital accumulation, and by this guaranteeing full employment. This can be done by the violence of a social revolution or by the introduction of the investment contribution as part of the wage. There is no alternative for humanity to “Crowding out Capitalism”.

Workers of All Countries Unite in Order to Control Capital!

Paris, March 26, 2014

Klaus Hagendorf

Appendix I

The Vector of Labour Values in a Leontief Input-Output Model

Leontief's Input-Output model is a simplified general economic equilibrium model which represents an equilibrium situation, considering multiple sectors. By relating the quantities of inputs to the outputs of the sectors a matrix of technical coefficients or capital-output ratios, **A**, as well as a vector of labour-output ratios, the labour-coefficients, $\mathbf{a_n}$ are constructed. Together with the rate of socially necessary capital accumulation, the ***k***-rate, one can construct a vector of labour values, ***v***.

$$\boldsymbol{v} = \boldsymbol{a_n} + (1 + \kappa)\boldsymbol{vA} \qquad \textbf{(AI.1)}$$

$\boldsymbol{V}$ – vector of labour values,

$\boldsymbol{a_n}$ – Vector of labour coefficients for the sectors,

κ - κ-rate of socially necessary capital accumulation,

A – Technology matrix (n x n square matrix of capital-output coefficients)

Orthodox Western Marxists do not include the socially necessary accumulation of capital and therefore their definition is equal to the bourgeois definition of labour values in which the **k**-rate is equal to zero.[32]

The solution for the vector of labour values, ***v***, is found as follows:

$$v - (1 + \kappa)vA = a_n \qquad \textbf{(AI.2)}$$

v - vector of labour values

[32] A presentation of the Leontief Model as well as the neo-Ricardian theory is in Luigi Pasinetti (1977), but he uses the orthodox bourgeois interpretation of labour values and in addition he rejects marginal reasoning. With other words, he ignores the Soviet economists in particular Victor V. Novozhilov's theoretical advances.

$$v\left[I-(1+\kappa)A\right]=a_n \qquad \textbf{(AI.3)}$$

$$v=a_n\left[I-(1+\kappa)A\right]^{-1} \qquad \textbf{(AI.4)}$$

v - vector of labour values

The vector of labour values represents the average labour values at the minimum of the average labour value functions where average labour values equal marginal labour values. The vector $\boldsymbol{v}$ represents marginal as well as average labour values.

$$v=a_n\left[I-(1+\kappa)A\right]^{-1}=\left[\partial L/\partial Q_1,\ldots,\partial L/\partial Q_n\right];\quad i=1,2,...,n \qquad \textbf{(AI.5)}$$

where $\frac{\partial L}{\partial Q_i}$ is the *marginal labour value* for the output of sector **i**.

The vector of labour values, $\boldsymbol{v}$, can also be represented as a power series and equation **(AI.5)** can be written as

$$v=\left[\partial L/\partial Q_1,\ldots,\partial L/\partial Q_n\right]=a_n\left[I-(1+\kappa)A\right]^{-1}=a_n+(1+\kappa)a_nA+(1+\kappa)^2a_nA^2+\ldots \qquad \textbf{(AI.6)}$$

in which the elements $\kappa a_n\boldsymbol{A}$; $\kappa^2 a_n\boldsymbol{A}^2$; ... represent surplus labour or the socially necessary labour devoted to capital accumulation.

We obtain the price vector, $\boldsymbol{p}$, by multiplication of the vector of labour values, $\boldsymbol{v}$, with the wage rate, the scalar $\boldsymbol{w}$.

$$p=wv=wa_n+(1+\kappa)wvA \qquad \textbf{(AI.7)}$$

v - vector of labour values

p - vector of prices of outputs.

$$p = wa_n + (1+\kappa)pA \qquad \textbf{(AI.8)}$$

p - vector of prices.

$$p[I - (1+\kappa)A] = wa_n \qquad \textbf{(AI.9)}$$

$$p = wv = wa_n[I - (1+\kappa)A]^{-1} \qquad \textbf{(AI.10)}$$

v - vector of labour values$p$ - vector of prices.

Equation **(A.10)** shows the proportionality of prices, ***p***, and labour values, ***v***.

$$p = wv = wa_n[I - (1+\kappa)A]^{-1} = [w\partial L/\partial Q_1, \ldots, w\partial L/\partial Q_n]; \quad i = 1,2,...,n \qquad \textbf{(AI.11)}$$

The price equation **(A.8)** can be decomposed as

$$p = wa_n + pA + \kappa pA \qquad \textbf{(AI.12)}$$

w wage rate,a_n - labour coefficients, p - vector of prices of n commodities,
$\kappa - \kappa$ - rate (rate of capital accumulation),

This price equation represents Marxian cost of production. The **κ**-rate represents the cost of using capital. The cost of production per unit of output consist of wages, $\mathbf{wa_n}$, the cost of the capital goods used up in production, **pA**, and the cost of accumulation of capital, **κpA**. In Marxian terminology $\mathbf{wa_n}$ is variable capital, **v**, **pA** is constant capital, **c**, and **κpA** surplus value, **s**, or profits per unit of output.

Appendix II

Numerical Example of the Calculation of Labour Values

In this appendix we present a numerical example how to calculate labour values for an economic production unit in an optimal economic system.

The production unit uses the following decreasing returns to scale Cobb-Douglas production function

Production Function: $\mathbf{Q = A\,K^a L^b}$ **(AII.1)**

with the

Parameters

A -	constant	5	
a -	production elasticity of capital	0,6	
b -	production elasticity of labor	0,25	
K -	value of capital	20,55	Euro
	(To simplify, it is assumed that all capital is used up,		
	that is the rate of depreciation is 100%).		
w -	wage rate	15,00	euro/hour
p -	market price of output	7,50	euro/unit

Cost minimization leads to the following level of output and labour values per unit of output:

Variables

Q -	output at minimum cost	**48**	units
$\frac{\partial L_v}{\partial Q}$ -	marginal labour value	**0,5**	hours/unit
$\frac{L_t}{Q}$ -	average labour value	**0,5**	hours/unit

Marxian variables in terms of labour time

We may now calculate the Marxian variables in terms of labour time and in terms of money. Because the labour theory of value holds prices are proportional to labour values and the monetary values are obtained simply by multiplication of the labour values with the wage rate.

Marxian variables in terms of labour values

$\mathbf{L_v}$ - direct labour	6	hours
$\mathbf{L_c}$ - labour value of constant capital	1,37	hours
$\mathbf{L_m}$ - surplus labour	16,63	hours
L - total labour value	24	hours

Marxian variables in terms of money (wage rate * labour time)

v – variable capital	90,00	euro	(15 * 6)
c - constant capital	20,55	euro	(15 * 1,37)
m - surplus value	249,45	euro	(15 * 16,63)
pQ – value of output	360	euro	(7,50 * 48)

Optimality condition for cost minimization: marginal labour value = average labour value

$$\frac{\partial L_v}{\partial Q}(\boldsymbol{Q^*}) = \frac{L}{Q}(\boldsymbol{Q^*}) \Rightarrow \frac{\partial L_v}{\partial Q} - \frac{L}{Q} = 0 \qquad 0{,}5 - 0{,}5 = 0 \qquad \textbf{(AII.2)}$$

$$Q = 5K^{0,6}L_v^{0,25} \qquad \mathbf{Q} = 48 \text{ units} \qquad \textbf{(AII.3)}$$

$$L_v = \left(\frac{1}{5}K^{-0,6}Q\right)^{\frac{1}{0,25}} \qquad \mathbf{L_v} = 6 \text{ hours} \qquad \textbf{(AII.4)}$$

$$\frac{\partial L_v}{\partial Q} = \frac{1}{0,25A}K^{-0,6}L_v^{1-0,25} \qquad \frac{\partial L_v}{\partial Q} \mathbf{= 0{,}5} \text{ hours/unit} \qquad \textbf{(AII.5)}$$

$\frac{\mathrm{L}}{Q} = \frac{L_v+L_c+L_m}{Q} = \frac{h(Q,\bar{K})+L_c+L_m}{Q}$ **L/Q = 0,5** hours/unit **(AII.6)**

$\frac{\partial L_v}{\partial Q*} = \frac{L_v+L_c+L_m}{Q*}$ $0,5 = \frac{6+1,37+16,63}{48}$ hours/unit **(AII.7)**

$p = w\frac{\partial L_v}{\partial Q*} = w\frac{L_v+L_c+L_m}{Q*} = \frac{v+c+m}{Q}$ **p** = 15 * 0,5 = 7,50 euro/unit **(AII.8)**

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